

Lab 6 Exercises

1 Convert range measurements to a point cloud

1.1 Consider a laser tape measure (LTM)¹ that projects a laser and measures a single distance to an object in the environment. Pretend it is affixed to the end effector of the Schunk robot.

```
robot = SchunkUTSv2_0();  
q = [0, pi/2, 0, 0, 0, 0];  
robot.plot3d(q);  
view(3);  
camlight;  
hold on;
```



1.2 Assume the LTM (max range 3m) has an identical start location as the end-effector location, and the ray is parallel to the Z axis. Plot a red line 1.9594m long from end effector parallel with the Z axis

```
tr = robot.fkine(q)  
startP = tr(1:3,4);  
endP = tr(1:3,4) + 1.9594 * tr(1:3,3);  
line1_h = plot3([startP(1),endP(1)], [startP(2),endP(2)], [startP(3),endP(3)], 'r');  
plot3(endP(1),endP(2),endP(3), 'r*');
```

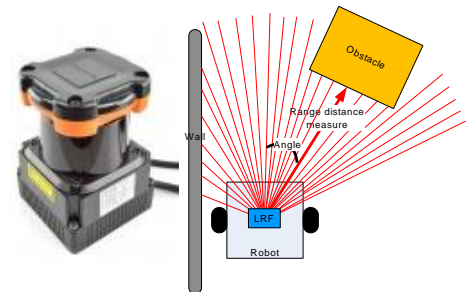
1.3 If the robot is in a pose $[0, \pi/2, 0, 0, 0, 0]$ and the laser tape measure returns a measurement (as plotted above), the robot is moved twice more and a measurement is taken. Plot this information

Robot Joint State	LTM Measurement
$[\pi/10, \pi/2, 0, 0, 0, 0]$	2.4861m
$[-\pi/10, 5\pi/12, 0, 0, 0, 0]$	1.9132m

1.4 Given now these three points, make a mesh of the possible location of a wall (only use value of the wall which are above 0m). Remember, where the triangle vertices are $[v1, v2, v3]$

```
triangleNormal = unit(cross((v1-v2), (v2-v3)))
```

1.5 Consider mounting a motor to rotate (roll) the LTM 40° around the X axis of the end effector, and look at the wall. Put the robot in a pose $[0, \pi/2, 0, 0, 0, 0]$ and rotate the LTM by increments of 1° from -20° to 20° (total of 41 readings). Essentially this is a Laser Range Finder (LRF)².



1.6 Consider mounting a second motor to rotate the LTM around both the X and the Y axis from -20° to 20° in each (i.e. 41 readings in each row and $41 \times 41 = 1681$ readings all together). Note the Field of View is 40° by 40°. Essentially this is a tilting LRF or 3D LiDAR³ used in many driverless cars. Use it to look at the wall you found.



2 More complex collision detection for 3-link planar robot

The textbook (Appendix E) talked about ellipses. Now, we will create a 3D ellipse called an ellipsoid. Note that the equations of this ellipsoid is

$$\left(\frac{x-x_c}{r_x}\right)^2 + \left(\frac{y-y_c}{r_y}\right)^2 + \left(\frac{z-z_c}{r_z}\right)^2 = 1$$

2.1 Create vertices that represent an ellipsoid with radii $(r_x=3, r_y=2, r_z=1)$ centered at $[x_c, y_c, z_c] = [0, 0, 0]$.

```
centerPoint = [0, 0, 0];  
radii = [3, 2, 1];  
[X, Y, Z] = ellipsoid(centerPoint(1), centerPoint(2), centerPoint(3), radii(1), radii(2), radii(3));
```

¹ Stanley TLM 100 FatMax Tru-Laser Distance Measurer 0020 <http://kk.org/cooltools/trulaser-distan/>

² Hokuyo UTM-30LX https://www.hokuyo-aut.jp/02sensor/07scanner/utm_30lx.html

³ Velodyne LiDAR <http://velodynelidar.com/news.php>

2.2 Now plot it

```
surf(X,Y,Z)
```

- 2.3 Put a cube with sides 1.5m in the environment that is centered at [2,0,-0.5]. Use mesh so as to create a high density mesh that has many vertices (either create in blender and load, or use 6 planes from `meshgrid`).

Note: create a single plane of a cube centered at the origin like follows:

```
[Y,Z] = meshgrid(-0.75:0.05:0.75,-0.75:0.05:0.75);  
X = repmat(0.75,size(Y,1),size(Y,2));
```

- 2.4 Check how many point and which points are inside the ellipsoid, using the equation. Note that points that are inside have an algebraic distance (AD) < 1, on the surface AD = 1 and outside AD > 1
- 2.5 Transform the ellipsoid by translating it [1,1,1], do this by changing the values of [xc,yc,zx], then check which points are inside the ellipsoid
- 2.6 This time, using the original centered-at-the-origin ellipse, notice how you can transform the points in the environment by `inv(transl(1,1,1))` and then check the original equation to see which have an algebraic distance less than 0. The points inside should be the same as when using the previous method.
- 2.7 Now, if the ellipsoid where transformed by `transl(1,1,1)*trotx(pi/4)`, which points are inside (note that you will need to transform the points in the environment instead of the ellipsoid formula
- 2.8 Now create a 3 link planar and use the 3 ellipsoids as the model points and faces. Now use teach to move it around so you should see the ellipsoids move around as well
- 2.9 **(Bonus)** For a given pose, work out the location of the ellipsoid of the end effector, using `fkine`, and the multiply the points in the environment by the inverse of this transform, and check the algebraic distance
- 2.10 **(Bonus)** Do this for each of the ellipsoids on the three links. Note: you will need to have your own forward kinematics routine so you can compute the location of each of the ellipsoids

3 Joint Interpolation vs Resolve Motion Rate Control

- 3.1 Moving from A to B with Joint Interpolation: Load a 2-Link Planar Robot with `mdl_planar2`;

3.2 Create two Transformation Matrices:

$$T_1 = \begin{bmatrix} I_3 & 1.5 \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} I_3 & 1.5 \\ \mathbf{0}_{1 \times 3} & -1 \end{bmatrix}$$

- 3.3 Use Inverse Kinematics to solve the joint angles required to achieve each pose.

```
M = [1 1 zeros(1,4)]; % Masking Matrix  
q1 = p2.ikine(T1,[0 0],M); % Solve for joint angles  
q2 = p2.ikine(T2,[0 0],M); % Solve for joint angles
```

- 3.4 Use joint interpolation to move between the two poses. Be sure to plot the end-effector path.

```
p2.plot(qMatrix,'trail','r-');
```

- 3.5 Moving from A to B with Resolved Motion Rate Control

- 3.6 Create two sets of points in the X-Y plane

```
x1 = [1.5 1]';  
x2 = [1.5 -1]';  
deltaT = ... % Discrete time step
```

- 3.7 Create a matrix of waypoints

```
x = zeros(2,steps); % Assign memory
```

```

s = linspace(0,1,steps); % Create interpolation scalar
for i = 1:steps
    x(:,i) = x1*(1-s(i)) + s(i)*x2; % Interpolate waypoints
end

```

3.8 Create a matrix of joint angles

```
qMatrix = nan(steps,2);
```

3.9 Set the Transformation for the 1st point, and solve for the joint angles $T_1 = \begin{bmatrix} I_3 & 1.5 \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}$

```
qMatrix(1,:) = p2.ikine(T1,[0 0],M);
```

3.10 Use Resolved Motion Rate Control to move the end-effector from x_1 to x_2 .

```

for i = 1:steps-1
    xdot = ... % Velocity to reach next waypoint
    J = p2.jacob0(qMatrix(i,:)); % Get Jacobian at current state
    J = J(1:2,:); % Take only first 2 rows
    qdot = ... % Solve the RMRC equation
    qMatrix(i+1,:)= ... % Update the joint state
end

```