

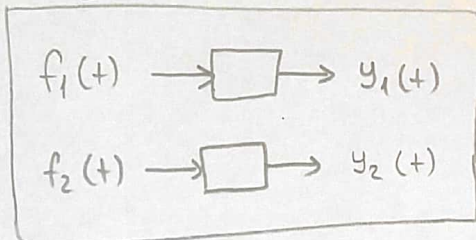
Homework 1

12.04.2021

① Determine whether the below systems are linear or non-linear.

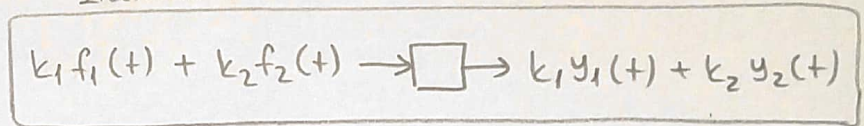
If a given system satisfies the superposition condition, then it is a linear system. Otherwise it is a non-linear system.

1. condition



Let $F_A'(t)$

2. condition



Let $F_B'(t)$

Apply the superposition condition;

$$a) \frac{dy}{dt} + 2y(t) = f^2(t)$$

1. condition

$$\left[\begin{aligned} \frac{dy_1}{dt} + 2y_1(t) = f_1^2(t) &\longrightarrow \frac{dk_1 y_1(t)}{dt} + 2k_1 y_1(t) = k_1 f_1^2(t) \\ \frac{dy_2}{dt} + 2y_2(t) = f_2^2(t) &\longrightarrow \frac{dk_2 y_2(t)}{dt} + 2k_2 y_2(t) = k_2 f_2^2(t) \\ \text{Add them} &+ \\ \frac{d}{dt} \left[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)} \right] + 2 \left[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)} \right] &= \underbrace{k_1 f_1^2(t) + k_2 f_2^2(t)}_{F_A'(t)} \end{aligned} \right.$$

2. condition

$$\left[\frac{d}{dt} \left[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)} \right] + 2 \left[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)} \right] = \left[\underbrace{k_1 f_1(t) + k_2 f_2(t)}_{F_B'(t)} \right]^2 \right.$$

Since $F_A'(t) \neq F_B'(t)$, this system is non-linear.

$$b) \frac{dy}{dt} + 3ty(t) = t^2 f(t)$$

1. condition

$$\left[\begin{aligned} \frac{dy_1(t)}{dt} + 3ty_1(t) &= t^2 f_1(t) \longrightarrow \frac{dk_1 y_1(t)}{dt} + 3tk_1 y_1(t) = t^2 k_1 f_1(t) \\ \frac{dy_2(t)}{dt} + 3ty_2(t) &= t^2 f_2(t) \longrightarrow \frac{dk_2 y_2(t)}{dt} + 3tk_2 y_2(t) = t^2 k_2 f_2(t) \end{aligned} \right.$$

Add them +

$$\frac{d}{dt} \left[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)} \right] + 3t \left[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)} \right] = t^2 \left[\underbrace{k_1 f_1(t) + k_2 f_2(t)}_{F'_A(t)} \right]$$

2. condition

$$\left[\frac{d}{dt} \left[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)} \right] + 3t \left[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y'(t)} \right] = t^2 \left[\underbrace{k_1 f_1(t) + k_2 f_2(t)}_{F'_B(t)} \right] \right.$$

Since $F'_A(t) = F'_B(t)$, this system is linear.

②

a) For the LTIC system with the below system equation, find the zero-input response ($y_0(t)$) where the initial conditions are $y_0(0) = 2$ and $\frac{dy_0(0)}{dt} = -1$.

$$(D^2 + 5D + 6)y(t) = (D + 1)f(t)$$

Find characteristic equation.

$$\lambda^2 + 5\lambda + 6 = 0$$

$$(\lambda + 2) \cdot (\lambda + 3) = 0$$

$$\boxed{\lambda_1 = -2} \quad \boxed{\lambda_2 = -3}$$

$$y_0(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} = c_1 e^{-2t} + c_2 e^{-3t}$$

$$y_0'(t) = \frac{dy_0(t)}{dt} = -2c_1 e^{-2t} - 3c_2 e^{-3t}$$

Use initial conditions;

$$y_0(0) = 2 \Rightarrow c_1 e^0 + c_2 e^0 = 2$$

$$\boxed{c_1 + c_2 = 2}$$

$$y_0'(0) = -1 \Rightarrow -2c_1 e^0 - 3c_2 e^0 = -1$$

$$\boxed{-2c_1 - 3c_2 = -1}$$

$$\rightarrow \begin{aligned} c_1 &= 5 \\ c_2 &= -3 \end{aligned}$$

Therefore

$$\boxed{y_0(t) = 5e^{-2t} - 3e^{-3t}}$$

b) For the LTIC system with the unit impulse response of $h(t) = e^{-t}u(t)$. Find the zero state response of the system $y(t)$ if input is $f(t) = u(t)$.

$$f(t) = u(t) \rightarrow \boxed{h(t) = e^{-t} \cdot u(t)} \rightarrow y(t) = ?$$

$$y(t) = h(t) * f(t)$$

$$= \int_{-\infty}^{\infty} f(\tau) \cdot h(t-\tau) \cdot d\tau$$

$$\boxed{f(\tau) = f(t) \Big|_{t=\tau} = u(\tau)}$$

$$\boxed{h(t-\tau) = h(t) \Big|_{t=t-\tau} = e^{T-t} \cdot u(t-\tau)}$$

$$y(t) = \int_0^t u(\tau) \cdot u(t-\tau) \cdot e^{T-t} d\tau$$

$$= \int_0^t e^{T-t} d\tau$$

$$= e^{-t} \int_0^t e^{\tau} d\tau$$

$$= e^{-t} \cdot e^{\tau} \Big|_0^t$$

$$= e^{-t} \cdot (e^t - 1)$$

$$= 1 - e^{-t}, \quad t \geq 0$$

$$y(t) = 0, \quad t < 0, \quad \text{then} \quad \boxed{y(t) = (1 - e^{-t})u(t)}$$

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③

a) Find the unit impulse response $h[k]$ of the following system: $y[k+1] + 2y[k] = f[k]$.

$$h[k] = \frac{b_0}{a_0} \delta[k] + y_0[k] u[k]$$

$$(E+2)y[k] = f[k]$$

Find characteristic equation:

$$(\gamma + 2) = 0$$

$$\gamma = -2$$

$$\begin{aligned} y_0[k] &= c(\gamma)^k \\ &= c(-2)^k \end{aligned}$$

$$a_0 = 2, b_0 = 1$$

Therefore,

$$h[k] = \frac{1}{2} \delta[k] + c(-2)^k \cdot u[k]$$

Find c , using iterative solution:

$$(E+2)h[k] = \delta[k]$$

$$h[k+1] + 2h[k] = \delta[k]$$

Put $k = -1$

$$h[-1] = \delta[-1] = 0$$

$$h[-1] = 0$$

$$h[0] = 0$$

use $h[0] = 0$ and $k = 0$:

$$h[k] = \frac{1}{2} + c \cdot (-2)^0 \rightarrow c = -\frac{1}{2}$$

So, the result is ; $h[k] = \frac{1}{2} \delta[k] - \frac{1}{2} (-2)^k \cdot u[k]$

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b) Determine the zero-state response of the LTID system with the unit impulse response of $h[k] = (-2)^k u[k]$ if the input $f[k] = e^{-k} u[k]$.

$$\left. \begin{array}{l} f[k] = e^{-k} u[k] \\ h[k] = (-2)^k u[k] \end{array} \right\} \rightarrow y[k] = f[k] * h[k]$$

↓
zero-state response

Using Table 3.1 of our textbook;

$$\left. \begin{array}{l} f_1[k] = \delta_1^k u[k] \\ f_2[k] = \delta_2^k u[k] \end{array} \right\} \rightarrow \boxed{\delta_1 = \frac{1}{e}} \quad \boxed{\delta_2 = -2}$$

$\delta_1 \neq \delta_2$

$$y[k] = f_1[k] * f_2[k] = \left[\frac{\delta_1^{k+1} - \delta_2^{k+1}}{\delta_1 - \delta_2} \right] u[k]$$

So, the result is

$$\boxed{\left[\frac{\left(\frac{1}{e}\right)^{k+1} - (-2)^{k+1}}{\frac{1}{e} + 2} \right] u[k]}$$

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