National University

of Computer & Emerging Sciences Peshawar Campus

Student Name: ____________________ Roll No: __________________

Program: BS (CS)
Semester: Spring-2018
Time Allowed: 3 hours
Course: MT104-Linear Algebra

Examination: FINAL
Total Marks: 100, Weightage: 50
Date: 15 / 05 / 2018
Instructor: Mr. Osama Sohrab

NOTE: Attempt all questions.

Q1.(a) Let $\mathbb{R}^3$ has the inner product defined by $<\boldsymbol{u},\boldsymbol{v}>= u_1v_1 + 2u_2v_2 + 3u_3v_3$. Use the Gram-Schmidt process to transform the basis vectors $\boldsymbol{u_1} = (1,1,1)$, $\boldsymbol{u_2} = (1,1,0)$, $\boldsymbol{u_3} = (1,0,\ 0)$ into an orthogonal basis $\{\boldsymbol{v_1},\boldsymbol{v_2},\boldsymbol{v_3}\}$, and then normalize the orthogonal basis vectors to obtain an orthonormal basis $\{\boldsymbol{w_1},\boldsymbol{w_2},\boldsymbol{w_3}\}$. **(15)**

(b) Show that the matrix $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$ is diagonalizable. Find a matrix P such that $P^{-1}AP$ is a diagonal matrix whose diagonal entries are eigenvalues of A. **(10)**

Q2.(a) Find the angle between the polynomials $\boldsymbol{p} = 1 + 2x + 3x^2$ and $\boldsymbol{q} = 6 + 5x + 4x^2$ with respect to the standard inner product on P_2, the vector space of all polynomials of degree less than or equal to 2. **(10)**

(b) Determine whether the set $S = \{x^2 - 2x,\ x^3 + 8,\ x^3 - x^2,\ x^2 - 4\}$ spans P_3, the vector space of all polynomials of degree ≤ 3. **(10)**

Q3. Find all eigenvalues and the corresponding eigenvectors of the matrix

$$M = \begin{bmatrix} -3 & 1 & -3 \\ 20 & 3 & 10 \\ 2 & -2 & 4 \end{bmatrix}.$$ **(15)**

Q4.(a) For any two matrices $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ in M_{22} show that the function defined by $< A, B >= a_{11}b_{11} + 2a_{12}b_{12} + 3a_{21}b_{21} + 4a_{22}b_{22}$ is an inner product. **(10)**

(b) Determine whether the function is a linear transformation

(i) $T: M_{22} \longrightarrow M_{22}$, $\quad T(A) = A^T$

(ii) $T: P_2 \longrightarrow P_2$, $\quad T(a_0 + a_1x + a_2x^2) = (a_0 + a_1 + a_2) + (a_1 + a_2)x + a_2x^2$. **(10)**

Q5.(a) Find the kernel of the linear transformation $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ defined by $T(\boldsymbol{x}) = A\boldsymbol{x}$, where

$$A = \begin{bmatrix} \frac{4}{9} & -\frac{4}{9} & \frac{2}{9} \\ -\frac{4}{9} & \frac{4}{9} & -\frac{2}{9} \\ \frac{2}{9} & -\frac{2}{9} & \frac{1}{9} \end{bmatrix}.$$ **(10)**

(b) Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the linear operator defined by the formula

$T(x_1, x_2, x_3) = (3x_1 + x_2,\ -2x_1 - 4x_2 + 3x_3,\ 5x_1 + 4x_2 - 2x_3)$

Determine whether T is one-to-one, if so, find $T^{-1}(x_1, x_2, x_3)$. **(10)**

****THE END****