1.0 Overview of Lectures on Robotics

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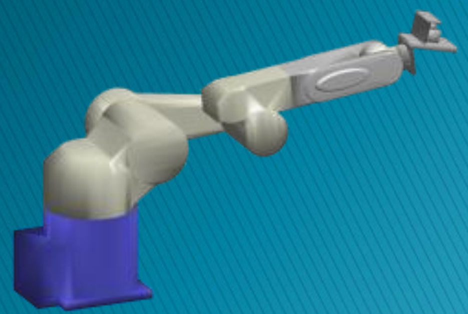


Roadmap

Rotation Matrices

$$\mathbf{R} = [\hat{\mathrm{x}} \quad \hat{\mathrm{y}} \quad \hat{\mathrm{z}}]$$

How can we describe the relative orientation between reference frames?

Roadmap

Rotation Matrices

$$\mathbf{R} = [\hat{x} \quad \hat{y} \quad \hat{z}]$$

→

Transformation Matrices

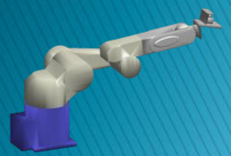
$$\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{p} \\ \mathbf{0} & 1 \end{bmatrix}$$

How can we describe the relative position **and** orientation between reference frames (pose)?

Roadmap

Rotation Matrices $\mathbf{R} = [\hat{x} \quad \hat{y} \quad \hat{z}]$ → Transformation Matrices $\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{p} \\ \mathbf{0} & 1 \end{bmatrix}$ → Forward Kinematics $\mathbf{x} = \mathbf{f}(\mathbf{q})$

Given a set of joint positions $\mathbf{q}$, what is the pose of the robot tool-tip $\mathbf{x}$?



Roadmap

Rotation Matrices

$$\mathbf{R} = [\hat{x} \quad \hat{y} \quad \hat{z}]$$

Transformation Matrices

$$\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{p} \\ \mathbf{0} & 1 \end{bmatrix}$$

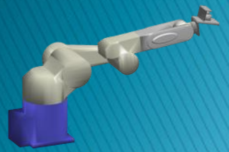
Forward Kinematics

$$\mathbf{x} = \mathbf{f}(\mathbf{q})$$

Inverse Kinematics

$$\mathbf{q} = \mathbf{f}^{-1}(\mathbf{x})$$

Given a pose of the robot tool-tip $\mathbf{x}$, what joint configurations $\mathbf{q}$ can achieve it?



Roadmap

Rotation Matrices

$$\mathbf{R} = [\hat{x} \quad \hat{y} \quad \hat{z}]$$

→

Transformation Matrices

$$\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{p} \\ \mathbf{0} & 1 \end{bmatrix}$$

→

Forward Kinematics

$$\mathbf{x} = \mathbf{f}(\mathbf{q})$$

↓

Inverse Kinematics

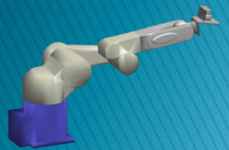
$$\mathbf{q} = \mathbf{f}^{-1}(\mathbf{x})$$

↓

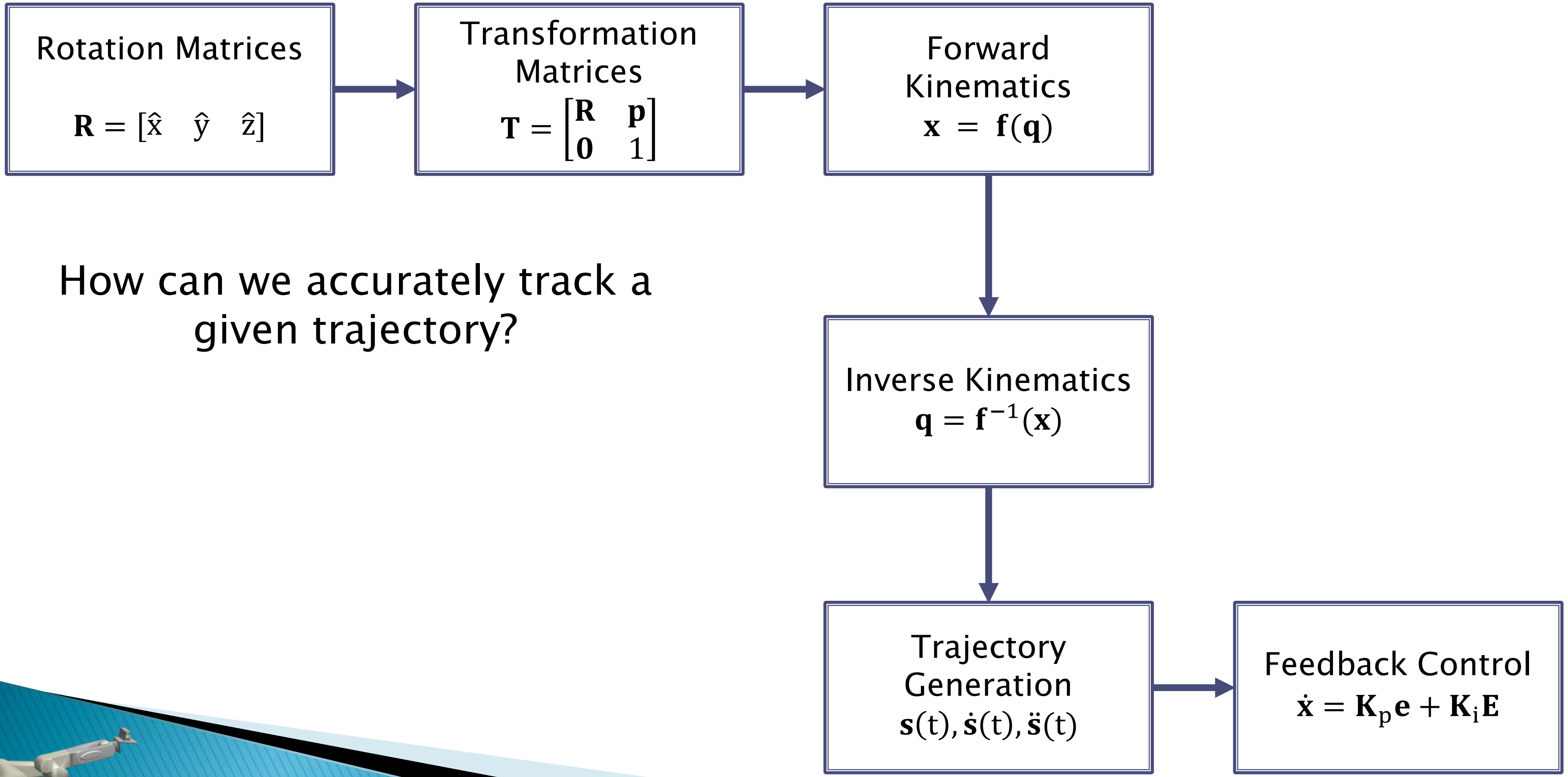
Trajectory Generation

$$\mathbf{s}(t), \dot{\mathbf{s}}(t), \ddot{\mathbf{s}}(t)$$

Given two joint configurations $\mathbf{q}_1$ and $\mathbf{q}_2$, how can we move between them?

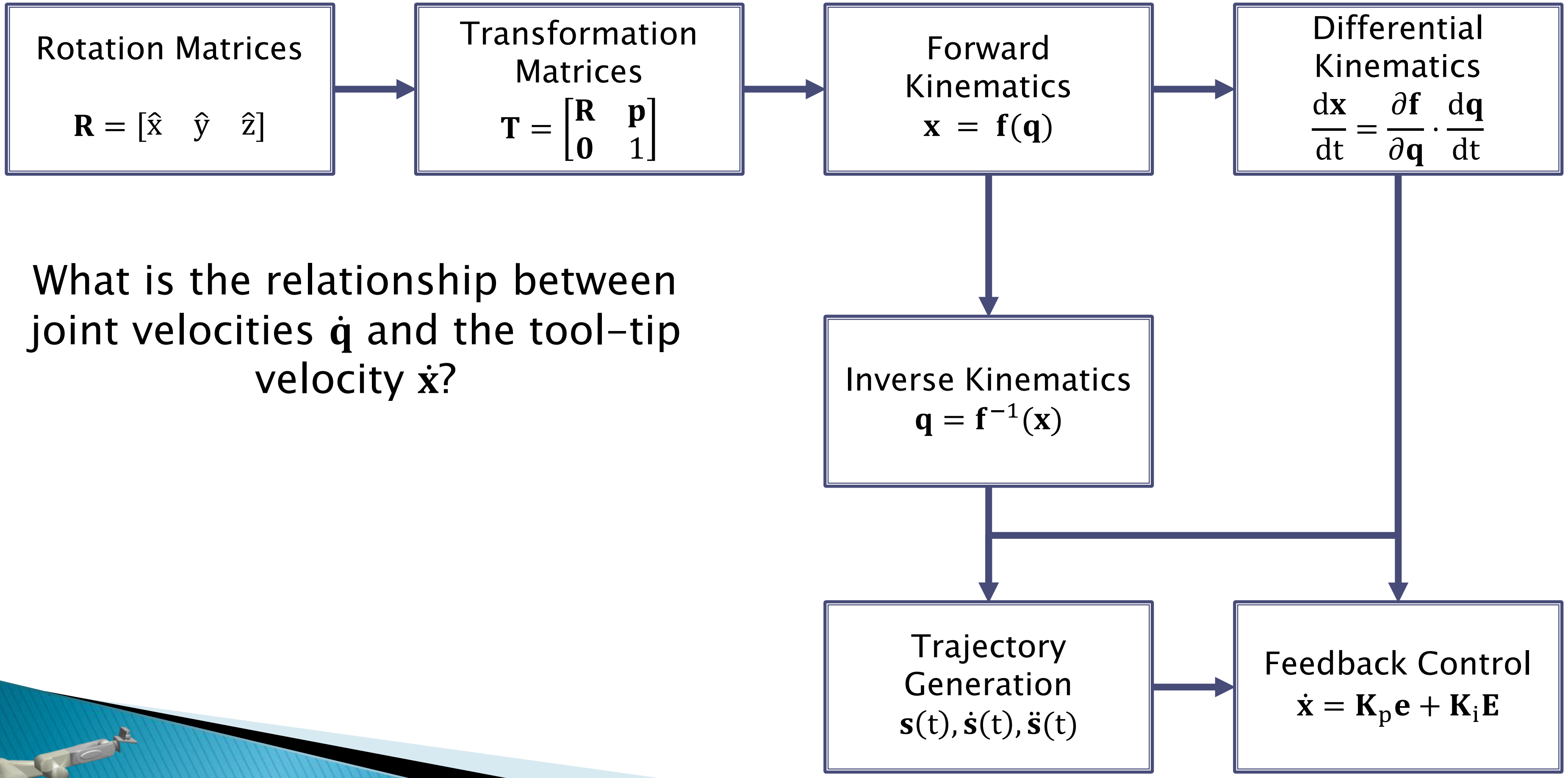


Roadmap

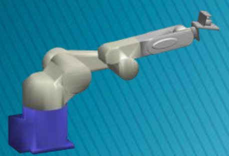


How can we accurately track a given trajectory?

Roadmap



What is the relationship between joint velocities $\dot{\mathbf{q}}$ and the tool-tip velocity $\dot{\mathbf{x}}$?



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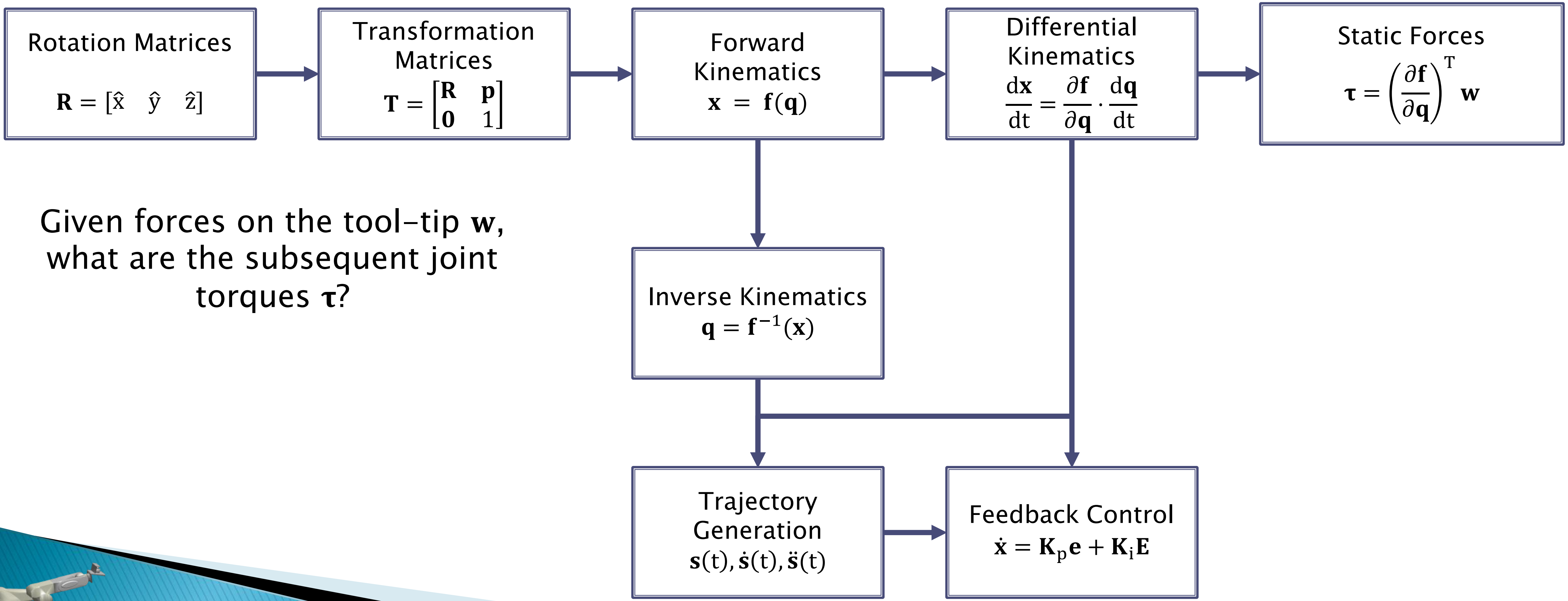
Roadmap



Given forces on the tool-tip $\mathbf{w}$, what are the subsequent joint torques $\boldsymbol{\tau}$?

Roadmap

Rotation Matrices
$\mathbf{R} = [\hat{x} \quad \hat{y} \quad \hat{z}]$

Transformation Matrices
$\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{p} \\ \mathbf{0} & 1 \end{bmatrix}$

Forward Kinematics
$\mathbf{x} = \mathbf{f}(\mathbf{q})$

Differential Kinematics
$\frac{d\mathbf{x}}{dt} = \frac{\partial \mathbf{f}}{\partial \mathbf{q}} \cdot \frac{d\mathbf{q}}{dt}$

Static Forces
$\boldsymbol{\tau} = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{q}}\right)^{T} \mathbf{w}$

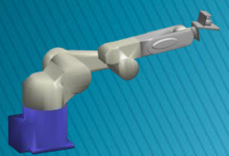
Inverse Kinematics
$\mathbf{q} = \mathbf{f}^{-1}(\mathbf{x})$

Dynamics
$\boldsymbol{\tau} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{g} + \mathbf{J}^{T}\mathbf{w}$

Trajectory Generation
$\mathbf{s}(t), \dot{\mathbf{s}}(t), \ddot{\mathbf{s}}(t)$

Feedback Control
$\dot{\mathbf{x}} = \mathbf{K}_p\mathbf{e} + \mathbf{K}_i\mathbf{E}$

What are the forces and torques governing the motion of a robot arm?



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Conventions for Vectors

- $\mathrm{x} \in \mathbb{R}$ — Scalar
- $\mathbf{x} = \begin{bmatrix} \mathrm{x}_1 \\ \vdots \\ \mathrm{x}_\mathrm{m} \end{bmatrix} \in \mathbb{R}^\mathrm{m}$ — Vector (bold font)
- $\hat{\mathrm{x}} = \mathbf{x}/||\mathbf{x}|| \in \mathbb{R}^\mathrm{m}$ — Unit vector (hat)
- $\mathbf{y}^\mathrm{T}\mathbf{x} = \sum_{\mathrm{i}=1}^{\mathrm{m}}(\mathrm{y}_\mathrm{i}\mathrm{x}_\mathrm{i}) \in \mathbb{R}$ — Sum of products (dot product)
- $\mathbf{x}^\mathrm{T}\mathbf{x} = \sum_{\mathrm{i}=1}^{\mathrm{m}}\left(\mathrm{x}_\mathrm{i}^2\right) \in \mathbb{R}$ — Sum of squares

Conventions for Matrices

- $\mathbf{X} = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \in \mathbb{R}^{m\times n}$

  Matrix (capital letter, bold font)

- $\mathbf{I} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{m\times m}$

  Identity Matrix

Vector derivatives:

- $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}) \quad \cdots \quad f_m(\mathbf{x})]^T \in \mathbb{R}^m$

  Vector function of $\mathbf{x}$

- $\mathbf{x} = [x_1 \quad \cdots \quad x_n]^T \in \mathbb{R}^n$

- $\partial\mathbf{f}/\partial\mathbf{x} = \begin{bmatrix} \partial f_1/\partial x_1 & \cdots & \partial f_1/\partial x_n \\ \vdots & \ddots & \vdots \\ \partial f_m/\partial x_1 & \cdots & \partial f_m/\partial x_n \end{bmatrix} \in \mathbb{R}^{m\times n}$

  Jacobian matrix