

## 41014 Sensors and Control for Mechatronic Systems

### Lecture-8: Visual Servoing

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## ❖ Discussion

- How to control the Fetch robot to pick up a box?



## ❖ Activity-1:

- How to control the Fetch robot to pick up a box?



## ❖ Activity-1:

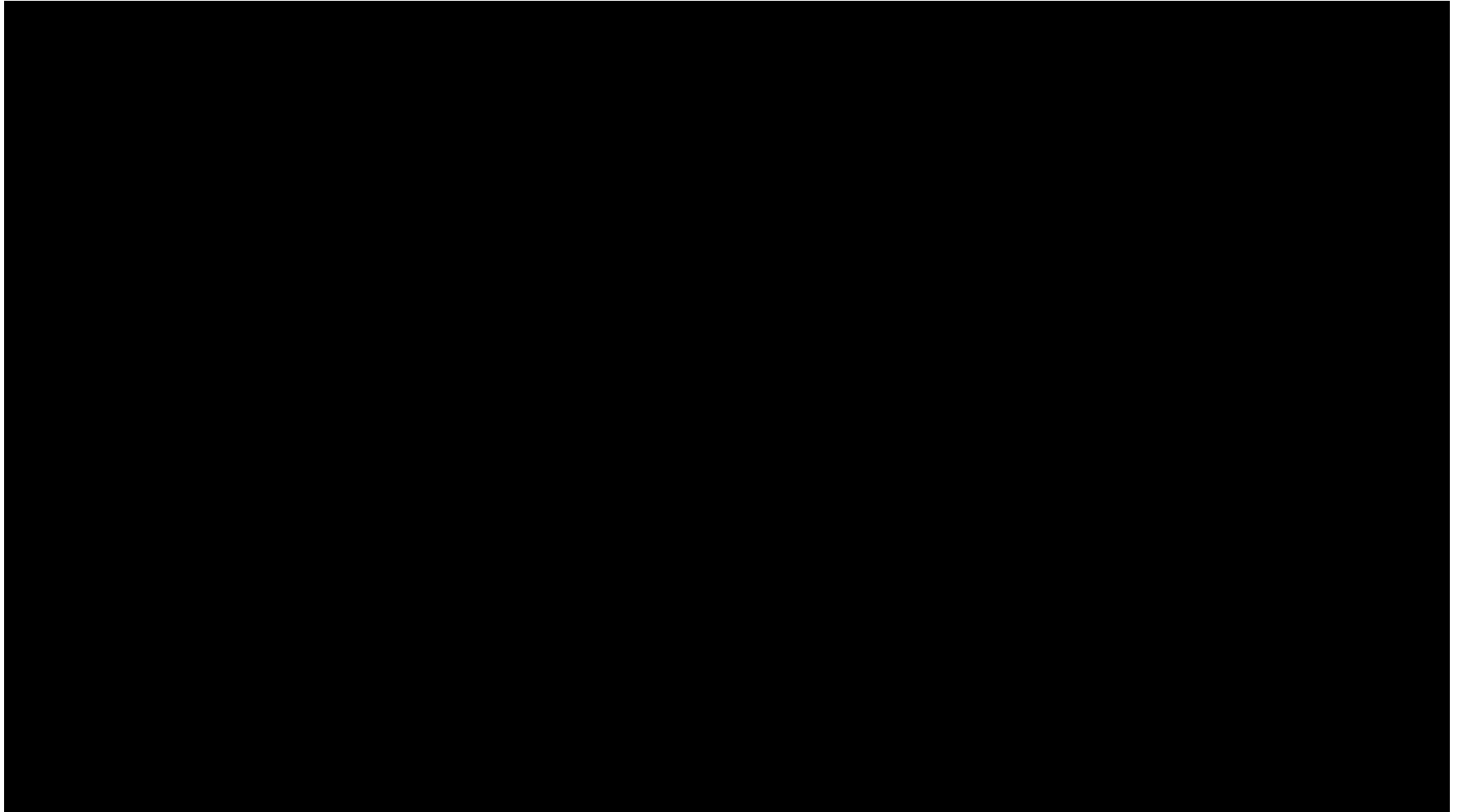
- How to control the Fetch robot to pick up a box?

## ❖ Method

- Other information
- Camera
- Detect the box
- Hand: where to go
- .....
- Error?



## Robotic Arm: Visual Servoing (Georgia Tech)



<https://www.youtube.com/watch?v=nLq9xbTuBpl>

## Ultrasound-guided robotic steering of a needle

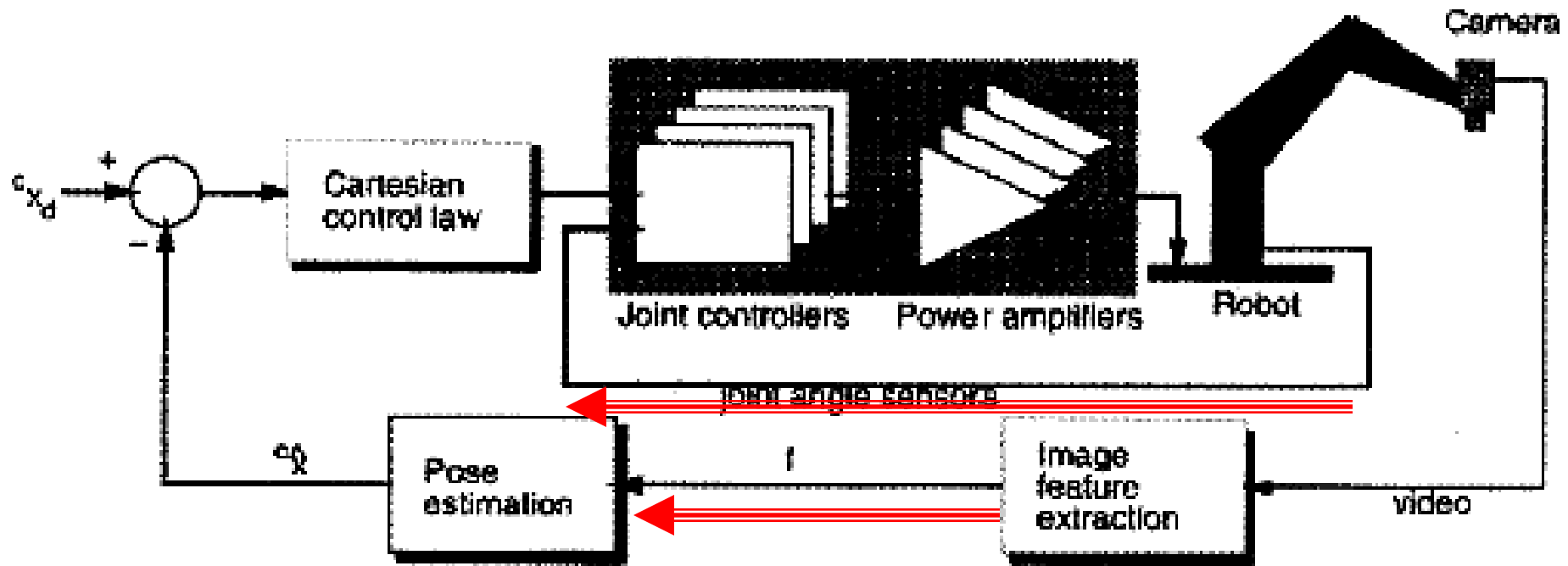
**3D ultrasound-guided robotic steering  
of a flexible needle via visual servoing**

**Pierre Chatelain  
Alexandre Krupa  
Nassir Navab**

<https://www.youtube.com/watch?v=8lyknL44n5s>

❖ What is **Visual Servoing**?

- Vision System operates in a **closed control loop**.
- Better Accuracy than “Look and Move” systems



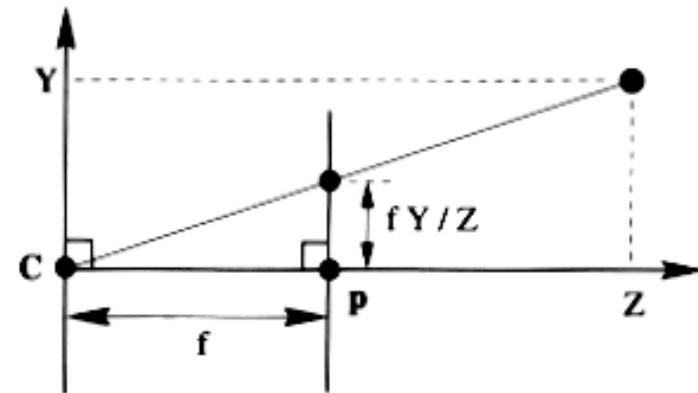
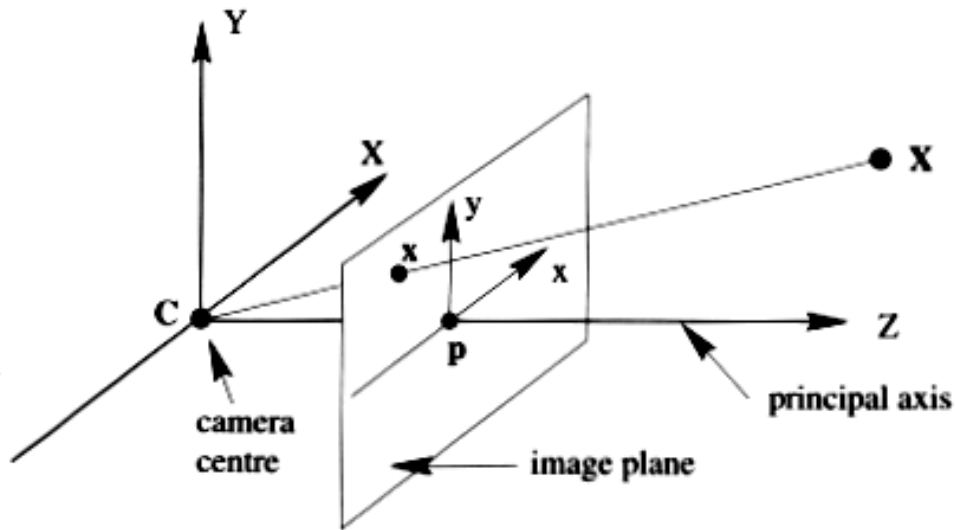
Figures from S.Hutchinson: A Tutorial on Visual Servo Control



## Vision Sensors

- Single Perspective Camera
- Multiple Perspective Cameras (e.g. Stereo Camera Pair)
- Laser Scanner
- Omnidirectional Camera
- Structured Light Sensor

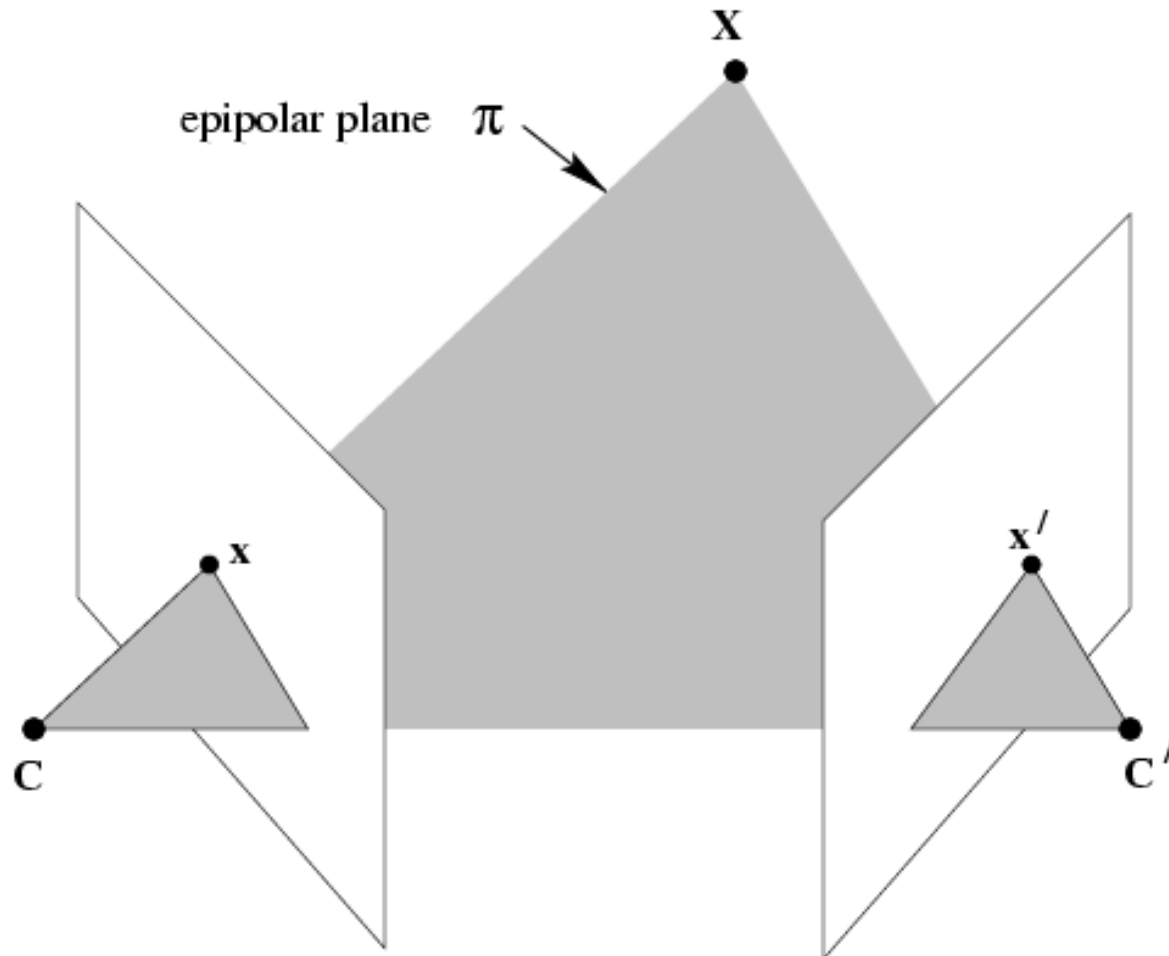
## Single Perspective Camera



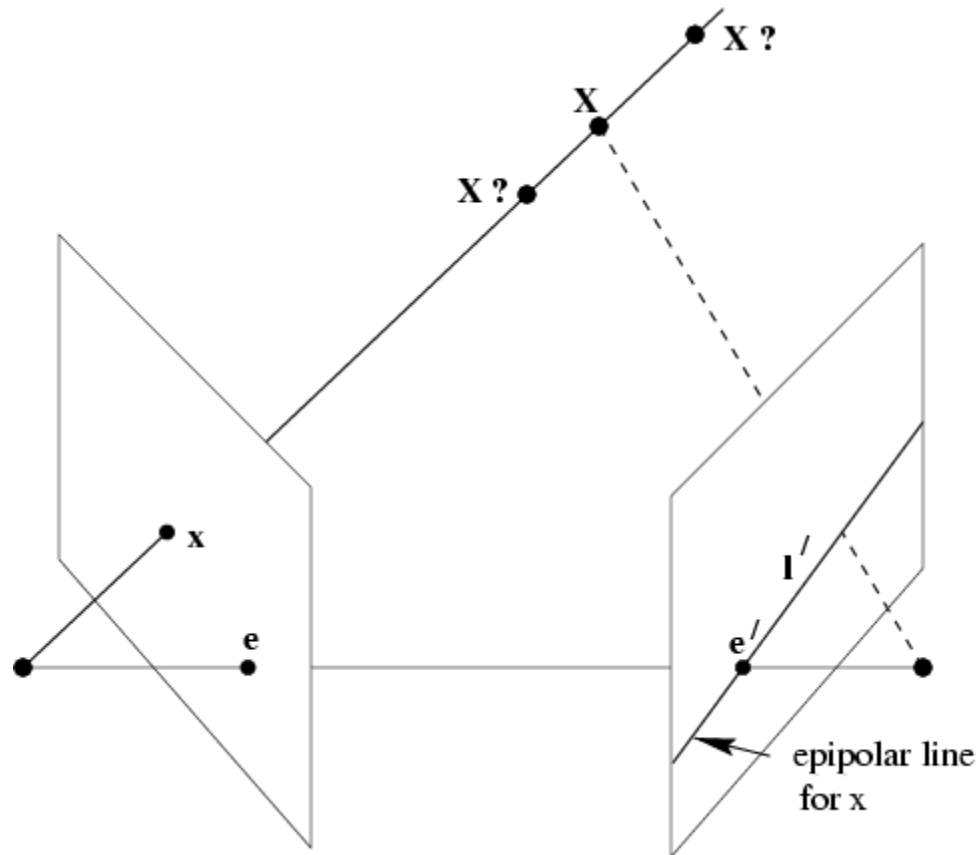
$$x = P_{3 \times 4} X$$

Single  
projection

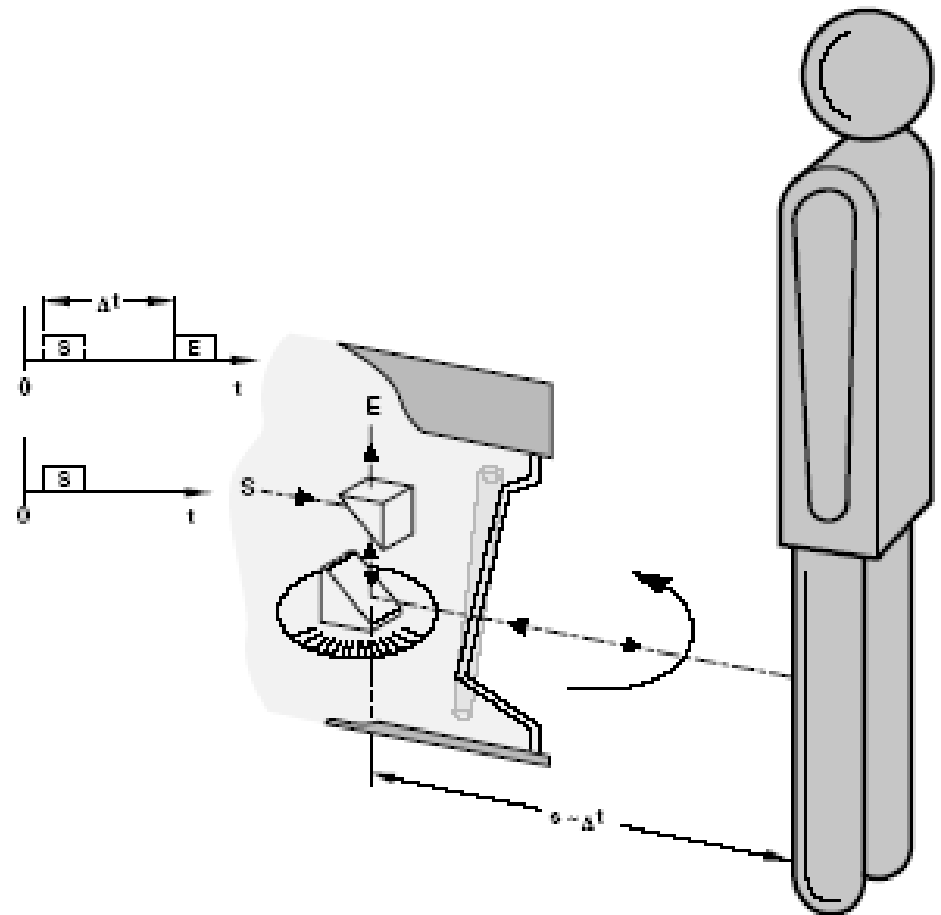
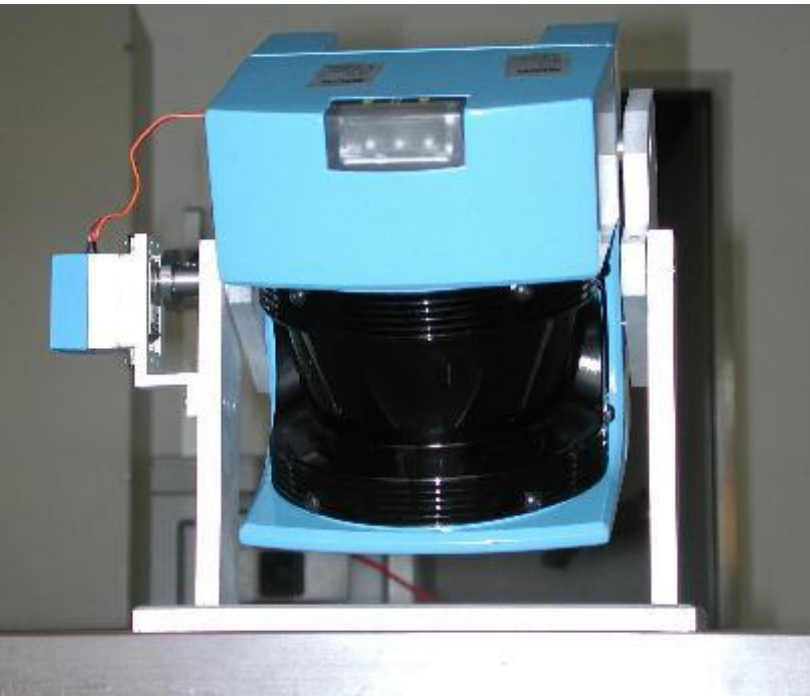
- **Multiple Perspective Cameras** (e.g. Stereo Camera Pair)



- Multiple Perspective Cameras (e.g. Stereo Camera Pair)



- Laser Scanner



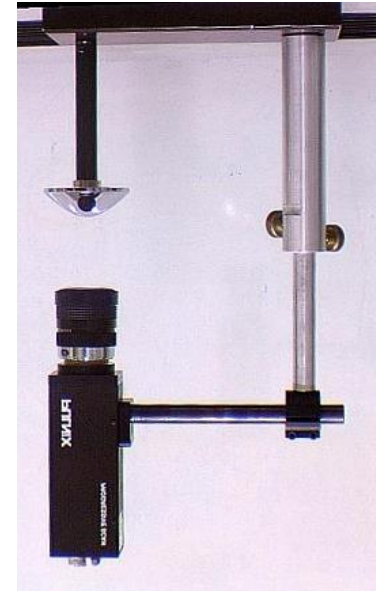
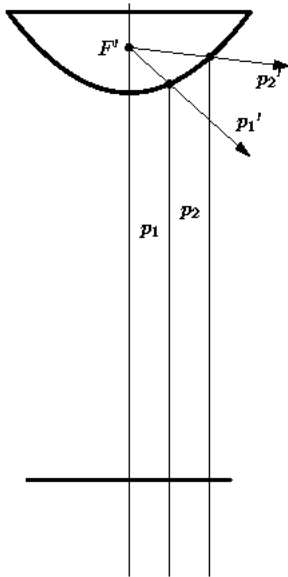
- Laser Scanner



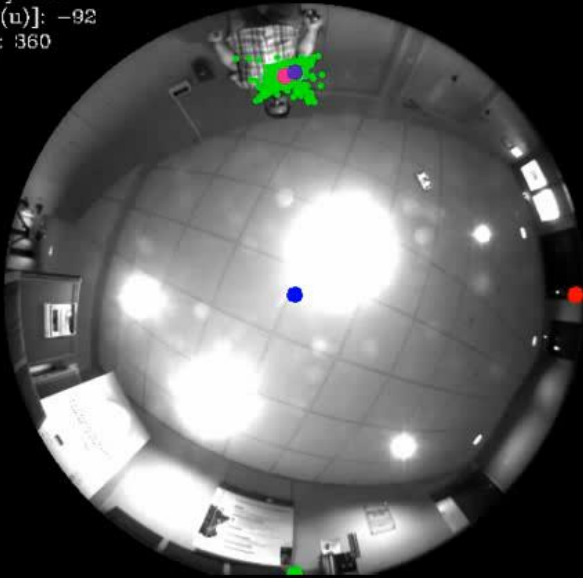
vrvis



- Omnidirectional Camera



```
[#keypts]: 666  
[#est az(u)]: -92  
[#est k]: 360
```

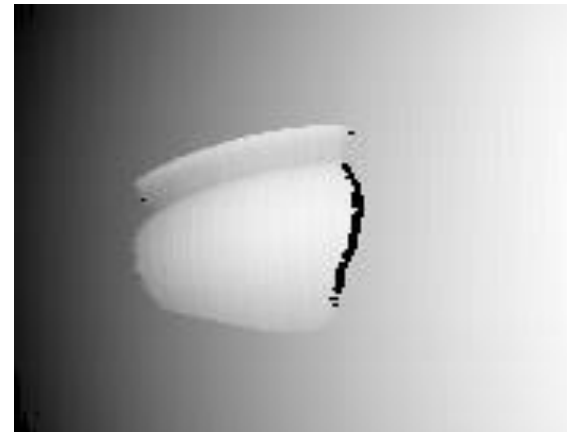
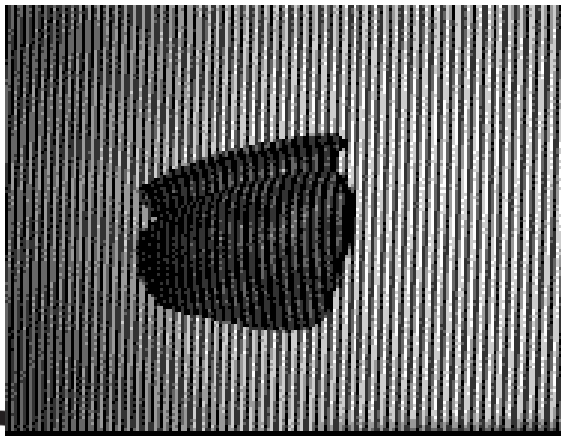
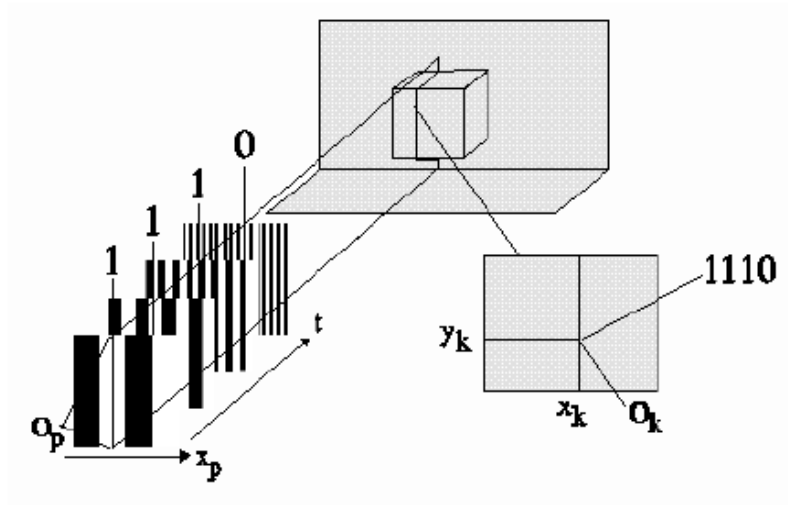


<https://www.youtube.com/watch?v=ndccTfSh9JY>

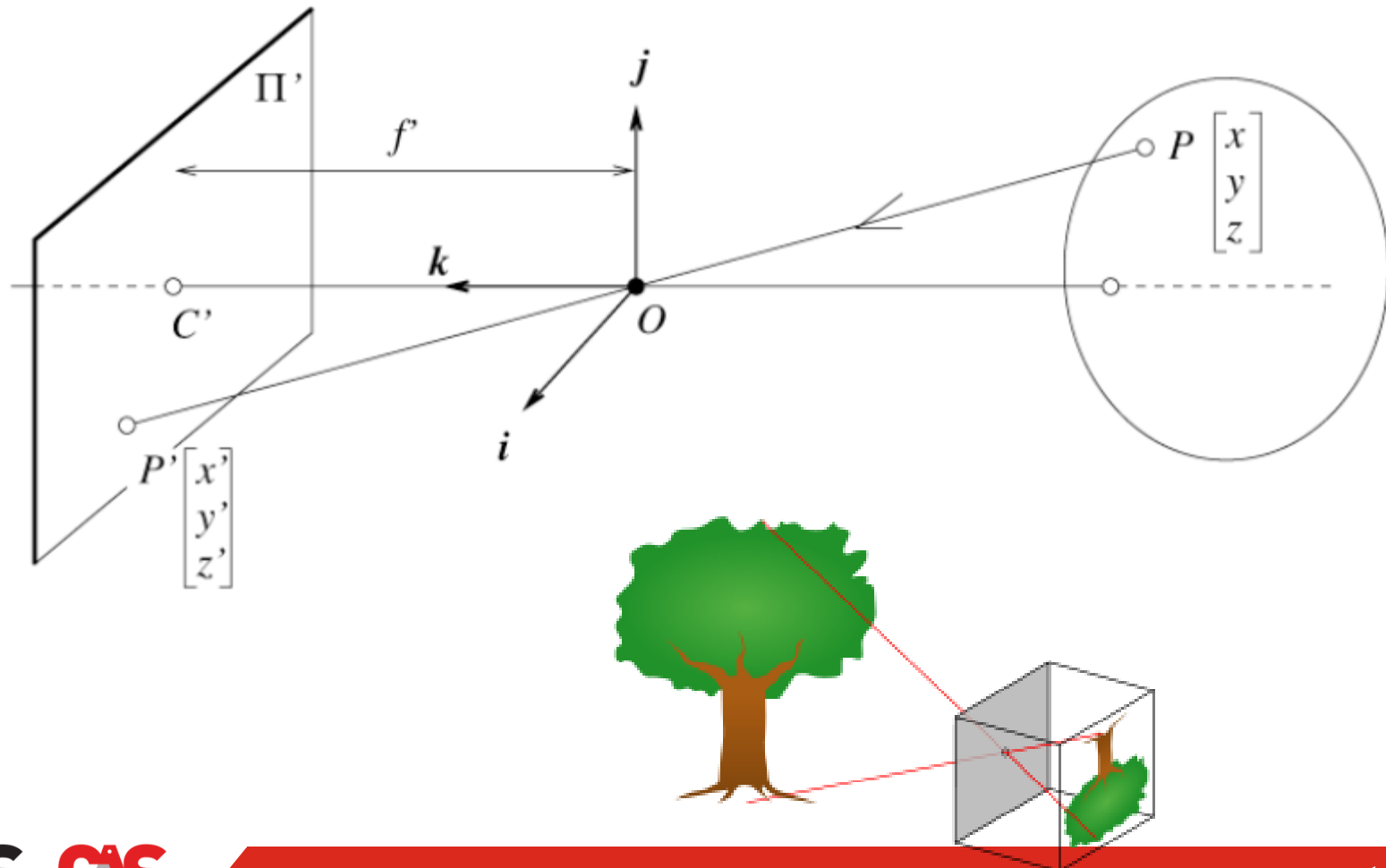
- Omnidirectional Camera



- Structured Light Sensor



- ❖ Single View Geometry
- ❖ Pinhole model



# Central projection with principle point offset

$$\begin{bmatrix} fx + zp_x \\ fy + zp_y \\ z \end{bmatrix} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$[p_x, p_y]$  is the coordinates of the principle points in image plane

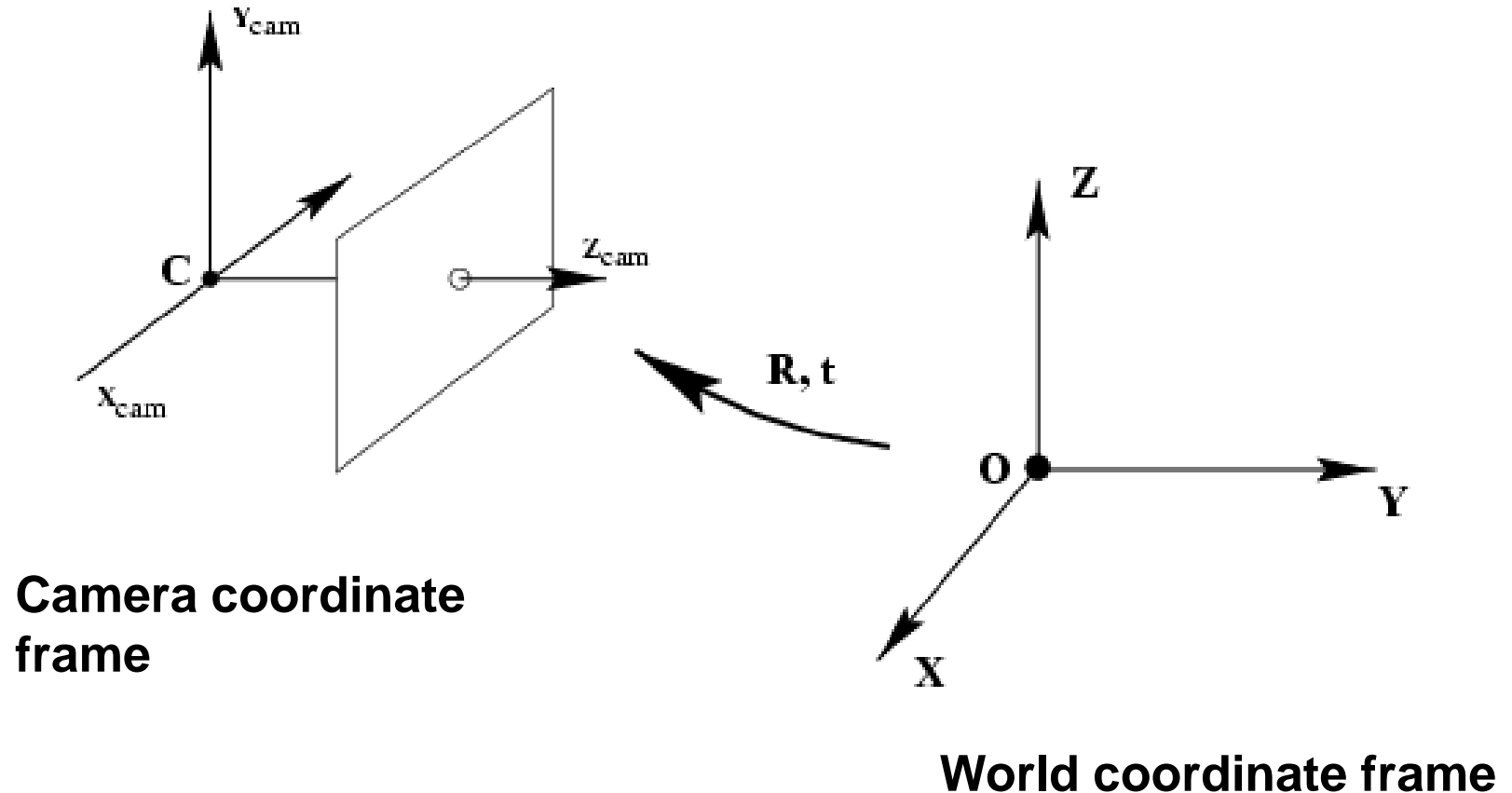
$$\mathbf{x} = \mathbf{K}[\mathbf{I}:0]\mathbf{X}_{\text{cam}}$$

homogeneous

$$\mathbf{K} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

**K** is camera calibration matrix; **X<sub>cam</sub>** is in camera coordinate frame

# Camera rotation and translation



**Camera is on a moving vehicle. Object is in a global reference frame.**

# Combine Projection and Transformation

$$\overline{X}_{\text{cam}} = R(\overline{X} - \overline{C}) \quad \text{inhomogeneous}$$

From world coordinate frame to camera coordinate frame

Matrix?

homogeneous

# Combine Projection and Transformation

$$\overline{X}_{cam} = R(\overline{X} - \overline{C}) \quad \text{inhomogeneous}$$

**From world coordinate frame to camera coordinate frame**

$$X_{cam} = \begin{bmatrix} R & -R\overline{C} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} R & -R\overline{C} \\ 0 & 1 \end{bmatrix} X \quad \text{homogeneous}$$

# Combine Projection and Transformation

- Step 0: Points are expressed in some coordinate system that is not the cameras (e.g. a model or a robot):

$$p = [x, y, z, 1]'$$

- Step 1: Transform the points into camera coordinates

$$q = [x', y', z', 1]' = T p$$

- Step 2: Project the points

$$u = f x'/z' ; v = f y'/z'$$

# Combine Projection and Transformation

$$\begin{aligned} \mathbf{x} &= \mathbf{K}[\mathbf{I}:\mathbf{0}]\mathbf{X}_{\text{cam}} \\ &= \mathbf{K}[\mathbf{I}:\mathbf{0}]\begin{bmatrix} \mathbf{R} & -\mathbf{R}\bar{\mathbf{C}} \\ \mathbf{0} & 1 \end{bmatrix}\mathbf{X} \end{aligned}$$

$$\begin{aligned} \mathbf{x} &= \mathbf{K}\mathbf{R}[\mathbf{I}:-\bar{\mathbf{C}}]\mathbf{X} \\ &= \mathbf{P}\mathbf{X} \end{aligned}$$

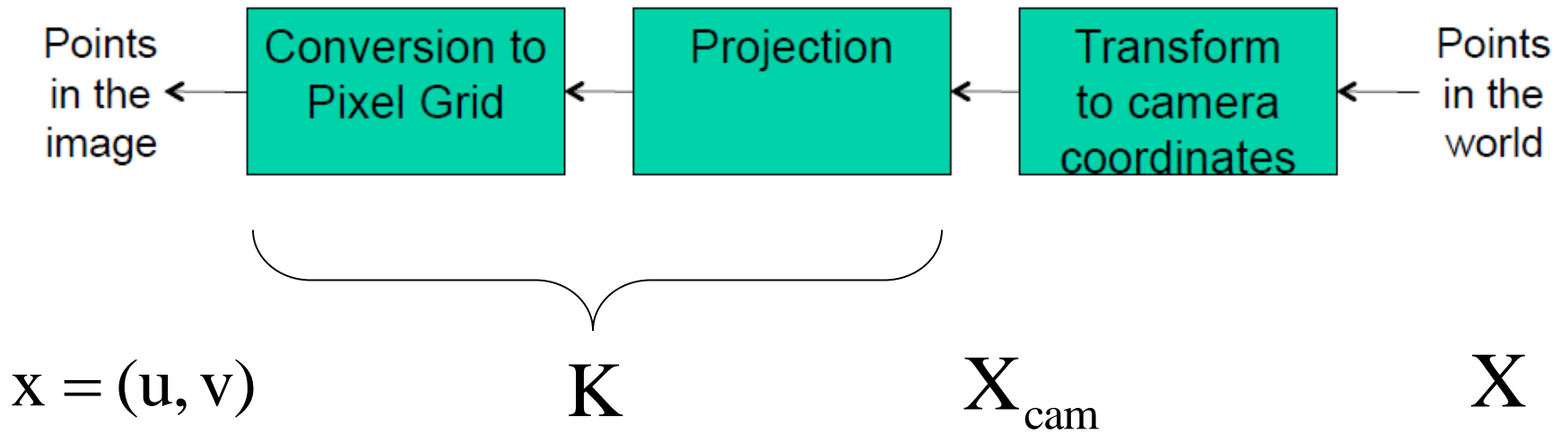
$$\mathbf{P} = \mathbf{K}\mathbf{R}[\mathbf{I}:-\bar{\mathbf{C}}]$$

**P** is camera projection  
matrix

**K:** camera intrinsic parameters; **R,C:** camera  
extrinsic parameters



# The Projection “Chain”



# Camera calibration matrix

$$K = \begin{bmatrix} f_x & 0 & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

**Non-square pixels**

$$K = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

**Square pixels**

$\alpha_x$  and  $\alpha_y$  represent the focal length in terms of pixel dimensions in x and y direction respectively in the image plane

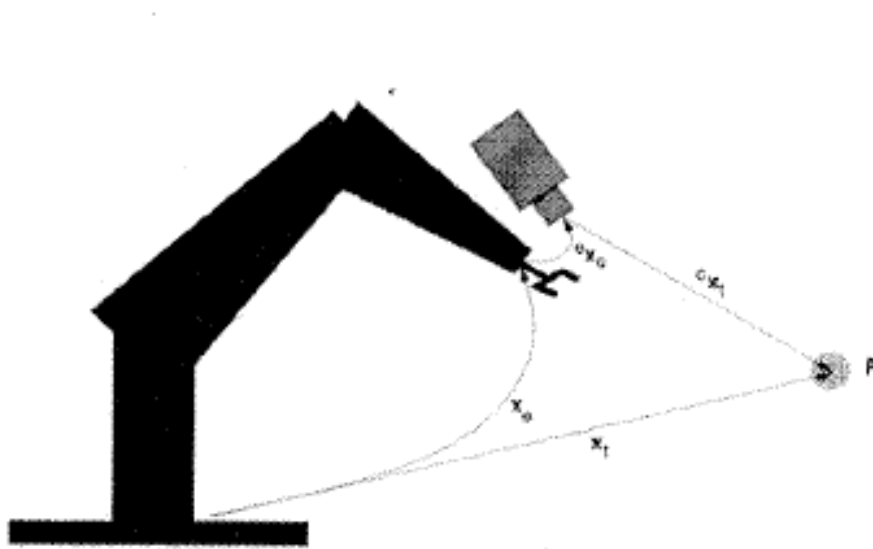
## ❖ Activity-2:

- Camera:
  - Image:  $800 \times 800$
  - Principle point: center
  - Focal length:  $(400, 400)$
  - Pose:  $r = (0, 0, 0) \Rightarrow R = I, T = [10, 20, 2]$
- 3D Point  $(25, 50, 80)$
- Image point  $(u, v)$ ?

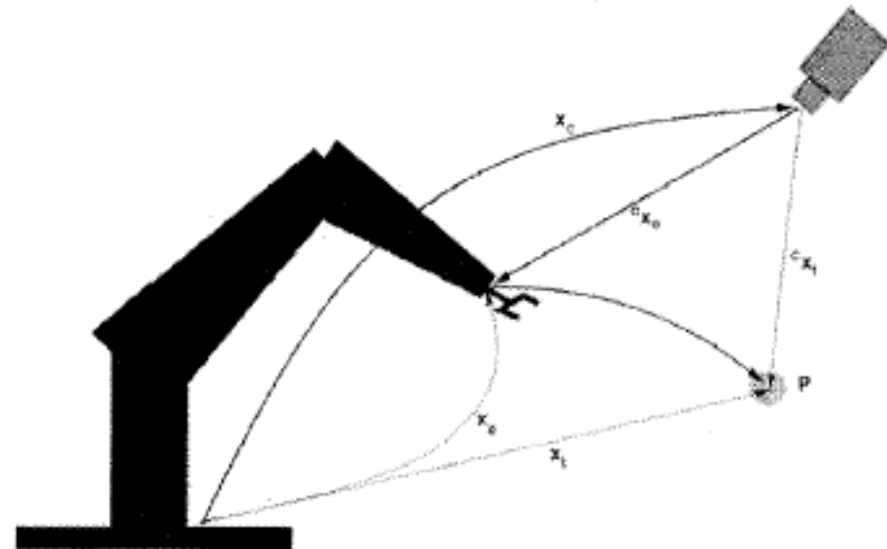
## ❖ Activity-2:

- Camera:
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  - Focal length:  $(400, 400)$
  - Pose:  $r = (0, 0, 0) \Rightarrow R = I, T = [10, 20, 2]$
- 3D Point  $(25, 50, 80)$
- Image point  $(u, v)$ ?
- **$(476.9231, 553.8462)$**

# Camera Configurations for Visual Servoing



End-Effector  
Mounted

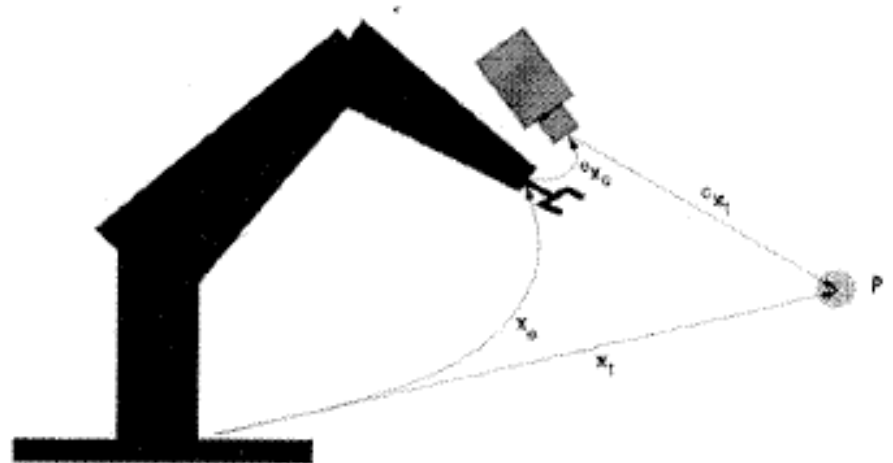


Fixed

Figures from S.Hutchinson: A Tutorial on Visual Servo Control

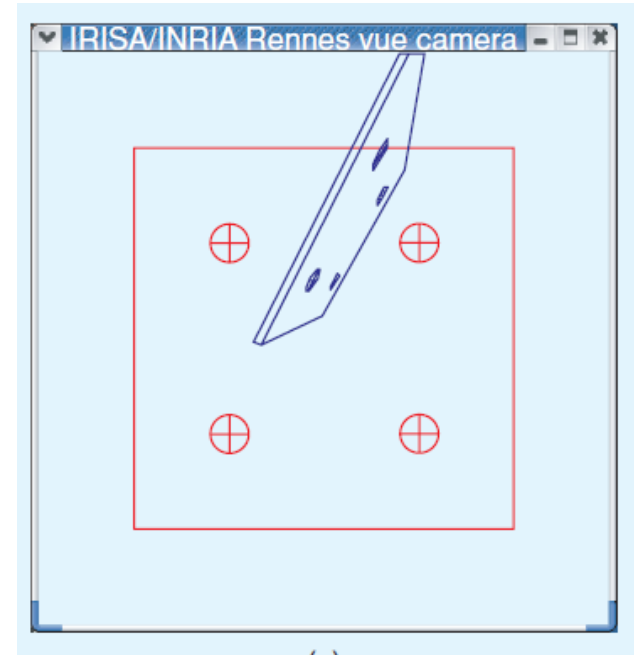
- End-Effector Mounted
- Basic Components of Visual Servoing
  - The aim of all vision-based control schemes is to minimize an error, which is typically defined by

$$\mathbf{e}(t) = \mathbf{s}(\mathbf{m}(t), \mathbf{a}) - \mathbf{s}^*$$



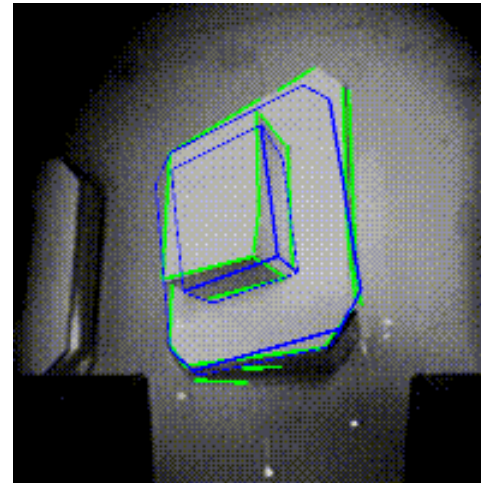
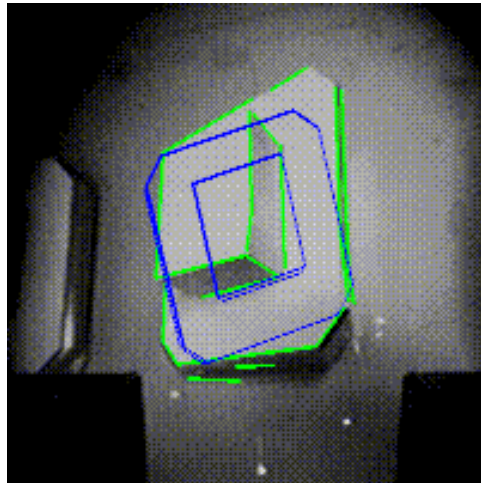
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- End-Effector Mounted
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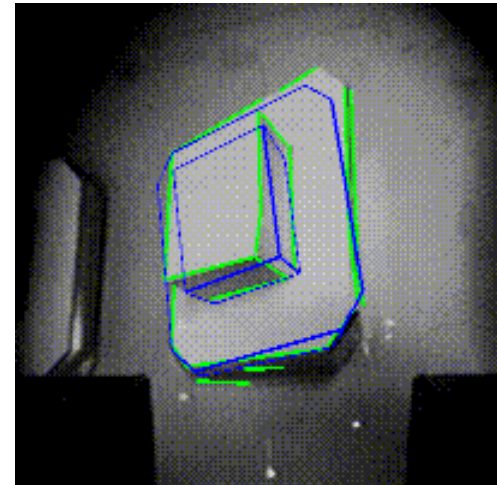
$$\mathbf{e}(t) = \mathbf{s}(\mathbf{m}(t), \mathbf{a}) - \mathbf{s}^*$$

$\mathbf{S}^*$ : desired values of the features

$\mathbf{S}(\ )$ : calculated values of the features

$\mathbf{M}(t)$ : the measurements from Image

$\mathbf{a}$ : parameters of the camera



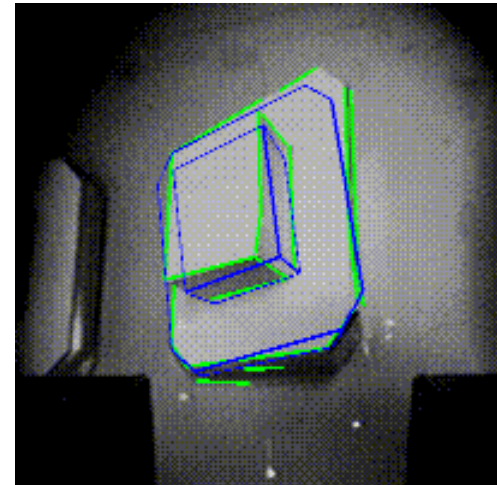
- End-Effector Mounted
- Basic Components of Visual Servoing
  - The aim of all vision-based control schemes is to minimize an error, which is typically defined by

$$\mathbf{e}(t) = \mathbf{s}(\mathbf{m}(t), \mathbf{a}) - \mathbf{s}^*$$

If  $\mathbf{S}^*$  is selected, velocity controller

$$\mathbf{v}_c = (v_c, \boldsymbol{\omega}_c) \quad \dot{\mathbf{s}} = \mathbf{L}_s \mathbf{v}_c$$

Where  $\mathbf{L}_s$  is the *interaction matrix* or *feature Jacobian*.



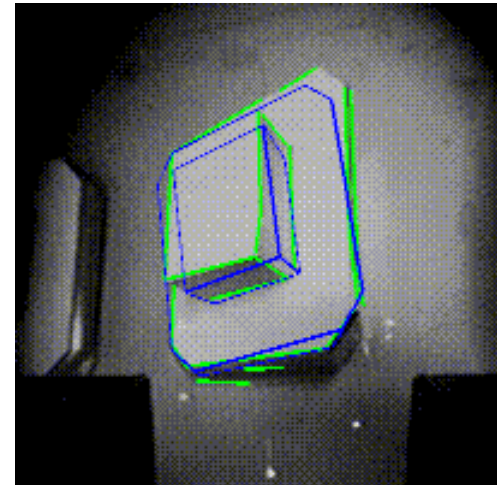
- End-Effector Mounted
- Basic Components of Visual Servoing
  - The aim of all vision-based control schemes is to minimize an error, which is typically defined by

$$\mathbf{e}(t) = \mathbf{s}(\mathbf{m}(t), \mathbf{a}) - \mathbf{s}^* \quad \dot{\mathbf{s}} = \mathbf{L}_s \mathbf{v}_c$$

Relationship between  $\mathbf{e}$  and  $\mathbf{v}_c$

$$\dot{\mathbf{e}} = \mathbf{L}_e \mathbf{v}_c$$

With  $\mathbf{L}_e = \mathbf{L}_s$ . Why?



- End-Effector Mounted
- Basic Components of Visual Servoing

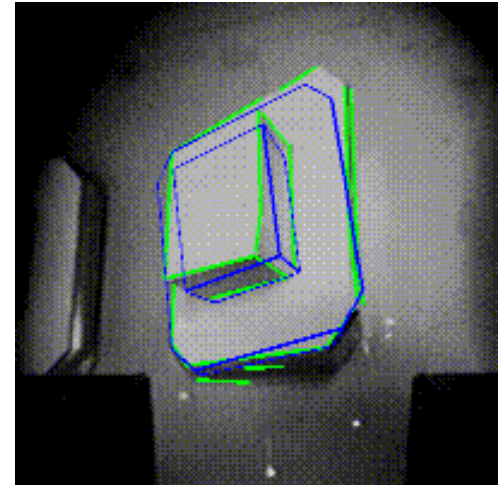
$$\dot{\mathbf{e}} = \mathbf{L}_e \mathbf{v}_c$$

How to solve

Derivative  $\mathbf{e}$  w.r.t  $t$

$$\dot{\mathbf{e}} = -\lambda \mathbf{e}$$

Linear Least Squares



- End-Effector Mounted
- Basic Components of Visual Servoing

$$\dot{\mathbf{e}} = \mathbf{L}_e \mathbf{v}_c$$

How to solve

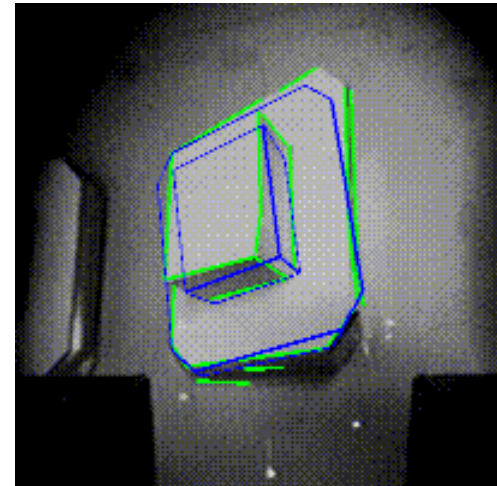
Linear Least Squares

$$\mathbf{v}_c = -\lambda \mathbf{L}_e^+ \mathbf{e}$$

Where

$$\mathbf{L}_e^+ = (\mathbf{L}_e^\top \mathbf{L}_e)^{-1} \mathbf{L}_e^\top$$

Is the Moore-Penrose pseudo-inverse of  $\mathbf{L}_e$ .



- End-Effector Mounted
- Basic Components of Visual Servoing

Camera system

3D point  $(X, Y, Z)$

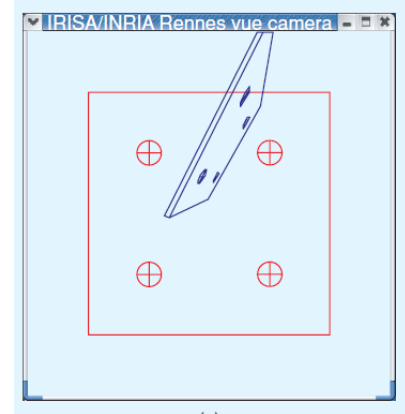
2D point  $(x, y)$

measurement  $m=(u, v)$

intrinsic parameters  $(c_u, c_v, f)$

How to compute  $(x, y)$ ?

$$\begin{cases} x &= X/Z = (u - c_u)/f \\ y &= Y/Z = (v - c_v)/f, \end{cases}$$



- End-Effector Mounted
- Basic Components of Visual Servoing

2D point (x,y)

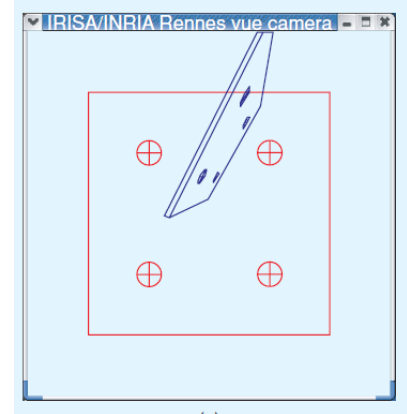
$$\begin{cases} x &= X/Z = (u - c_u)/f \\ y &= Y/Z = (v - c_v)/f, \end{cases}$$

$$\mathbf{e}(t) = \mathbf{s}(\mathbf{m}(t), \mathbf{a}) - \mathbf{s}^*$$

We take  $\mathbf{s} = (x, y)$

How to compute

$$\dot{\mathbf{s}} = \mathbf{L}_s \mathbf{v}_c$$



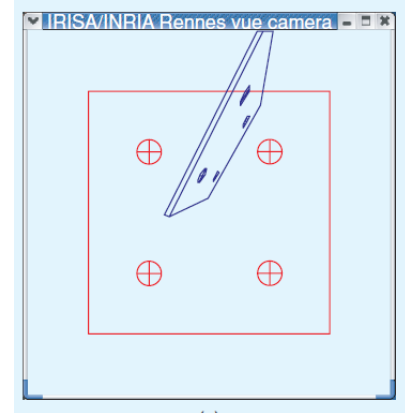
- End-Effector Mounted
- Basic Components of Visual Servoing

Derivatives

$$\begin{cases} \dot{x} &= \dot{X}/Z - X\dot{Z}/Z^2 &= (\dot{X} - x\dot{Z})/Z \\ \dot{y} &= \dot{Y}/Z - Y\dot{Z}/Z^2 &= (\dot{Y} - y\dot{Z})/Z. \end{cases}$$

We can relate the velocity of the 3-D point to the camera spatial velocity using the well-known equation

$$\dot{\mathbf{X}} = -\mathbf{v}_c - \boldsymbol{\omega}_c \times \mathbf{X} \Leftrightarrow \begin{cases} \dot{X} = -v_x - \omega_y Z + \omega_z Y \\ \dot{Y} = -v_y - \omega_z X + \omega_x Z \\ \dot{Z} = -v_z - \omega_x Y + \omega_y X. \end{cases}$$





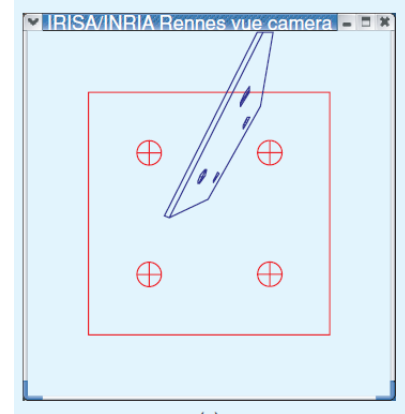
- End-Effector Mounted
- Basic Components of Visual Servoing

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We have

$$\begin{cases} \dot{x} = -v_x/Z + xv_z/Z + x\gamma\omega_x - (1+x^2)\omega_y + \gamma\omega_z \\ \dot{y} = -v_y/Z + yv_z/Z + (1+y^2)\omega_x - x\gamma\omega_y - x\omega_z \end{cases}$$



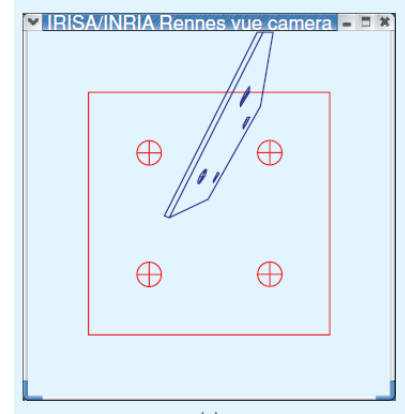
- End-Effector Mounted
- Basic Components of Visual Servoing

$$\begin{cases} \dot{x} &= \dot{X}/Z - X\dot{Z}/Z^2 &= (\dot{X} - x\dot{Z})/Z \\ \dot{y} &= \dot{Y}/Z - Y\dot{Z}/Z^2 &= (\dot{Y} - y\dot{Z})/Z. \end{cases}$$

$$\dot{\mathbf{X}} = -\mathbf{v}_c - \boldsymbol{\omega}_c \times \mathbf{X} \Leftrightarrow \begin{cases} \dot{X} = -v_x - \omega_y Z + \omega_z Y \\ \dot{Y} = -v_y - \omega_z X + \omega_x Z \\ \dot{Z} = -v_z - \omega_x Y + \omega_y X. \end{cases}$$

We have

$$\begin{cases} \dot{x} = -v_x/Z + xv_z/Z + x\gamma\omega_x - (1+x^2)\omega_y + \gamma\omega_z \\ \dot{y} = -v_y/Z + yv_z/Z + (1+y^2)\omega_x - x\gamma\omega_y - x\omega_z \end{cases}$$



- End-Effector Mounted
- Basic Components of Visual Servoing

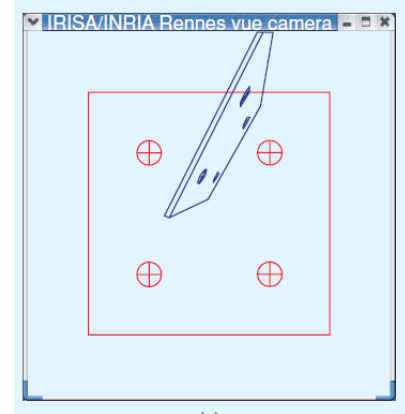
$$\begin{cases} \dot{x} = -v_x/Z + xv_z/Z + x\gamma\omega_x - (1+x^2)\omega_y + \gamma\omega_z \\ \dot{y} = -v_y/Z + \gamma v_z/Z + (1+\gamma^2)\omega_x - x\gamma\omega_y - x\omega_z \end{cases}$$

Which can be rewritten

$$\dot{\mathbf{s}} = \mathbf{L}_s \mathbf{v}_c$$

Where

$$\mathbf{L}_x = \begin{bmatrix} \frac{-1}{Z} & 0 & \frac{x}{Z} & x\gamma & -(1+x^2) & \gamma \\ 0 & \frac{-1}{Z} & \frac{\gamma}{Z} & 1+\gamma^2 & -x\gamma & -x \end{bmatrix}$$



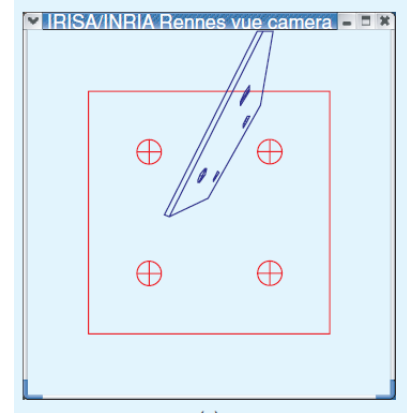
- End-Effector Mounted
- Basic Components of Visual Servoing

$$\mathbf{L}_x = \begin{bmatrix} \frac{-1}{Z} & 0 & \frac{x}{Z} & xy & -(1+x^2) & y \\ 0 & \frac{-1}{Z} & \frac{y}{Z} & 1+y^2 & -xy & -x \end{bmatrix}$$

Must estimate or approximate the value of  $Z$

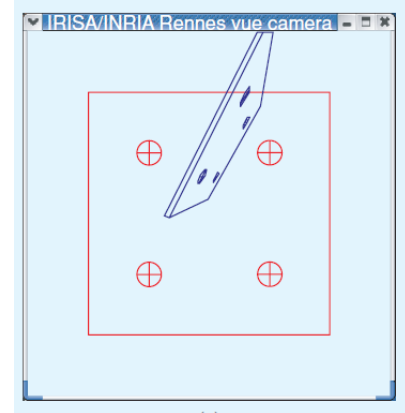
At least 3 points are necessary

$$\mathbf{L}_x = \begin{bmatrix} \mathbf{L}_{x_1} \\ \mathbf{L}_{x_2} \\ \mathbf{L}_{x_3} \end{bmatrix}$$



## ❖ Activity-3:

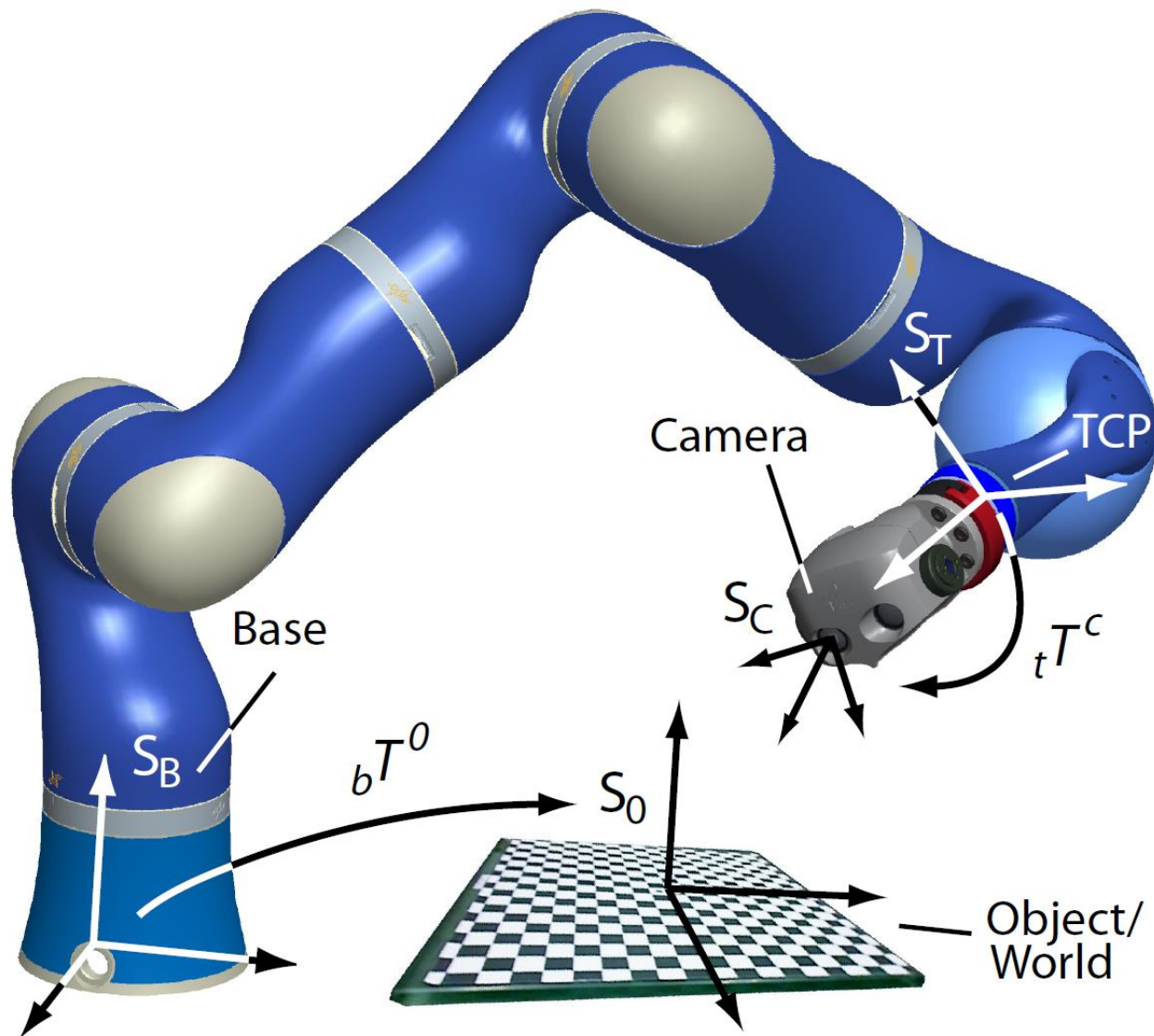
- Camera:
  - Image: 1000\*1000
  - Principle point: (400,400)
  - Focal length: (400,400)
- Pose:  $r = (0,0,0) \Rightarrow R = I, T = [10,20,2]$
- Desired features
  - (0,0), (800,0), (800,0),(800,800)
- Measurements
  - (0,0), (800,0), (800,0),(800,800) +50
- Assume  $Z = 50$
- Camera velocity  $v_c$ ?



# Hand-eye Calibration

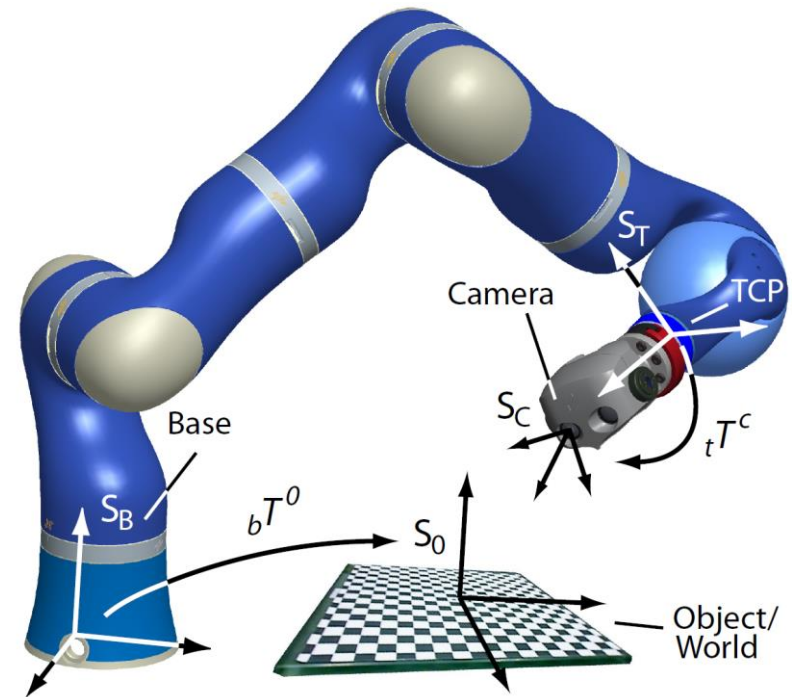


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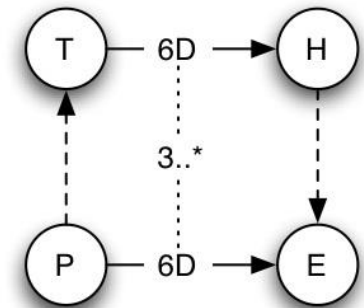
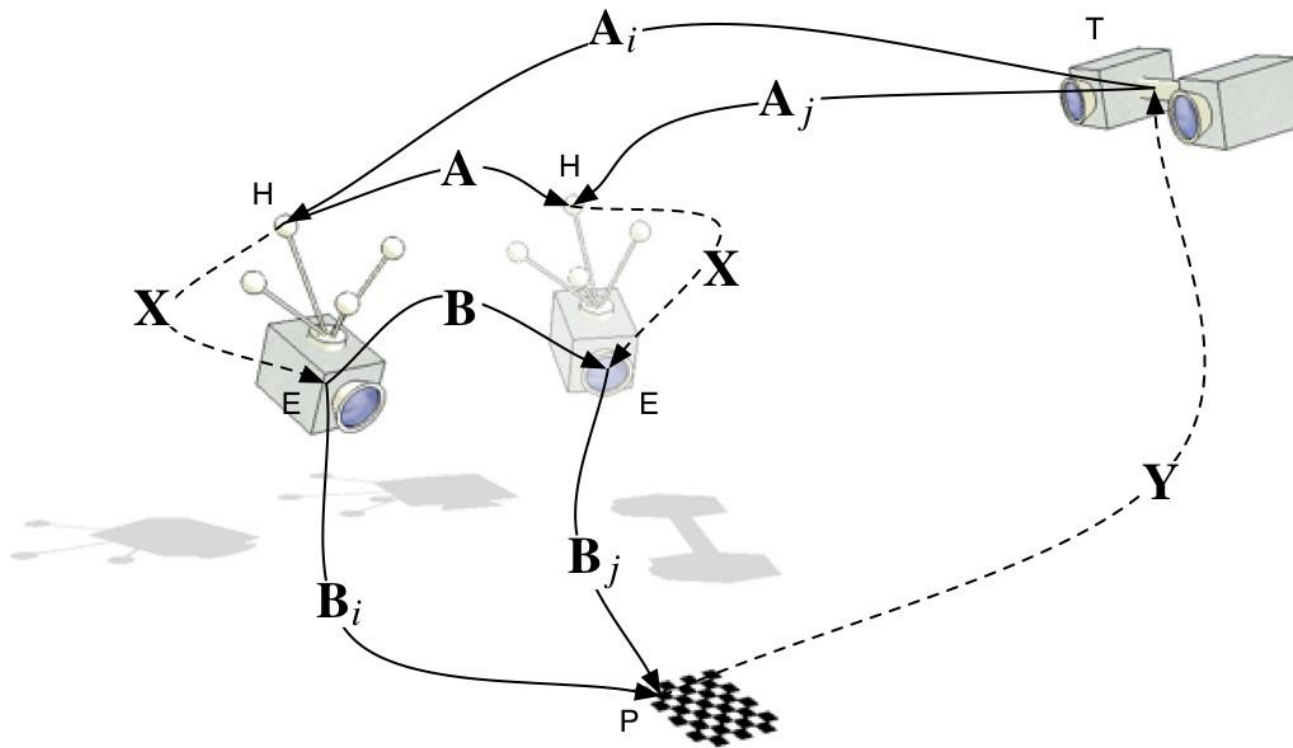


## ❖ Hand-eye Calibration

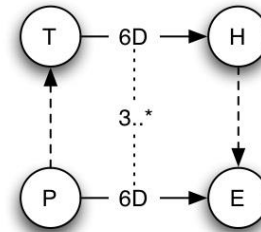
- Relative Pose
- Hand (end effector)
- Eye (Camera)



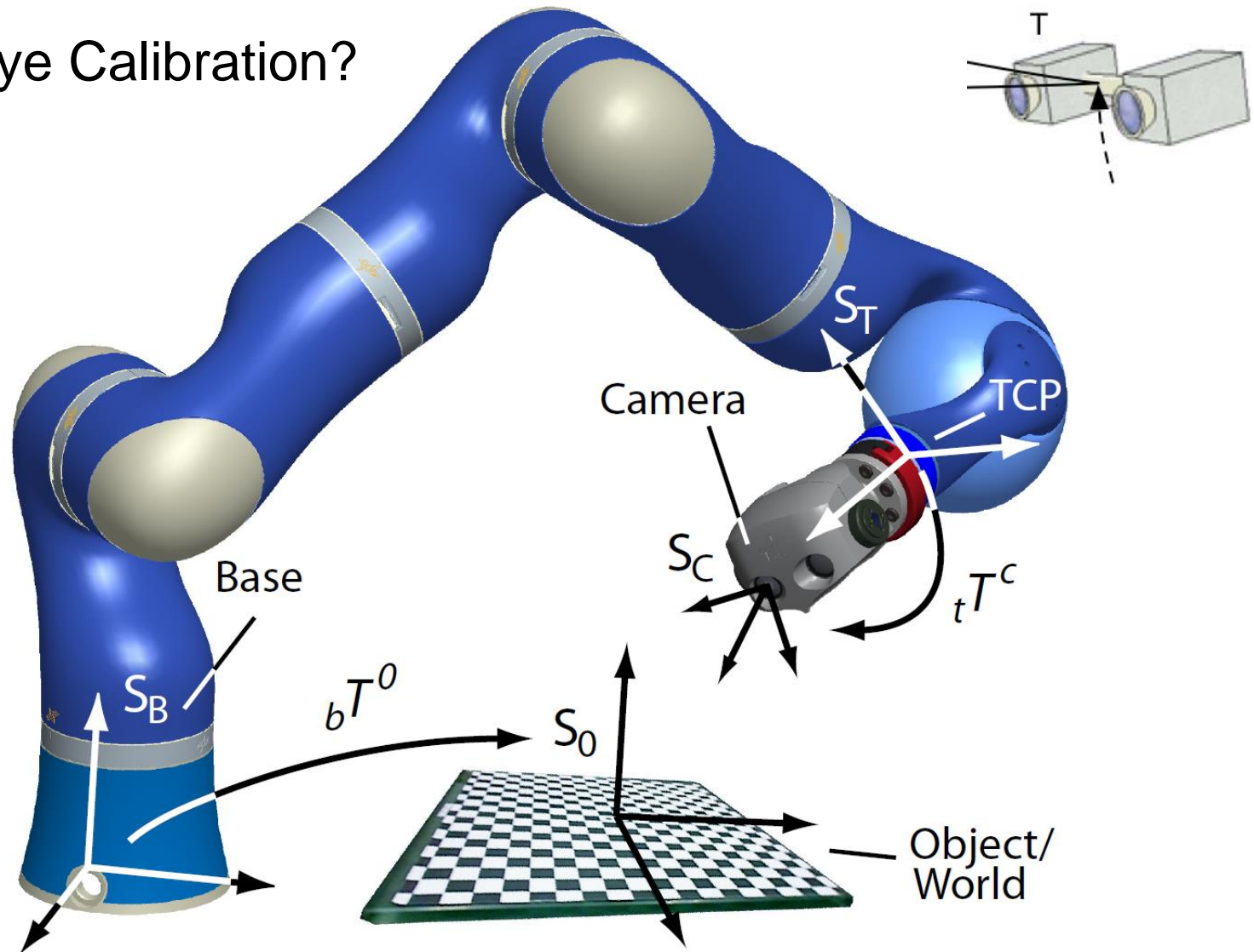
## ❖ Hand-eye Calibration



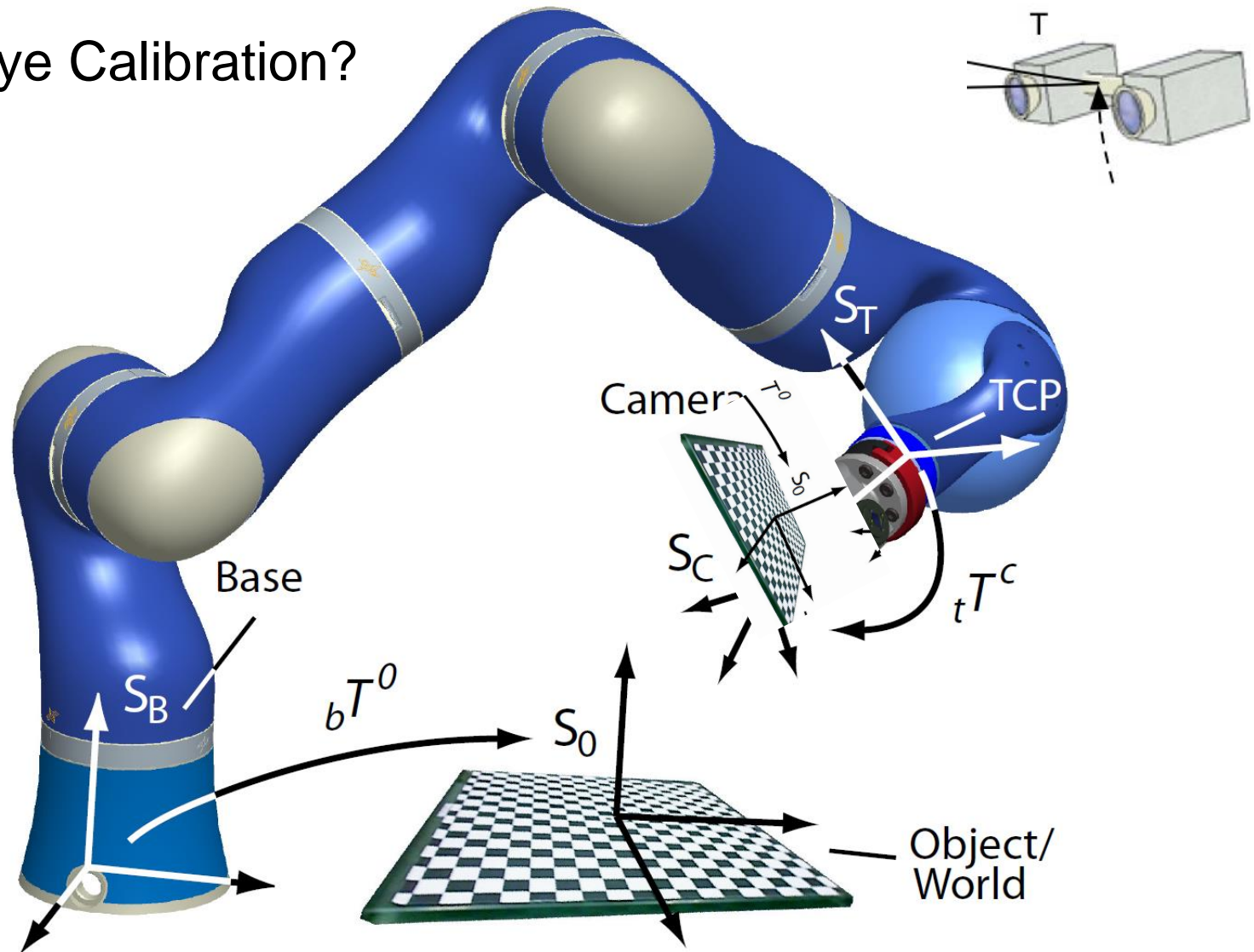


$$A^*X = X^*B$$


## ❖ Hand-eye Calibration?



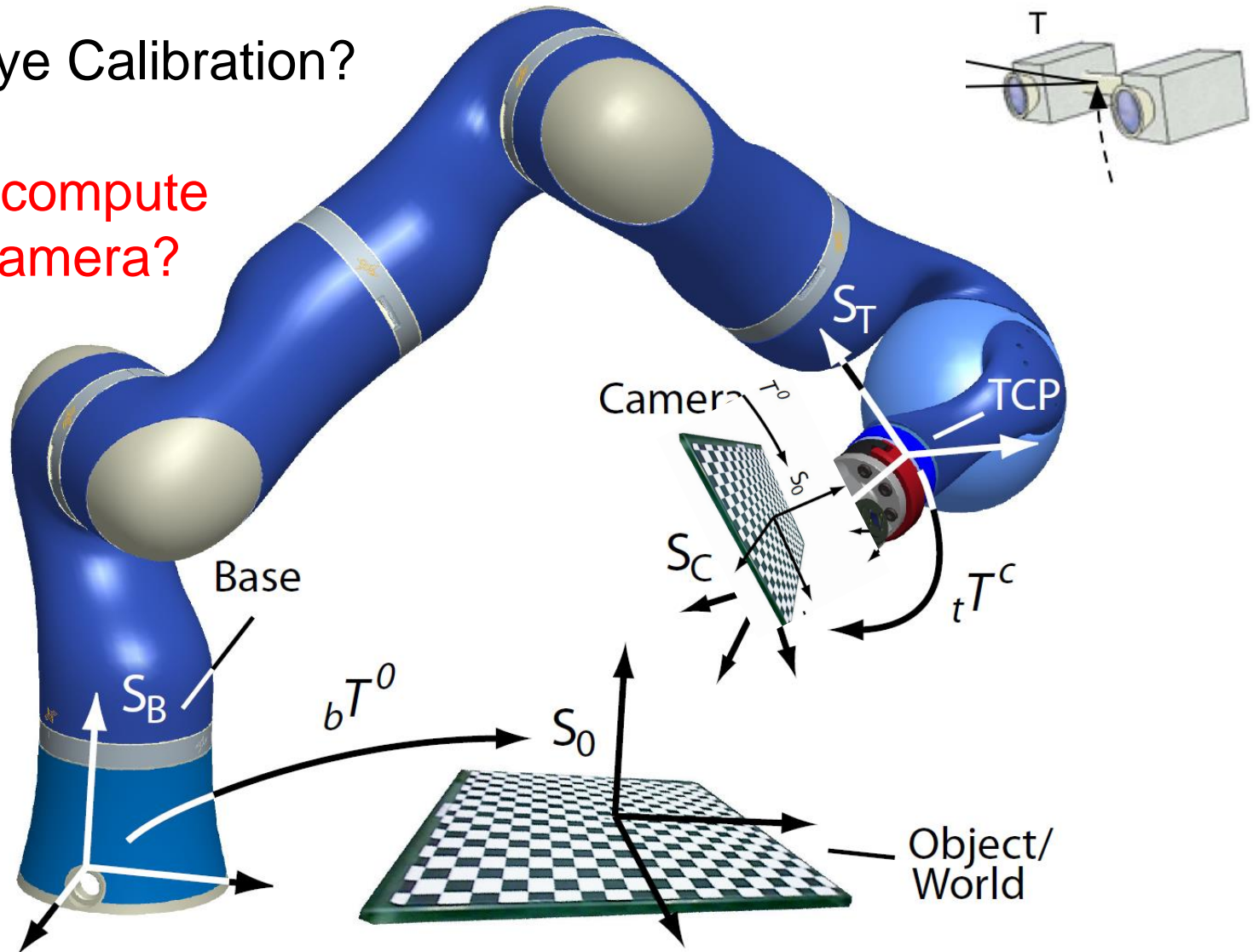
## ❖ Hand-eye Calibration?



❖ Hand-eye Calibration?

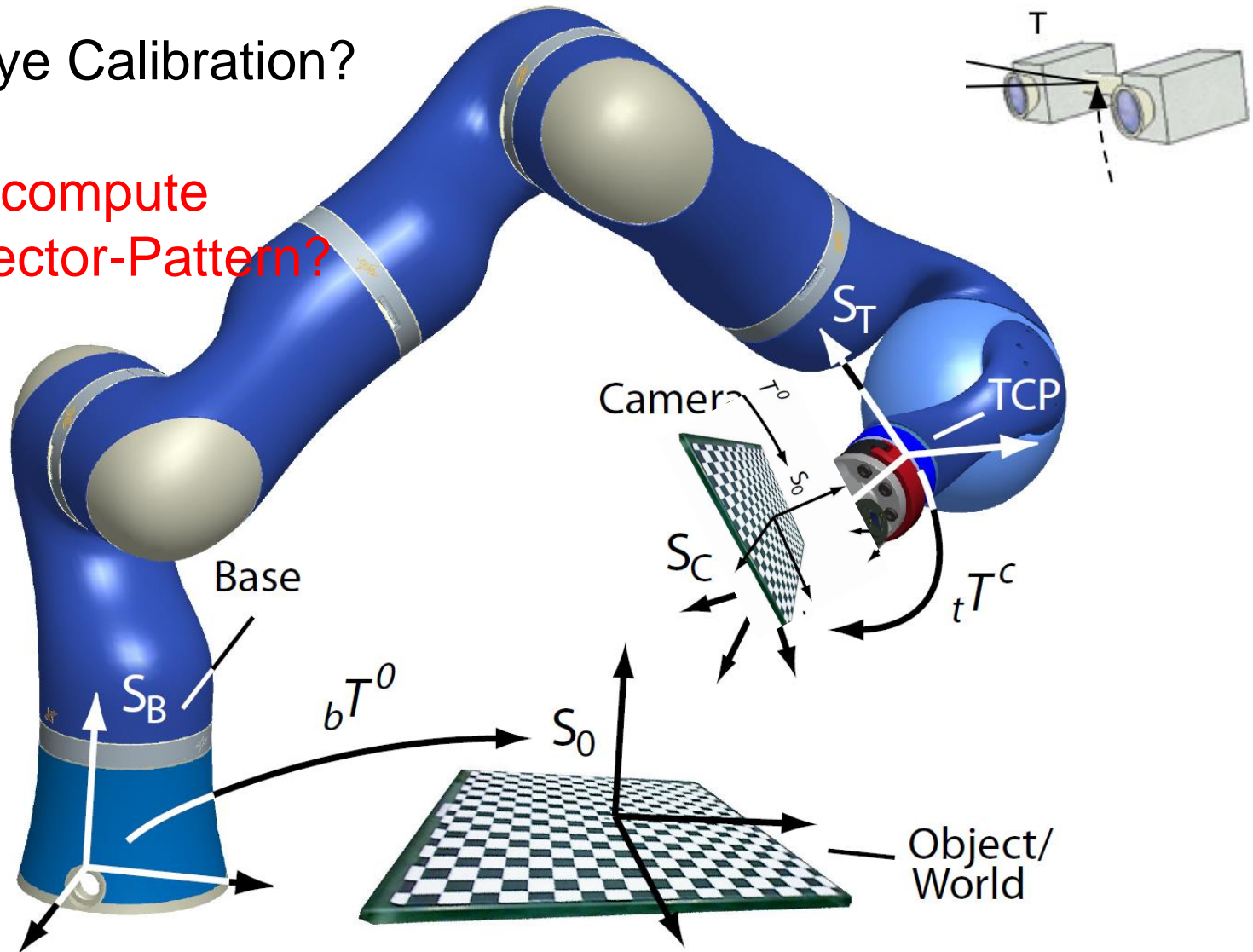
❖ How to compute

❖ Base-Camera?





- ❖ Hand-eye Calibration?
- ❖ How to compute
- ❖ End effector-Pattern?



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# *THANK YOU*

## Questions?



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## ***GROUP PROJECTS***



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