

Synchrony between time series.

The synchrony between two univariate time series of equal length is calculated as following:

Consider two time series X and Y of length N .

The vectors of changes, Cx and Cy , are created for each of the time series. The vector element $Cx[i]$, where $i = 1, 2, \dots, N-1$, is set to -1, 0 or 1, depending on the sign of the first derivative of X at the time point i :

$$\begin{aligned} Cx[i] &= 1, \text{ if } (X[i+1] - X[i]) > 0, \\ Cx[i] &= 0, \text{ if } (X[i+1] - X[i]) = 0, \\ Cx[i] &= -1, \text{ if } (X[i+1] - X[i]) < 0. \end{aligned}$$

The Cy vector is calculated in similar way.

The Cx and Cy vectors are compared and the result is stored in the vector of synchronies, Sxy . The vector element $Sxy[i]$, where $i = 1, 2, \dots, N-1$, is set to -1, 0 or 1, depending on the values in $Cx[i]$ and $Cy[i]$:

$$\begin{aligned} Sxy[i] &= 1, \text{ if } Cx[i] = Cy[i], \text{ positive synchrony case,} \\ Sxy[i] &= -1, \text{ if } Cx[i] = -Cy[i], \text{ negative synchrony case,} \\ Sxy[i] &= 0, \text{ if } \text{abs}(Cx[i]+Cy[i]) = 1, \text{ no synchrony case.} \end{aligned}$$

Since the $Cx[i]$ and $Cy[i]$ have 3 possible values each, the variants for the $Sxy[i]$ value can be placed in a 3x3 matrix:

$Cy[i]$ $Cx[i]$	-1	0	1
-1	1	0	-1
0	0	1	0
1	-1	0	1

The $Sxy[i]$ value depends on the $Cx[i]$ and $Cy[i]$ values. It is 1, if the X and Y time series changes are synchronous at the time point i .

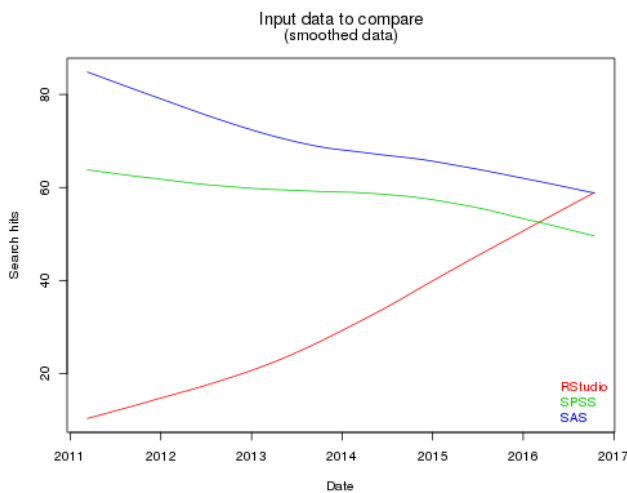
The **synchrony** between the X and Y time series, **S**, is equal to:

$$S = \frac{1}{N-1} \sum_{i=1}^{N-1} S_{xy}[i]$$

where N is the time series length, S_{xy} is the vector of synchronies between X and Y time series.

The synchrony value is varied from -1 to 1, where -1 is a perfect negative synchrony, 0 no synchrony and 1 a perfect positive synchrony.

Example for a positive and negative synchrony.



	Synchrony		
	RStud.	SPSS.	SAS.
RStud.		-1	-1
SPSS.			1
SAS.			