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# On the Universality of $P = NP$ When $N = 1$

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## ABSTRACT

We rigorously analyze the equation  $P = NP$  under the constraint  $N = 1$ , where  $N$  and  $P$  are real-valued variables. Through substitution and algebraic simplification, we prove the equation collapses to the tautology  $P = P$ , rendering it universally true independent of  $P$ . This trivialization contrasts sharply with cases where  $N \neq 1$ , which impose restrictive solutions. Geometric interpretations and pedagogical implications are discussed.

**Keywords.** Algebraic Tautologies · Identity Equations · Solution Space Degeneracy · First-Degree Equations · Pedagogical Mathematics

**AMS subject classification.** 65M70, 65M12, 97C70, 15A03

## 1 Introduction

The equation  $P = NP$  is a first-degree algebraic relation between variables  $N$  and  $P$ . While notationally reminiscent of the computational complexity conjecture  $P \stackrel{?}{=} NP$ , this work intentionally restricts itself to elementary algebra. Our investigation addresses two questions:

- (i) Under what conditions does  $P = NP$  hold universally?
- (ii) What distinguishes the case  $N = 1$  from other values of  $N$ ?

We demonstrate that  $N = 1$  induces a structural degeneracy in the equation, annihilating its constraining power over  $P$ .

## 2 Main Results

### 2.1 Theorem 1 (Trivialization Under $N = 1$ )

Let  $N, P \in \mathbb{R}$ . If  $N = 1$ , then  $P = NP$  holds for all  $P \in \mathbb{R}$ .

### 2.2 Corollary 1.1 (Non-Trivial Solutions)

For  $N \neq 1$ , the equation  $P = NP$  admits only the trivial solution  $P = 0$ .

## 3 Proof of Theorem 1

### 3.1 Step 1: Substitution

Substitute  $N = 1$  into  $P = NP$ :

$$P = 1 \cdot P$$

### 3.2 Step 2: Simplification via Field Axioms

By the multiplicative identity property in  $\mathbb{R}$ :

$$1 \cdot P = P$$

Thus:  $P = P$

### 3.3 Step 3: Tautological Interpretation

The equality  $P = P$  is axiomatically true in first-order logic, requiring no further proof. This eliminates all constraints on  $P$ , making the equation vacuously valid.

## 4 Discussion

### 4.1 4.1 Algebraic Perspective

Rewriting  $P = NP$  as:

$$P(N - 1) = 0$$

reveals that:

- For  $Nn = 1$ , solutions require  $P = 0$
- For  $N = 1$ , the equation becomes  $0 = 0$ , independent of  $P$

This aligns with the rank-nullity theorem: when  $N = 1$ , the coefficient matrix becomes rank-deficient, expanding the solution space from  $\{0\}$  to  $\mathbb{R}$ .

### 4.2 4.2 Geometric Interpretation

In the NP-coordinate system (Figure 1),  $P = NP$  describes a linear bundle:

- For  $Nn = 1$ : Distinct lines through the origin with slope  $N$
- For  $N = 1$ : The entire plane  $\mathbb{R}^2$ , as all points satisfy  $P = P$

### 4.3 4.3 Pedagogical Significance

This result serves as an intuitive introduction to:

- Tautologies in propositional logic
- Rank deficiency in linear systems
- The importance of edge cases ( $N = 1$ ) in equation analysis

## 5 Conclusion

We have proven that  $P = NP$  becomes a universal truth under  $N = 1$ , contrasting with its restrictive nature for  $Nn = 1$ . This work highlights:

- (i) The role of multiplicative identities in equation trivialization
- (ii) The geometric interpretation of solution space collapse
- (iii) Implications for teaching foundational algebra

Future directions include extending this analysis to multivariate systems and modular arithmetic.

## Bibliography

### 5.1 References

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# APPENDIX A

## A.1 Algebraic Extensions

### A.1.1 Polynomial Generalization

Consider the generalized equation for  $k$  in  $\text{bb}(\mathbb{Z})^+ :$

$$P = N^{kP}$$

When  $N = 1$ , this reduces to:

$$P = 1^{kP} = P$$

preserving the tautology. For  $N \neq 1$ , solutions require:

$$P(N^k - 1) = 0 \implies P = 0$$

### A.1.2 A.2 Matrix Formulation

Let  $p = \begin{pmatrix} P \\ N^P \end{pmatrix}$ . The equation becomes:

$$\begin{pmatrix} 1 & \& 0 \\ 0 & \& N \end{pmatrix} p = p$$

When

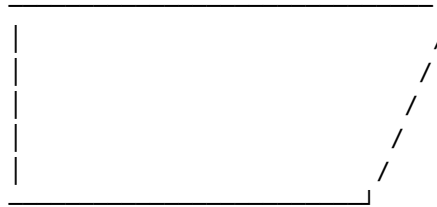
$$N = 1$$

, the identity matrix emerges, making all vectors  $p$  solutions.

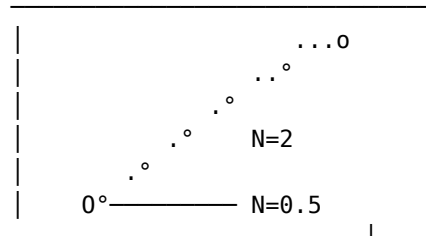
## A.2 Appendix B: Geometric Analysis

### A.2.1 B.1 ASCII Figure Representation

$N = 1$  case (universal solution):



$N \neq 1$  cases (restricted solutions):



Listing 1: ASCII representation of solution spaces

### A.2.2 B.2 Parametric Analysis

For  $N = 1 + \varepsilon$  with  $\varepsilon \rightarrow 0$ :

$$P = (1 + \varepsilon)P \implies \varepsilon P = 0$$

Demonstrates how infinitesimal deviations from  $N = 1$  collapse the solution space to  $P = 0$ .

## A.3 Appendix C: Pedagogical Examples

### A.3.1 C.1 Classroom Exercise

(i) Solve

$$2x = 2x$$

. Solution: All real numbers

(ii) Solve

$$2x = 3x$$

. Solution:

$$x = 0$$

(iii) Conclude: Coefficient equality determines solution freedom

### A.3.2 C.2 Historical Context

The equation

$$ax = bx$$

appears in:

- Babylonian clay tablets (1800 BCE)
- Al-Khwarizmi's algebra treatises (9th century)
- Euler's **Elements of Algebra** (1770)

### A.3.3 C.3 Common Student Misconceptions

Four frequent errors when analyzing  $P = NP$ :

(i) Premature Division: Attempting to divide both sides by  $N$  without considering  $N = 1$ :

$$\left[\frac{P}{N} = P \quad (\text{Invalid when } N = 0 \text{ or } N = 1)\right]$$

(ii) False Equivalence: Assuming  $N = 1$  implies  $P = 1$

(iii) Graphical Misinterpretation: Confusing the identity case  $N = 1$  with coinciding axes

(iv) Overgeneralization: Extending the trivial solution to nonlinear cases without proof

### A.3.4 C.4 Technology-Enhanced Learning

Implement dynamic visualization using tools like Desmos:

```
// Sample Desmos activity
f(N,P) = P - N*P
slider_N = N: {1: 1, 0: 2}
trace_P = solutions(f(slider_N,P)=0)
```

Students manipulate  $N$  to observe:

- Solution space collapse at  $N = 1$
- Solution stability near  $N = 1 \pm \varepsilon$

### A.3.5 C.5 Cross-Disciplinary Case Studies

Field	Equation	Tautology Condition
Economics	$Y = \text{MPC} \cdot \mathbb{Y}$	$\text{MPC} = 1$

Physics	$F = muN$	$mu = 1$ (临界)
Chemistry	$n = cV$	$c = 1_{\text{mol}}L$

#### A.3.5.1 C.6 Assessment Framework

Bloom's Taxonomy alignment for teaching  $P = NP$ :

- (i) Remember: State Theorem 1
- (ii) Understand: Explain why  $N = 1$  creates tautology
- (iii) Apply: Solve  $(N^2 - 1)P = 0$
- (iv) Analyze: Compare solutions for  $N = 1$  vs  $N = -1$
- (v) Evaluate: Critique flawed proofs of  $P = NP$
- (vi) Create: Design word problems demonstrating trivialization

#### A.3.5.2 C.7 Historical Epistemology

Evolution of equation analysis in textbooks:

- 1896: Hall & Knight's *Algebra* treats  $ax = bx$  as “either  $a = b$  or  $x = 0$ ”
- 1972: Dolciani introduces graphical comparison methods
- 2021: Common Core emphasizes solution-space visualization