

# Sensors and Control for Mechatronics Systems

## Tutorial 6

### Question 1 : The state space model

Consider the state space model for  $A = \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ . Using hand calculation;

1.1 : Compute the eigenvalues of A. Is the system stable?

1.2 : Check the system controllability.

1.3 : Perform pole placement to place the poles at -3, -4.

### Question 2 : Mass-spring-damper system

- Objective: Control the position  $x$
- Input: Force  $F$
- Output: position  $x$

Oscillatory force from spring

$$F_s = -kx$$

where  $k$  is the spring constant.

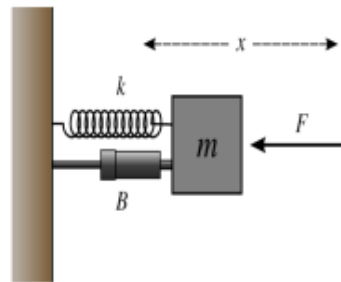
Damping force

$$F_d = -cv = -c \frac{dx}{dt} = -c\dot{x}$$

where  $c$  is damping coefficient.

Applying Newton's second law,

$$m\ddot{x} + c\dot{x} + kx = F. \quad \longrightarrow \quad \ddot{x} = -\frac{c}{m}\dot{x} - \frac{k}{m}x + \frac{1}{m}F.$$



2.1 : Define the state vector for the following system

2.2 : Define matrices A, B, C, D for the state space model

2.3 : For  $m = 1000$  kg,  $k = 50$  kgs-2, and  $c = 20$  kgs-1, use the Matlab function **ss** to create a state-space model of the open loop system.

Ref. : <https://au.mathworks.com/help/control/ref/ss.html>

2.4 : Plot the step response of the open loop system using **step** function.

<https://au.mathworks.com/help/control/ref/step.html>

2.5 : Use poles  $[-0.1 \ -0.3]$  and compute the gain matrix  $K$  using the Matlab function **place**, to stabilise the system using pole placement.

Ref. : <https://au.mathworks.com/help/control/ref/place.html>

2.6 : Using the state feedback control law  $u(t) = -Kx(t)$ , create the state-space model for the closed loop system.

2.7 : Use the **dcgain** function to scale the system appropriately.

### Question 3 : LQR control

3.1 : Consider a rotating mass in a frictionless space with the following system equations, design the state feedback control  $u(t) = -Kx(t)$  such that the performance index

$$J = \int_0^{\infty} (x^T Q x + u^T R U) dt \text{ is minimized.}$$

$$A = [0 \ 1; 0.01 \ 0]$$

$$B = [0; 1]$$

$$C = [1 \ 0];$$

$$D = 0;$$

$$Q = [1 \ 0; 0 \ 1]$$

$$R = 1$$

Download the helper code and complete it to observe the effect Q and R has on the system when starting from the initial conditions  $x_0 = [3; 0]$  to bring the system to equilibrium state  $x = [0; 0]$

For this purpose use Matlab function ***lqr*** to calculate the optimal gain matrix ***K*** and use this gain matrix K to create a closed loop system

Ref. : <https://au.mathworks.com/help/control/ref/lqr.html>

### Question 4 : Inverted Pendulum

4.1 : Simulate the state-space model of the inverted pendulum system (*refer to the notes provided for the model*). Use the following values for M, m, l and g.

$$m=0.1;$$

$$M=2;$$

$$l=0.5;$$

$$g=9.81;$$

4.2 : Check the system stability.

4.3 : Check the system controllability.

4.4 : Perform pole placement to place the poles to be  $[-1 \ -2 \ -3 \ -4]$ .

4.5 : Design LQR control using  $Q = I_4$  (Identity matrix of dimension 4) and  $R = 1$ .

4.6 : Compute and plot the state and control inputs for the arbitrarily placed poles and the optimal control solution.