SSY 230, System Identification
Project 3: Identification of a Real System

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1 Flexible Robot Arm

The system we have chosen to identify is a mechanical system, where a flexible robot arm have been installed on an electrcal motor. It is a SISO system where the input $u(t)$ is measured reaction torque and the output $y(t)$ is the acceleration of the flexible robot arm. The experimental set-up was performed using a periodic sinusodial sweep.

1.1 Data

As mentioned previously the input data is a periodic sinusodial sweep (see top plot of Figure 1). Due to the fact that the data was obtained using a periodic sinusodial sweep we split the data in half and use the first part as training data and the second part as validation data.

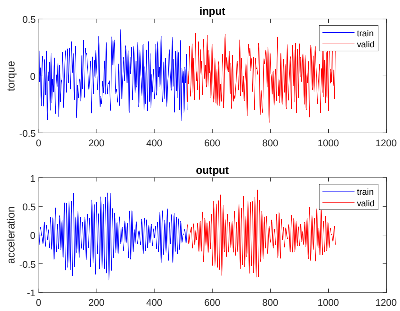
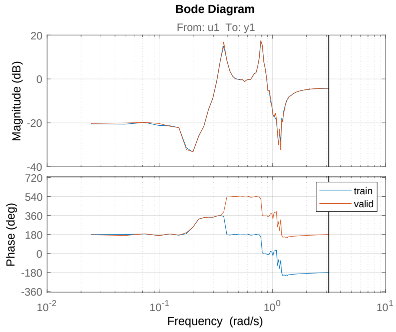


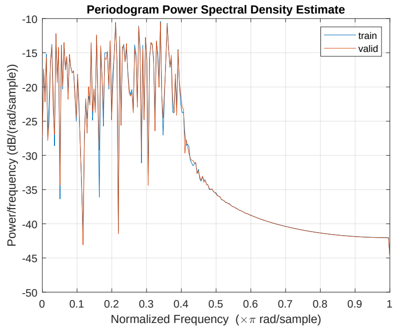
Figure 1: System data, input $u(t)$ (top) and output $y(t)$ (bottom).

To make sure that the frequency content in both the training and validation data are similar we use the *etfe* in MATLAB to find the Empirical Transfer Function Estimate of training- and validation data. The resulting bode-plot is shown in Figure 2a. Because the input signal is periodic, the empirical transfer function estimate is unbiased.

From analysing Figure 2a it is clear that the amplitude of the frequency content in both training- and validation data is very similar, while there is a 360 degree phase shift approximately starting from frequencies > 0.35 rad/s. However, it should be noted that a 360 degree phase shift means the validation data and training data are still in phase. Using the MATLAB build-in function *periodogram* it is clear that the frequency content of the training- and validation data is very similar.



(a) Bode-plot of training and validation data.



(b) Periodogram of training and validation data.

Figure 2: Analyzing training/validation split.

Also, it can be seen that for frequencies > 0.5 rad/s the power the amount of information for each frequency decreases. By examining the bode plot, we can have an initial guess of the number of poles and zeros we can observe that there are four sharp corners, each of which represent two conjugate poles or two conjugate zeros depending on its concave or convex. The first and last parts of the amplitude plot are ladder-shaped, which means that there are two more real poles. And we can also see that there is one hollow between two spikes, which represent a zero. Thus, there are six poles and five zeros in total. The high frequency assymptote amplitude is constant, which means that (for a linear system) there should be an equal amount of poles and zeroes.

1.2 Pre-Processing

In off-line situations, it is often better to remove trends in data before doing the system identification so that only the dynamic features are modelled. This can be done using the *getTrend* and *detrend* commands in MATLAB. However, in our case, the data before and after detrending look very similar since there is no obvious trend.

1.3 Model Estimation

1.3.1 Linear Models

In this section we search over the linear model space for candidate models. A few different linear models are tested and compared: ARX, OE, State Space and Transfer Function Estimation.

In Figure 3 the *mse* decreases for increasing model order but start to saturate for model orders above 8. As the high frequency assymptote from the bode plot of the ETFE tells us that there should be an equal ammount of poles and zeroes, and our more specific analysis resulted in 6 poles and 5 zeroes we first try models of order 6. From Figure 3 the *mse* shows that the models ARX, and State Space give the best results so lets focus on analysing them from now on.

Pole-Zero Plots Next, we show the pole/zero and bode plots of the ARX model and the state space model estimated using subspace method.

The left plot in Figure 4a shows the pole-zero map of an ARX model with model order na=nb=6, along with 95

Figure 4b shows the pole/zero plot of the ARX model with model order na = 6, and nb =5. We can see that by decreasing one model order in the numerator, the uncertainties of the left two complex conjugate poles are also decreased.

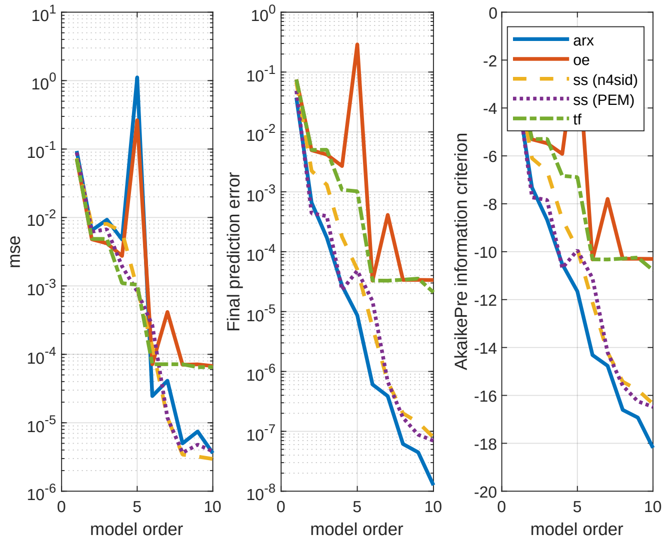
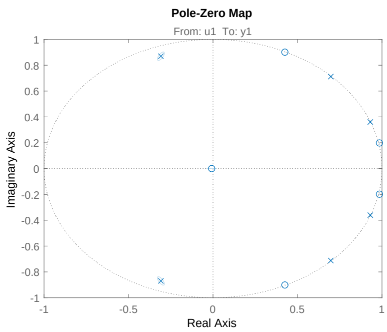
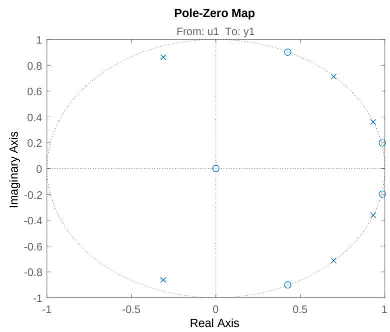


Figure 3: Searching over different model orders n, where the number of parameters p are $p = 2n+1$.

Figure 5 pole/zero plot corresponds to the state space model with order 6. Here can we can see that there is a real pole on the right half plane. This means that the system has an unstable gain. This can be also verified from this bode plot that for the state space model, the magnitude increases exponentially in the last part. Here, we can see that the gain margin for the estimated models when phase is at -180 degree, close to the Nyquist frequency varies a lot compared to the empirical transfer function estimate, which is almost constant. This can be further verified by examining the spectral density of the data, which has very low power in high frequency part, which means for the high frequency part, not enough information is provided to give a good model estimation.



(a) ARX na = 6 nb=6.



(b) ARX na=6 nb=5.

Figure 4: Pole/zero plots

The zero in the origin is an uncaptured time shift (positive or negative?) of the system dynamic.

In Figure 6, we can see that the gain margin when phase is at -180 degree, around the Nyquist frequency varies a lot compared to the empirical transfer function update, which is almost constant. This implies that we probability need some further data preprocessing step to filter out the high frequency component of the data.

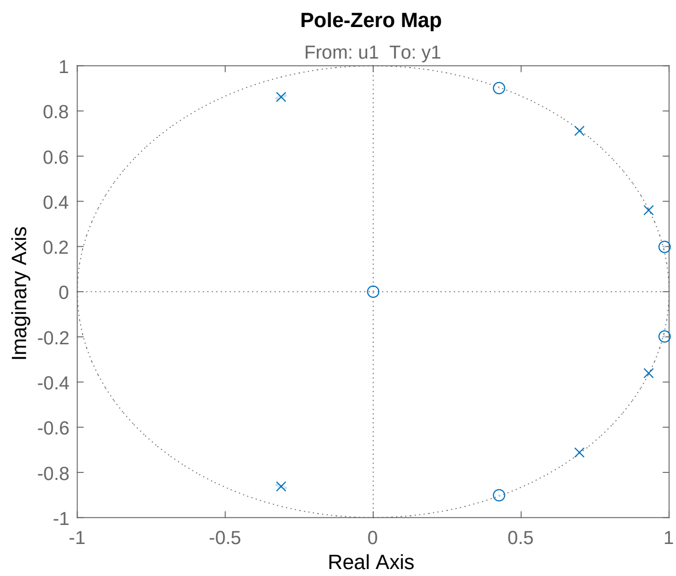


Figure 5: State Space ($n4sid$).

Simulations In the left plot of Figure 7 shows the simulation error of different models. Here, it is obvious to see that for the state space model estimated using prediction error method has the worst performance. Figure 8 shows the comparison of different models. For the plots on the diagonal, the simulated output is compared to true data. We can see that the OE model and the transfer function model are very similar. It is due to the fact that the only difference between these two models is that for the transfer function model, there is a feedthrough option that allows us to choose b0 in the numerator, while b0 is always zero in the OE model. Figure shows that the ARX model has less simulation error.

Predictions

Correlations Next, we did the residual analysis regarding its autocorrelation and its cross-correlation with the input. For a perfect-fitting model, the auto or cross correlation with the input should be zero expect when the lag is zero. In Figure 11, we can see that the ARX model and the state space model estimated using subspace method are better than the rests. However, from this cross-correlation plot, we can see that the state space model is not very good due to the correlation is not zero when the lag is large and that the normalised magnetite also often exceeds the 95

1.3.2 Non-linear models

In this subsection, we compare the linear model with several non-linear models regarding 5-step prediction error and simulation error in the terms of fitting rate, which is calculated as

$$fit\% = (1 - \frac{\sum(y(t) - \hat{y}(t))^2}{\sum(y(t) - \bar{y})^2}) * 100. \tag{1}$$

It can be seen from Figure 12 that the non-linear models estimated using the non-linear ARX model has higher fitting rate than those identified using the non-linear Hammerstein-Wiener model. For a non-linear model, it can be estimated either by directly using the estimated data or by adding non-linearity to an identified linear model for refinement. However, in our case, when we estimated

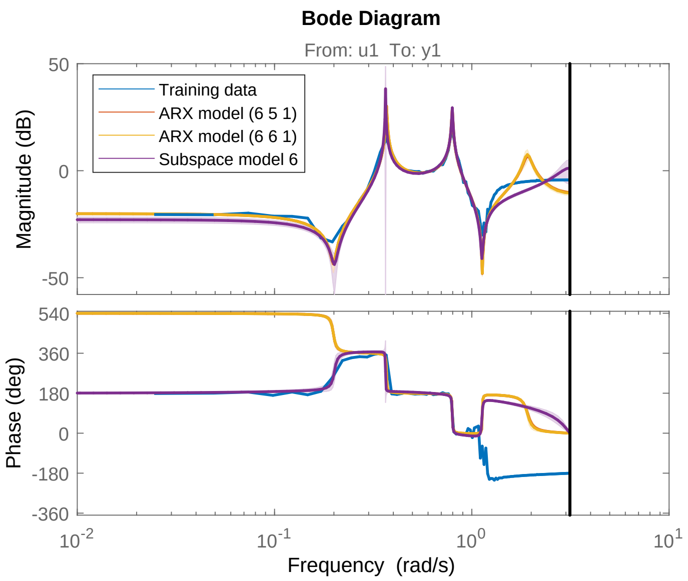
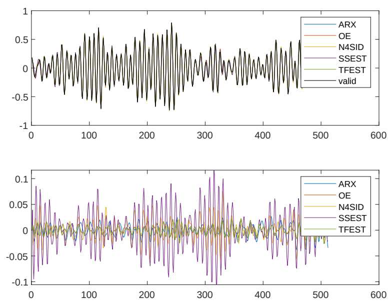


Figure 6: Bode plot of models.



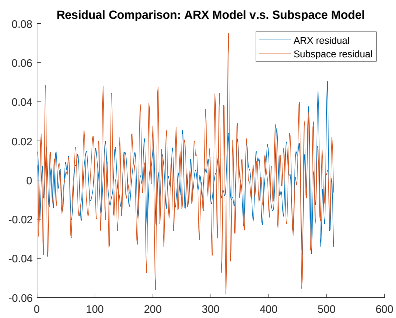


Figure 7: Simulations (left) and simulation error comparison of ARX and State Space model (right).

the default non-linear model using an input-output polynomial model of ARX or OE structure, the fitting rate did not increase compared to the result directly estimated using the linear model. Thus, we can draw the conclusion that there is little or no non-linearity in our data.

We also did some non-linear identification on our data. If we can obtain better performance using nonlinear model, then we can say that there are some non-linearities existing in our model. We tried the non-linear ARX model and the Hammerstein-Wiener model. Hammerstein-Wiener models describe dynamic systems using one or two static nonlinear blocks in series with a linear OE block, whereas in the non-linear ARX model, the regressors are followed by a non-linearity block. For the non-linear models, we can either choose to use the default option or we can initialize the non-linear system identification with an identified linear model, which means that the we start the model estimation with the linear model and begin the search by adding some non-linearities. However, from this figure we can see that, adding non-linearities to the linear model does not improve the model fitting rate. From this, we can infer that there is little or no non-linearity in the true model.

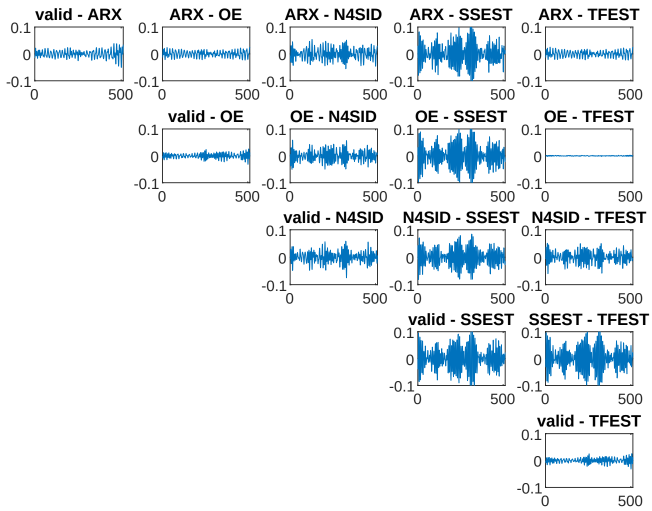


Figure 8: Model differences and simulation vs validation data error on diagonal.

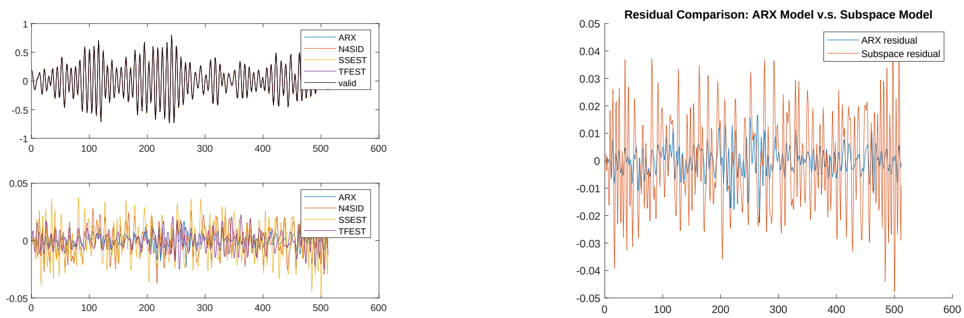


Figure 9: Predictions (left) and prediction error comparison of ARX and State Space model (right).

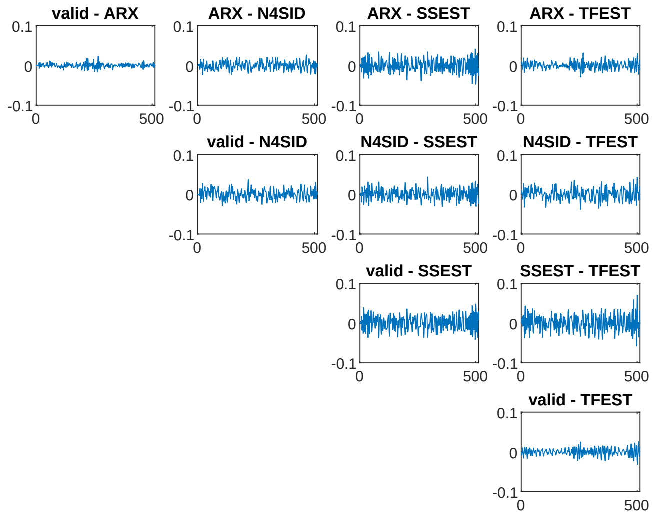


Figure 10: Model differences and prediction vs validation data error on diagonal.

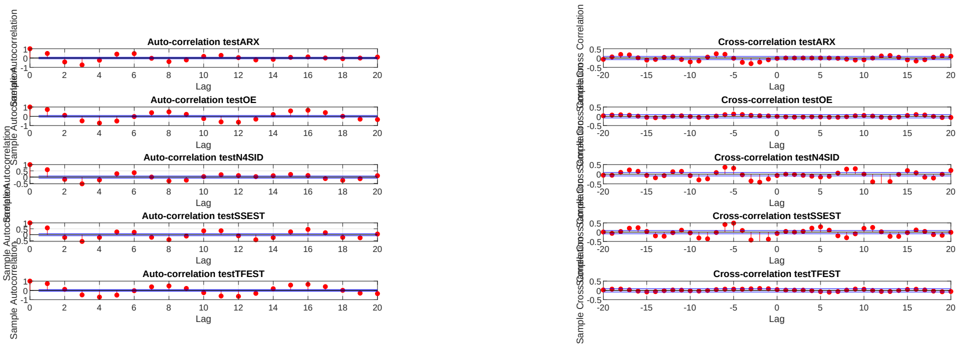


Figure 11: Auto-Correlation (left) and cross-correlation (right).

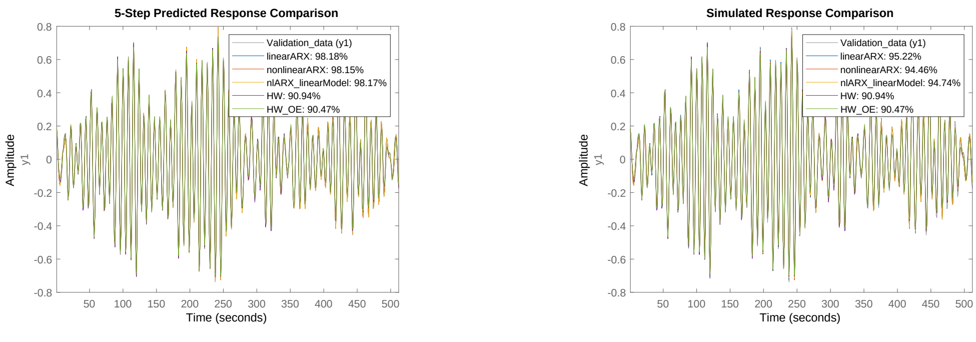


Figure 12: Non-linear model predictions (left) and simulations (right).