Hypergeometric Distribution

Esra Eryılmaz
171044046

MATH 118 - Gebze Technical University

26 June 2021

Introduction

Definition

In probability theory and statistics, the **hypergeometric distribution** is a discrete probability distribution that describes the probability of **k** successes (random draws for which the object drawn has a specified feature) in **n** draws, without replacement, from a finite population of size **N** that contains exactly **K** objects with that feature, wherein each draw is either a success or a failure.

Formula

- A random variable X follows the hypergeometric distribution if its probability mass function (pmf) is given by

$$\boxed{P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}}$$

where

- **N** is the population size,
- **n** is the number of draws
- **K** is the number of success states in the population,
- **k** is the number of observed successes.

Problem 1

Assume that a bag contains 8 blue, 12 red balls,

a) What is the probability of getting red ball among 5 of the 8 selected balls, without replacement?

b) What is the probability of getting at most 1 red ball among selected 4 balls, without replacement?

Problem 1 (a)

a) What is the probability of getting red ball among 5 of the 8 selected balls, without replacement?

Problem 1 (a) Solution

a) What is the probability of getting red ball among 5 of the 8 selected balls, without replacement?

$$P(X=k)=\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

- N = 20 (Total 20 balls exist)
- n = 8 (8 balls are drawn from the bag)
- K = 12 (Since the red ball will be drawn, 12 red balls exist in bag)
- k = 5 (Question asks the probability of 5 selected balls being red)

$$P(X=5)=\frac{\binom{12}{5}\binom{20-12}{8-5}}{\binom{20}{8}}=\frac{\binom{12}{5}\binom{8}{3}}{\binom{20}{8}}=\frac{\frac{12!}{5!7!}\cdot\frac{8!}{3!5!}}{\frac{20!}{8!12!}}=\frac{792\cdot 56}{125970}\approx 0,352$$

Problem 1 (b)

b) What is the probability of getting at most 1 red ball among selected 4 balls, without replacement?

Problem 1 (b) Solution

b) What is the probability of getting at most 1 red ball among selected 4 balls, without replacement?

- N = 20 (Total 20 balls exist)
- n = 4 (Choose 4 balls from the bag)
- K = 12 (Since the red ball will be drawn, 12 red balls exist in bag)
- k = 0 and 1 (Question asks the probability of at most 1 selected balls being red)

$$P(X \leq 1) = [P(X=0) + P(X=1)] = \left[\frac{\binom{12}{0}\binom{20-12}{4-0}}{\binom{20}{4}} + \frac{\binom{12}{1}\binom{20-12}{4-1}}{\binom{20}{4}} \right]$$

$$= \frac{1 \cdot 70 + 12 \cdot 56}{4845} \approx 0,153$$

References

- Walpole, Myers, Myers, Ye, Probability & Statistics for Engineers & Scientists - *9th Edition*
- Wikipedia: Hypergeometric distribution
- BUders : http://www.buders.com/