Example of state space model

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} . \qquad \dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}$$

$$\frac{dx_1(t)}{dt} = \dot{x}_1(t) = x_1(t) + 2x_2(t)$$

$$\frac{dx_2(t)}{dt} = \dot{x}_2(t) = 3x_1(t) + 4x_2(t)$$

which is

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

This is

$$\dot{x}(t) = A\ x(t)$$

where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Mass-spring-damper system

$$\ddot{x} = -\frac{c}{m}\dot{x} - \frac{k}{m}x + \frac{1}{m}F$$

$$X = \begin{pmatrix} x \\ \dot{x} \end{pmatrix} \text{ ——— } \begin{matrix} x_1 = x \\ x_2 = \dot{x} \end{matrix} \qquad \dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}$$

we have $\dot{x} = \dot{x} = 0x + \dot{x} + 0 \cdot F$

$$\ddot{x} = -\frac{k}{m}x - \frac{c}{m}\dot{x} + \frac{1}{m}F$$

That is $\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$

So we have $\dot{X} = AX + BF$

where $A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}$. $B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$

Transfer function of cruise control example

$$\boxed{T(s) = C(sI - A)^{-1} B + D}$$

$A = -\frac{b}{m}$, $B = \frac{1}{m}$, $C = 1$. $D = 0$

$$T(s) = 1 \cdot \left(s + \frac{b}{m}\right)^{-1} \cdot \frac{1}{m} + 0$$

$$= \frac{1}{m\left(s + \frac{b}{m}\right)}$$

$$= \frac{1}{ms + b}$$

If $m = 100$. $b = 50$.

then $T(s) = \frac{1}{100s + 50} = \frac{0.01}{s + 0.5}$