HOMEWORK-2

1)

The Legendre Polynomials can be calculated by means of the formulas

$P_0(x) = 1$ and $P_1(x) = x$,

$$P_n(\boldsymbol{x}) = \left[\left(\frac{2n-1}{n}\right)\right]\boldsymbol{x}P_{n-1}(\boldsymbol{x}) - \left[\left(\frac{n-1}{n}\right)\right]P_{n-2}(\boldsymbol{x})$$

Where n=2,3,4,… and x is any floating number between -1 and 1. (Note that Legendre Polynomials are floating-point quantities.)

- **a)** Evaluate the first 10 terms (n=0,1,…,9) of polynomials and store them in an array . Let x be a imput parameter. Do not use the x value given in the input text file. The array should be like {1,x,P_2(x),…,P_9(x)}.
- **b)** Determine the sum of first 10 terms (n=0,1,…,9) of the polynomials for the given x value in input textfile and print it on the screen.
- **c)** The asymptotic expansion of Legendre Polynomials for large numbers of n is given as,

$$P_n(x) = \sqrt{\frac{2}{\pi n \sin\left(\arccos(x)\right)}}\left\{\cos\left[\left(n+0.5\right)\arccos\left(x\right) - \frac{\pi}{4}\right]\right\}$$

  - i) Calculate $P_n(\boldsymbol{x})$ for n=50 and compare it with the asympotic expansion. Print both values on the screen.
  - ii) Evaluate the error between them. Print it on the screen
  - iii) Find the appropriate n value that makes the absolute error is less then 10^{-4}. Print the appropriate n value on the screen.
- **d)** Let $\vec{r}$=a$\hat{\imath}$+b$\hat{\jmath}$+c$\hat{k}$ and $\vec{r}'$=x$\hat{\imath}$+y$\hat{\jmath}$+z$\hat{k}$ two vectors in cartesian coordinates. And the inverse distance $\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$ between vectors can be calculated for $|\vec{r}| > |\vec{r}'|$. Note that θ is the angle between the vectors.

$$\frac{1}{\left|\vec{r}-\vec{r}'\right|} = \sum_{n=0}^{\infty}\frac{\left|\vec{r}'\right|^n}{\left|\vec{r}\right|^{n+1}}P_n\left(\cos\theta\right)$$

i) Evaluate the angle θ and cos(θ) and print it on the screen.

ii) Calculate the $\frac{1}{|\vec{r}-\vec{r}'|}$ and print on the screen.(Check for $|\vec{r}| > |\vec{r}'|$).

iii) Then, evalute the series sum for an appropriate n value that makes the absolute error is less then 10^{-4}. Print all the results on the screen.

Note: <u>You are allowed to use math.h library. Do not use pointer !!! Your code should be properly commented. Uncommented code will get partial credit. You</u>