# Boolean Algebra

## Exercises:

1. Determine the values of A, B, C & D that make the sum term $A + \overline{B} + C + \overline{D}$ equal to 0 ?

soln;
$$A + \overline{B} + C + \overline{D} = 0$$
$$\Rightarrow \quad 0 \quad 0 \quad 0 \quad 0$$
so $A = 0$, $B = 1$, $C = 0$, $D = 1$

what about $A\overline{B}C\overline{D} = 1$
$$1 \quad 1 \quad 1 \quad 1$$
so $A = 1$, $B = 0$, $C = 1$, $D = 0$

2. Apply DeMorgan's Theorem to each of the following expressions:

- $\overline{(A+B+C)D} = \overline{A}\,\overline{B}\,\overline{C} + \overline{D}$

- $\overline{A\overline{B} + \overline{C}D + EF} = \overline{A\overline{B}} \cdot \overline{(\overline{C}D)} \cdot \overline{EF}$
$$= (\overline{A} + B)(C + \overline{D})(\overline{E} + \overline{F})$$

- $\overline{(A+B)\overline{C}\overline{D} + E + \overline{F}} = \overline{(A+B)\overline{C}\overline{D}} \cdot \overline{E} \cdot \overline{\overline{F}}$
$$= (\overline{(A+B)} + C + D)\,\overline{E} \cdot F$$
$$= (\overline{A}\,\overline{B} + C + D)\,\overline{E} \cdot F$$

3. Use De Morgan's Law & other Boolean laws to develope an expression for the exclusive - NOR gate.

soln: exclusive OR gate : XOR : $A \oplus B = A\bar{B} + \bar{A}B$

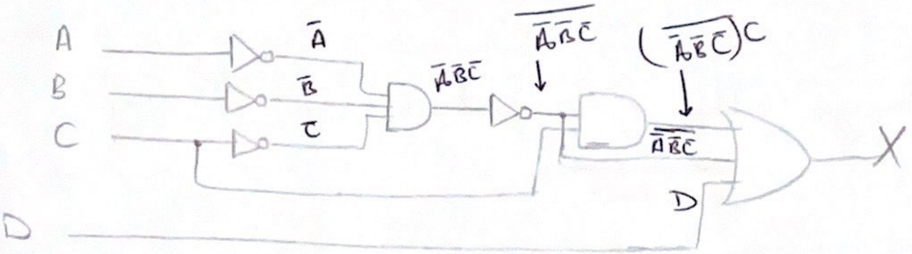
NOR gate : $\overline{A\bar{B} + \bar{A}B} = \overline{A\bar{B}} \cdot \overline{\bar{A}B}$

$$= (\bar{A} + B)(A + \bar{B})$$

$$= \underbrace{\bar{A}A}_{0} + \bar{A}\bar{B} + BA + \underbrace{B\bar{B}}_{0}$$

$$= \bar{A}\bar{B} + AB$$ *

4. Reduce the following logic circuit to a minimum form.




$= (\overline{\bar{A}\bar{B}\bar{C}})C + \overline{\bar{A}\bar{B}\bar{C}} + D$

Simply X to a minimum form:

$$X = (\overline{\bar{A}\bar{B}\bar{C}})C + \overline{\bar{A}\bar{B}\bar{C}} + D$$

$$= \underbrace{(A+B+C)C} + \underbrace{(A+B+C)} + D$$

$$= (A+B+C)(\underbrace{C+1}_{1}) + D$$

$$= A+B+C+D$$

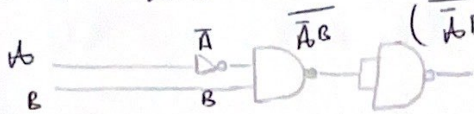
Equivalent to

⑤ Use <u>NAND gates</u> to implement each expression: (you can use negation gate).

(i) $X = \overline{A} + B$


$\overline{A\overline{B}} = \overline{A} + B$

(ii) $X = \overline{A}\,B$




$\overline{(\overline{\overline{A}B})(\overline{A}B)}$

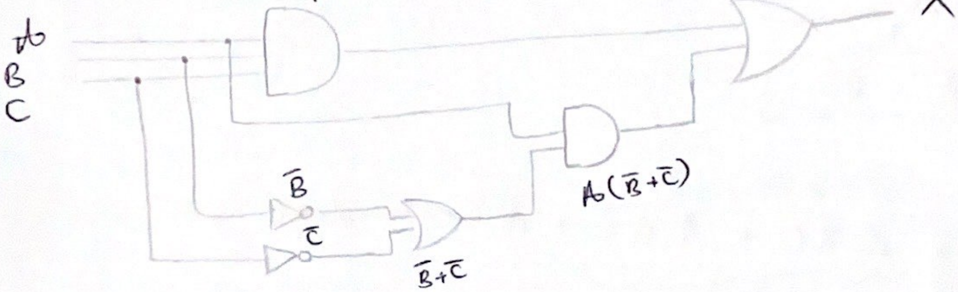
$\downarrow$

$\overline{(A+\overline{B})(A+\overline{B})}$

$\overline{(A+\overline{B})} = \overline{A}\,B$

⑥ Find the Boolean algebra expression for the following system:

$ABC + A(\overline{B}+\overline{C})$




Simplify & construct the truth table for this logic circuit.

$ABC + A(\overline{B}+\overline{C})$

$ABC + A(\overline{BC})$

$A(\underbrace{BC + \overline{BC}}_{1})$

$A \cdot 1$

$= \underline{\underline{A}}$

⑦ Simplify : ① $(A\bar{B}(C+BD)+\bar{A}\bar{B})C$

Soln: $(A\bar{B}C + \underbrace{A\underbrace{\bar{B}B}_{0}D}_{0} + \bar{A}\bar{B})C$

$$(A\bar{B}C+\bar{A}\bar{B})C = A\bar{B}\underbrace{CC} + \bar{A}\bar{B}C$$

$$= A\bar{B}C + \bar{A}\bar{B}C$$

$$= \underbrace{(A+\bar{A})}_{1}\bar{B}C = \bar{B}C$$

② $\overline{AB+AC} + \bar{A}\bar{B}C$

$(\bar{A}+\bar{B})(\bar{A}+\bar{C}) + \bar{A}\bar{B}C$

$\underbrace{\bar{A}\bar{A}} + \bar{A}\bar{C} + \bar{B}\bar{A} + \bar{B}\bar{C} + \bar{A}\bar{B}C$

$\underbrace{\bar{A}} + \underbrace{\bar{A}\bar{C}} + \underbrace{\bar{B}\bar{A}} + \bar{B}\bar{C} + \underbrace{\bar{A}\bar{B}C}$

$\bar{A}(\underbrace{1+\bar{C}+\bar{B}+\bar{A}\bar{B}}_{1}) + \bar{B}\bar{C}$

$$\boxed{\bar{A}+\bar{B}\bar{C}}$$

③ $\bar{A}BC + \underbrace{A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}} + A\bar{B}C + ABC$

$\underbrace{(A+\bar{A})}_{1}\bar{B}\bar{C}$

$\bar{B}\bar{C}$

$\underbrace{(\bar{A}+A)}_{1}BC$

$BC$

∴ $\underbrace{\bar{B}\bar{C}} + BC + \underbrace{A\bar{B}C}$

$\bar{B}(\underbrace{\bar{C}+AC}) + BC$

$\downarrow$ law

$\bar{B}(\bar{C}+A)+BC$

$\bar{B}\bar{C}+\bar{B}A+BC$

Recall : ① The Sum of Product Form SOP

examples: $AB + ABC \rightarrow$ domain $A, B, C$

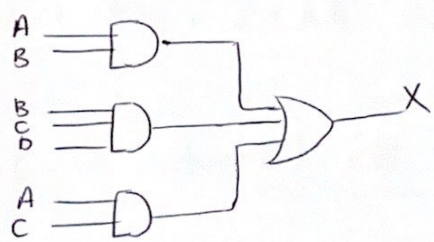
$ABC + CD\bar{E} \rightarrow$ domain $A, B, C, D$ & $E$.

$\bar{B}C\bar{D} + AB + DC \rightarrow$ domain $A, B, C, D$

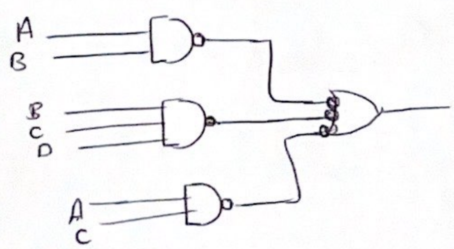
Note that SOP can contain single complement $\bar{A}$ not $\overline{AB}$

* AND/OR implementation of an SOP expression :

$$X = AB + BCD + AC$$




* NAND/NOR implementation of $X = AB + BCD + AC$




Def: A standard SOP expression is one in which all the variables in the domain appear in each product term in the expression.

e.g: $\bar{A}BC\bar{D} + A\bar{B}\bar{C}D + ABC\bar{D}$

* Convert the following to standard SOP :

$$\underbrace{A\bar{B}C}_{D\text{ is missing}} + \underbrace{\bar{A}\bar{B}}_{C\ \&\ D\text{ are missing}} + A\bar{B}CD$$

Soln: we use trick $(C + \bar{C} = 1)$

$$A\bar{B}C\,(D+\bar{D}) = A\bar{B}CD + A\bar{B}C\bar{D}$$

$$\bar{A}\bar{B}\,(C+\bar{C}) = [\underbrace{\bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}}](D+\bar{D})$$

$$= \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

Hence, $\longrightarrow$ $A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$

* Binary Representation of a SOP (standard form)

→ Determine the binary values for which the following SOP expression is equal to 1.

$$\underbrace{ACBD}_{=1} + \underbrace{A\bar{B}\bar{C}D}_{=1} + \underbrace{\bar{A}\bar{B}\bar{C}\bar{D}}_{=1} = 1$$

| | | |
|---|---|---|
| ∴ A = 1 | A = 1 | A = 0 |
| B = 1 | B = 0 | B = 0 |
| C = 1 | C = 0 | C = 0 |
| D = 1 | D = 1 | D = 0 |

SOP equal 1 when $\underline{\text{any}}$ or $\underline{\text{all}}$ of the three product terms is 1 .

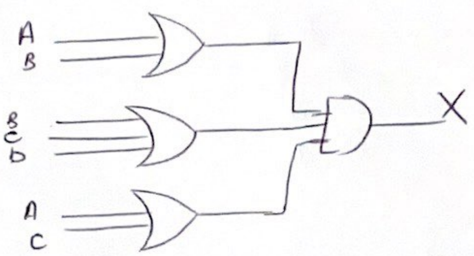
## 2 The Product of Sums (POS)

examples: $(\bar{A}+B)(A+\bar{B}+C)$ ⇝ domain A, B, C

$(\bar{A}+\bar{B}+\bar{C})(C+\bar{D}+E)(\bar{B}+C+D)$ ⇝ domain A, B, C, D & E

$X = (A+B)(B+C+D)(A+C)$

AND implementation:




The Standard POS Form:

Def: A standard POS expression is one in which all the variables in the domain appear in each term in the expression.

* Convert to the standard POS form:

$$(\underbrace{A+\bar{B}+C}_{+D\bar{D}})(\underbrace{\bar{B}+C+\bar{D}}_{+A\bar{A}})(\underbrace{A+\bar{B}+\bar{C}+D}_{\checkmark})$$

$$(A+\bar{B}+C+D\bar{D})(\bar{B}+C+\bar{D}+A\bar{A})$$

$$(A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})(\bar{B}+C+\bar{D}+A)(\bar{B}+C+\bar{D}+\bar{A})(A+\bar{B}+\bar{C}+D)$$

* Binary representation for POS :-

Determine the binary values of the variables for which the following **standard** POS expression is equal to 0:

$$\underbrace{(A+B+C+D)}_{0}\ \underbrace{(A+\bar{B}+\bar{C}+D)}_{0}\ \underbrace{(\bar{A}+\bar{B}+\bar{C}+\bar{D})}_{0}$$

| | | |
|---|---|---|
| $A=0$ | $A=0$ | $A=1$ |
| $B=0$ | $B=1$ | $B=1$ |
| $C=0$ | $C=1$ | $C=1$ |
| $D=0$ | $D=0$ | $D=1$ |

The POS expression equals 0 when any of the three terms equals 0.