CSE 211: Discrete Mathematics **(Due: 27/10/19)**

Homework #1

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework1 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Conditional Statements (5+5+5=15 points)

State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If it snows tonight, then I will stay at home.
(Solution)

Converse: *If I stay at home, it will snow tonight.*

Contrapositive: *If I dont stay at home, it wont snow tonight.*

Inverse: *If it doesnt snow tonight, I wont stay at home.*

(b) I go to the beach whenever it is a sunny summer day.
(Solution)

Converse: *Whenever its sunny summer day, I go to the beach.*

Contrapositive: *Whenever its not sunny summer day, I dont go to the beach.*

Inverse: *I dont go to the beach if its not a suny summer day.*

(c) When I stay up late, it is necessary that I sleep until noon.
(Solution)

Converse: *It is necessary that I sleep until noon when I stay up late.*

Contrapositive: *Its not necessary that I sleep until noon when I dont stay up late.*

Inverse: *When I dont stay up late, it is not necessary that I sleep until noon.*

Problem 2: Truth Tables For Logic Operators (5+5+5=15 points)

Construct a truth table for each of the following compound propositions.

(a) (p $\oplus$ ¬ q)

(Solution)

p	q	$\neg q$	$p \oplus \neg q$
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

(b) (p $\iff$ q) $\oplus$ (¬ p $\iff$ ¬ r)

(Solution)

p	q	r	$p \iff q$	$\neg p \iff \neg r$	$(p \iff q) \oplus (\neg p \iff \neg r)$
T	T	T	T	T	F
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	T	F	T
F	F	F	T	T	F

(c) (p $\oplus$ q) $\Rightarrow$ (p $\oplus$ ¬ q)

(Solution)

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \Rightarrow (p \oplus \neg q)$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

Problem 3: Logic in Algorithms (10+10+10=30 points)

If x = 1 before the statement is reached, what is the value of x after each of these statements is encountered in a computer program? Why? Show your work step by step.

(a) for i $\Leftarrow$ 1 **to** 10 **do**

if x + 2 = 3 **then** x := x + 1

end

At the beginning of the loop i = 1 and x = 1. If statement is true so we increment x by one. So we have i=2 and x=2 , but if statement is not true ,then we dont increment x by one. It goes like this.

i:=1 x:=1 if (x+2=3) $\equiv$ 1 then x:=x+1
i:=2 x:=2 if (x+2=3) $\equiv$ 0
i:=3 x:=2 if (x+2=3) $\equiv$ 0
i:=4 x:=2 if (x+2=3) $\equiv$ 0
i:=5 x:=2 if (x+2=3) $\equiv$ 0
i:=6 x:=2 if (x+2=3) $\equiv$ 0
i:=7 x:=2 if (x+2=3) $\equiv$ 0
i:=8 x:=2 if (x+2=3) $\equiv$ 0
i:=9 x:=2 if (x+2=3) $\equiv$ 0
i:=10 x:=2 if (x+2=3) $\equiv$ 0

(b) for i $\Leftarrow$ 1 **to** 5 **do**

if (x + 1 = 2) XOR (x + 2 = 3) **then** x := x + 1

end

At the beginning of the loop i = 1 and x = 1. If statement is false so we dont increment x by one. Then we have i=2 and x=1 then the same thing again, it goes like this.

i:=1 x:=1 if (x+1=2)XOR(x+2=3) $\equiv$ (1$\oplus$1) $\equiv$ 0
i:=2 x:=1 if (x+1=2)XOR(x+2=3) $\equiv$ (1$\oplus$1) $\equiv$ 0
i:=3 x:=1 if (x+1=2)XOR(x+2=3) $\equiv$ (1$\oplus$1) $\equiv$ 0
i:=4 x:=1 if (x+1=2)XOR(x+2=3) $\equiv$ (1$\oplus$1) $\equiv$ 0
i:=5 x:=1 if (x+1=2)XOR(x+2=3) $\equiv$ (1$\oplus$1) $\equiv$ 0

(c) for i $\Leftarrow$ 1 **to** 4 **do**

if (2x + 3 = 5) AND (3x + 4 = 7) **then** x := x + 1

end

At the beginning of the loop i = 1 and x = 1. If statement is true so we increment x by one. So we have i=2 and x=2 , but if statement is not true ,then we dont increment x by one. It goes like this.

i:=1 x:=1 if (2x+3=5)AND(3x+4=7) $\equiv$ (1$\wedge$1) $\equiv$ 1 then x:=x+1
i:=2 x:=2 if (2x+3=5)AND(3x+4=7) $\equiv$ (0$\wedge$0) $\equiv$ 0
i:=3 x:=2 if (2x+3=5)AND(3x+4=7) $\equiv$ (0$\wedge$0) $\equiv$ 0
i:=4 x:=2 if (2x+3=5)AND(3x+4=7) $\equiv$ (0$\wedge$0) $\equiv$ 0

Problem 4: Proof by contradiction (20 points)

Show that at least three of any 25 days chosen must fall in the same month of the year using a proof by contradiction. Explain your work step by step.
(Solution)

The idea is to assume that the statement we want to prove is false, then show that this assumption is meaningless. To conclude that we were wrong to assume the statement was false, so the statement must be true.

- If we assume that there are at most two days falling in the same month,
- Then we have at most $2 \cdot 12 = 24$ days, because we have twelve months.
- So we choose 24 days, then we have to choose one more day ,
- So at least three day must fall in the same month.

Problem 5: Proof by contraposition (20 points)

Show that if $n^3 + 5$ is odd, then n is even using a proof by contraposition. Explain your work step by step.
Note: Assume that n is an integer.
(Solution)

Let;
p=$n^3 + 5$ is odd
q=n is even

1) We prove the contrapositive : if n is odd ,then $n^3 + 5$ is even,
   $\neg p$=$n^3 + 5$ is even
   $\neg q$=n is odd
   $\neg p \Rightarrow \neg q$
2) n=2k+1 is odd for some integer k (definition of odd numbers)
3) $n^3 + 5 = (2k + 1)^3 + 5 = 8k^3 + 12k^2 + 6$k $+6 = 2$ ($4k^3$+$6k^2$+3k+3)
4) Since $n^3 + 5$ is 2 times an integer , it is even.
5) Since $\neg p \Rightarrow \neg q$, we have that $p \Rightarrow q$