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- CSE 351 -

Homework 1

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1) Determine whether the below systems are linear or non-linear.

If a given system satisfies the superposition condition, then it is a linear system. Otherwise it is a non-linear system.

1. condition

$$\boxed{\begin{array}{l} f_1(t) \longrightarrow \boxed{} \longrightarrow y_1(t) \\ f_2(t) \longrightarrow \boxed{} \longrightarrow y_2(t) \end{array}}$$

↳ Let $F_A'(t)$

2. condition

$$\boxed{k_1 f_1(t) + k_2 f_2(t) \longrightarrow \boxed{} \longrightarrow k_1 y_1(t) + k_2 y_2(t)}$$

↳ Let $F_B'(t)$

Apply the superposition condition;

a) $$\frac{dy}{dt} + 2y(t) = f^2(t)$$

1. condition

$$\frac{dy_1}{dt} + 2y_1(t) = f_1^2(t) \longrightarrow \frac{d k_1 y_1(t)}{dt} + 2k_1 y_1(t) = k_1 f_1^2(t)$$

$$\frac{dy_2}{dt} + 2y_2(t) = f_2^2(t) \longrightarrow \frac{d k_2 y_2(t)}{dt} + 2k_2 y_2(t) = k_2 f_2^2(t)$$

Add them +

$$\frac{d}{dt}\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)}\Big] + 2\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)}\Big] = \underbrace{k_1 f_1^2(t) + k_2 f_2^2(t)}_{F_A'(t)}$$

2. condition

$$\frac{d}{dt}\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)}\Big] + 2\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)}\Big] = \underbrace{\Big[k_1 f_1(t) + k_2 f_2(t)\Big]^2}_{F_B'(t)}$$

Since $F_A'(t) \neq F_B'(t)$, this system is <u>non-linear.</u>

b) $\frac{dy}{dt} + 3t\,y(t) = t^2 f(t)$

1. condition

$$\frac{dy_1(t)}{dt} + 3t\,y_1(t) = t^2 f_1(t) \longrightarrow \frac{d k_1 y_1(t)}{dt} + 3t k_1 y_1(t) = t^2 k_1 f_1(t)$$

$$\frac{dy_2(t)}{dt} + 3t\,y_2(t) = t^2 f_2(t) \longrightarrow \frac{d k_2 y_2(t)}{dt} + 3t k_2 y_2(t) = t^2 k_2 f_2(t)$$

Add them $+$

$$\frac{d}{dt}\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)}\Big] + 3t\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)}\Big] = t^2\Big[\underbrace{k_1 f_1(t) + k_2 f_2(t)}_{F_A'(t)}\Big]$$

2. condition

$$\frac{d}{dt}\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)}\Big] + 3t\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)}\Big] = t^2\Big[\underbrace{k_1 f_1(t) + k_2 f_2(t)}_{F_B'(t)}\Big]$$

Since $F_A'(t) = F_B'(t)$, this system is linear.

2

a) For the LTIC system with the below system equation, find the zero-input response ($y_0(t)$) where the initial conditions are $y_0(0) = 2$ and $\frac{dy_0(0)}{dt} = -1$.

$$(D^2 + 5D + 6)\, y(t) = (D+1)\, f(t)$$

Find characteristic equation.

$$\lambda^2 + 5\lambda + 6 = 0$$

$$(\lambda + 2) \cdot (\lambda + 3) = 0$$

$$\boxed{\lambda_1 = -2} \qquad \boxed{\lambda_2 = -3}$$

$$\boxed{y_0(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} = c_1 e^{-2t} + c_2 e^{-3t}}$$

$$\boxed{y_0'(t) = \frac{d y_0(t)}{dt} = -2c_1 e^{-2t} - 3c_2 e^{-3t}}$$

Use initial conditions;

$$y_0(0) = 2 \implies c_1 e^0 + c_2 e^0 = 2$$

$$\boxed{c_1 + c_2 = 2}$$

$$y_0'(0) = -1 \implies -2c_1 e^0 - 3c_2 e^0 = -1$$

$$\boxed{-2c_1 - 3c_2 = -1}$$

$\rightarrow$ $c_1 = 5$, $c_2 = -3$

Therefore

$$\boxed{y_0(t) = 5e^{-2t} - 3e^{-3t}}$$

b) For the LTIC system with the unit impulse response of $h(t) = e^{-t}u(t)$. Find the zero state response of the system $y(t)$ if input is $f(t) = u(t)$.

$$f(t) = u(t) \longrightarrow \boxed{h(t) = e^{-t} \cdot u(t)} \longrightarrow y(t) = ?$$

$$y(t) = h(t) * f(t)$$

$$= \int_{-\infty}^{\infty} f(\tau) \cdot h(t-\tau) \cdot d\tau$$

$$\boxed{f(\tau) = f(t)\Big|_{t=\tau} = u(\tau)}$$

$$\boxed{h(t-\tau) = h(t)\Big|_{t=t-\tau} = e^{\tau - t} \cdot u(t-\tau)}$$

$$y(t) = \int_{0}^{t} u(\tau) \cdot u(t-\tau) \cdot e^{\tau - t} \, d\tau$$

$$= \int_{0}^{t} e^{\tau - t} \, d\tau$$

$$= e^{-t} \int_{0}^{t} e^{\tau} \, d\tau$$

$$= e^{-t} \cdot e^{\tau} \Big|_{0}^{t}$$

$$= e^{-t} \cdot (e^{t} - 1)$$

$$= 1 - e^{-t} , \quad t \geq 0$$

$y(t) = 0$, $t < 0$, then $\boxed{y(t) = (1 - e^{-t}) u(t)}$

3

a) Find the unit impulse response $h[k]$ of the following system : $y[k+1] + 2y[k] = f[k]$.

$$\boxed{h[k] = \frac{b_0}{a_0}\delta[k] + y_0[k]u[k]}$$

$$(E+2)y[k] = f[k]$$

Find characteristic equation : $(\gamma+2) = 0$

$$\boxed{\gamma = -2}$$

$$y_0[k] = c(\gamma)^k$$
$$= c(-2)^k$$

$$\boxed{a_0 = 2, \ b_0 = 1}$$

Therefore,

$$\boxed{h[k] = \frac{1}{2}\delta[k] + c(-2)^k \cdot u[k]}$$

Find c, using iterative solution :

$$(E+2)h[k] = \delta[k]$$
$$h[k+1] + 2h[k] = \delta[k]$$

Put $k=-1$

$$h[-1] = \delta[-1] = 0$$

$$\boxed{h[-1] = 0} \quad \boxed{h[0] = 0}$$

use $h[0] = 0$ and $k=0$:

$$\underset{0}{h[k]} = \frac{1}{2} + c\cdot(-2)^0 \longrightarrow \boxed{c = \frac{-1}{2}}$$

So, the result is ; $\boxed{h[k] = \frac{1}{2}\delta[k] - \frac{1}{2}(-2)^k \cdot u[k]}$

b) Determine the zero-state response of the LTID system with the unit impulse response of $h[k] = (-2)^k u[k]$ if the input $f[k] = e^{-k} u[k]$.

$$\left.\begin{array}{l} f[k] = e^{-k} u[k] \\ h[k] = (-2)^k u[k] \end{array}\right\} \longrightarrow \underbrace{y[k] = f[k] * h[k]}_{\text{zero-state response}}$$

using Table 3.1 of our text book;

$$\left.\begin{array}{l} f_1[k] = \gamma_1^k u[k] \\ f_2[k] = \gamma_2^k u[k] \end{array}\right\} \longrightarrow \boxed{\gamma_1 = \frac{1}{e}} \quad \boxed{\gamma_2 = -2}$$

$$\gamma_1 \neq \gamma_2$$

$$y[k] = f_1[k] * f_2[k] = \left[\frac{\gamma_1^{k+1} - \gamma_2^{k+1}}{\gamma_1 - \gamma_2}\right] u[k]$$

So, the result is $\boxed{\left[\dfrac{\left(\frac{1}{e}\right)^{k+1} - (-2)^{k+1}}{\frac{1}{e} + 2}\right] u[k]}$