Math 338: Exam 1

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a)

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#ELIOT
#EXERCISE ONE
#PART A

numbers = 1:52

montmort = replicate(10000,{
  #shuffle the deck
  deck = sample(numbers,52,replace=FALSE)

  #check for matches
  matches = deck==numbers

  #count number of matches
  #return true if there's at least one match
  sum(matches)>0
})

#find the probability
print(mean(montmort))
```

```
> print(mean(montmort))
[1] 0.6272
```

The simulated probability of winning is .627

b)

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#EXERCISE ONE
#PART B

geese = 1:30

simmons = replicate(10000,{
  #sample 20 geese, "tag" them
  first = sample(geese,20,replace=FALSE)

  #sample 10 geese
  second = sample(geese,10,replace=FALSE)

  #count number of geese in second sample who were tagged in first
  #return true if more than 8 geese in second sample were tagged
  sum(second%in%first)>8
})

#find the probability
print(mean(simmons))
```

```
> print(mean(simmons))
[1] 0.0645
```

The simulated probability that at least 9 geese in the new sample were previously tagged is .0645

$\boxed{2}$

L = Anthony is lying $\quad \bar{L}$ = Anthony is telling the truth

F = Anthony fails the polygraph $\quad \bar{F}$ = Anthony passes the polygraph

$P(\bar{F} \mid \bar{L}) = .8 \quad P(F \mid L) = .8$

a) $P(\bar{F} \mid L)$? $\quad P(F \mid \bar{L})$?

$$P(\bar{F} \mid L) + P(F \mid L) = 1 = P(F \mid \bar{L}) + P(\bar{F} \mid \bar{L})$$

$$P(\bar{F} \mid L) + .8 = 1 = P(F \mid \bar{L}) + .8$$

$$P(\bar{F} \mid L) = .2 = P(F \mid \bar{L})$$

b) $P(L) = .05 \quad P(\bar{L}) = 1 - P(L) = .95$

$$P(L \mid F)? = \frac{P(F \mid L)\, P(L)}{P(F)} = \frac{(.8)(.05)}{P(F)}$$

$$P(F) = P(F \mid L)\, P(L) + P(F \mid \bar{L})\, P(\bar{L})$$
$$= (.8)(.05) + (.2)(.95) = .23$$

$$\frac{(.8)(.05)}{.23} = .174$$

The probability Anthony is actually lying given that he failed the test is .174

c) .8 is the probability that the test is correct given that we know whether or not someone lied. We cannot equate this to the probability that someone lied given that we know the test results. (prosecutor's fallacy).

$P(L \mid F) \neq P(F \mid L)$

d) $P(L)$ and $P(F)$ independent?

$\hookrightarrow P(L \mid F) = P(L)$? $\quad P(F \mid L) = P(F)$?

$.174 \neq .05 \quad , \quad .8 \neq .23$

The events L and F are not independent because knowing that F is true changes the probability of L, and vice versa. $P(L)$ alone is .05 but if we know F is true, the probability of L is .174 ($P(L \mid F)$)

e) $P(L|F) = .7 \quad P(F|L)? \quad P(L) = .05$

$$P(F) = P(F|L)P(L) + P(F|\bar{L})P(\bar{L})$$

$$= (.05)P(F|L) + (.95)(1 - P(F|L))$$

$$= (.05)P(F|L) - (.95)P(F|L) + .95$$

$$= .95 - (.9)P(F|L)$$

$$.7 = \frac{(.05)P(F|L)}{.95 - (.9)P(F|L)}$$

$.665 - .63\,P(F|L) = .05\,P(F|L)$

$.665 = .68\,P(F|L)$

$P(F|L) = \frac{.665}{.68} = .978$

The polygraph test must be 97.8% reliable in order for the probability Anthony is lying given that he failed the test to be 70%.

3

$$E(Y) = \sum_y y\,p(y)$$

a)

Y	p(y)
16	1

$E(Y) = \$16$

b)

y	p(y)
1	.5
32	.375
64	.125

$.5 \cdot .25 = .125$
$.5 \cdot .75 = .375$

$$E(Y) = .5 + 32(.375) + 64(.125)$$
$$= .5 + 12 + 8 = \$20.50$$

c)

Y	p(Y)
1	.75
32	.125
64	.125

$.25 \cdot .5 = .125$

$$E(Y) = .75 + 32(.125) + 64(.125)$$
$$= .75 + 4 + 8 = \$12.75$$

d) $Var(Y) = E(Y^2) - [E(Y)]^2 \qquad E(Y) = 16$

y^2	p(y)
256	1

$E(Y^2) = 256 \cdot 1 = 256$

$Var(Y) = 256 - 16^2 = 0$

The variance of the winnings in scenario (a) is 0 because there is only one possible outcome for the winnings. Therefore Y, the winnings, doesn't vary at all.

4

$n = 100 \quad p = .01$

Y = # of adults in sample who have side effects

a) Y is a binomial distribution with a probability of .01 and size of 100.

$Y \sim \text{Binomial}(100, .01)$

PMF: $p(Y=y) = \binom{100}{y}(.01)^y(.99)^{100-y} \quad y = 0,1,2,\ldots,100$

└ support of Y

b) $p(Y \geq 3) = 1-(p(Y=0)+p(Y=1)+p(Y=2))$

$p(Y=0) = \binom{100}{0}(.01)^0(.99)^{100} = .366$

$p(Y=1) = \binom{100}{1}(.01)^1(.99)^{99} = .370$

$p(Y=2) = \binom{100}{2}(.01)^2(.99)^{98} = .185$

$P(Y \geq 3) = 1-(.366+.370+.185) = 1-.921 = .079 = P(Y \geq 3)$

c) $P(Y=5 \mid Y \geq 3) = \dfrac{P(Y=5 \wedge Y \geq 3)}{P(Y \geq 3)} = \dfrac{P(Y=5)}{P(Y \geq 3)}$

$P(Y=5) = \binom{100}{5}(.01)^5(.99)^{95} = .00290$

$\dfrac{.00290}{.079} = .0365 = P(Y=5 \mid Y \geq 3)$

d) $E(Y) = np = (100)(.01) = 1$

$Var(Y) = np(1-p) = 1(.99) = .99$

e) $X \sim \text{Poisson}(\lambda = np) \quad \lambda = np = 1 \quad X \sim \text{Poisson}(1)$

$P(X=x) = \dfrac{1^x}{x!}e^{-1} \quad x = 0,1,\ldots$

$P(X \geq 3) = 1-(P(X=0)+P(X=1)+P(X=2))$

$P(X=0) = e^{-1} \quad P(X=1) = e^{-1} \quad P(X=2) = \frac{1}{2}e^{-1}$

$P(X \geq 3) = 1 - \frac{5}{2}e^{-1} = .08$

This answer is very similar to the answer from part b, which was .079.