

Assignment 1 Solutions (Stat 497)

Q1) a) $(1 - 0.8B) Y_t = (1 - 0.7B - 0.6B^2) a_t$

\downarrow $\phi(B)$ \downarrow $\theta(B)$
 AR polynomials MA polynomials

AR parameter : $\phi = 0.8$ MA parameters : $\theta_1 = 0.7$ $\theta_2 = 0.6$

so the process is ARMA(1,2)

b) For stationarity, look the AR polynomial

The stationarity condition for AR(1) is " $|\phi| < 1$ "

$\phi = 0.8$ is less than 1 so Y_t is STATIONARY

For invertibility, look the MA polynomials

The invertibility condition for MA(2) is $\theta_1 + \theta_2 < 1$

$$\theta_2 - \theta_1 < 1$$

$$|\theta_2| < 1$$

$$\theta_1 = -0.6 \quad \theta_2 = 0.7 \Rightarrow \theta_1 + \theta_2 = 1.3 > 1 \quad \text{X}$$

so Y_t is not INVERTIBLE

c) In order to write the model in MA representation (RSF), the series should be stationary and $\phi(B)$ & $\theta(B)$ should not share same common root. We know that Y_t is stationary. Now let us check if $\phi(B)$ and $\theta(B)$ share same common root or not

$$(1 - 0.8B) = 0$$

$$B = \frac{1}{0.8} = 1.25 //$$

$$\theta(B) = 1 - 0.7B - 0.6B^2 = 0$$

$$\begin{matrix} -1.2B \\ + 0.5B \end{matrix}$$

$$(1 - 1.2B)(1 + 0.5B) = 0$$

$$B = \frac{1}{1.2} = 0.83$$

$$1 + 0.5B = 0$$

$$B = \frac{-1}{0.5} = -2$$

That is, $\phi(B)$ and $\theta(B)$ do not share same root

So, we can rewrite the model in Random Shock Form

To obtain RSF, leave a_t alone

$$(1 - 0.8B) Y_t = (1 - 0.7B - 0.6B^2) a_t$$

$$\frac{(1 - 0.8B)}{(1 - 0.7B - 0.6B^2)} Y_t = a_t$$

\downarrow
 $\pi(B)$

Pure MA representation (P.P.B + M.M.) condition 1: invertibility

$$Y_t = \psi(B) a_t \quad \text{where } \psi(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots \quad (10)$$

To find $\psi(B)$, we use the equation $\psi(B)\pi(B) = 1$ where

$$\pi(B) = \frac{1 - 0.8B}{1 - 0.7B - 0.6B^2}$$

$$\psi(B) \pi(B) = 1$$

$$\psi(B) \cdot \frac{(1 - 0.8B)}{1 - 0.7B - 0.6B^2} = 1$$

$$\psi(B) \cdot (1 - 0.8B) = 1 - 0.7B - 0.6B^2$$

$$(1 + \psi_1 B + \psi_2 B^2 + \psi_3 B^3 + \dots)(1 - 0.8B) = 1 - 0.7B - 0.6B^2$$

$$1 + \psi_1 B + \psi_2 B^2 + \psi_3 B^3 + \dots - 0.8B - 0.8\psi_1 B^2 - 0.8\psi_2 B^3 + \dots = 1 - 0.7B - 0.6B^2$$

$$1 + B(\psi_1 - 0.8) + B^2(\psi_2 - 0.8\psi_1) + B^3(\psi_3 - 0.8\psi_2) + \dots = 1 - 0.7B - 0.6B^2$$

$$\psi_1 - 0.8 = -0.7$$

$$\psi_1 = 0.1$$

$$\psi_2 - 0.8\psi_1 = -0.6$$

$$\psi_2 - 0.08 = -0.6$$

$$\psi_2 = -0.52$$

$$\psi_3 - 0.8\psi_2 = 0$$

$$\psi_3 = 0.8\psi_2$$

$$\psi_3 = 0.8 \cdot (-0.52)$$

$$\psi_3 = -0.416$$

so, The RSF is $Y_t = \psi(B) a_t$

$$Y_t = (1 + \psi_1 B + \psi_2 B^2 + \psi_3 B^3 + \dots) a_t$$

$$Y_t = (1 + 0.1B - 0.52B^2 - 0.416B^3 + \dots) a_t$$

d) In order to write the model in AR representation (inverted form), the series should be invertible. However, Y_t is not invertible so we cannot write the model in AR representation.

$$Q2) Y_t = 1.4Y_{t-1} - 0.1Y_{t-2} - 0.3Y_{t-3} + a_t - 0.8a_{t-1} - 0.1a_{t-2}$$

$$(1 - 1.4B + 0.1B^2 + 0.3B^3) Y_t = a_t (1 - 0.8B - 0.1B^2)$$

\downarrow $\phi(B)$ \downarrow $\theta(B)$

a) Since finding the root of $\phi(B)$ is not easy task, to check whether the series is stationary or not, we need to look whether the series have unit root or not. For this, we divide AR polynomials by $(1-B)$

$$\begin{array}{r|l} 1 - 1.4B + 0.1B^2 + 0.3B^3 & 1 - B \\ \hline - & -0.3B^2 + 0.3B^3 \\ \hline 1 - 1.4B + 0.4B^2 & \\ - & -0.4B + 0.4B^2 \\ \hline 1 - B & \\ - & 1 - B \\ \hline 0 & \end{array}$$

so AR polynomial is equal to $(1-B)(1-0.4B-0.3B^2)$

so we can write the model

$$(1 - 0.4B - 0.3B^2)(1-B)Y_t = a_t (1 - 0.8B - 0.1B^2)$$

\downarrow

$B_1 = 1 \Rightarrow$ there exists unit root //

so Y_t is NOT STATIONARY

For invertibility, we look MA polynomials

MA parameters: $\theta_1 = 0.8$ $\theta_2 = 0.1$

Invertibility condition $\rightarrow \theta_1 + \theta_2 < 1 \quad \rightarrow$ Let us check $0.8 + 0.1 = 0.9 < 1$
 $\theta_2 - \theta_1 < 1$ $0.1 - 0.8 = -0.7 < 1$
 $|\theta_2| < 1$ $\theta_2 = 0.1 < 1$

so Y_t is invertible.

b) Since the series Y_t is not stationary because of having unit root, we cannot calculate the ACF directly. Existence of unit root means that there exist "stochastic trend". To remove stochastic trend, we need to take differences of Y_t series. Let us apply

$$(1 - 0.4B - 0.3B^2)(1-B)Y_t = a_t (1 - 0.8B - 0.1B^2)$$

$\underbrace{(1-B)}_{\text{unit root}}$

∇Y_t is called differenced series $\Rightarrow \nabla Y_t = Y_t - Y_{t-1} = (1-B)Y_t$

$$(1 - 0.4B - 0.3B^2) \nabla Y_t = a_t (1 - 0.8B - 0.1B^2)$$

Let W_t denote ∇Y_t

$$(1 - 0.4B - 0.3B^2)W_t = a_t (1 - 0.8B - 0.1B^2)$$

$\underbrace{(1 - 0.4B - 0.3B^2)}_{\phi(B)} \quad \underbrace{(1 - 0.8B - 0.1B^2)}_{\theta(B)}$

$\phi_1 = 0.4 \quad \phi_2 = 0.3 \quad \theta_1 = 0.8 \quad \theta_2 = 0.1$

For stationary condition $\Rightarrow \phi_1 + \phi_2 = 0.4 + 0.3 = 0.7 < 1$
 $\phi_2 - \phi_1 = 0.3 - 0.4 = -0.1 < 1$
 $|\phi_2| = 0.3 < 1$

so W_t is stationary

When we look the root of $\phi(B)$ and $\theta(B)$, $\theta(B)$ and $\phi(B)$ do not share some common roots. Therefore, the suggested model is $W_t \sim \text{ARMA}(2, 2)$

$$W_t = 0.4W_{t-1} + 0.3W_{t-2} + a_t - 0.8a_{t-1} - 0.1a_{t-2}$$

After taking differences, we can calculate ACF directly since stationarity condition is satisfied. Therefore, let us calculate ACF

Now firstly calculate autocovariance function and variance of W_t

$$\begin{aligned} \text{Var}(W_t) &= \text{Cov}(W_t, W_t) = \text{Cov}(0.4W_{t-1} + 0.3W_{t-2} + a_t - 0.8a_{t-1} - 0.1a_{t-2}, W_t) \\ &= 0.4\text{Cov}(W_{t-1}, W_t) + 0.3\text{Cov}(W_{t-2}, W_t) + \underbrace{\text{Cov}(a_t, W_t)}_{(*)} - \underbrace{0.8\text{Cov}(a_{t-1}, W_t)}_{(**)} - \underbrace{0.1\text{Cov}(a_{t-2}, W_t)}_{(***)} \end{aligned}$$

$$\begin{aligned} \text{Cov}(a_t, W_t) &= (*) = \text{Cov}(a_t, 0.4W_{t-1} + 0.3W_{t-2} + a_t - 0.8a_{t-1} - 0.1a_{t-2}) \\ &= \underbrace{\text{Cov}(a_t, 0.4W_{t-1})}_0 + \underbrace{\text{Cov}(a_t, 0.3W_{t-2})}_0 + \text{Var}(a_t) - 0.8\text{Cov}(a_t, a_{t-1}) - 0.1\text{Cov}(a_t, a_{t-2}) \\ &\quad \text{present error is indep from past values of series} \quad \text{because present error is indep from past values of series} \quad \text{0 because of indep of errors} \quad \text{0 because of indep of errors} \end{aligned}$$

$$(*) = \text{Var}(a_t) = 1$$

$$\begin{aligned} \text{Cov}(a_{t-1}, W_t) &= (**) = \text{Cov}(a_{t-1}, 0.4W_{t-1} + 0.3W_{t-2} + a_t - 0.8a_{t-1} - 0.1a_{t-2}) \\ &= 0.4\text{Cov}(a_{t-1}, W_{t-1}) + 0.3\text{Cov}(a_{t-1}, W_{t-2}) + \underbrace{\text{Cov}(a_{t-1}, a_t)}_0 - 0.8\text{Var}(a_{t-1}) - \underbrace{0.1\text{Cov}(a_{t-1}, a_{t-2})}_0 \\ &= 0.4\text{Cov}(a_{t-1}, 0.4W_{t-2} + 0.3W_{t-3} + a_{t-1} - 0.8a_{t-2} - 0.1a_{t-3}) - 0.8 \\ &\quad \text{bc of indep of errors} \quad \text{bc of indep} \\ &= 0.4\text{Cov}(a_{t-1}, a_{t-1}) - 0.8 \\ &= 0.4\text{Var}(a_{t-1}) - 0.8 = 0.4 - 0.8 = -0.4 \end{aligned}$$

$$(**) = -0.4$$

$$\begin{aligned} \text{Cov}(a_{t-2}, W_t) &= (***) = \text{Cov}(a_{t-2}, 0.4W_{t-1} + 0.3W_{t-2} + a_t - 0.8a_{t-1} - 0.1a_{t-2}) \\ &= 0.4\text{Cov}(a_{t-2}, W_{t-1}) + 0.3\text{Cov}(a_{t-2}, W_{t-2}) - 0.1\text{Cov}(a_{t-2}, a_{t-2}) \\ &= 0.4\text{Cov}(a_{t-2}, 0.4W_{t-2} + 0.3W_{t-3} + a_{t-1} - 0.8a_{t-2} - 0.1a_{t-3}) \\ &\quad + 0.3\text{Cov}(a_{t-2}, 0.4W_{t-3} + 0.3W_{t-4} + a_{t-2} - 0.8a_{t-3} - 0.1a_{t-4}) \\ &\quad - 0.1\text{Var}(a_{t-2}) \\ &= 0.16\text{Cov}(a_{t-2}, W_{t-2}) - 0.32\text{Var}(a_{t-2}) + 0.3\text{Var}(a_{t-2}) - 0.1\text{Var}(a_{t-2}) \\ &\quad \text{Var}(a_{t-2}) \\ &= 0.16\text{Var}(a_{t-2}) - 0.32\text{Var}(a_{t-2}) + 0.3\text{Var}(a_{t-2}) - 0.1\text{Var}(a_{t-2}) = 0.04 // \end{aligned}$$

$$Var(W_t) = \gamma_0 = 0.4 \text{ cov}(W_{t-1}, W_t) + 0.3 \text{ cov}(W_{t-2}, W_t) + \text{cov}(a_t, W_t) - 0.8 \text{ cov}(a_{t-1}, W_t) - 0.1 \text{ cov}(a_{t-2}, W_t)$$

$$y_0 = 0.4y_1 + 0.3y_2 + 1 - 0.8 * (-0.4) - 0.1 * (0.04)$$

$$y_0 = 0.4x_1 + 0.3x_2 + 1 + 0.32 - 0.004$$

$$y_0 = 0.4y_1 + 0.3y_2 + 1.316$$

$$\begin{aligned} \gamma_1 &= \text{cov}(w_t, w_{t-1}) = \text{cov}(0.4w_{t-1} + 0.3w_{t-2} + a_t - 0.8a_{t-1} - 0.1a_{t-2}, w_{t-1}) \\ &= 0.4\text{cov}(w_{t-1}, w_{t-1}) + 0.3\text{cov}(w_{t-2}, w_{t-1}) + 0.8\text{cov}(a_{t-1}, w_{t-1}) - 0.1\text{cov}(a_{t-2}, w_{t-1}) \\ &= 0.4\gamma_0 + 0.3\gamma_{-1} - 0.8\underbrace{\text{cov}(a_{t-1}, w_{t-1})}_{\text{Var}(a_{t-1})} - 0.1\underbrace{\text{cov}(a_{t-2}, w_{t-1})}_{\text{cov}(a_{t-2}, 0.4w_{t-2}) + \text{cov}(a_{t-2}, 0.8a_{t-2})} \\ &\quad \underbrace{\phantom{\text{cov}(a_{t-2}, w_{t-1})}}_{0.4\text{Var}(a_{t-2}) + 0.8\text{Var}(a_{t-2})} \end{aligned}$$

$$Y_1 = 0.4Y_0 + 0.3Y_1 - 0.8 \times 4 - 0.1(0.4 - 0.8) = 0.4Y_0 + 0.3Y_1 - 0.76$$

$$0.7x_1 = 0.4y_0 - 0.76 //$$

$$\begin{aligned} \gamma_2 &= \text{cov}(w_t, w_{t-2}) = \text{cov}(0.4w_{t-1} + 0.3w_{t-2} + a_t - 0.8a_{t-1} - 0.1a_{t-2}, w_{t-2}) \\ &= 0.4 \text{cov}(w_{t-1}, w_{t-2}) + 0.3 \text{cov}(w_{t-2}, w_{t-2}) - 0.1 \underbrace{\text{cov}(a_{t-2}, w_{t-2})}_{\text{Var}(a_{t-2})} \\ &= 0.4 \gamma_1 + 0.3 \gamma_0 - 0.1 // \end{aligned}$$

After finding γ_1 & $\gamma_2 \Rightarrow$ let us calculate γ_0

$$\begin{aligned}\gamma_0 &= 0.4\gamma_1 + 0.3\gamma_2 + 1.316 = 0.4\gamma_1 + 0.3(0.4\gamma_1 + 0.3\gamma_0 - 0.1) + 1.316 \\ &= 0.4\gamma_1 + 0.12\gamma_1 + 0.09\gamma_0 - 0.03 + 1.316 = 0.52\gamma_1 + 0.09\gamma_0 + 1.286\end{aligned}$$

$$0.9180 = 0.5281 + 1.286$$

$$0.91\gamma_o = 0.52 \left(\frac{0.4\gamma_o - 0.76}{0.7} \right) + 1.286$$

$$0.9\gamma_0 = 0.2971\gamma_0 - 0.5646 + 1.286$$

$$0.6029 \gamma_0 = 0.7214$$

$$x_0 = 1.1966 //$$

$$x_1 = \frac{0.4x_0 - 0.7b}{0.7} = \frac{0.4 * 1.1966 - 0.7b}{0.7} = -0.4019$$

$$x_2 = 0.4x_1 + 0.3x_0 - 0.1 = 0.4 * (-0.4019) + 0.3 * (1.1966) - 0.1$$
$$= 0.09822$$

$$\gamma_3 = \text{cov}(w_t, w_{t-3}) = \text{cov}(0.4w_{t-1}, 0.3w_{t-2} + a_t - 0.8a_{t-1} - 0.1a_{t-2}, w_{t-3})$$

$$= 0.4 \text{cov}(w_{t-1}, w_{t-3}) + 0.3 \text{cov}(w_{t-2}, w_{t-3})$$

$$= 0.4 \gamma_2 + 0.3 \gamma_1$$

$$\begin{aligned}\gamma_4 &= \text{cov}(w_t, w_{t-4}) = \text{cov}(0.4w_{t-1}, 0.3w_{t-2} + a_t - 0.8a_{t-1} - 0.1a_{t-2}, w_{t-4}) \\ &= 0.4 \text{cov}(w_{t-1}, w_{t-4}) + 0.3 \text{cov}(w_{t-2}, w_{t-4}) \\ &= 0.4 \gamma_3 + 0.3 \gamma_2\end{aligned}$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

For $k > 2$, $\gamma_k = 0.4 \gamma_{k-1} + 0.3 \gamma_{k-2}$

$$\gamma_w(k) = \begin{cases} 1.1966 & k=0 \\ -0.4019 & k=1 \\ 0.09822 & k=2 \\ 0.4 \gamma_{k-1} + 0.3 \gamma_{k-2} & k > 2 \end{cases}$$

ACF is

$$\rho_w(k) = \begin{cases} 1 & k=0 \\ -0.4019 / 1.1966 & k=1 \\ 0.09822 / 1.1966 & k=2 \\ 0.4 \rho_w(k-1) + 0.3 \rho_w(k-2) & k > 2 \end{cases}$$

$$\gamma_k = 0.4 \gamma_{k-1} + 0.3 \gamma_{k-2} \quad \text{for } k > 2$$

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{-0.4019}{1.1966}$$

$$\frac{\gamma_k}{\gamma_0} = 0.4 \frac{\gamma_{k-1}}{\gamma_0} + 0.3 \frac{\gamma_{k-2}}{\gamma_0} \quad \text{for } k > 2$$

$$\rho_2 = \frac{\gamma_2}{\gamma_0} = \frac{0.09822}{1.1966}$$

$$\rho_k = 0.4 \rho_{k-1} + 0.3 \rho_{k-2} \quad \text{for } k > 2$$

Q3) $z_t = 0.6z_{t-1} + 0.3z_{t-2} + a_t$
 $z_t = 0.6Bz_t + 0.3B^2z_t + a_t \Rightarrow (1 - 0.6B - 0.3B^2)z_t = a_t$

a) $(1 - 0.6B - 0.3B^2)z_t = a_t$
 \downarrow
 $\phi(B)$

$\phi_1 = 0.6 \quad \phi_2 = 0.3 \Rightarrow$

$\phi_1 + \phi_2 = 0.9 < 1$
 $\phi_2 - \phi_1 = -0.3 < 1$
 $|\phi_2| < 1$ } so the series is stationary

b) $E(z_t) = 0.6E(z_{t-1}) + 0.3E(z_{t-2}) + E(a_t)$

Since z_t is stationary, $E(z_t) = 0.6E(z_t) + 0.3E(z_t) + 0$
 $E(z_t) = 0$

$Var(z_t) = Var(0.6z_{t-1} + 0.3z_{t-2} + a_t)$
 $= 0.36 Var(z_{t-1}) + 0.09 Var(z_{t-2}) + Var(a_t) + 2 \times 0.3 \times 0.6 \times cov(z_{t-1}, z_{t-2})$
 $+ 2 \times 0.3 \times cov(z_{t-2}, a_t) + 2 \times 0.6 \times cov(z_{t-1}, a_t)$
 $\underbrace{0}_{\text{because there is no common term when we open } z_{t-2}}$ $\underbrace{0}_{\text{because there is no common term when we open } z_{t-1}}$

Since z_t is stationary, $Var(z_t) = 0.36 Var(z_t) + 0.09 Var(z_t) + Var(a_t) + 0.36 cov(z_{t-1}, z_{t-2})$

$\gamma_0 = 0.36 \gamma_0 + 0.09 \gamma_0 + \sigma_a^2 + 0.36 \gamma_1$

$0.55 \gamma_0 = \sigma_a^2 + 0.36 \gamma_1$

$\gamma_2(1) = cov(z_t, z_{t-1}) = cov(0.6z_{t-1} + 0.3z_{t-2} + a_t, z_{t-1})$

$\gamma_1 = 0.6 cov(z_{t-1}, z_{t-1}) + 0.3 cov(z_{t-2}, z_{t-1}) + \underbrace{cov(a_t, z_{t-1})}_0$

$\gamma_1 = 0.6 \gamma_0 + 0.3 \gamma_1 =$

$0.7 \gamma_1 = 0.6 \gamma_0 \Rightarrow \gamma_1 = \frac{0.6 \gamma_0}{0.7}$

$0.55 \gamma_0 = \sigma_a^2 + 0.36 \times \frac{0.6}{0.7} \gamma_0 \Rightarrow \gamma_0 = \frac{\sigma_a^2}{0.24} //$

c) $\gamma_2(\rho) = \frac{\sigma_a^2}{0.24}$

$\gamma_1 = 0.6 \gamma_0 + 0.3 \gamma_1$

$cov(z_t, z_{t-2}) = \gamma_2 = cov(0.6z_{t-1} + 0.3z_{t-2} + a_t, z_{t-2})$

$\gamma_2 = 0.6 cov(z_{t-1}, z_{t-2}) + 0.3 cov(z_{t-2}, z_{t-2}) + cov(a_t, z_{t-2})$

$\gamma_2 = 0.6 \gamma_1 + 0.3 \gamma_0$

$cov(z_t, z_{t-3}) = cov(0.6z_{t-1} + 0.3z_{t-2} + a_t, z_{t-3})$

$\gamma_3 = 0.6 cov(z_{t-1}, z_{t-3}) + 0.3 cov(z_{t-2}, z_{t-3}) + cov(a_t, z_{t-3})$

$\gamma_3 = 0.6 \gamma_2 + 0.3 \gamma_1$

$\gamma_2(k) = 0.6 \gamma_2(k-1) + 0.3 \gamma_2(k-2)$

$$\gamma_z(k) = \begin{cases} \sigma_a^2 / 0.24 & k=0 \\ 0.6\gamma_z(k-1) + 0.3\gamma_z(k-2) & k = \pm 1, \pm 2, \dots \end{cases}$$

$$\rho_z(k) = \frac{\gamma_z(k)}{\gamma_z(0)}$$

$$\frac{\gamma_z(k)}{\gamma_z(0)} = 0.6 \frac{\gamma_z(k-1)}{\gamma_z(0)} + 0.3 \frac{\gamma_z(k-2)}{\gamma_z(0)} \quad \text{for } k = \pm 1, \pm 2, \dots$$

$$\rho_k = 0.6 \rho_{k-1} + 0.3 \rho_{k-2}$$

$$\rho_k = \begin{cases} 1 & \text{if } k=0 \\ 0.6\rho_{k-1} + 0.3\rho_{k-2} & \text{if } k = \pm 1, \pm 2, \dots \end{cases}$$

Q4) A) (i) SARIMA (2,1,2) (1,2,0)₁₂

$$\underbrace{(1 - \phi_1 B - \phi_2 B^2)}_{\text{AR}(2)} \underbrace{(1-B)}_{\text{I}(1)} \underbrace{(1-B^{12})^2}_{\text{SI}(2)} \underbrace{(1 - \Phi B^{12})}_{\text{SAR}(1)} y_t = \underbrace{(1 - \theta_1 B - \theta_2 B^2)}_{\text{MA}(2)} a_t$$

(ii) SARIMA (3,0,1) (0,2,2)₄

$$\underbrace{(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)}_{\text{AR}(3)} \underbrace{(1-B^4)^2}_{\text{SI}(2)} y_t = \underbrace{(1 - \theta_1 B)}_{\text{MA}(1)} \underbrace{(1 - \Theta_1 B^4 - \Theta_2 B^8)}_{\text{SMA}(2)} a_t$$

B) (i) SARIMA (1,2,2) (1,1,1)₁₂

(ii) SARIMA (0,0,1) (3,2,2)₁₂

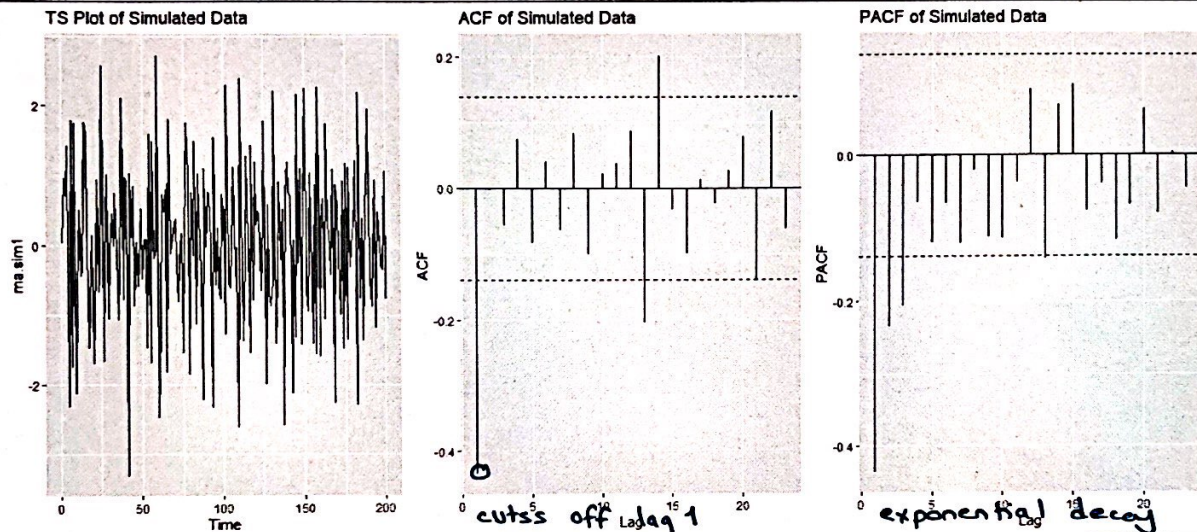
ASSIGNMENT 1 (STAT 497-IAM 526)
(Due date: 29.11.2022 - Tuesday at 12:00)

5. (15 pt) Consider the following stochastic processes:

- i) $Y_t = 0.7Y_{t-1} + 0.3Y_{t-2} + a_t - 0.7a_{t-1} - 0.8a_{t-2} \Rightarrow \phi_1 = 0.7, \phi_2 = 0.3, \phi_1 + \phi_2 = 1 \rightarrow \text{not stationary}$
- ii) $Y_t = -248410 + 126t + a_t \Rightarrow \text{The series have deterministic trend, so D}$
- iii) $Y_t = 0.2Y_{t-1} + 0.4Y_{t-2} + a_t \Rightarrow \phi_1 = 0.2, \phi_2 = 0.4 \rightarrow \text{Stationary series \& AR(2) B}$
- iv) $Y_t = a_t + 0.75a_{t-1} \Rightarrow \text{MA(1) process is stationary, ACF cuts off at lag 1 so A, PACF has exponential decay}$

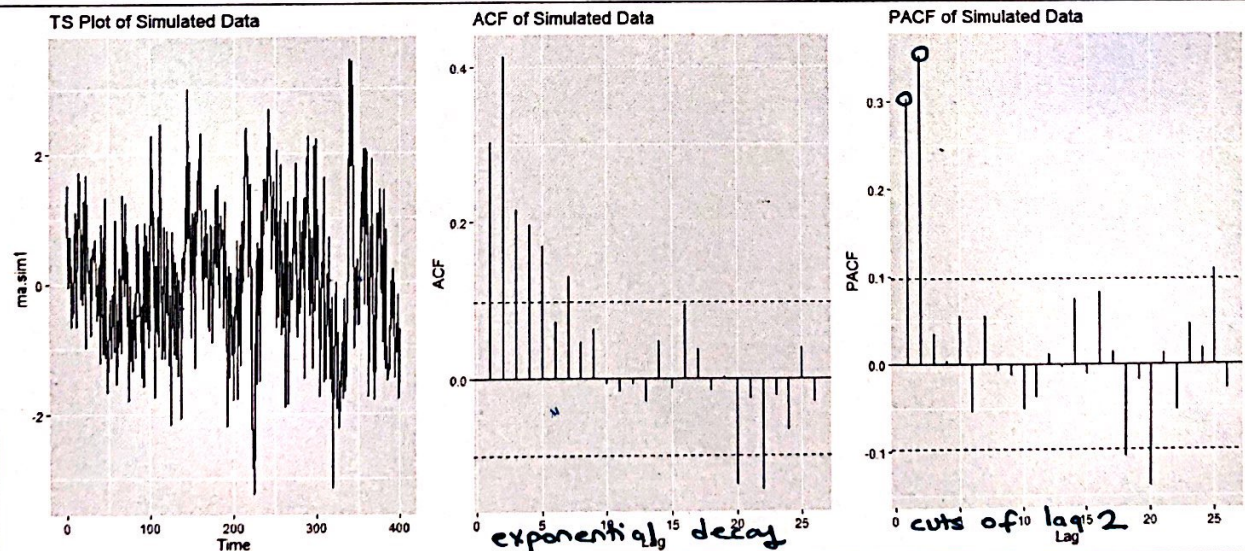
(ii)

A



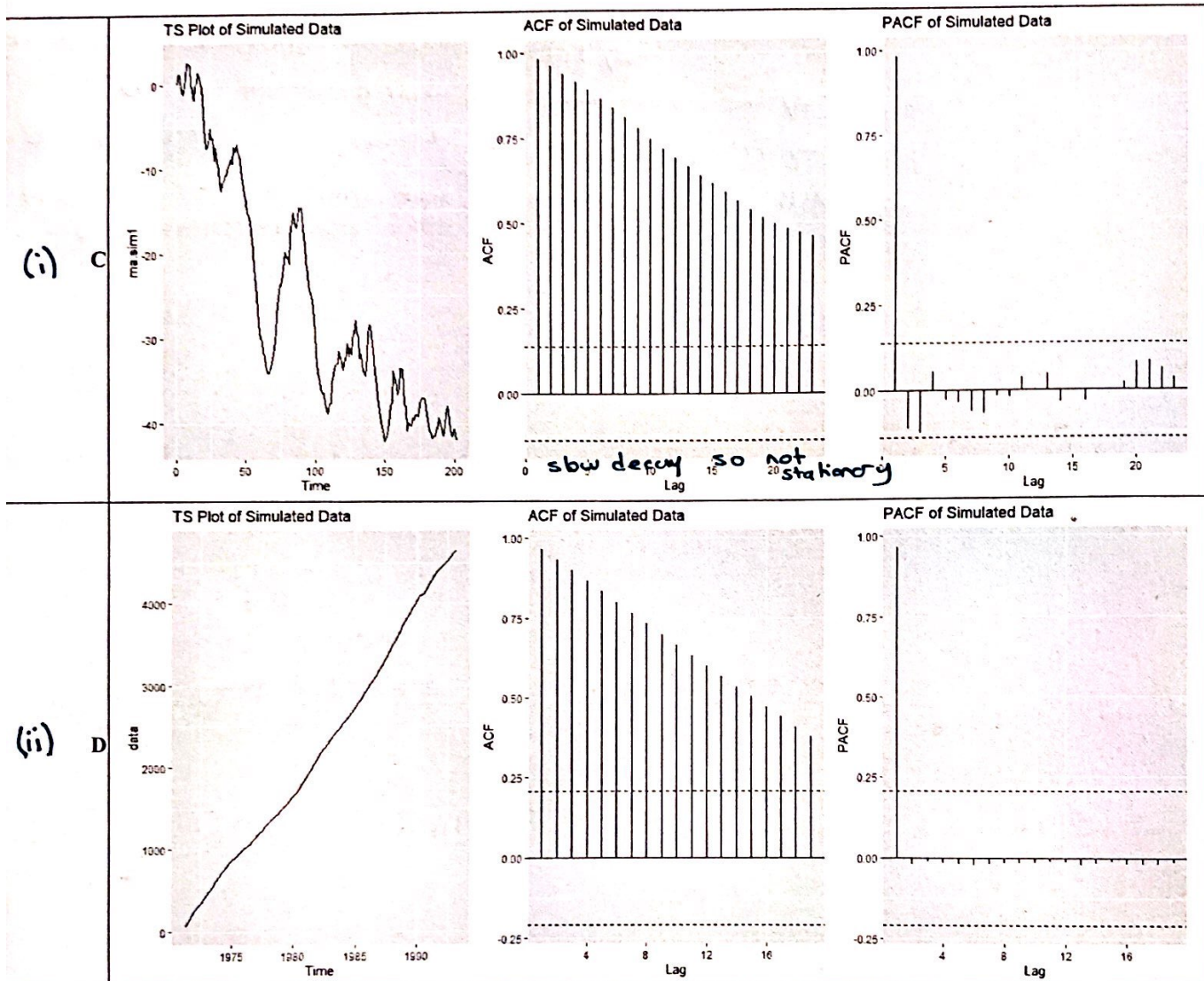
(iii)

B



$$(1 - 0.7B - 0.3B^2) Y_t =$$

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Match the presented time series plots A to D with the generating stochastic processes given by i) to vi). Explain the reasons of your identification.