

# CSE231

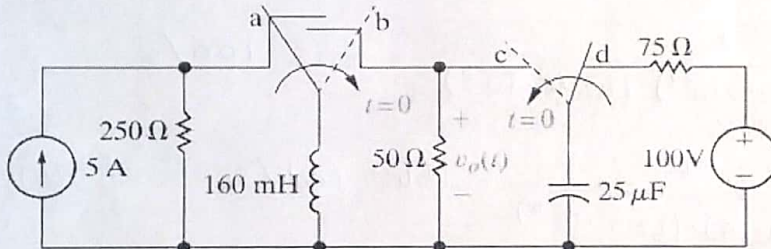
## - Homework 3 -

31.12.2020

8.11

**8.11** The two switches in the circuit seen in Fig. P8.11 operate synchronously. When switch 1 is in position a, switch 2 is in position d. When switch 1 moves to position b, switch 2 moves to position c. Switch 1 has been in position a for a long time. At  $t = 0$ , the switches move to their alternate positions. Find  $v_o(t)$  for  $t \geq 0$ .

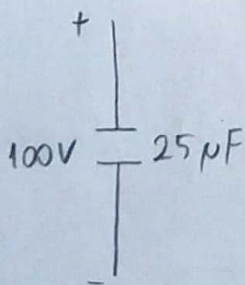
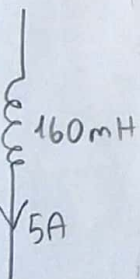
Figure P8.11



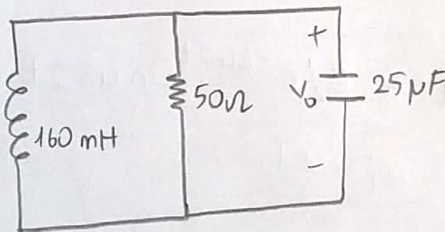
$t < 0$

$$V_o = 100V$$

$$I_o = 5A$$



$t > 0$



$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 50 \cdot (25 \cdot 10^{-6})} = 400 \text{ rad/s}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(160 \cdot 10^{-3}) \cdot (25 \cdot 10^{-6})}} = 500$$

$\alpha^2 < \omega_0^2 \rightarrow$  Underdamped response

$$\omega_d = \sqrt{500^2 - 400^2} = 300$$

$$V_o = B_1 e^{-400t} \cos 300t + B_2 e^{-400t} \sin 300t$$

$$V_o(0) = B_1 = 100$$

$$\frac{dV_o(0)}{dt} = -\alpha B_1 + \omega_d B_2$$

$$= \frac{1}{C} \left( -I_o - \frac{V_o}{R} \right)$$

$$\Rightarrow -400 \cdot 100 + 300 B_2 = \frac{1}{25 \cdot 10^{-6}} \left( -5 - \frac{100}{50} \right)$$

$$B_2 = -800$$

Result

$t \geq 0$

$$V_o = 100 e^{-400t} \cos 300t - 800 e^{-400t} \sin 300t$$

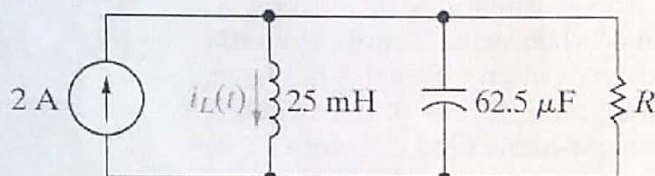
volt

8.27

**8.27** Assume that at the instant the 2 A dc current source is applied to the circuit in Fig. P8.27, the initial current in the 25 mH inductor is 1 A, and the initial voltage on the capacitor is 50 V (positive at the upper terminal). Find the expression for  $i_L(t)$  for  $t \geq 0$  if  $R$  equals 12.5  $\Omega$ .

PSPICE  
MULTISIM

Figure P8.27



$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(25 \cdot 10^{-3}) \cdot (62.5 \cdot 10^{-6})}} = 800 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot (12.5) \cdot (62.5 \cdot 10^{-6})} = 640 \text{ rad/s}$$

$$\alpha^2 < \omega_0^2$$

underdamped response

$$\omega_d = \sqrt{800^2 - 640^2} = 480 \quad I_f = 2 \text{ A}$$

$$i_L = 2 + B_1' e^{-640t} \cos 480t + B_2' e^{-640t} \sin 480t$$

$$i_L(0) = 2 + B_1' = 1 \rightarrow B_1' = -1$$

$$\frac{di_L}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \frac{V_0}{L}$$

$$-640(-1) + 480 B_2' = \frac{50}{25 \cdot 10^{-3}} \rightarrow B_2' = 2.83$$

$$t \geq 0 \quad i_L(t) = 2 - e^{-640t} \cos 480t + 2.83 \cdot e^{-640t} \sin 480t \quad \text{Ampere}$$

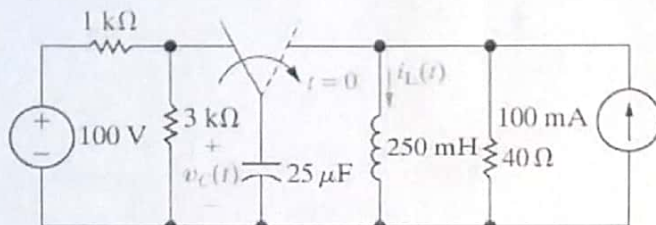


8.35

8.35 The switch in the circuit in Fig. P8.35 has been in the left position for a long time before moving to the right position at  $t = 0$ . Find

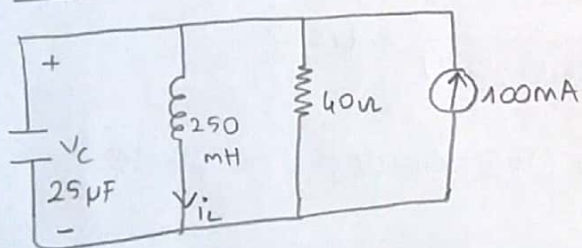
- a)  $i_L(t)$  for  $t \geq 0$ ,  
b)  $v_C(t)$  for  $t \geq 0$ .

Figure P8.35

 $t < 0$ 

$$V_o = V_o(0^-) = V_o(0^+) = \frac{3000}{4000} 100 = 75 \text{ V}$$

$$I_o = i_L(0^-) = i_L(0^+) = 100 \text{ mA}$$

 $t > 0$ 

$$\alpha = \frac{1}{2RC} = \frac{1}{2(40)(25 \cdot 10^{-6})} = 500 \text{ rad/s}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(250 \cdot 10^{-3}) \cdot (25 \cdot 10^{-6})}} = 400$$

$\alpha^2 > \omega_0^2 \rightarrow$  overdamped response

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$s_{1,2} = -500 \pm \sqrt{500^2 - 400^2}$$

$$= -200, -800$$

$$a) i_L = I_f + A_1 e^{-200t} + A_2 e^{-800t}$$

$$I_f = 100 \text{ mA}$$

$$i_L(0) = 0.1 + A_1 + A_2 = 0.1$$

$$\Rightarrow A_1 + A_2 = 0$$

$$\frac{di_L}{dt}(0) = -200A_1 - 800A_2 = \frac{V_o}{L}$$

$$= \frac{75}{0.25} = 300$$

$$A_1 = 0.5 \quad A_2 = -0.5$$

 $t \geq 0$ 

$$i_L(t) = 0.1 + 0.5e^{-200t} - 0.5e^{-800t} \text{ Amper}$$

$$b) v_C(t) = v_L(t) = L \cdot \frac{di_L}{dt}$$

$$= (0.25) \cdot (-100e^{-200t} + 400e^{-800t})$$

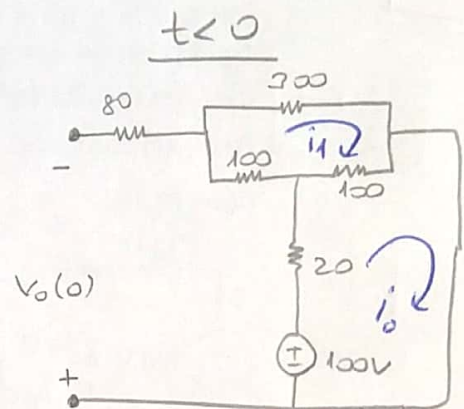
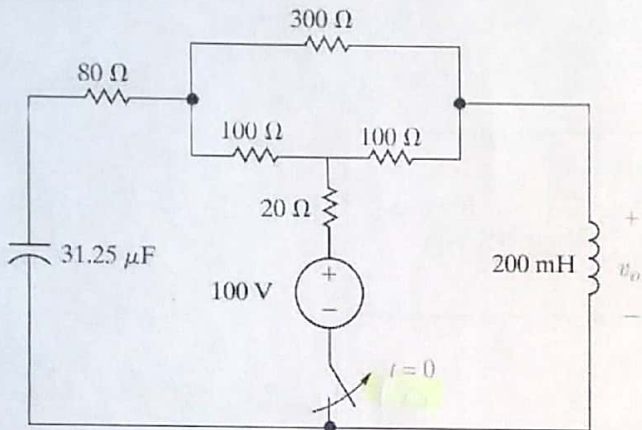
 $t \geq 0$ 

$$v_C(t) = -25e^{-200t} + 100e^{-800t} \text{ Volt}$$

8.47

8.47 The switch in the circuit shown in Fig. P8.47 has been closed for a long time. The switch opens at  $t = 0$ . Find  $v_o(t)$  for  $t \geq 0^+$ .

Figure P8.47

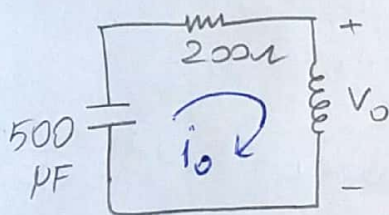


$$\begin{aligned} 500 i_1 - 100 i_0 &= 0 \\ -100 i_1 + 120 i_0 &= 100 \end{aligned}$$

$$i_1 = 0.2 \text{ A} \quad i_0 = 1 \text{ A}$$

$$V_o = -100 + 20 i_0 + 100 i_1 = -60 \text{ V}$$

$t > 0$



$$\alpha = \frac{R}{2L} = \frac{200}{2(0.2)} = 500$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.2)(31.25 \cdot 10^{-6})}} = 400$$

$\alpha^2 > \omega_0^2 \rightarrow$  Overdamped response

$$\begin{aligned} s_{1,2} &= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \\ &= -500 \pm \sqrt{500^2 - 400^2} \end{aligned}$$

$$s_{1,2} = -200, -800 \text{ rad/s}$$

$$i_o = A_1 e^{-200t} + A_2 e^{-800t}$$

$$i_o(0) = A_1 + A_2 = 1$$

$$\frac{di_o}{dt}(0) = -200A_1 - 800A_2 = \frac{1}{L}(-V_o - RI_o) = -700$$

$$\begin{aligned} A_1 &= 166.67 \text{ mA} \\ A_2 &= 833.33 \text{ mA} \end{aligned}$$

$$i_o(t) = 166.67 e^{-200t} + 833.33 e^{-800t} \text{ mA}$$

$$V_o(t) = L \cdot \frac{di_o}{dt} = (0.2) \cdot [(-200) \cdot 0.16667 e^{-200t} + (-800) \cdot 0.83333 e^{-800t}]$$

$t > 0$

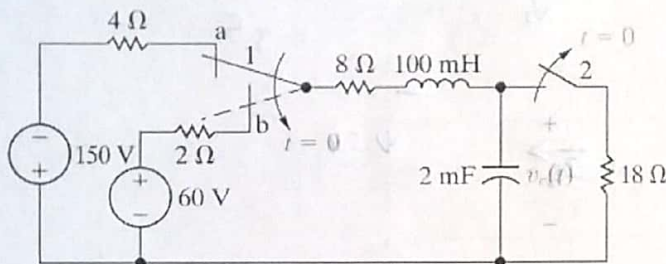
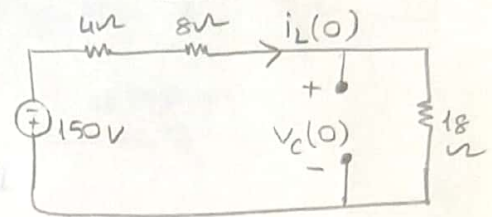
$$V_o(t) = -6.67 e^{-200t} - 133.33 e^{-800t} \text{ V}$$



8.54

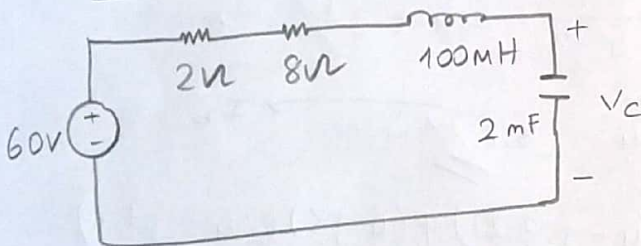
**8.54** The two switches in the circuit seen in Fig. P8.55 operate synchronously. When switch 1 is in position a, switch 2 is closed. When switch 1 is in position b, switch 2 is open. Switch 1 has been in position a for a long time. At  $t = 0$ , it moves instantaneously to position b. Find  $v_c(t)$  for  $t \geq 0$ .

Figure P8.54

 $t < 0$ 

$$i_L(0) = \frac{-150}{30} = -5 \text{ A}$$

$$v_c(0) = 18 i_L(0) = -90 \text{ V}$$

 $t > 0$ 

$$\alpha = \frac{R}{2L} = \frac{10}{2 \cdot (0.1)} = 50 \text{ rad/s}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.1) \cdot (2 \cdot 10^{-3})}} = \sqrt{5000}$$

$$\omega_0^2 > \alpha^2 \rightarrow \text{underdamped}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -50 \pm j50$$

$$v_c = 60 + B_1' e^{-50t} \cos 50t + B_2' e^{-50t} \sin 50t$$

$$v_c(0) = -90 = 60 + B_1' \Rightarrow B_1' = -150$$

$$C \cdot \frac{dv_c}{dt}(0) = -5 \rightarrow \frac{dv_c}{dt}(0) = \frac{-5}{2 \cdot 10^{-3}} = -2500$$

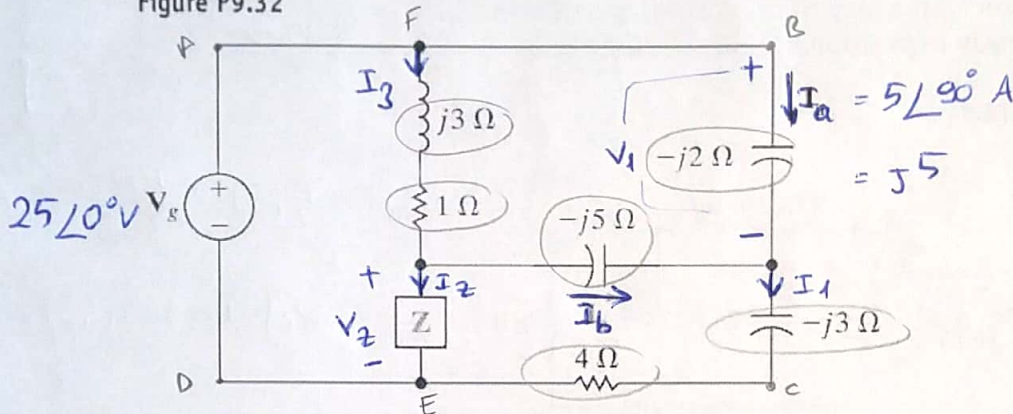
$$\frac{dv_c}{dt}(0) = -50 B_1' + 50 B_2' = -2500 \Rightarrow B_2' = -200$$

$$t \geq 0, \quad v_c(t) = 60 - 150 e^{-50t} \cos 50t - 200 e^{-50t} \sin 50t \text{ Volt}$$

9.32

9.32 Find  $I_b$  and  $Z$  in the circuit shown in Fig. P9.32 if  $V_g = 25 \angle 0^\circ$  V and  $I_a = 5 \angle 90^\circ$  A.

Figure P9.32



$$V_1 = j5 \cdot (-j2) = 10 \text{ V}$$

ABCD Mesh  $\rightarrow$

$$-25 + 10 + (4 - j3)I_1 = 0 \Rightarrow I_1 = \frac{15}{4 - j3} = 2.4 + j1.8 \text{ Amper}$$

$$I_b = I_1 - j5 = (2.4 + j1.8) - j5 = 2.4 - j3.2 \text{ Amper} \rightarrow I_b$$

$$V_2 = -j5 I_2 + (4 - j3) I_1 = -j5 (2.4 - j3.2) + (4 - j3) (2.4 + j1.8) = -1 - j12 \text{ Volt}$$

AFED Mesh  $\rightarrow$

$$-25 + (j3 + 1) I_3 + (-1 - j12) = 0 \Rightarrow I_3 = 6.2 - j(6.6) \text{ Amper}$$

$$I_2 = I_3 - I_1 = (6.2 - j(6.6)) - (2.4 - j3.2) = (3.8 - j(3.4)) \text{ Amper}$$

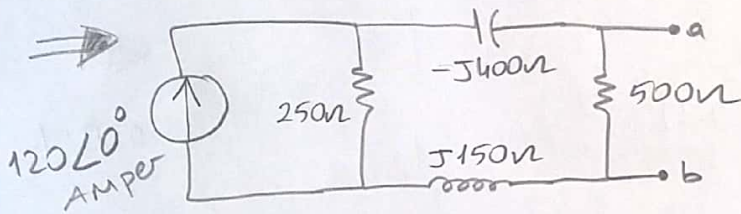
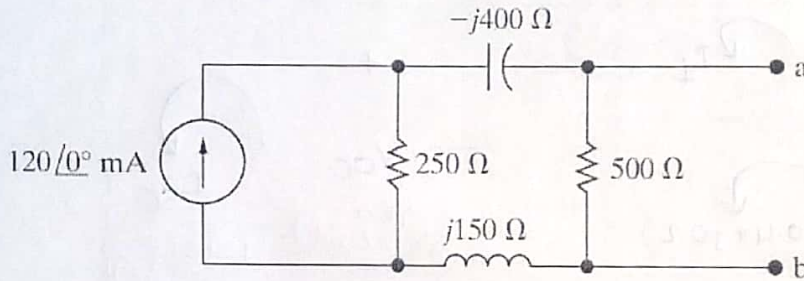
$$Z = \frac{V_2}{I_2} = \frac{-1 - j12}{(3.8 - j(3.4))} = 1.42 - j(1.88) \Omega$$



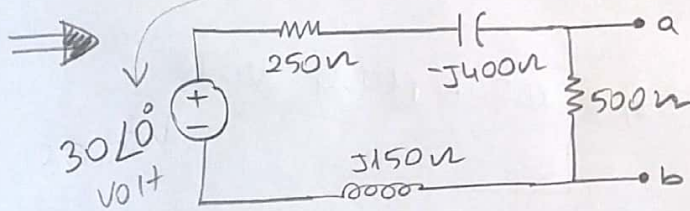
9.45

9.45 Use source transformations to find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.45.

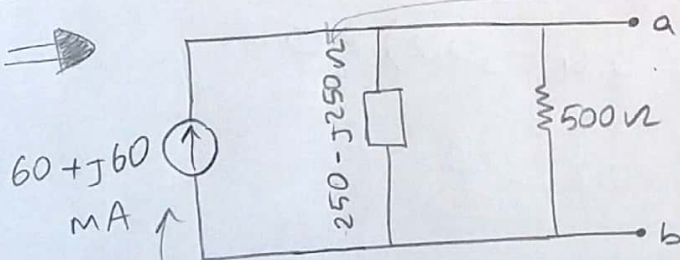
Figure P9.45



$$I \quad R \quad V \\ (0.12 \angle 0^\circ) \cdot (250) = 30 \angle 0^\circ \text{ VOLT}$$

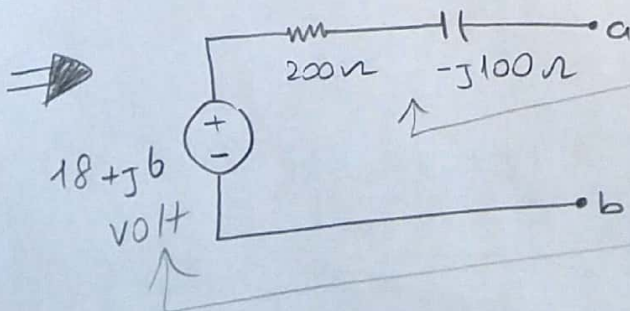


$$250 - j400 + j150 = 250 - j250 \Omega$$



$$\frac{30 \angle 0^\circ}{250 - j250} = 60 - j60 \text{ mA}$$

$$(250 - j250) \parallel 500 = 200 - j100 \Omega$$

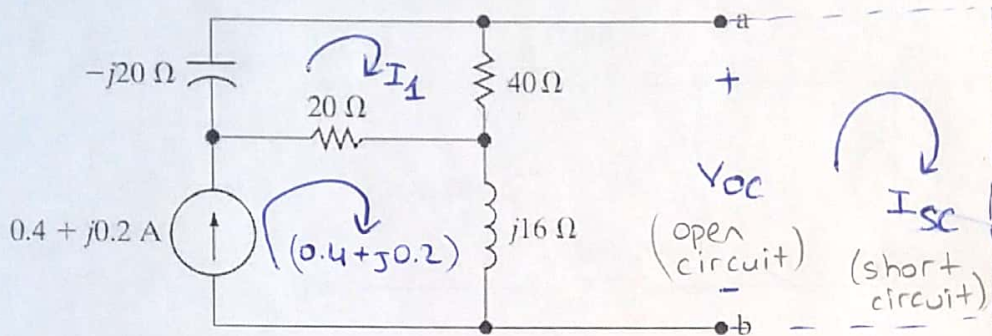


$$(200 - j100)(0.06 - j0.06) = 18 - j6 \text{ V}$$

9.46

9.46 Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.46.

Figure P9.46



For Open Circuit:

$$-j20I_1 + 40I_1 + 20(I_1 - 0.4 - j0.2) = 0$$

$$I_1 = \frac{20(0.4 + j0.2)}{60 - j20} = 0.1 + j0.1 \text{ Ampere}$$

$$V_{oc} = 40I_1 + j16(0.4 + j0.2) = 0.8 + j10.4 \text{ Volt}$$

For short circuit:

MESH

$$-j20I_1 + 40(I_1 - I_{sc}) + 20(I_1 - 0.4 - j0.2) = 0$$

$$40(I_{sc} - I_1) + j16(I_{sc} - 0.4 - j0.2) = 0$$

$$I_{sc} = 0.3 + j0.5 \text{ AMPERE}$$

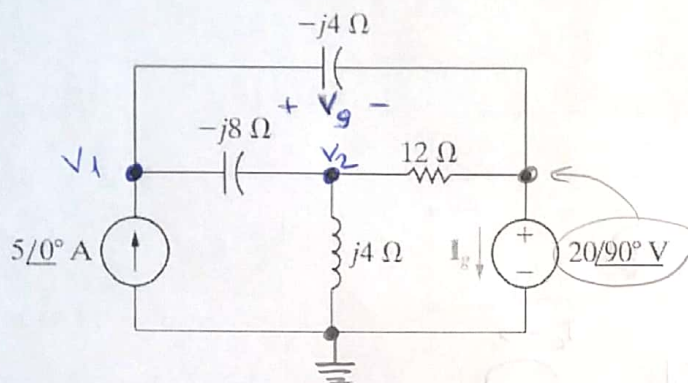
$$Z_N = \frac{V_{oc}}{I_{sc}} = \frac{0.8 + j10.4}{0.3 + j0.5} = 16 + j8 \Omega$$



9.55

9.55 Use the node-voltage method to find the phasor voltage  $V_g$  in the circuit shown in Fig. P9.55.

Figure P9.55



For  $V_1$  :

$$-5\angle 0^\circ + \frac{V_1 - V_2}{-j8} + \frac{V_1 - 20\angle 90^\circ}{-j4} = 0$$

For  $V_2$  :

$$\frac{V_2 - V_1}{-j8} + \frac{V_2}{j4} + \frac{V_2 - 20\angle 90^\circ}{12} = 0$$

← solve

$$V_1 = \frac{-8}{3} + j\frac{4}{3}$$

$$V_2 = -8 + j4$$

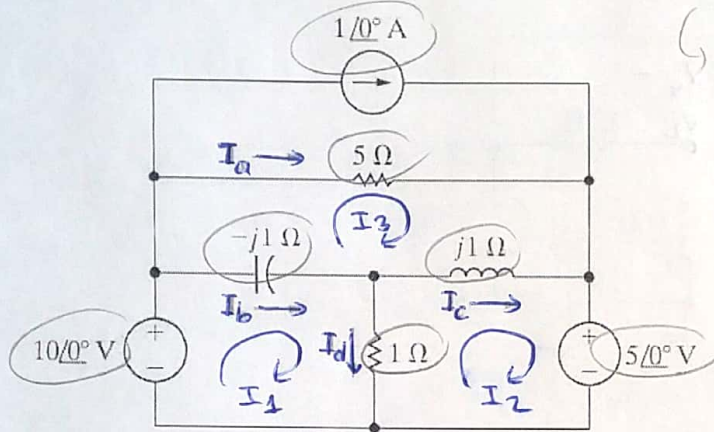
$$V_g = V_1 - 20\angle 90^\circ$$

$$V_g = -\frac{8}{3} - j\frac{56}{3} \text{ V}$$

9.62

9.62 Use the mesh-current method to find the branch currents  $I_a$ ,  $I_b$ ,  $I_c$ , and  $I_d$  in the circuit shown in Fig. P9.62.

Figure P9.62



$$\begin{aligned}
 & -10\angle 0^\circ + (-j1) \cdot (I_1 - I_3) + 1 \cdot (I_1 - I_2) = 0 \\
 & -10\angle 0^\circ - jI_1 + jI_3 + I_1 - I_2 = 0 \\
 & -5\angle 0^\circ + 1(I_2 - I_3) + j1(I_2 - I_3) = 0 \\
 & -5\angle 0^\circ + I_2 - I_3 + jI_2 - jI_3 = 0 \\
 & -1\angle 0^\circ + j1(I_3 - I_2) + (-j1) \cdot (I_3 - I_1) = 0 \\
 & -1\angle 0^\circ + jI_3 - jI_2 - jI_3 + jI_1 = 0
 \end{aligned}$$

$$\begin{aligned}
 & -10\angle 0^\circ + (1 - j1)I_1 - 1 \cdot I_2 + j1 \cdot I_3 = 0 \\
 & -5\angle 0^\circ - 1I_1 + (1 + j1)I_2 - j1 \cdot I_3 = 0 \\
 & -1\angle 0^\circ + j1 \cdot I_1 - j1 \cdot I_2 + I_3 = 0
 \end{aligned}$$

$$\begin{aligned}
 I_1 &= 11 + j10 \text{ Amper} \\
 I_2 &= 11 + j5 \text{ Amper} \\
 I_3 &= 6 \text{ Amper}
 \end{aligned}$$

$$I_a = I_3 - 1 = 5 \text{ Amper}$$

$$I_b = I_1 - I_3 = 5 + j10 \text{ Amper}$$

$$I_c = I_2 - I_3 = 5 + j5 \text{ Amper}$$

$$I_d = I_1 - I_2 = j5 \text{ Amper}$$

solve