STATS 210 (Session 4, Spring 2022)
Final Project

1. Logistics

This project is worth 15% of your course grade. You should work in groups of 3 or 4. We strongly recommend forming teams as soon as possible. Of course, if you need help with forming the team, you can contact the instructor.

We provide two project topics for you to choose. Each team should ONLY work on one project topic. Each topic has different requirements and these are listed under the topics in more detail.

You have 7 days to form a team and **make the decision topic selection**. Each team should have one person to mail the TA your teammates and the topic which your team choose by April 28.

Hint: *The first project topic "Auction" requires you to finish the coding in Python, so this topic is suitable for students with good Python background. However, the second project topic, "Red Envelope", is suitable for all students.*

After finishing the project, you are expected to submit two reports (each team only needs to submit one copy) and do one live presentation on Zoom:

1. **One written report:** Generally, in this report, you should summarize your solutions to the tasks in the project assignment, present the ideas, data/experiment results and conclusions.
2. **One presentation report:** You will submit the PPT for the presentation. In the PPT, each team should clearly present your team work, including the problem you are solving, the ideas/solutions you propose to solve the problem, the data results you get from the experiments, etc.
3. **Live presentation on Zoom**: You will do the presentation on Zoom in Week 8 (tentatively on May 12). In the meeting, each group member will present his/her own piece of work. The presentation time for each group is 10-15 minutes.

2. Timeline

- April 21: Project topics are released
- April 28: Due date for teammates and topic decision
- May 12 (tentative): Live presentation on Zoom.
- May 13: Reports due.

3. Topic 1: Auction

Be it frequency bandwidth or ads that we "see" every day, we're constantly asking ourselves how to distribute goods in a "fair" manner. Optimal allocation of rival goods is a hot question. If we want to use math and probability to gain some insight into this dilemma, we must first set up a theoretical model of an **auction** - which is basically a mechanism.
An auction has a **seller**, who is trying to sell **one** item. Multiple **buyers** (synonymous with "bidder") - there are n of them - are interested in purchasing this item, but the item can only be sold to one buyer. Each buyer associates a value to an item (how much is this item worth *to them*). These values usually differ and may or may not be independent of one another. Call buyer i's value of an item x_i. Each buyer then bids $\beta_i\ (x_i)$, a function of x_i. This is person i's bidding function. The seller then sells the item to one of the buyers according to some mechanism M.

We will attempt to answer some basic questions about auctions. We are interested in:

- Seller revenue, i.e. how much does the seller make?
- Buyer profit - what, if any, is the difference between the winning bid and the buyer's value of an item?
- Difference between mechanisms - what difference does the type of auction have on the buyer? What about the seller? How do reserves affect them?

3.1 Task 1

This task explores auction theory mathematically. In auction theory, n bidders (n is a positive integer) have valuations which represent how much they value an item; we will make the simplifying assumption that the valuations are i.i.d. (independent identically distributed) with continuous density f.

- In the first-price auction, the bidder who makes the highest bid wins the item and pays his/her bid.
- In the second-price auction, the bidder who makes the highest bid wins the auction, and pays an amount equal to the second-highest bid.

A strategy for the auction is a bidding function β, which is a function of the bidder's valuation. The bidding function determines how much to bid as a function of the bidder's valuation, and the goal is to find a bidding function $\beta(\cdot)$ which maximizes your expected utility -- 0 if you do not win, and your valuation minus the amount of money you bid if you do win.

a) For the first-price auction, consider the following scenario: each person draws his/her valuation uniformly from the interval $(0, 1)$ (so $f(x) = 1$ for $x \in (0, 1)$). Suppose that the other bidders bid their own valuations (they use $\beta(x) = x$, the identity bidding function). Consider the case where there is only one other bidder. What is your optimal bidding function? Try to prove it.
b) Consider the same situation as the previous part, but now assume that there are n other bidders. Again, what is your optimal bidding function? Try to prove it.

3.2 Task 2

For this task, we provide a python file "Auction.ipynb". Read the instruction in the file carefully. It focuses on the simulation with your derivate result. There're about 3 questions.

- Question Q1 asks you to simulate the first-price auction with uniform and exponential distribution. You can use your bidding function (from Task 1) for uniform distribution and explore your optimal bidding function for exponential distribution.
- Question Q2 discusses the second-price auction from buyer and seller's view respectively.
- In Question Q3, you are asked to develop a strategy for the seller to maximize revenue selling k identical items over a auctions with a reserve price for each auction.

Please finish code and answer the questions in the python file.

3.3 Report

Your report includes your solution to Task 1, content of finished Task 2 in ipynb file (in pdf format), and your PPTs for the presentation.

3.4 Submission

Please submit your finished codes and your reports by sending them to the TA via email at bohuan.lin@dukekunshan.edu.cn.

4. Topic 2: Red Envelope

Red envelope (or red packet) is popular in WeChat among both young and senior generations. There is a type of red envelope called *random amount red packets*: multiple money takers share one packet offered by the sponsor. To investigate the rationales behind the distribution of random amounts, you are required to conduct experiments on WeChat and get enough data for further analyses.

4.1 Task 1

Conduct experiments of grabbing red envelope on WeChat and record the results. Recommended number of trials is larger than 100 times for the same gross money and same number of money takers. Ensure that the number of money takers is larger than 5. you can corporate with another team that chooses this topic, or seek help from your friends.

4.2 Task 2

From your results, would the order of grabbing (early or late) affect the expectation of the money you get? For each order of money taker (from 1st to nth), plot the distributions of the grabbed money values. Since these values are discrete, you might need to choose a proper length of bin to count the experiment results, for example, if the value ranges from 0 to 5 RMB, a bin of 0.5-1.0 RMB might be proper (you can use the function "histogram" in Matlab). What is the difference between the first money taker and latter ones according to these distributions?

4.3 Task 3

It is easy to preassign random values for each money taker grabbing one envolope; However, since WeChat is dealing with thousands and hundreds of envelope grabbing at the same time, the cost of accessing database with such a high frequency is really high. Therefore, WeChat took the strategy like this: the money you will grab is not generated until you send the command of "grab".

Given the above information, design your red envelope distribution mechanism that satisfies:

a) the amount you will get is generated based on the remaining gross money and remaining number of money takers at the time you send "grab" command;
b) the results follow the distributions in Task 2.

4.4 Task 4

According to what you have got for Task 3, write your own codes (in MATLAB or Python) to simulate the process of grabbing envelopes and compare the simulated results with your experiments.

4.5 Task 5

Present any additional analyses, simulations or findings during your work. You will get bonus score from this task.

4.6 Reports

Your report includes your solutions to task1-5, pdf content of your codes, and your PPTs for the presentation.

4.7 Submission

Please submit your finished codes and your reports by sending them to the TA via email at bohuan.lin@dukekunshan.edu.cn.