

State space model from third order differential equation

$$\ddot{x} = 2\ddot{x} + 3\dot{x} + 4x$$

Define $X = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}$ $\begin{matrix} x_1 = x \\ x_2 = \dot{x} \\ x_3 = \ddot{x} \end{matrix}$

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dddot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}$$

$$\dot{x}_1 = \dot{x} = x_2$$

$$\dot{x}_2 = \ddot{x} = x_3$$

$$\begin{aligned} \dot{x}_3 = \dddot{x} &= 2\ddot{x} + 3\dot{x} + 4x \\ &= 4x_1 + 3x_2 + 2x_3 \end{aligned}$$

So we have $\begin{aligned} \dot{x}_1 &= 0x_1 + 1x_2 + 0x_3 \\ \dot{x}_2 &= 0x_1 + 0x_2 + 1x_3 \\ \dot{x}_3 &= 4x_1 + 3x_2 + 2x_3 \end{aligned}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\dot{X} = AX$$

where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 3 & 2 \end{bmatrix}$