SSY 230, System Identification
Project 3: Identification of a Real System

Yuxuan Xia
yuxuan.xia@chalmers.se
Emil Staf
emil.staf@chalmers.se

May 23, 2018

1 Flexible Robot Arm

The system we have chosen to identify is a mechanical system, where a flexible robot arm have been installed on an electrcal motor. It is a SISO system where the input $u(t)$ is measured reaction torque and the output $y(t)$ is the acceleration of the flexible robot arm. The experimental set-up was performed using a periodic sinusodial sweep.

1.1 Data

As mentioned previously the input data is a periodic sinusodial sweep (see top plot of Figure 1). Due to the fact that the data was obtained using a periodic sinusodial sweep we split the data in half and use the first part as training data and the second part as validation data.

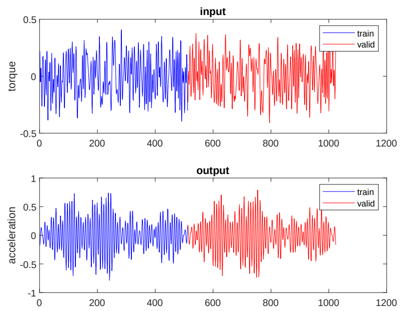
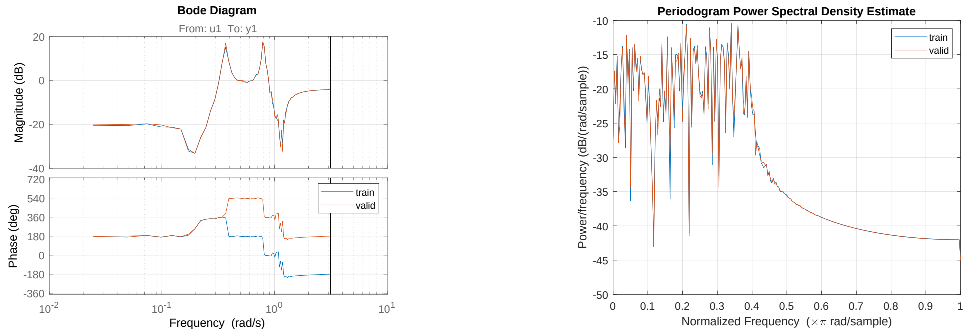


Figure 1: System data, input $u(t)$ (top) and output $y(t)$ (bottom).

To make sure that the frequency content in both the training and validation data are similar we use the *etfe* in MATLAB to find the Empirical Transfer Function Estimate of training- and validation data. The resulting bode-plot is shown in Figure 2a. Because the input signal is periodic, the empirical transfer function estimate is unbiased.

From analysing Figure 2a it is clear that the amplitude of the frequency content in both training- and validation data is very similar, while there is a 360 degree phase shift approximately starting from frequencies > 0.35 rad/s. However, it should be noted that a 360 degree phase shift means the validation data and training data are still in phase. Using the MATLAB build-in function *periodogram* it is clear that the frequency content of the training- and validation data is very similar.



(a) Bode-plot of training and validation data.

(b) Periodogram of training and validation data.

Figure 2: Analyzing training/validation split.

Also, it can be seen that for frequencies > 0.5 rad/s the power the amount of information for each frequency decreases. By examining the bode plot, we can have an initial guess of the number of poles and zeros we can observe that there are four sharp corners, each of which represent two conjugate poles or two conjugate zeros depending on its concave or convex. The first and last parts of the amplitude plot are ladder-shaped, which means that there are two more real poles. And we can also see that there is one hollow between two spikes, which represent a zero. Thus, there are six poles and five zeros in total. The high frequency assymptote amplitude is constant, which means that (for a linear system) there should be an equal amount of poles and zeroes.

1.2 Pre-Processing

In off-line situations, it is often better to remove trends in data before doing the system identification so that only the dynamic features are modelled. This can be done using the *getTrend* and *detrend* commands in MATLAB. However, in our case, the data before and after detrending look very similar since there is no obvious trend.

1.3 Model Estimation

1.3.1 Linear Models

In this section we search over the linear model space for candidate models. A few different linear models are tested and compared: ARX, OE, State Space and Transfer Function Estimation.

In Figure 3 the *mse* decreases for increasing model order but start to saturate for model orders above 8. As the high frequency assymptote from the bode plot of the ETFE tells us that there should be an equal ammount of poles and zeroes, and our more specific analysis resulted in 6 poles and 5 zeroes we first try models of order 6. From Figure 3 the *mse* shows that the models ARX, and State Space give the best results so lets focus on analysing them from now on.

Pole-Zero Plots In Figure 6, we can see that the gain margin when phase is at -180 degree, around the Nyquist frequency varies a lot compared to the empirical transfer function update, which is almost constant. This implies that we probability need some further data preprocessing step to filter out the high frequency component of the data.

Simulations

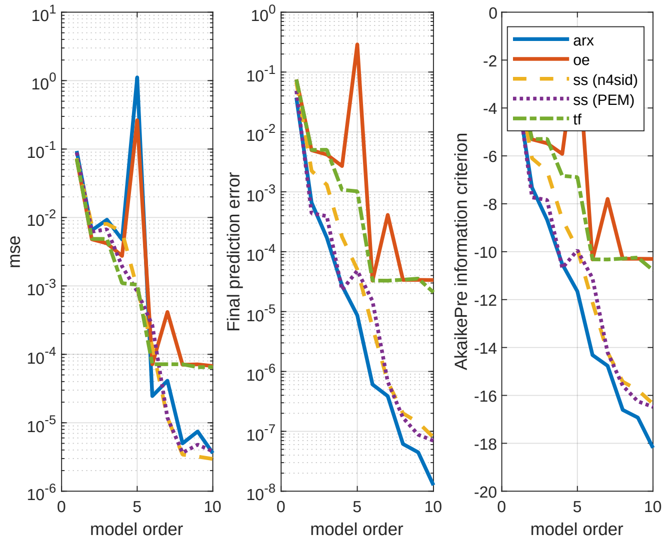
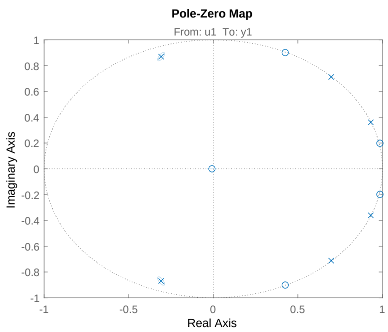
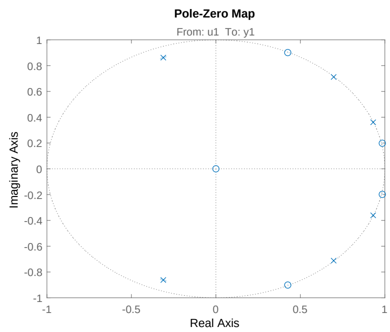


Figure 3: Searching over different model orders n, where the number of parameters p are $p = 2n+1$.



(a) ARX na = 6 nb=6.



(b) ARX na=6 nb=5.

Pole-Zero Plots

Predictions

Pole-Zero Plots

correlations

1.3.2 Non-linear models

In this subsection, we compare the linear model with several non-linear models regarding 5-step prediction error and simulation error in the terms of fitting rate, which is calculated as

$$fit\% = (1 - \frac{\sum(y(t) - \hat{y}(t))^2}{\sum(y(t) - \bar{y})^2}) * 100. \tag{1}$$

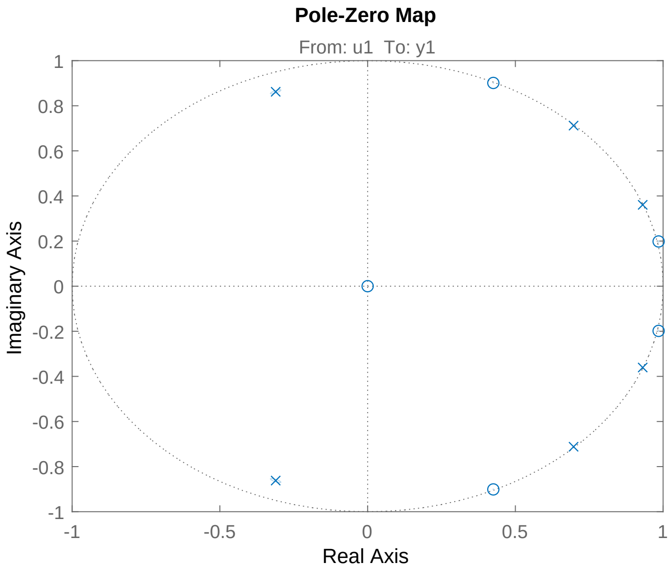


Figure 5: State Space ($n4sid$).

It can be seen from Figure 12 that the non-linear models estimated using the non-linear ARX model has higher fitting rate than those identified using the non-linear Hammerstein-Wiener model. For a non-linear model, it can be estimated either by directly using the estimated data or by adding non-linearity to an identified linear model for refinement. However, in our case, when we estimated the default non-linear model using an input-output polynomial model of ARX or OE structure, the fitting rate did not increase compared to the result directly estimated using the linear model. Thus, we can draw the conclusion that there is little or no non-linearity in our data.

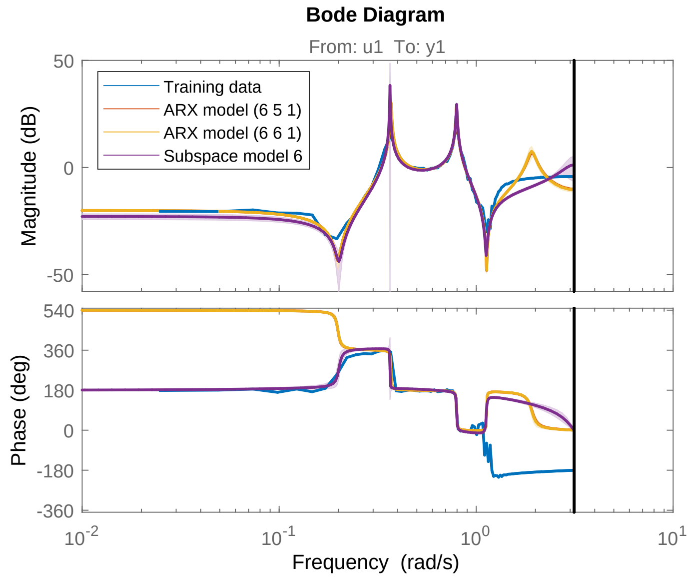
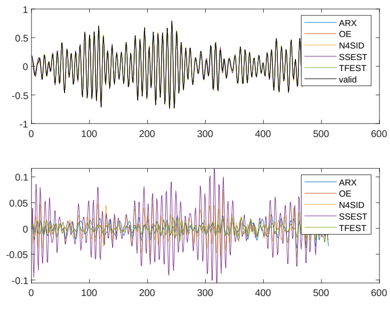


Figure 6: Bode plot of models.



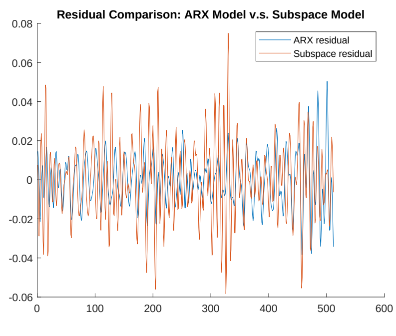


Figure 7: Simulations (left) and simulation error comparison of ARX and State Space model (right).

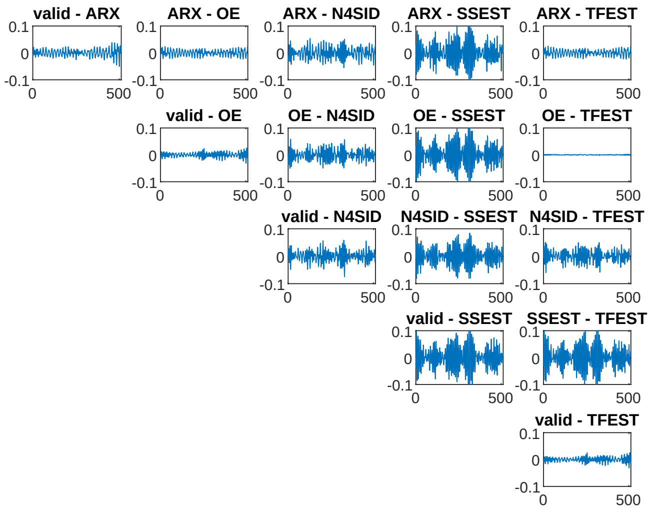


Figure 8: Model differences and simulation vs validation data error on diagonal.

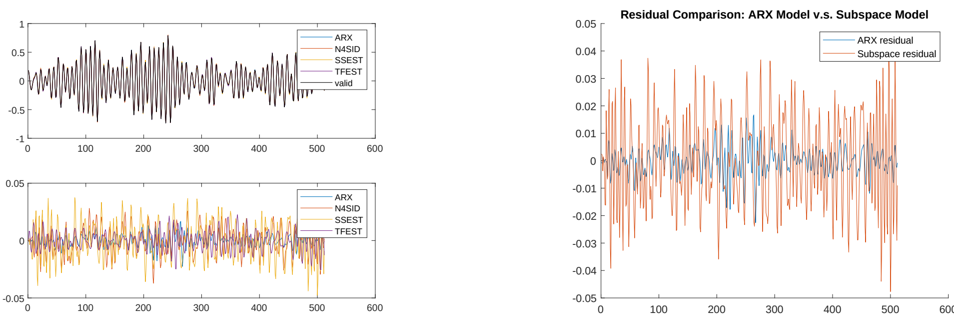


Figure 9: Predictions (left) and prediction error comparison of ARX and State Space model (right).

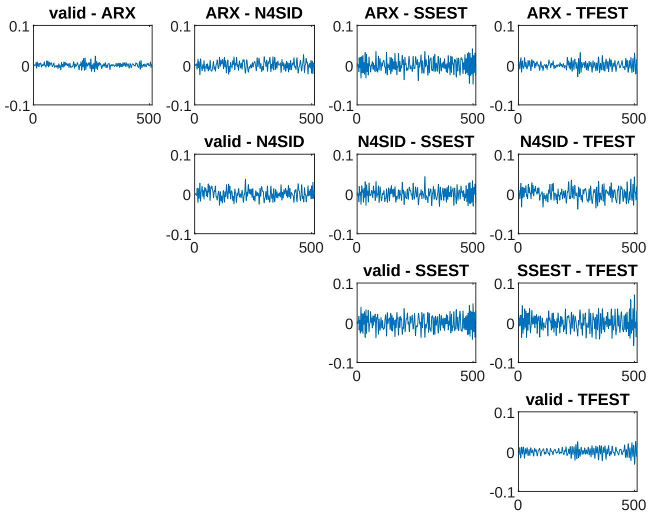


Figure 10: Model differences and prediction vs validation data error on diagonal.

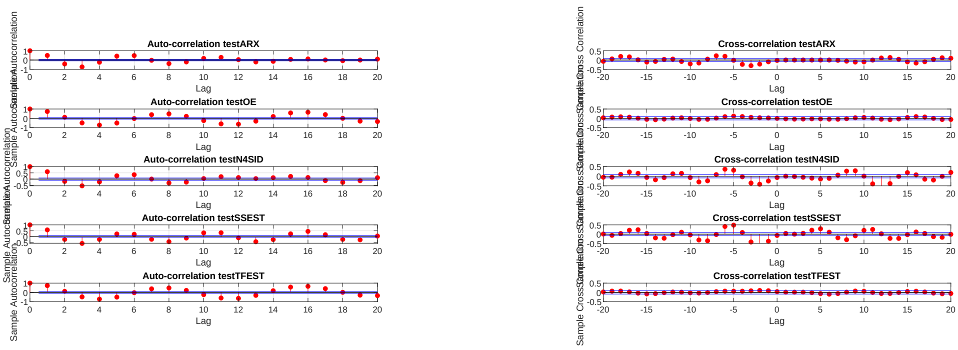


Figure 11: Auto-Correlation (left) and cross-correlation (right).

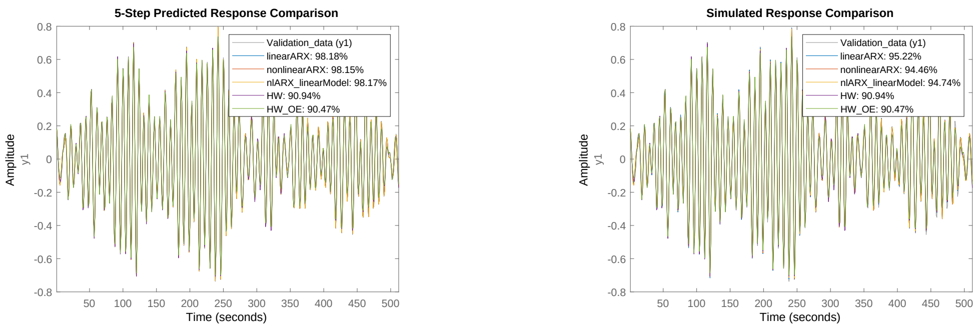


Figure 12: Non-linear model predictions (left) and simulations (right).