CSE 211: Discrete Mathematics (Due: 15/12/19)

Homework #3

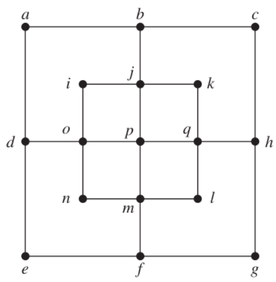
Instructor: Dr. Zafeirakis Zafeirakopoulos *Name:Esra Eryılmaz* *Student Id:171044046*
Assistants: Gizem Süngü, Baak Karaka

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

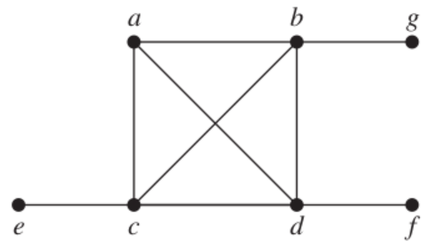
- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework3 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Hamilton Circuits (10+10+10=30 points)

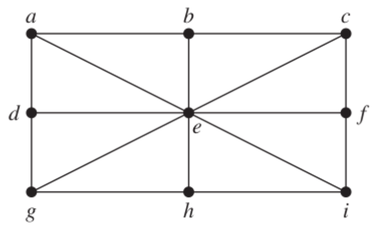
Determine whether there is a Hamilton circuit for each given graph (See Figure 1a, Figure 1b, Figure 1c). If the graph has a Hamilton circuit, show the path with its vertices which gives a Hamilton circuit. If it does not, explain why no Hamilton circuit exists.



(a) The graph G_1



(b) The graph G_2



(c) The graph G_3

Figure 1: The graphs to find Hamilton circuits for Problem 1

•***HAMILTON PATH :*** *In graph theory, an Hamilton path is a path which visits every vertex exactly once.*
•***HAMILTON CIRCUIT :*** *It is a Hamilton path that starts and stops at the same vertex.*

There are a few theorems about hamilton circuit , for example:

•***DIRAC'S THEOREM :*** *A simple graph G with n vertices ($n \geq 3$) has a Hamilton circuit,if the degree of every vertex in G is at least n/2.*

(a) ***(Solution)***

deg(a)=2 , deg(b)=3 , deg(c)=2 , deg(d)=3 , deg(e)=2 , deg(f)=3 , deg(g)=2 , deg(h)=3 , deg(i)=2 , deg(j)=4 , deg(k)=2 , deg(l)=2 , deg(m)=4 , deg(n)=2 , deg(o)=4 , deg(p)=4 , deg(q)=4

- For the Dirac's theorem no Hamilton circuit exists, because all degrees are less than n/2 = 17/2. But this is not means that no Hamilton circuit exists.
- But, when we try to find the Hamilton circuit, we cannot go back to the starting vertex without passing through the same vertex more than ones.
For example if we choose a,b,c,h,g,f,e,d,o,n,m,l,q,k,j,p and then we should pass j again that means that ;
- Hamilton circuit does not exist for this graph, because we should pass some vertex more than ones.

(b) ***(Solution)***

deg(a)=3 , deg(b)=4 , deg(c)=4 , deg(d)=4 , deg(e)=1 , deg(f)=1 , deg(g)=1

- For the Dirac's theorem no Hamilton circuit exists , because some degrees are less than n/2 = 7/2. But this is not means that no Hamilton circuit exists.
- But, when we try to find the Hamilton circuit; e ,g and f has only one edge. For example if we want to pass through f we should go back to the d again, it is not possible to pass every vertex without passing through the same vertex twice.
For example if we choose g,b,a,c,e and for returning we should pass c again so ;
- Because of these reasons, Hamilton circuit does not exist for this graph.

(c) ***(Solution)***

deg(a)=3 , deg(b)=3 , deg(c)=3 , deg(d)=3 , deg(e)=8 , deg(f)=3 , deg(g)=3 , deg(h)=3 , deg(i)=3

- For the Dirac's theorem no Hamilton circuit exists , because some degrees are less than n/2 = 9/2. But this is not means that no Hamilton circuit exists.
- In this graph, it is easy to find a Hamilton circuit.
- If we starts from a and go to the b ,c ,f ,i ,h ,g ,d ,e and back to the a.
- Hamilton circuit exists and path is {a ,b ,c ,f ,i ,h ,g ,d ,e ,a}

Problem 2: Graph Isomorphism (10+10+10=30 points)

Determine whether each pair of graphs (see Figure 2, Figure 3, Figure 4) is isomorphic or not.
Note: If you answer only "isomorphic" or "not isomorphic", you cannot get points. Show your work.

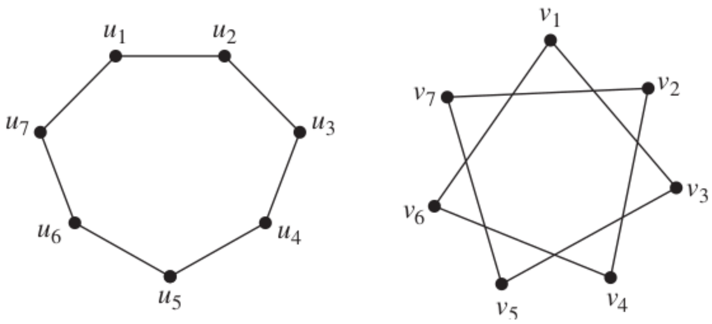


Figure 2: The graphs G_{a1}(left) and G_{a2}(right) to find isomorphism for Problem 2(a)

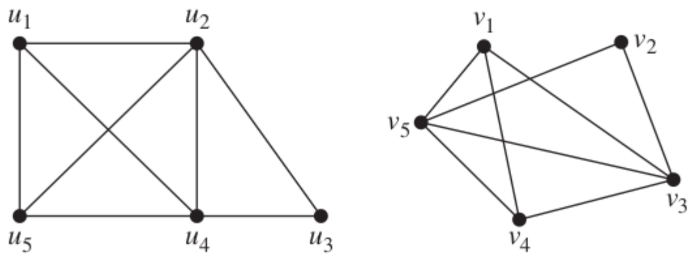


Figure 3: The graphs G_{b1}(left) and G_{b2}(right) to find isomorphism for Problem 2(b)

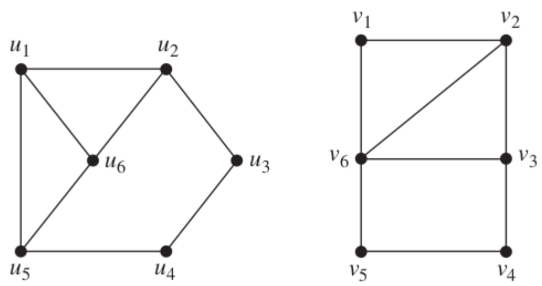


Figure 4: The graphs G_{c1}(left) and G_{c2}(right) to find isomorphism for Problem 2(c)

•Isomorphic graphs should have the same number of vertices, same number of edges and same degree sequences. Then we compare this vertices and edges.

(a) ***(Solution)***

- G_{a1} and G_{a2} both have seven vertices.

deg(u_1)=2, deg(u_2)=2, deg(u_3)=2, deg(u_4)=2, deg(u_5)=2, deg(u_6)=2, deg(u_7)=2
deg(v_1)=2, deg(v_2)=2, deg(v_3)=2, deg(v_4)=2, deg(v_5)=2, deg(v_6)=2, deg(v_7)=2
- Degree sequence of G_{a1} and G_{a2} are same.

- Set of vertices for G_{a1} is $\{u_1,\ u_2\ ,u_3\ ,u_4\ ,u_5\ ,u_6\ ,u_7\}$
- Set of vertices for G_{a2} is $\{v_1,\ v_2\ ,v_3\ ,v_4\ ,v_5\ ,v_6\ ,v_7\}$
- Set of edges for G_{a1} is $\{(u_1, u_2), (u_2, u_3), (u_3, u_4), (u_4, u_5), (u_5, u_6), (u_6, u_7), (u_7, u_1)\}$
- Set of edges for G_{a2} is $\{(v_1, v_3), (v_3, v_5), (v_5, v_7), (v_7, v_2), (v_2, v_4), (v_4, v_6), (v_6, v_1)\}$

- By comparing this vertices and edges we can say that
f(u_1)=v_1
f(u_2)=v_3
f(u_3)=v_5
f(u_4)=v_7
f(u_5)=v_2
f(u_6)=v_4
f(u_7)=v_6

u_1 and u_2 are adjacent when v_1 and v_3 are adjacent,
u_2 and u_3 are adjacent when v_3 and v_5 are adjacent,
u_3 and u_4 are adjacent when v_5 and v_7 are adjacent,
u_4 and u_5 are adjacent when v_7 and v_2 are adjacent,
u_5 and u_6 are adjacent when v_2 and v_4 are adjacent,
u_6 and u_7 are adjacent when v_4 and v_6 are adjacent,
u_7 and u_1 are adjacent when v_6 and v_1 are adjacent.

- This graphs are isomorphic.

(b) ***(Solution)***

- G_{b1} and G_{b2} both have five vertices.

deg(u_1)=3, deg(u_2)=4, deg(u_3)=2, deg(u_4)=4, deg(u_5)=3
deg(v_1)=3, deg(v_2)=2, deg(v_3)=4, deg(v_4)=3, deg(v_5)=4
- Degree sequence of G_{b1} and G_{b2} are same.

- Set of vertices for G_{b1} is $\{u_1,\ u_2\ ,u_3\ ,u_4\ ,u_5\}$
- Set of vertices for G_{b2} is $\{v_1,\ v_2\ ,v_3\ ,v_4\ ,v_5\}$
- Set of edges for G_{b1} is $\{(u_1, u_2), (u_1, u_4), (u_1, u_5), (u_2, u_3), (u_2, u_4), (u_2, u_5), (u_3, u_4), (u_4, u_5\}$
- Set of edges for G_{b2} is $\{(v_1, v_3), (v_1, v_4), (v_1, v_5), (v_2, v_3), (v_2, v_5), (v_3, v_4), (v_3, v_5), (v_4, v_5)\}$

- By comparing this vertices and edges we can say that
f(u_1)=v_1
f(u_2)=v_5
f(u_3)=v_2
f(u_4)=v_3
f(u_5)=v_4

u_1 and u_2 are adjacent when v_1 and v_5 are adjacent,
u_1 and u_4 are adjacent when v_1 and v_3 are adjacent,
u_1 and u_5 are adjacent when v_1 and v_4 are adjacent,
u_2 and u_3 are adjacent when v_5 and v_2 are adjacent,
u_2 and u_4 are adjacent when v_5 and v_3 are adjacent,
u_2 and u_5 are adjacent when v_5 and v_4 are adjacent,
u_3 and u_4 are adjacent when v_2 and v_3 are adjacent,
u_4 and u_5 are adjacent when v_3 and v_4 are adjacent,

- This graphs are isomorphic.

(c) ***(Solution)***

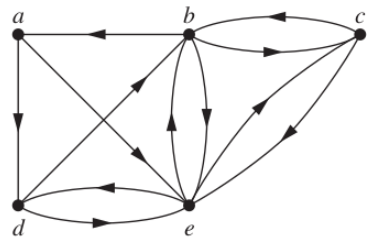
- G_{c1} and G_{c2} both have six vertices.

deg(u_1)=3, deg(u_2)=3, deg(u_3)=2, deg(u_4)=2, deg(u_5)=3, deg(u_6)=3
deg(v_1)=2, deg(v_2)=3, deg(v_3)=3, deg(v_4)=2, deg(v_5)=2, deg(v_6)=4
- Degree sequence of G_{c1} and G_{c2} are not same.

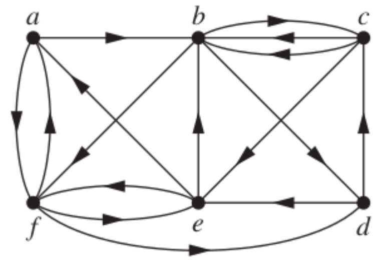
- For that reason this two graphs are not isomorphic.

Problem 3: Euler Circuits (10+10=20 points)

Determine whether there is a Euler circuit for each given graph (See Figure 6, Figure 5b). If the graph has a Euler circuit, show the path with its vertices which gives a Euler circuit. If it does not, explain why no Euler circuit exists.



(a) The graph G_{3a}



(b) The graph G_{3b}

Figure 5: The graphs to find Euler circuits for Problem 3

(Solution)

•***EULER PATH :*** *In graph theory, Euler path is a path which visits every edge of a graph exactly once.*
•***EULER CIRCUIT :*** *It is a Euler path that starts and stops at the same vertex.*
•***THEOREM :*** *If the input and output degrees are the same in directional graphs, we can say that Euler circuit exists.*

(a)

vertex	*in-degrees*	*out-degrees*
a	1	2
b	3	3
c	2	2
d	2	2
e	4	3

• *Vertex a and e : in-degrees and out-degrees are not same.*
•*Number of in-degrees and out-degrees should be same ,for example I started from a then back to the a, then pass to the a again but there is no way back to the a again ,for that reason ;*
•*Euler circuit does not exist. But Euler path exists.*

(b)

vertex	*in-degrees*	*out-degrees*
a	2	2
b	4	3
c	2	3
d	2	2
e	3	3
f	3	3

• *Vertex b and c : in-degrees and out-degrees are not same.*
•*Number of in-degrees and out-degrees should be same ,for example I started from c then back to the c twice , then pass to the c again but there is no way back to the c again ,for that reason ;*
• *Euler circuit does not exist. But Euler path exists.*

Problem 4: Applications on Graphs (20 points)

Schedule the final exams for Math 101, Math 243, CSE 333, CSE 346, CSE 101, CSE 102, CSE 273, and CSE 211, using the fewest number of different time slots, if there are no students who are taking:

- both Math 101 and CS 211,
- both Math 243 and CS 211,
- both CSE 346 and CSE 101,
- both CSE 346 and CSE 102,
- both Math 101 and Math 243,
- both Math 101 and CSE 333,
- both CSE 333 and CSE 346

but there are students in every other pair of courses together for this semester.
Note: Assume that you have only one classroom.

Hint 1: Solve the problem with respect to your problem session notes.
Hint 2: Check the website

(Solution)

I put 1 if there can be students in pair of courses and I put 0 if there cannot be students in pair of courses.

	MATH101	MATH243	CSE333	CSE346	CSE101	CSE102	CSE273	CSE211
MATH101	-	0	0	1	1	1	1	0
MATH243	0	-	1	1	1	1	1	0
CSE333	0	1	-	0	1	1	1	1
CSE346	1	1	0	-	0	0	1	1
CSE101	1	1	1	0	-	1	1	1
CSE102	1	1	1	0	1	-	1	1
CSE273	1	1	1	1	1	1	-	1
CSE211	0	0	1	1	1	1	1	-

•**GREEN -** *CSE273 is connected to all courses so I choose green just for CSE273.*
•**PURPLE -** *For CSE101, I choose purple and CSE101 connected to all courses except CSE346, so CSE101 and CSE346 are purple.*
•**RED -** *For CSE102, connected to all courses except CSE346 but CSE346 is purple and CSE102 is connected to CSE101(which is purple too) I cannot choose purple I choose red for CSE102.*
•**YELLOW -** *For CSE211 , connected to CSE273(green) ,CSE102(red) and CSE101(purple).So I choose new color yellow for CSE211.*
For MATH101 and MATH243 ,they are connected to green purple and red, but we can choose yellow because they are not connected CSE211.
•**BLUE -** *For CSE333 , I cannot choose green,purple,red,yellow and blue so I choose different color blue.*

We need 5 different colors for this graph.
So we need 5 different time slots for the final exams.
1)CSE273
2)CSE101 and CSE346
3)CSE102
4)CSE211, MATH101 and MATH243
5)CSE333

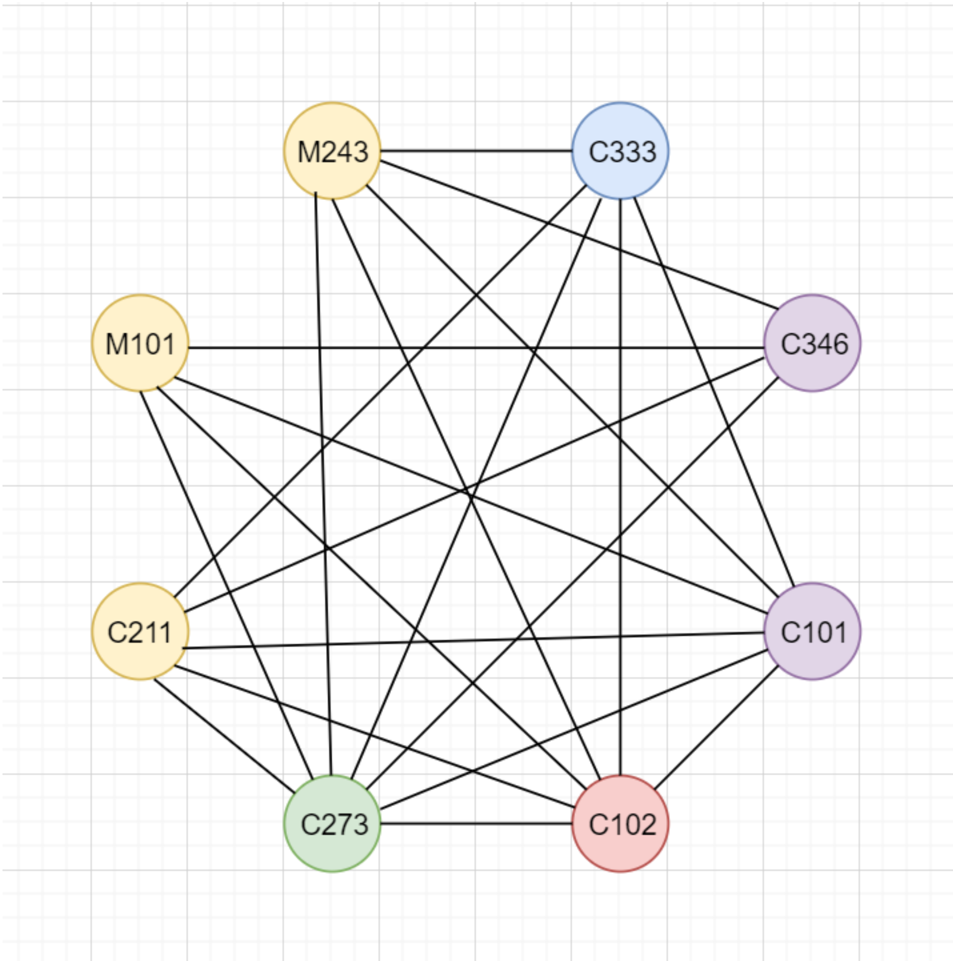


Figure 6: The graph of courses