SSY 230, System Identification
Project 1: Estimating functions from noisy data

Yuxuan Xia
yuxuan.xia@chalmers.se
Emil Staf
emil.staf@chalmers.se

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1 ARX estimator

1.1 (a) arxfit

The *arxfit* fucntion can beneficially be implemented using the Linear Regression code written in project 1. The ARX model is given by

$$A(q^{-1})y(t) = B(q^{-1})u(t) + e(t) \tag{1}$$
$$\theta = (a_1, \dots a_{na}, b_1, \dots, b_{nb})^{\mathsf{T}} \tag{2}$$

according to (6.13a) in S & S. This can be rewritten on a Linear Regression format

$$y(t) = \phi(t)\theta + e(t) \tag{3}$$

where

$$\phi(t) = (-y(t-1), \dots, -y(t-na), u(t-1-nk), \dots, u(t-1-nk-nb))^{\mathsf{T}} \tag{4}$$

The function arxfit is implemented accordingly. It was verified against the function

$$y(t) = 0.2y(t-1) - 0.3y(t-2) + 0.4u(t-2) - 0.2u(t-3) \tag{5}$$

without any noise, and the correct model was obtained.

1.2 (b) id2tf

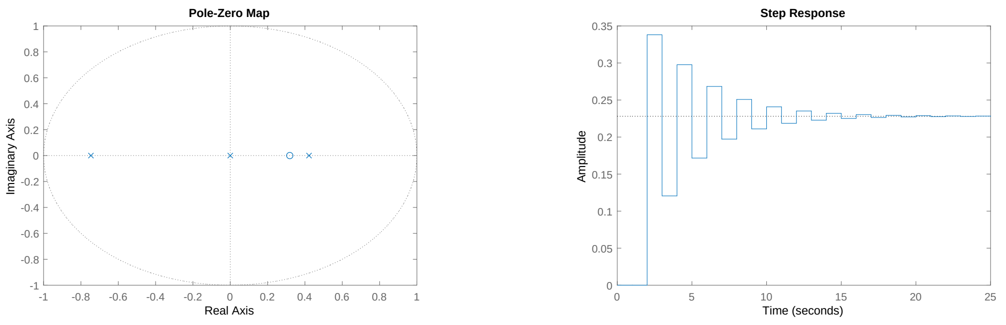
The function *id2tf* is implemented using MATLABs build in function *tf(Numerator, Denominator, -1)* where *Numerator* $= (\hat{b}_1, \dots, \hat{b}_{nb})$ and *Denominator* $= (1, \hat{a}_1, \dots, \hat{a}_{na})$.

1.3 (c) idpredict and idsimulate

A k-step predictor for an ARX model can be fund by studying the following equations.

$$\hat{y}(t|t-1) = -a_1 y(t-1) - a_2 y(t-2) \dots \tag{6}$$
$$\hat{y}(t|t-2) = -a_1 \hat{y}(t-1|t-2) - a_2 y(t-2) \dots \tag{7}$$
$$\hat{y}(t|t-3) = -a_1 \hat{y}(t-1|t-3) - a_2 \hat{y}(t-2|t-3) \dots \tag{8}$$

It is possible to use the 1-step ahead predictor to calculate the 2-step predictor, the 1-step and 2-step predictor to calculate the 3-step predictor and so on. It is the regressor matrix $\Phi_i = [Y_i, U]$



(a) pzmap using ltiview (b) Step response using ltiview

Figure 1: MATLABs build in function *ltiview* analyzing the found ARX model for the system in (5) including noise.

that needs to be updated in a clever way in order to achieve fast matrix calculations which is shown below. The U matrix is constant and given by

$$U = \begin{bmatrix} u(0-n_k) & u(-1-n_k) & \dots & u(1-n_b-n_k) \\ u(1-n_k) & u(0-n_k) & \dots & u(2-n_b-n_k) \\ u(2-n_k) & u(1-n_k) & \dots & u(3-n_b-n_k) \\ \vdots & \vdots & \vdots & \vdots \\ u(n-1-n_k) & u(n-2-n_k) & \dots & u(n-n_b-n_k) \end{bmatrix} \tag{9}$$

while

$$Y_1 = \begin{bmatrix} y(0) & y(-1) & \dots & y(1-n_a) \\ y(1) & y(0) & \dots & y(2-n_a) \\ y(2) & y(1) & \dots & y(3-n_a) \\ \vdots & \vdots & \vdots & \vdots \\ y(n-1) & y(n-2) & \dots & y(n-n_a) \end{bmatrix} \tag{10}$$

$$Y_2 = \begin{bmatrix} \hat{y}(0|-1) & y(-1) & \dots & y(1-n_a) \\ \hat{y}(1|0) & y(0) & \dots & y(2-n_a) \\ \hat{y}(2|1) & y(1) & \dots & y(3-n_a) \\ \vdots & \vdots & \vdots & \vdots \\ \hat{y}(n-1|n-2) & y(n-2) & \dots & y(n-n_a) \end{bmatrix} \tag{11}$$

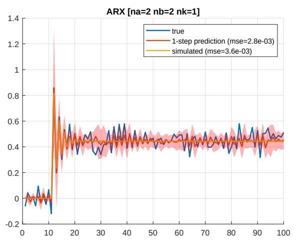
$$Y_3 = \begin{bmatrix} \hat{y}(0|-2) & \hat{y}(-1|-2) & \dots & y(1-n_a) \\ \hat{y}(1|-1) & \hat{y}(0|-1) & \dots & y(2-n_a) \\ \hat{y}(2|0) & \hat{y}(1|0) & \dots & y(3-n_a) \\ \vdots & \vdots & \vdots & \vdots \\ \hat{y}(n-1|n-3) & \hat{y}(n-2|n-1) & \dots & y(n-n_a) \end{bmatrix} \tag{12}$$

where $\hat{y}(t|t-k) = \Phi_k\hat{\theta}$ and the values for both y and u prior to $t=1$ are initialized to zero.

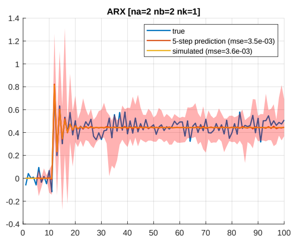
Simulation can be performed by setting $k > n$, i.e. the predictor is forced not to use any of the measured outputs.

1.4 (d) idcompare

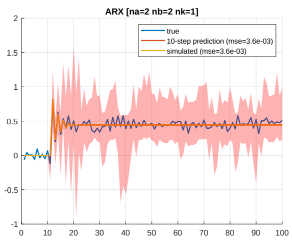
Some results for the same system as before (5) are compared using *idcompare* in Figure 2.



(a) Horizon 1



(b) Horizon 5



(c) Horizon 10

Figure 2: Some results using the implemented *idcompare* function. It is clear that the model uncertainty is increasing for increasing horizons, which means that less data is allowed in the prediction step. The mse on validation data is also increasing as the horizon is increasing.

2 OE estimator

2.1 (a) oefit

Need to include explanation of why it is ok to use arxfit to and filter to obtain OE model!

2.2 (b) id2tf

No changes from ARX.

2.3 (c) idpredict

According to the lecture notes a k-step predictor can be written on the form

$$\hat{y}(t|t-k) = \bar{H}_k(q)H^{-1}(q)G(q)u(t) + (1 - \bar{H}_k(q)H^{-1}(q))y(t) \tag{13}$$

which for an OE-model simplifies to

$$\hat{y}(t|t-k) = G(q)u(t) \tag{14}$$

since $H^{-1}(q) = 1$ and $\bar{H}_k^{-1}(q) = 1$ for all $k > 1$. Therefore prediction is the same as simulation for the OE-model and the *idpredict* function calls the *idsimulate* function in the case of an OE-model.

2.4 (d) idsimulate

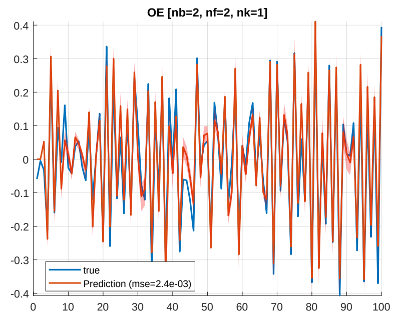
Using the MATLAB build in function *lsim* after creating the OE transfer function by calling *id2tf*.

2.5 (e) idcompare

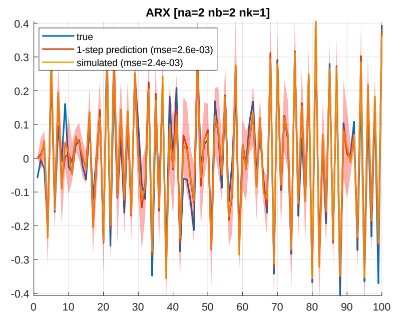
No changes necessary?

Showing only the prediction since simulation is the same as stated above. This can be seen in the Figure 3, where OE-data was created and an ARX-model is compared to an OE-model.

3 Identify two systems

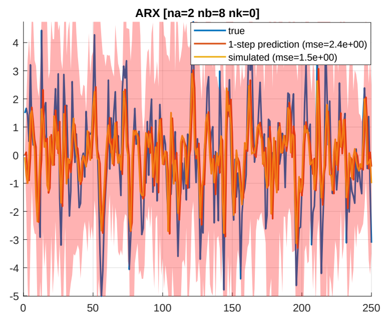


(a) OE-model

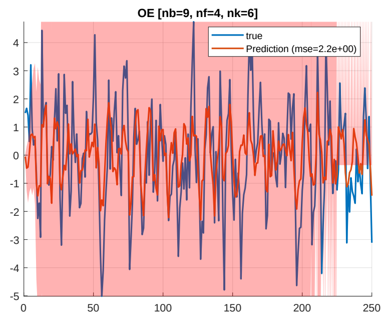


(b) ARX-model

Figure 3: OE-model vs ARX-model on OE system of order nb = 2, nf = 2, nk = 1.

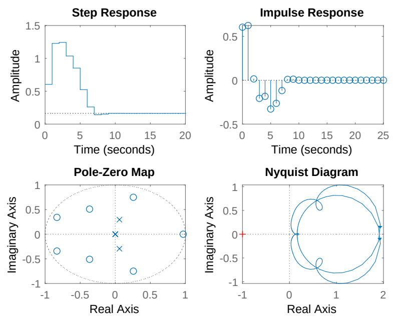


(a) ARX

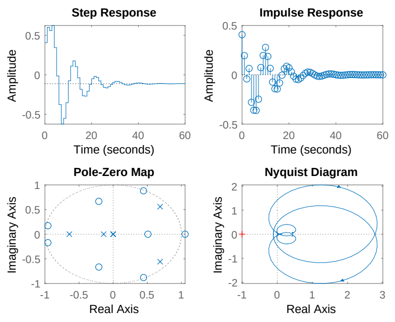


(b) OE

Figure 4: Dataset exercise1

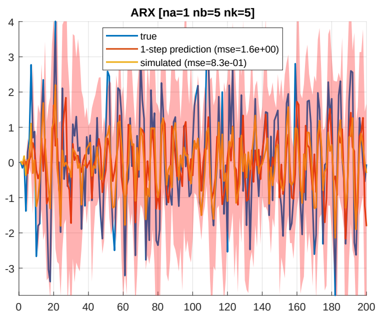


(a) ARX

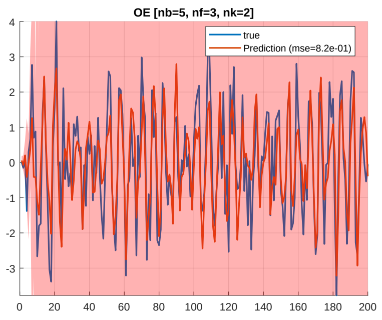


(b) OE

Figure 5: Dataset exercise1, using build-in MATLAB plotting functionality.

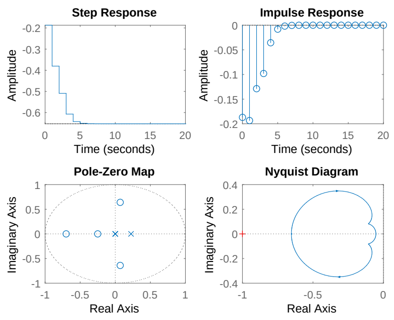


(a) ARX

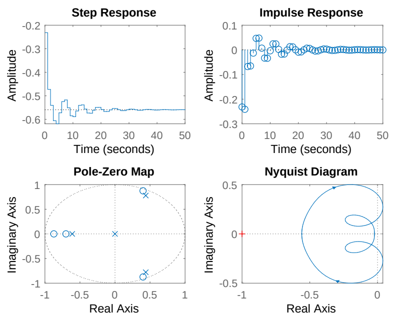


(b) OE

Figure 6: Dataset exercise2



(a) ARX



(b) OE

Figure 7: Dataset exercise2, using build-in MATLAB plotting functionality.