Introduction to Statistics and Data Science using *eStat*

Chapter 7 Testing Hypothesis for Single Population

7.2 Testing Hypothesis for a Population Variance

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7.2 Testing Hypothesis for a Population Variance

- **Examples for testing hypothesis of population variances.**

- **Bolts of a company that currently supplies bolts to an automaker have an average diameter of 7mm and a variance of 0.25. Recently, rival companies have been applying for the supply, claiming that their companies' bolts have an average diameter of 7 millimeters and a variance of 0.16. How can I find out if this claim is true?**

- **The variance of math score of the last year's college scholastic aptitude test was 100. This year's math problem is said to be much easier than last year's. How can I find out if the variance of math score of this year test is smaller than last year?**

7.2 Testing Hypothesis for a Population Variance

Table 7.2.1 Testing hypothesis of population variance
- population is normally distributed -

Type of Hypothesis	Decision Rule
1) $H_0 : \sigma^2 = \sigma_0^2$: $H_1 : \sigma^2 > \sigma_o^2$:	If $\dfrac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1;\alpha}$, then reject H_0, else accept H_0
2) $H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 < \sigma_o^2$	If $\dfrac{(n-1)S^2}{\sigma_0^2} < \chi^2_{n-1;\cancel{\alpha}}$, then reject H_0, else accept H_0 $1-\alpha$
3) $H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 \neq \sigma_o^2$	If $\dfrac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1;\alpha/2}$ or $\dfrac{(n-1)S^2}{\sigma_0^2} < \chi^2_{n-1;1-\alpha/2}$, then reject H_0, else accept H_0

Note: In 1) the null hypothesis can be written as $H_0 : \sigma^2 \leq \sigma_o^2$, in 2) $H_0 : \sigma^2 \geq \sigma_o^2$.

7.2 Testing Hypothesis for a Population Variance

[Example 7.2.1] One company produces bolts for an automobile. If the average diameter of bolts is 15mm and its variance is less than 0.10^2, it can be delivered to the automobile company.

- Twenty-five of the most recent products were randomly sampled and their variance was 0.15^2.
- Assuming that the diameter of a bolt follows a normal distribution,

1) Conduct testing hypothesis at the 5% significance level to determine if the product can be delivered to the automotive company.
2) Check the result using 『eStatU』

7.2 Testing Hypothesis for a Population Variance

<Answer of Ex 7.2.1>

1) Hypothesis is $H_0 : \sigma^2 \leq 0.1^2$, $H_1 : \sigma^2 \geq 0.1^2$ and its decision rule is as follows:

'If $\frac{(n-1)S^2}{\sigma_0{}^2} > \chi^2{}_{n-1;\alpha}$, then reject H_0 '

$S^2 = 0.15^2 = 0.0225,$

$\frac{(n-1)S^2}{\sigma_0{}^2} = \frac{(25-1)\, 0.15^2}{0.10^2} = 54$

$\chi^2{}_{n-1;\alpha} = \chi^2{}_{25-1;0.05} = 36.42.$

Therefore, H_0 is rejected.

7.2 Testing Hypothesis for a Population Variance

2) Select 'Testing Hypothesis ' in 『eStatU』. Enter $\sigma_0{}^2$ = 0.01, select the right sided test and the 5% significance level in the input box. Then enter the sample size n = 25 and sample variance $s^2 = 0.15^2$ = 0.0225.

Testing Hypothesis σ²

Menu

[Hypothesis] $H_0 : \sigma^2 = \sigma_0{}^2$ 0.01 (> 0)

○ $H_1 : \sigma^2 \neq \sigma_0{}^2$ ◉ $H_1 : \sigma^2 > \sigma_0{}^2$ ○ $H_1 : \sigma^2 < \sigma_0{}^2$

[Test Type] χ^2 *test*

Significance Level α = ◉ 5% ○ 1%

[Sample Data] *Input either sample data using BSV or sample statistics at the next boxes*

[Sample Statistics]

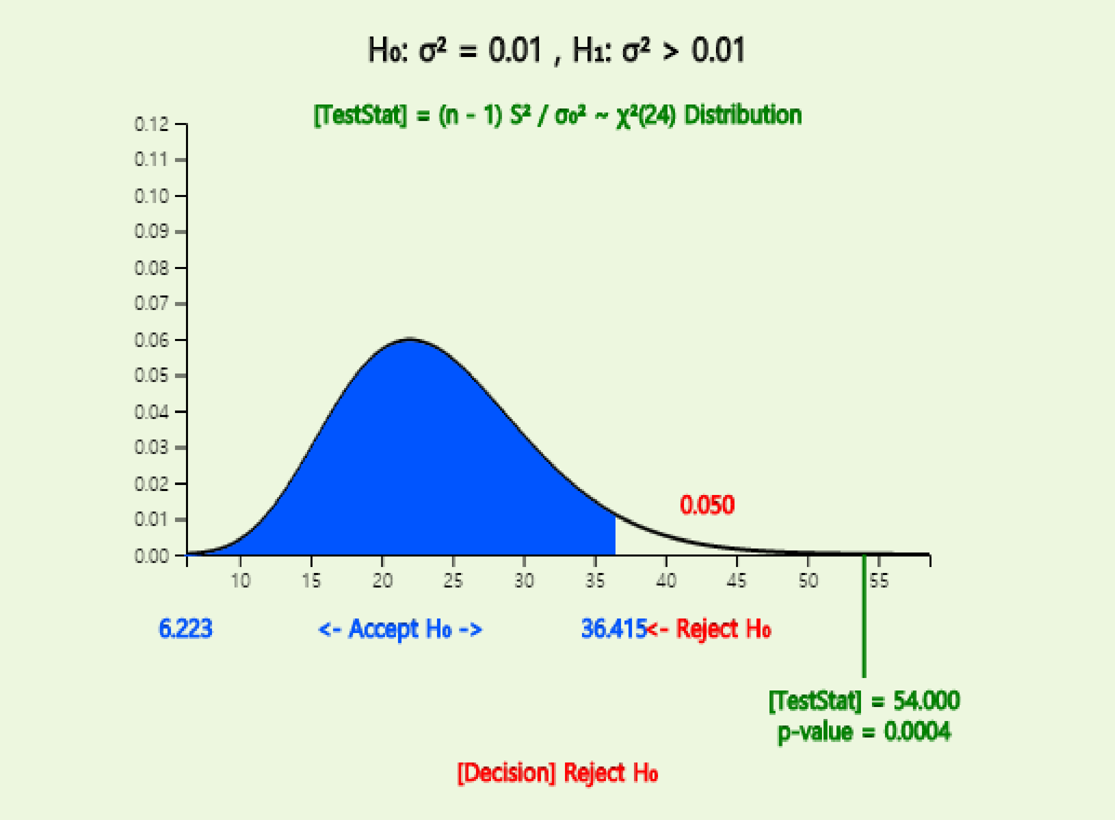
Sample Size n = 25 (> 1)

Sample Variance s^2 = 0.0225 (> 0)

[Confidence Interval]

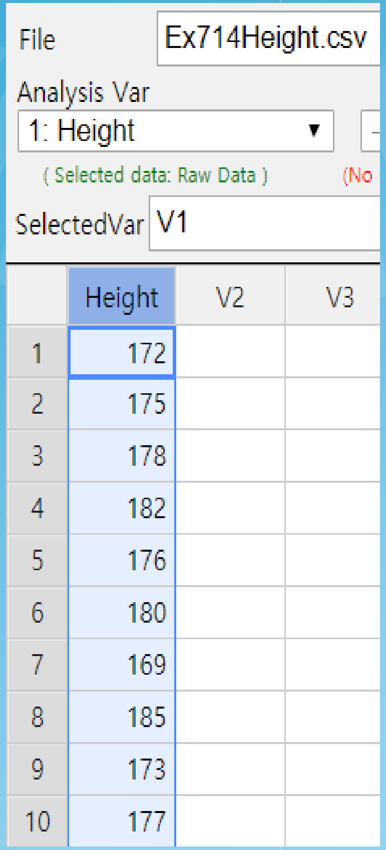
$((n-1)S^2 / \chi^2_{n-1\,;\,\alpha/2}\ ,\ (n-1)\,S^2 / \chi^2_{n-1\,;\,1-\alpha/2})$ ⇔ (0.014 , 0.044)

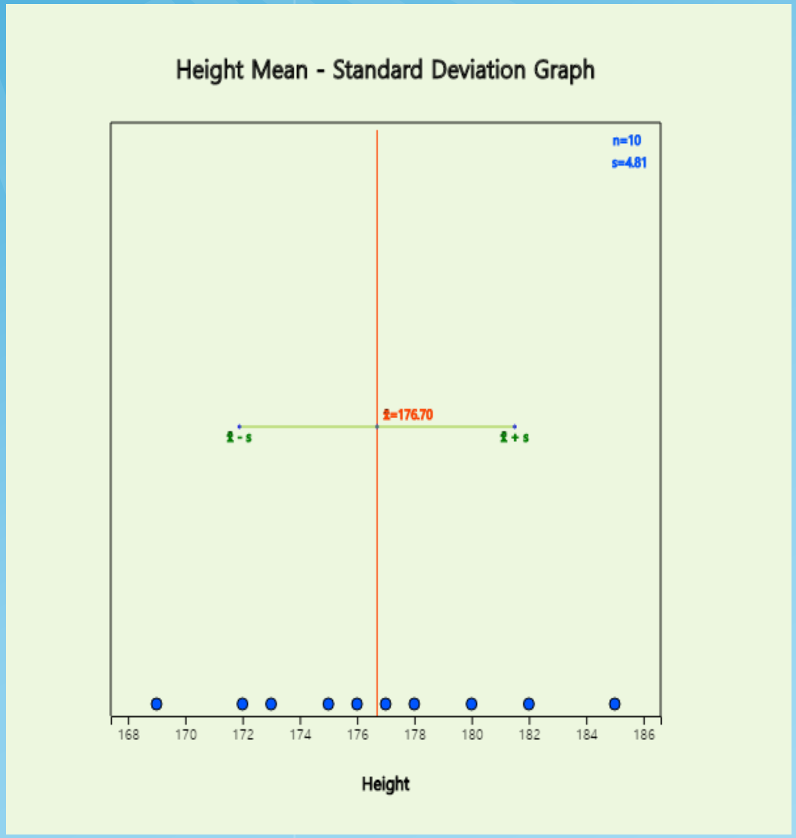
Execute

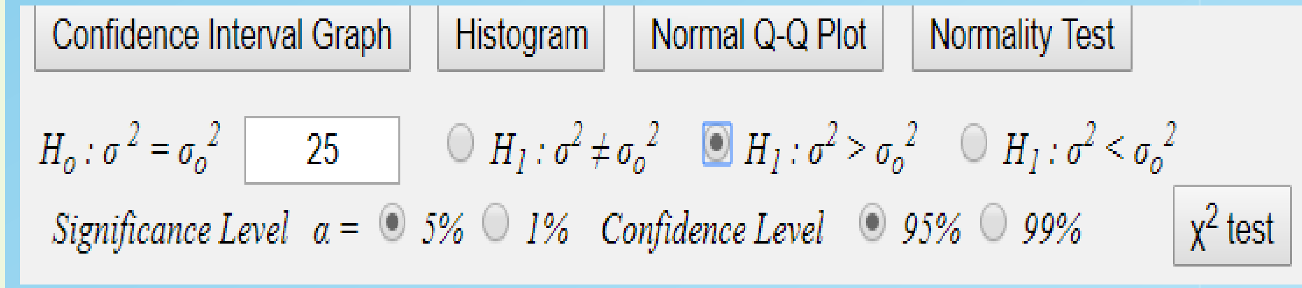


7.2 Testing Hypothesis for a Population Variance

[Ex 7.2.2] By using the height data of 10 male college students in [Ex 7.1.4], 172 175 178 182 176 180 169 185 173 177, test the hypothesis whether the population variance is greater than 25 at a significant level of 5%.

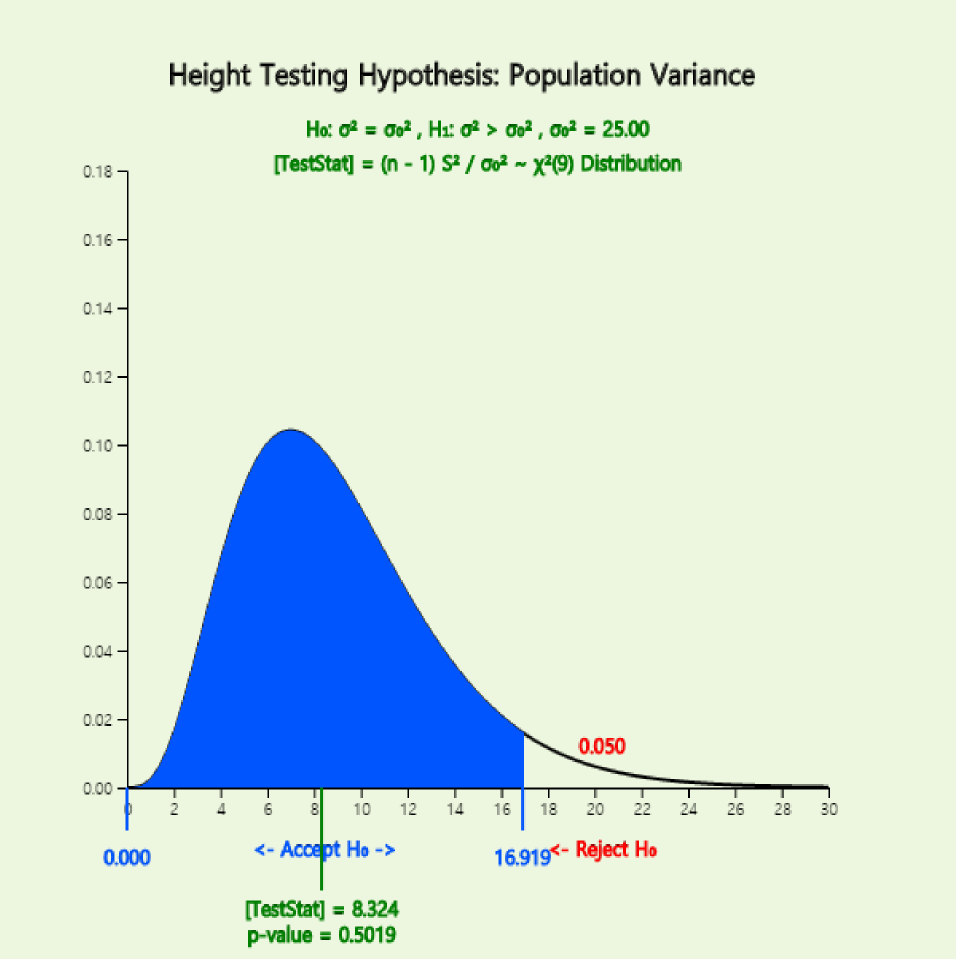






7.2 Testing Hypothesis for a Population Variance

<Answer of Ex 7.2.2>



Testing Hypothesis: Population Variance	Analysis Var	Height			
Statistics	Observation	Mean	Std Dev	std err	Population Variance 95% Confidence Interval
	10	176.700	4.809	1.521	(10.940, 77.063)
Missing Observations	0				
Hypothesis					
$H_0 : \sigma^2 = \sigma_0^2$	σ_0^2	[TestStat]	ChiSq value	p-value	
$H_1 : \sigma^2 > \sigma_0^2$	25.00	$(n-1)\ S^2 / \sigma_0^2$	8.324	0.5019	

Thank you