CSE 211: Discrete Mathematics **(Due: 12/11/19)**

Homework #2

Instructor: Dr. Zafeirakis Zafeirakopoulos *Name:Esra Eryılmaz* *Student Id:171044046*
Assistant: Gizem Süngü

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework1 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Sets

(2+2+2+2+2=10 points)

Which of the following sets are equal? Show your work step by step.

(a) {t : t is a root of x^2 6x + 8 = 0}

(b) {y : y is a real number in the closed interval [2, 3]}

(c) {4, 2, 5, 4}

(d) {4, 5, 7, 2} - {5, 7}

(e) {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}

(Solution)

a) Let this set name "A"
x^2 - 6x + 8 = 0
(x-2)(x-4) = 0
(x = 2) $\vee$ (x = 4)
A = {2,4}

b) Let this set name "B"
B={y $\in$ R | 2 $\leq$ y $\leq$ 3 }

c) Let this set name "C"
C = {2,4,5}
Note that:Sets cannot contain duplicate elements

d) Let this set name "D"
D = {4,2}

e) Let this set name "E"
4 is number of sides of a rectangle and 2 is number of digits
E = {4,2}

So set A, D and E are equal

Problem 2: Cartesian Product of Sets (15 points)

Explain why (A × B) × (C × D) and A × (B × C) × D are not the same.

(Solution)

In the sets the order of elements are immaterial; for example; {x,y}={y,x}. In some circumstances, however, order is significant.For instance,in coordinate system (1,2) and (2,1) respectively are distinct.

Definition:
The Cartesian product, X × Y ,of two sets X and Y is the set of all ordered pairs (x,y) where x belongs to X and y belongs to Y:
X × Y = {(x,y) : x ∈ X and y ∈ Y}.
• *The cartesian product is not associative :* (A × B) × C ≠ A × (B × C)

So due to the importance of the processing order of elements;
(A × B) × (C × D) and A × (B × C) × D are not the same.

Problem 3: Cartesian Product of Sets in Algorithms (25 points)

Let A, B and C be sets which have different cardinalities. Let (p, q, r) be each triple of A × B × C where p ∈ A, q ∈ B and r ∈ C. Design an algorithm which finds all the triples that are satisfying the criteria: p ≤ q and q ≥ r. Write the pseudo code of the algorithm in your solution.

For example: Let the set A, B and C be as A = { 3, 5, 7 }, B = { 3, 6 } and C = { 4, 6, 9 }. Then the output should be : { (3, 6, 4), (3, 6, 6), (5, 6, 4), (5, 6, 6) }.

(Note: Assume that you have sets of A, B, C as an input argument.)

(Solution)

Algorithm 1: Pseudo Code of Your Algorithm

```
Input: The sets of A, B, C
Get A, B, C
Set s(A) = a , s(B) = b , s(C) = c
Set Cartesian product array = CarPro()
Element of CP = 'element' and initial value is=0
for (i = 0 ; i < a ; ++i) do
    for (j = i ; j < b ; ++j) do
        for (k = j ; k < c ; ++k) do
            if (A(i) ≤ B(j)) then
                Statements
                if (B(j) ≥ C(k)) then
                    Set CarPro(element)
                    ++element
                else
                    continue
                end
            else
                continue
            end
        end
    end
end
print CarPro() on the screen
```

Problem 4: Relations (3+3+3+3+3+3+3=21 points)

Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where (x, y) $\in$ R if and only if

(a) x $\neq$ y.

(Solution)

• *The relation is not reflexive because we cannot have (0,0),(1,1)..*

• *The relation is symmetric because $x \neq y$ and $y \neq x$, if we have (1,2) than we can have (2,1) too.*

• *The relation is not antisymmetric because we have $x \neq y$ and $y \neq x$*

• *The relation is not transitive because we can have (1,2) and (2,1) but not (1,1)*

(b) xy $\geq$ 1.

(Solution)

• *The relation is not reflexive because we cant have (0,0)*

• *The relation is symmetric because we have xy= yx such that (1,1),(1/2, 2) (2, 1/2)..*

• *The relation is not antisymmetric because we have (1,2) (2,1)*

• *The relation is transitive because if we have (a, b) $\in$ R and (b, c) $\in$ R, then we can have (a, c) $\in$ R*

(c) x = y + 1 or x = y - 1.

(Solution)

• *The relation is not reflexive because we cannot have (1,1)*

• *The relation is symmetric because we have x = y + 1 and y = x - 1 and they are equivalent*

• *The relation is not antisymmetric because we have (1,2) and (2,1)*

• *The relation is not transitive because if we have (1,2) and (2,1) then we cannot have (1,1)*

(d) x is a multiple of y.

(Solution)

• *The relation is reflexive because we can have (1,1), (2,2)..*

• *The relation is not symmetric because we have (2,1) but we cannot have (1,2)*

• *The relation is not antisymmetric because we have (-1, 1) and (1, -1)*

• *The relation is transitive if x is multiple of y and if y is multiple of z then x is also multiple of z ; (x,y) (y,z) (x,z)*

(e) x and y are both negative or both nonnegative.

(Solution)

• *The relation is reflexive because x always has same sign (1,1) (-1,-1)..*

• *The relation is symmetric because x and y both negative or both nonnegative and also again same thing with y and x (1,2) (2,1) (-1,-2) (-2,-1)*

• *The relation is not antisymmetric because (1,2) (2,1)*

• *The relation is transitive because if x and y both negative or both nonnegative and also if y and z both negative or both nonnegative then x and z also both negative or both nonnegative (x,y) (y,z) (x,z)*

(f) x $\geq y^2$.

(Solution)

• *The relation is not reflexive because we cannot have (2,2)*

• *The relation is not symmetric because we can have (4,2) but cannot have (2,4)*

• *The relation is antisymmetric because except (0,0) and (1,1) each integer goes to another integer but not in reverse*

• *The relation is transitive because if $x \geq y^2$ and $y \geq z^2$, then $x \geq z^2$*

(g) x = y^2.
(Solution)
• *The relation is not reflexive because we cannot have (2,2)*
• *The relation is not symmetric because we can have (2,4) but cannot have (4,2))*
• *The relation is antisymmetric because except (0,0) and (1,1) each integer goes to another integer but not in reverse*
• *The relation is not transitive because we have (16,4) and (4,2) but we cannot have (16,2)*

Problem 5: Functions (15 points)

If f and f ∘ g are one-to-one, does it follow that g is one-to-one? Justify your answer.
(Solution)
I prove that by direct proof.
Let $g : A \rightarrow B$ *and let* $f : B \rightarrow C$
Lets assume that $g(a) = g(b)$
Take the function f both sides of the equation $f(g(a)) = f(g(b))$
This equation equals to $(f \circ g)(a) = (f \circ g)(b)$
Since $f \circ g$ *is one-to-one then a=b*
So $g(a) = g(b)$ *,that means that g is one-to-one.*

Problem 6: Inverse of Functions (7+7=14 points)

Let f be the function from $\mathbb{R}$ to $\mathbb{R}$ defined by f(x) = x^2. Find
(a) f^{-1} ({ x | 0 < x < 1 })
(Solution)
$y = x^2$
$x = \sqrt{y}$
Inverse of function is ; $f^{-1}(x) = \sqrt{x}$
$0 < f(x) < 1$
$0 < x^2 < 1$
$-1 < x < 1$ and $x \neq 0$
f^{-1} ({ x | $0 < x < 1$ }) = {x | $-1 < x < 1) \wedge (x \neq 0)$ }

(b)f^{-1} ({ x | x > 4 })
(Solution)
$f(x) > 4$
$x^2 > 4$
$(x<-2) \vee (x>2)$
f^{-1} ({ x | $x > 4$ }) = {x | $(x < -2) \vee (x > 2)$ }