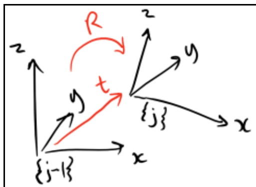
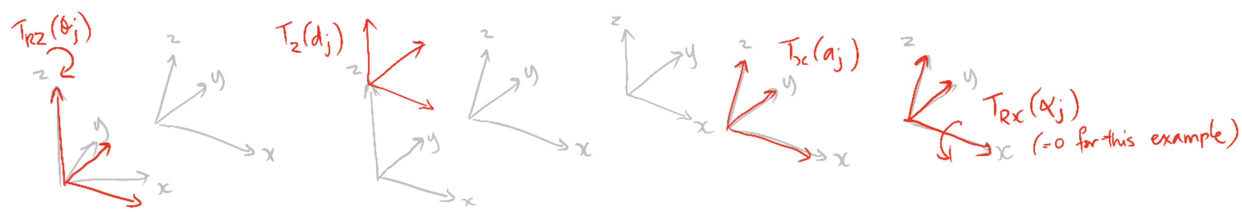
DH Parameters Quick Reference Guide

Detailed explanations of DH Parameters are in the textbook and the online lecture. This reference guide is just a brief reminder.

DH Parameters are an efficient way of calculating the forward transformation matrix between joints on a robotic manipulator. From these, we can express the relative position and orientation between the joint reference frames. DH Parameters also provide a concise description of the manipulator geometry that is universally understood.

<table>
<tr><td>

The transformation matrix between two consecutive reference frames *j-1* → j is given by:</td><td colspan="3">$$\boldsymbol{T}_{j-1}^{j} = \begin{bmatrix} \boldsymbol{R}_{j-1}^{j} & \boldsymbol{t}_{j-1}^{j} \\ \boldsymbol{0}_{1\times3} & 1 \end{bmatrix}$$</td></tr>
<tr><td>Where $\boldsymbol{R}_{j-1}^{j}$ is the relative rotation matrix from *j-1* to *j*, and $\boldsymbol{t}_{j-1}^{j}$ is the translation vector in Euclidean space from *j-1* to *j*. This can be partitioned in to functions of the 4 DH Parameters:</td><td colspan="3">$$\boldsymbol{T}_{j-1}^{j} = \boldsymbol{T}_{Rz}(\theta_j)\boldsymbol{T}_z(d_j)\boldsymbol{T}_x(a_j)\boldsymbol{T}_{Rx}(\alpha_j)$$</td></tr>
<tr><td colspan="4">
</td></tr>
<tr><td>We can then concatenate multiple consecutive joints to express the end-effector relative to the base of the manipulator as a product of all transformations:</td><td colspan="3">$$\boldsymbol{T}_0^n = \prod_{j=1}^{n} \boldsymbol{T}_{Rz}(\theta_j)\boldsymbol{T}_z(d_j)\boldsymbol{T}_x(a_j)\boldsymbol{T}_{Rx}(\alpha_j)$$</td></tr>
<tr><td colspan="4">Where $\boldsymbol{I}_3$ is the 3×3 identity matrix, each of the individual transformations is given by:</td></tr>
<tr><td>$$\boldsymbol{T}_{Rz}(\theta_j) = \begin{bmatrix} \cos(\theta_j) & -\sin(\theta_j) & 0 & \\ \sin(\theta_j) & \cos(\theta_j) & 0 & \boldsymbol{0}_{3\times1} \\ 0 & 0 & 1 & \\ & \boldsymbol{0}_{1\times3} & & 1 \end{bmatrix}$$</td><td>$$\boldsymbol{T}_z(d_j) = \begin{bmatrix} & 0 \\ \boldsymbol{I}_3 & 0 \\ & d_j \\ \boldsymbol{0}_{1\times3} & 1 \end{bmatrix}$$</td><td>$$\boldsymbol{T}_x(a_j) = \begin{bmatrix} & a_j \\ \boldsymbol{I}_3 & 0 \\ & 0 \\ \boldsymbol{0}_{1\times3} & 1 \end{bmatrix}$$</td><td>$$\boldsymbol{T}_{Rx}(\alpha_j) = \begin{bmatrix} 1 & 0 & 0 & \\ 0 & \cos(\alpha_j) & -\sin(\alpha_j) & \boldsymbol{0}_{3\times1} \\ 0 & \sin(\alpha_j) & \cos(\alpha_j) & \\ & \boldsymbol{0}_{1\times3} & & 1 \end{bmatrix}$$</td></tr>
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