

Introduction to Statistics and Data Science using *eStat*

## Chapter 7 Testing Hypothesis for Single Population

# 7.2 Testing Hypothesis for a Population Variance

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## 7.2 Testing Hypothesis for a Population Variance

- Examples for testing hypothesis of population variances.
- Bolts of a company that currently supplies bolts to an automaker have an average diameter of 7mm and a variance of 0.25. Recently, rival companies have been applying for the supply, claiming that their companies' bolts have an average diameter of 7 millimeters and a variance of 0.16. How can I find out if this claim is true?
- The variance of math score of the last year's college scholastic aptitude test was 100. This year's math problem is said to be much easier than last year's. How can I find out if the variance of math score of this year test is smaller than last year?

## 7.2 Testing Hypothesis for a Population Variance

Table 7.2.1 Testing hypothesis of population variance  
- population is normally distributed -

Type of Hypothesis	Decision Rule
1) $H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 > \sigma_0^2$	If $\frac{(n-1)S^2}{\sigma_0^2} > \chi_{n-1;\alpha}^2$ , then reject $H_0$ , else accept $H_0$
2) $H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 < \sigma_0^2$	If $\frac{(n-1)S^2}{\sigma_0^2} < \chi_{n-1;\alpha}^2$ , then reject $H_0$ , else accept $H_0$ / -X
3) $H_0 : \sigma^2 = \sigma_0^2$ $H_1 : \sigma^2 \neq \sigma_0^2$	If $\frac{(n-1)S^2}{\sigma_0^2} > \chi_{n-1;\alpha/2}^2$ or $\frac{(n-1)S^2}{\sigma_0^2} < \chi_{n-1;1-\alpha/2}^2$ , then reject $H_0$ , else accept $H_0$

Note: In 1) the null hypothesis can be written as  $H_0 : \sigma^2 \leq \sigma_0^2$ , in 2)  $H_0 : \sigma^2 \geq \sigma_0^2$ .

## 7.2 Testing Hypothesis for a Population Variance

**[Example 7.2.1]** One company produces bolts for an automobile. If the average diameter of bolts is 15mm and its variance is less than  $0.10^2$ , it can be delivered to the automobile company.

- Twenty-five of the most recent products were randomly sampled and their variance was  $0.15^2$ .
- Assuming that the diameter of a bolt follows a normal distribution,
  - 1) Conduct testing hypothesis at the 5% significance level to determine if the product can be delivered to the automotive company.
  - 2) Check the result using 『eStatU』

## 7.2 Testing Hypothesis for a Population Variance

### <Answer of Ex 7.2.1>

1) Hypothesis is  $H_0 : \sigma^2 \leq 0.1^2$  ,  $H_1 : \sigma^2 \geq 0.1^2$  and its decision rule is as follows:

'If  $\frac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1;\alpha}$  , then reject  $H_0$  '

$$S^2 = 0.15^2 = 0.0225,$$

$$\frac{(n-1)S^2}{\sigma_0^2} = \frac{(25-1) 0.15^2}{0.10^2} = 54$$

$$\chi^2_{n-1;\alpha} = \chi^2_{25-1;0.05} = 36.42.$$

Therefore,  $H_0$  is rejected.

## 7.2 Testing Hypothesis for a Population Variance

2) Select 'Testing Hypothesis' in 『eStatU』. Enter  $\sigma_0^2 = 0.01$ , select the right sided test and the 5% significance level in the input box. Then enter the sample size  $n = 25$  and sample variance  $s^2 = 0.15^2 = 0.0225$ .

### Testing Hypothesis $\sigma^2$

[Menu](#)

[Hypothesis]  $H_0: \sigma^2 = \sigma_0^2$   ( $> 0$ )

☐  $H_1: \sigma^2 \neq \sigma_0^2$  ☒  $H_1: \sigma^2 > \sigma_0^2$  ☐  $H_1: \sigma^2 < \sigma_0^2$

[Test Type]  $\chi^2$  test

Significance Level  $\alpha =$  ☒ 5% ☐ 1%

[Sample Data] *Input either sample data using BSV or sample statistics at the next boxes*

[Sample Statistics]

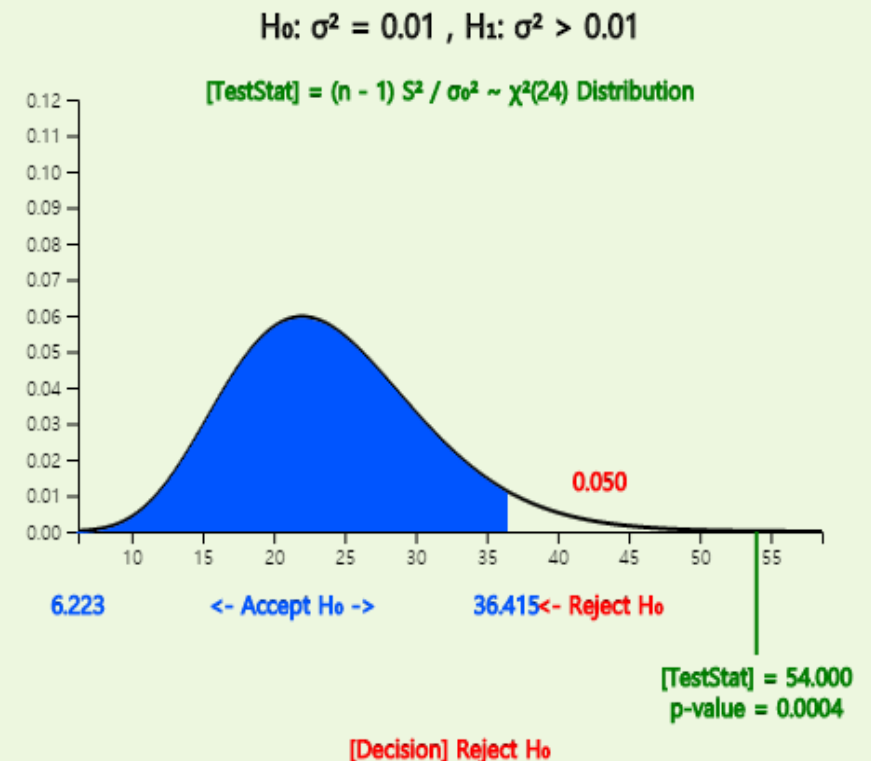
Sample Size  $n =$   ( $> 1$ )

Sample Variance  $s^2 =$   ( $> 0$ )

[Confidence Interval]

$((n-1)S^2 / \chi^2_{n-1; \alpha/2}, (n-1)S^2 / \chi^2_{n-1; 1-\alpha/2}) \Leftrightarrow$  (  ,  )

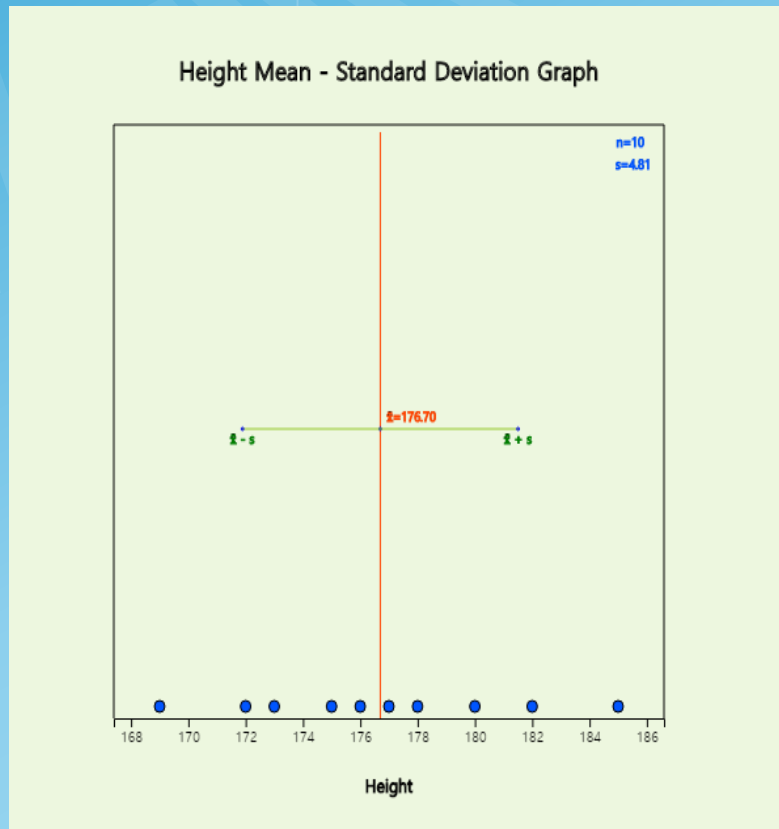
Execute



## 7.2 Testing Hypothesis for a Population Variance

[Ex 7.2.2] By using the height data of 10 male college students in [Ex 7.1.4], 172 175 178 182 176 180 169 185 173 177, test the hypothesis whether the population variance is greater than 25 at a significant level of 5%.

File	Ex714Height.csv		
Analysis Var	1: Height		
	( Selected data: Raw Data ) (No		
SelectedVar	V1		
	Height	V2	V3
1	172		
2	175		
3	178		
4	182		
5	176		
6	180		
7	169		
8	185		
9	173		
10	177		



Confidence Interval Graph   Histogram   Normal Q-Q Plot   Normality Test

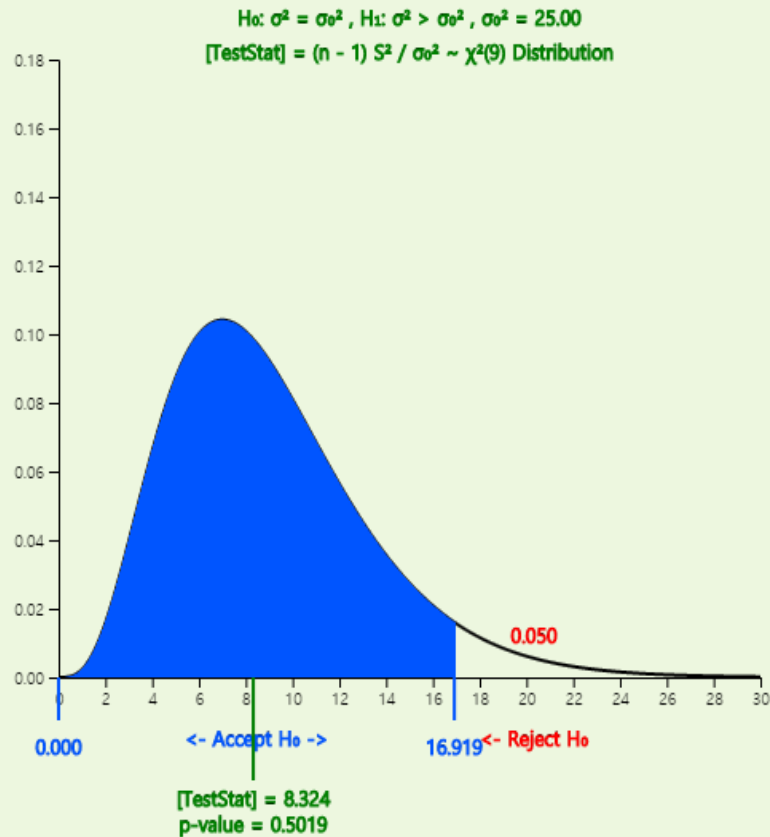
$H_0: \sigma^2 = \sigma_0^2$    25   ☐  $H_1: \sigma^2 \neq \sigma_0^2$    ☒  $H_1: \sigma^2 > \sigma_0^2$    ☐  $H_1: \sigma^2 < \sigma_0^2$

Significance Level  $\alpha =$  ☒ 5%   ☐ 1%   Confidence Level ☒ 95%   ☐ 99%

## 7.2 Testing Hypothesis for a Population Variance

### <Answer of Ex 7.2.2>

Height Testing Hypothesis: Population Variance



Testing Hypothesis: Population Variance	Analysis Var	Height			
Statistics	Observation	Mean	Std Dev	std err	Population Variance 95% Confidence Interval
	10	176.700	4.809	1.521	(10.940, 77.063)
Missing Observations	0				
Hypothesis					
$H_0 : \sigma^2 = \sigma_0^2$	$\sigma_0^2$	[TestStat]	ChiSq value	p-value	
$H_1 : \sigma^2 > \sigma_0^2$	25.00	$(n-1) S^2 / \sigma_0^2$	8.324	0.5019	





Thank you