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CS 3130

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Project 1 - Part E

Recursive vs.. Iterative for The Fibonacci Sequence

Objective:

The objective of this project to was to compare the efficiency of calculating Fibonacci numbers recursively vs. iteratively, and to further our understanding of order of growth. We do this by first calculating the theoretical order of growth for both algorithms, then run our programs for find the experimental order of growth, which can be found in Part C of this project.

Theoretical order of growth for Recursive and Iterative algorithms for the Fibonacci Sequence:

Recursive Implementation

```python
def main():
    userNum = -1        #Initilaize user input variable and set it to -1
    while userNum < 0:      #Verification loop to make sure user inputs a valid number
        userNum = int(input("Enter a positive number: "))
        if userNum < 0:
            print("Invalid number")
    print(fib(userNum))     #Run code

def fib(userNum):
    if userNum == 0:        #First fib number is 0
        return 0
    elif userNum == 1:      #Second fib bumber is 1
        return 1
    elif userNum == 2:      #Third fib number is 1
        return 1
    else:
        return(fib(userNum - 1) + fib(userNum - 2))     #Fib equation with recursion

main()
```

The time complexity of the recursive algorithm is: **Tn = Tn-1 + Tn-2 ; Tn ∈ O(Φ)n**

The recursive implementation of the Fibonacci sequence takes an exponential amount of time to run since it will repeat itself many times over (recalling itself/function) multiple times. For example, whenever we recursively call the Fibonacci sequence and we let n = 5 we get:

```
                        F(5) = 5
                   /               \
              F(4)=3               F(3)=2
              /    \               /    \
         F(3)=2    F(2)=1      F(2)=1   F(1)=1
         /    \
    F(2)=1    F(1)=1
```

In this we see that F(3)=2 is calculated twice, which wastes time. From this, you can imagine that with an exponential n, that the amount of wasted time will grow rapidly.

Iterative Implementation

```python
def main():
    userNum = -1        #Initialize variable for user input

    while userNum < 0:      #Verification loop for user input
        userNum = int(input("Enter a positive number: "))
        if userNum < 0:
            print("Invalid number")

    print(fibIter(userNum))     #fibIter functuion call

def fibIter(userNum):
    if userNum == 0:        #first fib number is 0
        return 0
    elif userNum == 1:      #Second fib number is 1
        return 1
    elif userNum == 2:      #Third fib number is 1
        return 1
    elif userNum == 3:      #4th fib number is 2
        return 2
    elif userNum > 3:       # Iterative loop for fib sequence
        fn = 0
        fn1 = 1
        fn2 = 2
        for i in range(3, userNum):     #Loop for numbers between 3 and user input
            fn = fn1 + fn2              #Fib sequence definition
            fn1 = fn2                   #Swap
            fn2 = fn
        return fn
    else:
        return -1

main()
```

The time complexity of an iterative algorithm for the Fibonacci sequence is **$T_n \in O(n)$**.

	Code run	**Cost**	**# of times run**
1	**If** n **is** 0	C1	1
2	**Return** 0	C2	1
3	**If** n **is** 1	C3	1
4	**Return** 1	C4	1
5	**If** n **is** 2	C5	1
6	**Return** 1	C6	1
7	**If** n **is** 3	C7	1
8	**Return** 2	C8	1
9	Fn = 0	C9	1
10	Fn1 = 1	C10	1
11	Fn2 = 2	C11	1
12	**For** I **to** n	C12	(n-1)+1 =n
13	Fn = fn1 + fn2	C13	N
14	Fn1 = fn2	C14	N

15	Fn2 = fn	C15	N
16	**Return** fn	C16	1

C1+C2+C3+C4+C5+C6+C7+C8+C9+C10+C11+n(C12+C13+C14+C15)+C16 $\in$ **O(n)**

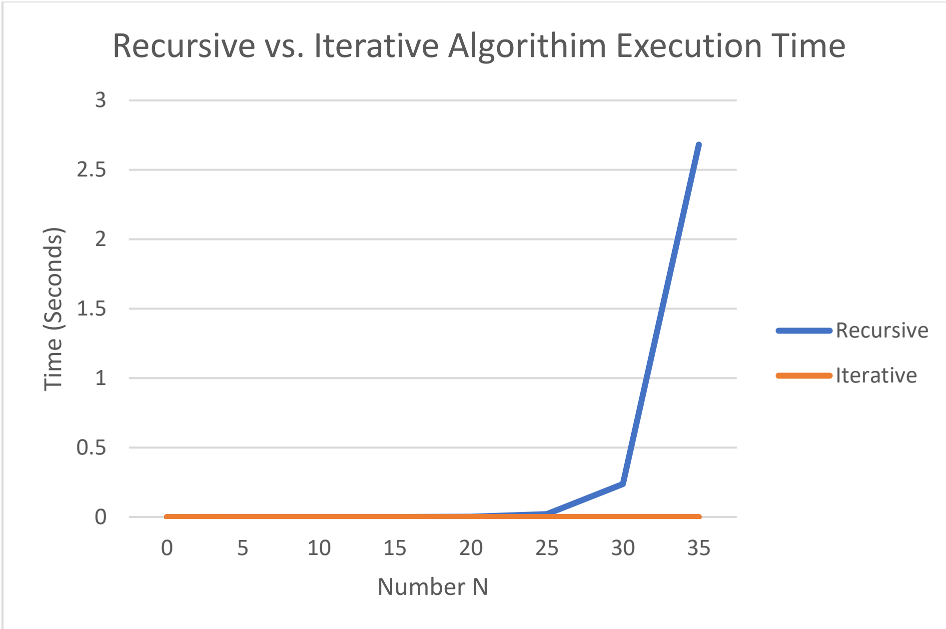
The iterative algorithm/function will store the two most recently calculated numbers, then use those two numbers to calculate the next number in the Fibonacci sequence. By doing this, we eliminate the wasted time that we would normally have in a recursive algorithm/function, due to the reduced amount of calculations.

Experimental Results for Recursive Algorithm:

N	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
0	0	0	0	0	0	0
5	3.814697265625e-06	3.337860107421875e-06	2.384185791015625e-06	2.6226043701171875e-06	2.6226043701171875e-06	2.956390380859375e-06
10	1.8358230590820312e-05	1.6689300537109375e-05	1.6689300537109375e-05	1.621246337890625e-05	1.6450881958007812e-05	1.6880035400390624e-05
15	0.0001780986785888672	0.0001766681671142578	0.0001761913299560547	0.00017595291137695312	0.00017499923706054688	0.00017638206481933593
20	0.00017638206481933593	0.0018641948699951172	0.001851797103881836	0.0018737316131591797	0.0018537044525146484	0.0018609523773193359
25	0.020853042602539062	0.02068805694580078	0.02251887321472168	0.021716833114624023	0.0206148624420166	0.02127833366394043
30	0.2355027198791504	0.23641037940979004	0.23279619216918945	0.24962329864501953	0.23133563995361328	0.23713364601135253
35	2.5994772911071777	2.6273508071899414	2.746936798095703	2.6630730628967285	2.7718207836151123	2.6817317485809324

Experimental Results for Iterative Algorithms:

N	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
0	0	0	0	0	0	0
5	3.5762786865234375e-06	2.6226043701171875e-06	2.1457672119140625e-06	2.1457672119140625e-06	1.6689300537109375e-06	2.4318695068359373e-06
10	4.291534423828125e-06	2.6226043701171875e-06	2.384185791015625e-06	1.9073486328125e-06	1.9073486328125e-06	2.6226043701171875e-06
15	4.76837158203125e-06	3.337860107421875e-06	2.86102294921875e-06	2.6226043701171875e-06	2.384185791015625e-06	3.1948089599609376e-06
20	5.4836273193359375e-06	3.814697265625e-06	3.0994415283203125e-06	2.86102294921875e-06	2.384185791015625e-06	3.528594970703125e-06
25	5.0067901611328125e-06	4.291534423828125e-06	3.5762786865234375e-06	3.0994415283203125e-06	2.6226043701171875e-06	3.719329833984375e-06
30	5.4836273193359375e-06	4.291534423828125e-06	3.814697265625e-06	3.337860107421875e-06	3.337860107421875e-06	4.0531158447265625e-06
35	5.7220458984375e-06	4.0531158447265625e-06	4.0531158447265625e-06	3.5762786865234375e-06	3.5762786865234375e-06	4.1961669921875e-06



Based on the results seen in the graph above, we can conclude that the recursive algorithm has an exponential time complexity, while the iterative function has a linear time complexity.

Conclusion:

From this research we can conclude that the recursive algorithm grows faster due to it having to perform the same computations multiple times, while the iterative function does not, meaning its execution time is a lot less. So in all, recursive **O(Φ)n** and iterative **O(Φ)n**