Intersect

1 Question 1

1. It's easier to use camera space, this is because the film plane is perpendicular to the look vector in camera space. You could use world space, but the film plane wouldn't be properly oriented so mapping the pixels would be much more difficult. You can't use screen space because there's no inverse of the unhinging perspective transformation.

2. $$p_{film}\left(\frac{2x}{x_{max}}-1, \frac{-2y}{y_{max}}+1, -1\right)$$

3. $$M_{film\_to\_world} = M_{trans}^{-1} M_{rot}^{-1} M_{scale}^{-1}$$

   This is because:

   $$\begin{aligned}
   M_{trans}^{-1} M_{rot}^{-1} M_{scale}^{-1} M_{scale} M_{rot} M_{trans} * p_{world} \\
   M_{trans}^{-1} M_{rot}^{-1} * I * M_{rot} M_{trans} * p_{world} \\
   M_{trans}^{-1} M_{rot}^{-1} M_{rot} M_{trans} * p_{world} \\
   M_{trans}^{-1} * I * M_{trans} * p_{world} \\
   I * p_{world} \\
   p_{world}
   \end{aligned}$$

   We don't have have to invert the perspective transform because we haven't done the perspective transformation yet (and I don't think it has an inverse).

4. $ray = p_{eye} + t * \frac{p_{world}-p_{eye}}{|p_{world}-p_{eye}|}$

2 Question 2

1. Cone body; A cone can be thought of a series of stacked circles. We know from the shapes algo that the slope of the unit cone $m = 2$, also we know that the radius of a cone is determined by the height of the cone and the height of the slice.

   $$\begin{aligned}
   y &= top_y - m * r \\
   r &= \frac{top_y - y}{m} \\
   r &= \frac{.5 - y}{2}
   \end{aligned}$$

   We also know the equation for circle along the edge of the cone at a given height is: $x^2 + z^2 = r^2$. So:

   $$x^2 + z^2 = \left(\frac{.5 - y}{2}\right)^2$$

We know that the parameterization of the ray is:

$$< x, y, z >= r(t) =< P_x + t * d_x, P_y + t * d_y, P_z + t * d_z >$$

We can now plug in and isolate t:

$$(P_x + t * d_x)^2 + (P_z + t * d_z)^2 = \left(\frac{.5 - (P_y + t * d_y)}{2}\right)^2$$

$$P_x^2 + 2*P_x*d_x*t + d_x^2*t^2 + P_z^2 + 2*P_z*d_z*t + d_z^2*t^2 - \frac{t^2 * d_y^2}{4} - \frac{t * P_y * d_y}{2} + \frac{t * d_y}{4} - \frac{P_y^2}{4} + \frac{P_y}{4} - 0.0625 = 0$$

$$\left(d_z^2 + d_x^2 - \frac{d_y^2}{4}\right) * t^2 + \left(2 * P_x * d_x + 2 * P_z * d_z - \frac{P_y * d_y}{2} + \frac{d_y}{4}\right) t + (P_x^2 + P_z^2 - \frac{P_y^2}{4} + \frac{P_y}{4} - .0625) = 0$$

By the quadratic formula:

$$A = \left(d_z^2 + d_x^2 - \frac{d_y^2}{4}\right)$$

$$B = \left(2 * P_x * d_x + 2 * P_z * d_z - \frac{P_y * d_y}{2} + \frac{d_y}{4}\right)$$

$$C = (P_x^2 + P_z^2 - \frac{P_y^2}{4} + \frac{P_y}{4} - .0625)$$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

2. The bottom cap of the cone is at $-.5$, so y must equal $-.5$.

$$-.5 = y$$

$$-.5 = P_y + d_y * t$$

$$t = \frac{-.5 - P_y}{d_y}$$

3 Question 3

1. $n_{world} = \left(M^{-1}\right)^T * n_{object}$ (textbook section 10.12)

2. $n * L$ represents the cosine function, more specifically the cosine of theta, where theta is the angle between the surface normal and the reflected light ray. It is the diffuse intensity of the light. As we saw in the Phong shading lab, "the diffuse component makes surfaces that are facing towards the light source appear brighter. We will represent how much a surface is facing the light source with the expression $n * L$, where n is the normal vector and L is the normalized vector from the surface point to the light source." This is part of the Lambertian model.

4 Question 4

1. Illumination is "the process of computing the intensity and color of a sample point in a scene as seen by a view", where as shading is "interpolation of color between known points of lighting or illumination, most often vertices."