SSY 230, System Identification
Project 1: Estimating functions from noisy data

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1 ARX estimator

1.1 (a) arxfit

The *arxfit* fucntion can beneficially be implemented using the Linear Regression code written in project 1. The ARX model is given by

$$A(q^{-1})y(t) = B(q^{-1})u(t) + e(t) \tag{1}$$
$$\theta = (a_1, \dots a_{na}, b_1, \dots, b_{nb})^\mathsf{T} \tag{2}$$

according to (6.13a) in S & S. This can be rewritten on a Linear Regression format

$$y(t) = \phi(t)\theta + e(t) \tag{3}$$

where

$$\phi(t) = (-y(t-1), \dots, -y(t-na), u(t-1-nk), \dots, u(t-1-nk-nb))^\mathsf{T} \tag{4}$$

The function arxfit is implemented accordingly. It was verified against the function

$$y(t) = 0.2y(t-1) - 0.3y(t-2) + 0.4u(t-2) - 0.2u(t-3) \tag{5}$$

without any noise, and the correct model was obtained.

1.2 (b) id2tf

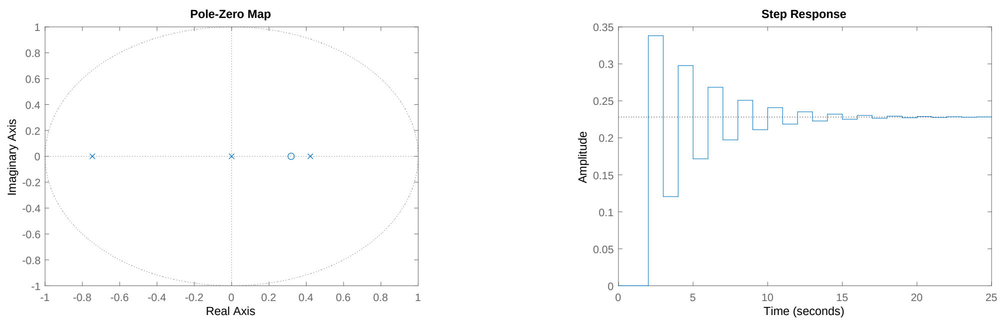
The function *id2tf* is implemented using MATLABs build in function *tf(Numerator, Denominator, -1)* where *Numerator* $= (\hat{b}_1, \dots, \hat{b}_{nb})$ and *Denominator* $= (1, \hat{a}_1, \dots, \hat{a}_{na})$.

1.3 (c) idpredict and idsimulate

A k-step predictor for an ARX model can be fund by studying the following equations.

$$\hat{y}(t|t-1) = -a_1 y(t-1) - a_2 y(t-2) \dots \tag{6}$$
$$\hat{y}(t|t-2) = -a_1 \hat{y}(t-1|t-2) - a_2 y(t-2) \dots \tag{7}$$
$$\hat{y}(t|t-3) = -a_1 \hat{y}(t-1|t-3) - a_2 \hat{y}(t-2|t-3) \dots \tag{8}$$

It is possible to use the 1-step ahead predictor to calculate the 2-step predictor, the 1-step and 2-step predictor to calculate the 3-step predictor and so on. It is the regressor matrix $\Phi_i = [Y_i, U]$



(a) pzmap using ltiview

(b) Step response using ltiview

Figure 1: MATLABs build in function *ltiview* analyzing the found ARX model for the system in (5) including noise.

that needs to be updated in a clever way in order to achieve fast matrix calculations which is shown below. The U matrix is constant and given by

$$U = \begin{bmatrix} u(0-n_k) & u(-1-n_k) & \dots & u(1-n_b-n_k) \\ u(1-n_k) & u(0-n_k) & \dots & u(2-n_b-n_k) \\ u(2-n_k) & u(1-n_k) & \dots & u(3-n_b-n_k) \\ \vdots & \vdots & \vdots & \vdots \\ u(n-1-n_k) & u(n-2-n_k) & \dots & u(n-n_b-n_k) \end{bmatrix} \tag{9}$$

while

$$Y_1 = \begin{bmatrix} y(0) & y(-1) & \dots & y(1-n_a) \\ y(1) & y(0) & \dots & y(2-n_a) \\ y(2) & y(1) & \dots & y(3-n_a) \\ \vdots & \vdots & \vdots & \vdots \\ y(n-1) & y(n-2) & \dots & y(n-n_a) \end{bmatrix} \tag{10}$$

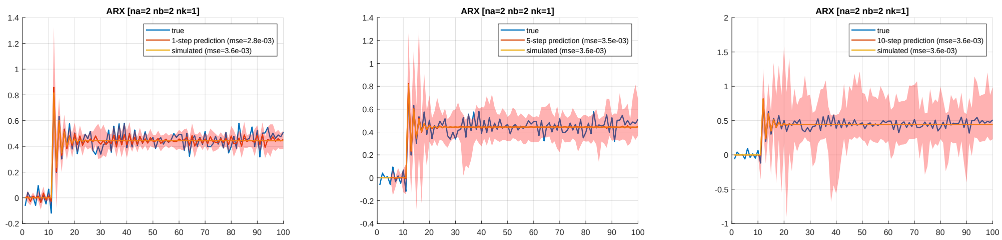
$$Y_2 = \begin{bmatrix} \hat{y}(0|-1) & y(-1) & \dots & y(1-n_a) \\ \hat{y}(1|0) & y(0) & \dots & y(2-n_a) \\ \hat{y}(2|1) & y(1) & \dots & y(3-n_a) \\ \vdots & \vdots & \vdots & \vdots \\ \hat{y}(n-1|n-2) & y(n-2) & \dots & y(n-n_a) \end{bmatrix} \tag{11}$$

$$Y_3 = \begin{bmatrix} \hat{y}(0|-2) & \hat{y}(-1|-2) & \dots & y(1-n_a) \\ \hat{y}(1|-1) & \hat{y}(0|-1) & \dots & y(2-n_a) \\ \hat{y}(2|0) & \hat{y}(1|0) & \dots & y(3-n_a) \\ \vdots & \vdots & \vdots & \vdots \\ \hat{y}(n-1|n-3) & \hat{y}(n-2|n-1) & \dots & y(n-n_a) \end{bmatrix} \tag{12}$$

where $\hat{y}(t|t-k) = \Phi_k\hat{\theta}$ and the values for both y and u prior to $t = 1$ are initialized to zero.

1.4 (d) idcompare

Some results for the same system as before (5) are compared using *idcompare* in Figure 2.



(a) Horizon 1 (b) Horizon 5 (c) Horizon 10

Figure 2: Some results using the implemented *idcompare* function. It is clear that the model uncertainty is increasing for increasing horizons, which means that less data is allowed in the prediction step. The mse on validation data is also increasing as the horizon is increasing.

2 OE estimator

2.1 (a) oefit

2.2 (b) id2tf

No changes from ARX.

2.3 (c) idpredict

Changes necessary.

2.4 (d) idsimulate

2.5 (e) idcompare

3 Identify two systems