Esra Eryılmaz
171044046

1

- CSE 351 -

Homework 2

23.04.2021

① Consider a system with the following transfer function,

$$H(s) = (2s+3) / (s^2+5s+6)$$

a) Determine it's zero-state response if the input $f(t) = e^{-3t}u(t)$.

$$f(t) = e^{-3t}u(t) \iff F(s) = \frac{1}{s+3}$$

$$y(s) = H(s)\cdot F(s) = \frac{2s+3}{(s+3)\cdot(s^2+5s+6)} = \frac{2s+3}{(s+2)\cdot(s+3)^2}$$

$$= \frac{k}{s+2} + \frac{a_0}{(s+3)^2} + \frac{a_1}{s+3}$$

$$k = \left.\frac{2s+3}{(s+3)^2}\right|_{s=-2} = -1 \qquad a_0 = \left.\frac{2s+3}{s+2}\right|_{s=-3} = 3$$

$$y(s) = s/ \frac{-1}{s+2} + \frac{3}{(s+3)^2} + \frac{a_1}{s+3} = s/ \left.\frac{2s+3}{(s+2)\cdot(s+3)^2}\right|_{s\to\infty}$$

$$-1 + 0 + a_1 = 0$$

$$a_1 = 1$$

$$y(s) = \frac{-1}{s+2} + \frac{3}{(s+3)^2} + \frac{1}{s+3}$$

$$y(t) = \left(-e^{-2t} + (1+3t)e^{-3t}\right)u(t)$$

b) Write down the differential equation relating output $y(t)$ to the input $f(t)$.

$$Y(s) = \left(\frac{(2s+3)}{(s^2+5s+6)}\right) \cdot F(s)$$

$$(s^2+5s+6) \cdot Y(s) = (2s+3) \cdot F(s)$$

$$(D^2+5D+6)\, y(t) = (2D+3)\, f(t)$$

$$\boxed{\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y(t) = 2\frac{df}{dt} + 3f(t)}$$

c) Find the inverse Laplace transform of $(s+2)/s(s+1)^2$

$$H(s) = \frac{s+2}{s(s+1)^2} = \frac{k}{s} + \frac{a_0}{(s+1)^2} + \frac{a_1}{s+1}$$

Partial fraction expansion $\rightarrow$

$$k = \frac{s+2}{s(s+1)^2} \cdot s \Big|_{s=0} \Rightarrow \boxed{k = 2} \qquad a_0 = \frac{s+2}{s(s+1)^2} \cdot (s+1)^2 \Big|_{s=-1} \Rightarrow \boxed{a_0 = -1}$$

$$H(s) = s \,\Big/ \frac{s+2}{s\cdot(s+1)^2} = s \,\Big/\!\!\Big/ \frac{2}{s} - \frac{1}{(s+1)^2} + \frac{a_1}{s+1} \Bigg|_{s\to\infty}$$

$$0 = 2 + 0 + a_1$$

$$\boxed{a_1 = -2}$$

$$H(s) = \frac{2}{s} - \frac{1}{(s+1)^2} - \frac{2}{s+1}$$

$$h(t) = \left(2 - (2+t)e^{-t}\right) u(t)$$

(2) For a an LTID system with the following differential equation,

$$2y[k+2] - 3y[k+1] + y[k] = 4f[k+2] - 3f[k-1],$$

Find the output $y[k]$ if the input is $f[k] = (4)^{-k} u[k]$ and initial conditions are $y[-1] = 0$ and $y[-2] = 1$.

Delay form:

$$\boxed{2y[k] - 3y[k-1] + y[k-2] = 4f[k] - 3f[k-3]}$$

use initial conditions:

$$y[k] \Leftrightarrow y[z]$$

$$y[k-1] \Leftrightarrow \frac{1}{z} y[z] + \cancel{y[-1]}^{\,0} = \frac{1}{z} y[z]$$

$$y[k-2] \Leftrightarrow \frac{1}{z^2} y[z] + \underbrace{\cancel{y[-2]}}_{1} = \frac{1}{z^2} y[z] + 1$$

$$f[k] = (4)^{-k} \cdot u[k] = \left(\frac{1}{4}\right)^k u[k] = (0.25)^k u[k] \Rightarrow \frac{z}{z - 0.25}$$

$$2y[z] - \frac{3}{z} y[z] + \frac{1}{z^2} y[z] + 1 = \frac{4z^3 - 3}{z^2(z - 0.25)}$$

$$\left(2 - \frac{3}{z} + \frac{1}{z^2}\right) y[z] = \frac{3z^3 + 0.25z^2 - 3}{z^3 - 0.25z^2}$$

$$\frac{y[z]}{z} = \frac{3z^3 + 0.25z^2 - 3}{(2z^2 - 3z + 1) \cdot (z^2 - 0.25z)} = \frac{3z^2 + 0.25z^2 - 3}{(2z-1)(z-1)(z-0.25) \cdot z}$$

$$= \frac{k1}{z} + \frac{k2}{(2z-1)} + \frac{k3}{(z-1)} + \frac{k4}{(z-0.25)}$$

Using Partial fraction expansion

$$k_1 = 12$$

$$k_2 = 41$$

$$k_3 = \frac{1}{3}$$

$$k_4 = \frac{-94}{3}$$

$$\frac{y(z)}{z} = \frac{12}{k} + \frac{41}{(2z-1)} + \frac{1/3}{(z-1)} - \frac{94/3}{(z-0.25)}$$

$$y(z) = 12 + \frac{41z}{(2z-1)} + \frac{z}{3(z-1)} - \frac{94 \cdot z}{3(z-0.25)}$$

$$\boxed{y[k] = \left(41\,(0.5)^k + \frac{1}{3} - \frac{94}{3}(0.25)^k\right) u[k]}$$

3) Find the inverse z-transform of $z(-5z+22)/(z+1)(z-2)^2$.

$$z\,\frac{H[z]}{z} = z\,\frac{-5z+22}{(z+1)(z-2)^2} = z\left(\frac{3}{z+1} + \frac{k}{z-2} + \frac{4}{(z-2)^2}\right)\Bigg|_{z\to\infty}$$

$$0 = 3 + k + 0$$

$$\boxed{k = -3}$$

$$H[z] = \frac{3z}{z+1} - \frac{3z}{z-2} + \frac{4z}{(z-2)^2}$$

$$H[z] \Longleftrightarrow h[k]$$

$$h[k] = \left(3(-1)^k - 3(2)^k + 2k(2)^k\right)u[k]$$