

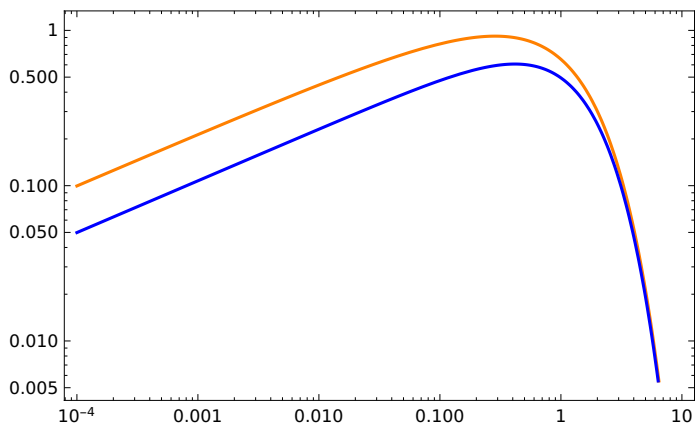
2 + 2

4

$$f[x_] = x \int_x^{\infty} \text{BesselK}[5/3, x] \, dx;$$

$$g[x_] = x \text{BesselK}[2/3, x];$$

LogLogPlot[{f[x], g[x]}, {x, 10⁻⁴, 10}, Frame → True, PlotStyle → {Orange, Blue}]



Asymptotic expansion of modified Bessel functions for small/large z

$$KS[v_, z_] = \frac{\text{Gamma}[v]}{2} \left(\frac{2}{z} \right)^v;$$

$$KL[v_, z_] = \sqrt{\frac{\pi}{2z}} e^{-z} \left(1 + \frac{4v^2 - 1}{8z} + \frac{(4v^2 - 1)(4v^2 - 9)}{2! \times 8z^2} \right);$$

First the G (x) functions

$$gS[z_] = \text{FullSimplify}[z KS[2/3, z]]$$

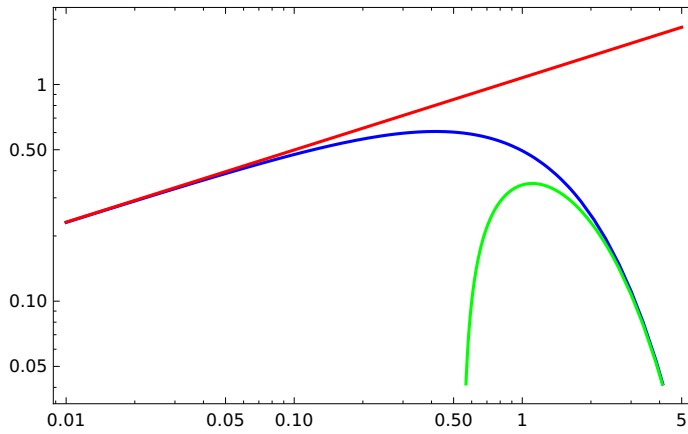
$$gL[z_] = \text{FullSimplify}[z KL[2/3, z]]$$

$$\frac{\text{Gamma}\left[\frac{2}{3}\right]}{2^{1/3} \left(\frac{1}{z}\right)^{1/3}}$$

$$e^{-z} \sqrt{\frac{\pi}{2}} \left(\frac{1}{z}\right)^{3/2} (-455 + 18z(7 + 72z))$$

1296

```
LogLogPlot[{g[x], gS[x], gL[x]}, {x, 10-2, 5},
Frame → True, PlotStyle → {Blue, Red, Green}]
```



Now the F(x) functions

$$fS[z_]=\text{FullSimplify}\left[z\int_z^\infty \text{KS}\left[\frac{5}{3},x\right]dx,\text{Assumptions}\rightarrow\{z\in\text{Reals},z>0\}\right]$$

$$fL[z_]=\text{FullSimplify}\left[z\int_z^\infty \text{KL}\left[\frac{5}{3},x\right]dx,\text{Assumptions}\rightarrow\{z\in\text{Reals},z>0\}\right]$$

$$2^{2/3}z^{1/3}\Gamma\left[\frac{2}{3}\right]$$

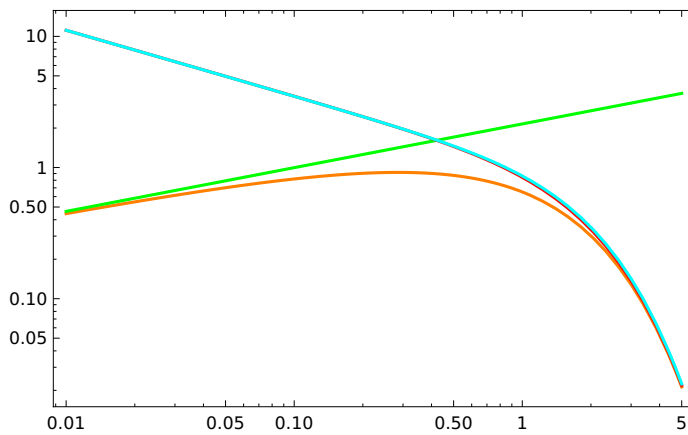
$$\frac{\sqrt{\frac{\pi}{2}}z\left(\frac{91e^{-z}(19+16z)}{z^{3/2}}+488\sqrt{\pi}\text{Erfc}\left[\sqrt{z}\right]\right)}{1944}$$

Now introduce an expansion of the Erfc function

$$fL2[z_]=\frac{\sqrt{\frac{\pi}{2}}z\left(\frac{91e^{-z}(19+16z)}{z^{3/2}}+488\sqrt{\pi}\left(\frac{1}{6}e^{-z}+\frac{1}{2}e^{-(4/3)z}\right)\right)}{1944}$$

$$\frac{\sqrt{\frac{\pi}{2}}z\left(488\left(\frac{1}{2}e^{-4z/3}+\frac{e^{-z}}{6}\right)\sqrt{\pi}+\frac{91e^{-z}(19+16z)}{z^{3/2}}\right)}{1944}$$

```
LogLogPlot[{f[x], fS[x], fL[x], fL2[x]}, {x, 10-2, 5},
  Frame → True, PlotStyle → {Orange, Green, Red, Cyan}]
```



This are the small/large x expressions that are going to be used in the code

For F (x)

```
ExpandAll[fS[x]] // N
```

```
FullSimplify[ExpandAll[fL2[x]] // N]
```

$$2.14953 x^{1/3}$$

$$0.278823 e^{-1.33333 x} x + \frac{e^{-1. x} (1.1147 + 0.938696 x + 0.092941 x^{3/2})}{\sqrt{x}}$$

For G (x)

```
ExpandAll[gS[x]] // N
```

```
FullSimplify[ExpandAll[gL[x]] // N]
```

$$\frac{1.07476}{\left(\frac{1}{x}\right)^{1/3}}$$

$$1.25331 e^{-1. x} \left(\frac{1}{x}\right)^{3/2} (-0.35108 + x (0.0972222 + 1. x))$$

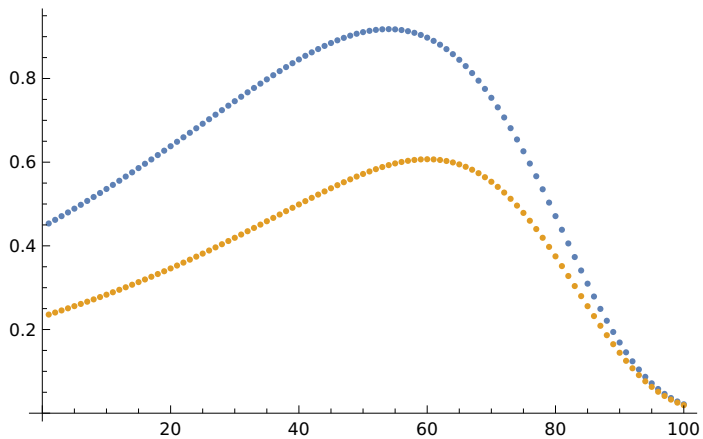
Now Lets do tables for a range in x between 0.01 and 5

Log spaced numbers

```
{a, b, c} = {0.01, 100, 5};
t = (c / a) ^ (1 / b) // N;
Xlist = a * t^Range[b];
Flist := {}
Glist := {}

For[i = 1, i < 101, i++,
  {AppendTo[Flist, f[Xlist[[i]]]], AppendTo[Glist, g[Xlist[[i]]]]}]

ListPlot[{Flist, Glist}]
```



Write to textfiles

```
Export["Xlist.txt", Xlist]
Export["Flist.txt", Flist]
Export["Glist.txt", Glist]

Xlist.txt

Flist.txt

Glist.txt
```