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STAT 497 - Assignment 2, due: 29 Dec 2022

1) $Y_t = Y_{t-1} + 0.6Y_{t-2} - 0.6Y_{t-3} + a_t$, where $a_t \sim WN(0,1)$

a-
$$(1 - B - 0.6B^2 + 0.6B^3)Y_t = a_t$$

- In order to investigate whether the process has unit root or not, let us divide AR polynomial by $(1-B)$

$$\begin{array}{l|l} 1-B-0.6B^2+0.6B^3 & 1-B \\ \hline -\ (1-B) & 1-0.6B^2 \\ \hline -0.6B^2+0.6B^3 & \\ -\ (-0.6B^2+0.6B^3) & \\ \hline 0 & \end{array}$$

$$(1-0.6B^2)\underbrace{(1-B)}Y_t = a_t$$

There exist unit root $(\phi = 1)$

- We found out that the process is not stationary, since it has unit root, we need to apply differencing.

∇Y_t is called differenced series $\Rightarrow \nabla Y_t = Y_t - Y_{t-1} = (1-B)Y_t$

$$(1-0.6B^2)\nabla Y_t = a_t$$

Let W_t denote ∇Y_t

$$\underbrace{(1-0.6B^2)}_{\phi(B)} W_t = a_t$$

$\Rightarrow$ So W_t is stationary.

- AR(2) with $\phi = 0.6 < 1$, after differencing the series become stationary.

b- $n=100$, mean $= -0.066$ and variance $= 1.594$

$$WN = \pm\frac{2}{\sqrt{n}} = \pm 0.2$$

- Differenced series seem AR(2), PACF cuts off lag 2

$$W_t = (1-B)Y_t \sim AR(2)$$
$$Y_t \sim ARIMA(2,1,0)$$

b) Yule - Walker estimates:

$$\tilde{\phi}_1 = \frac{\hat{\rho}_1(1-\hat{\rho}_2)}{1-\hat{\rho}_1^2} = \frac{r_1(1-r_2)}{1-r_1^2} = \frac{(-0.132)(1-0.539)}{1-(-0.132)^2} = -0.062$$

$$\tilde{\phi}_2 = \frac{\hat{\rho}_2-\hat{\rho}_1^2}{1-\hat{\rho}_1^2} = \frac{r_2-r_1^2}{1-r_1^2} = \frac{(0.539)-(-0.132)^2}{1-(-0.132)^2} = 0.531$$

$$\tilde{\sigma}^2 = \hat{\gamma}_0(1-\tilde{\phi}_1\hat{\rho}_1 - \tilde{\phi}_2\hat{\rho}_2) = 1.594\left(1-[(-0.062)\cdot(-0.132)]-[(0.531)\cdot(0.539)]\right)$$

$$= 1.1247$$

$$\tilde{c} = \bar{x}(1-\tilde{\phi}_1-\tilde{\phi}_2) = (-0.066)(1-(-0.062)-(0.531))$$

$$= -0.035046$$

c) AR(2)

$$Y_t = -0.035 - 0.062\,Y_{t-1} + 0.531\,Y_{t-2} + a_t$$

$$\hat{Y}_{100}(1) = -0.035 - 0.062(1178.77) + 0.531(1190.31) = 27.93587$$

95% Prediction interval for Y_{101}

$$\hat{Y}_{100}(1) \mp 1.96\sqrt{\tilde{\sigma}^2}$$

$$= 27.936 \mp 1.96\sqrt{1.12}$$

$(25.86 \ , \ 30.01)$

$$\hat{Y}_{100}(2) = -0.035 - 0.062(27.936) + 0.531(1178.77) = 624.159$$

$$= \hat{Y}_{100}(2) \mp 1.96\sqrt{\tilde{\sigma}^2(1+(0.062)^2)}$$

$$= 624.159 \mp 1.96\sqrt{1.12(1+0.062^2)}$$

$(622.082, 626.237)$

$$e_{100}(2) = 0.062\,\underbrace{e_{100}(1)}_{a_{111}} + a_{102}$$

$$V(e_{100}(2)) = \sigma^2(1+0.062^2)$$