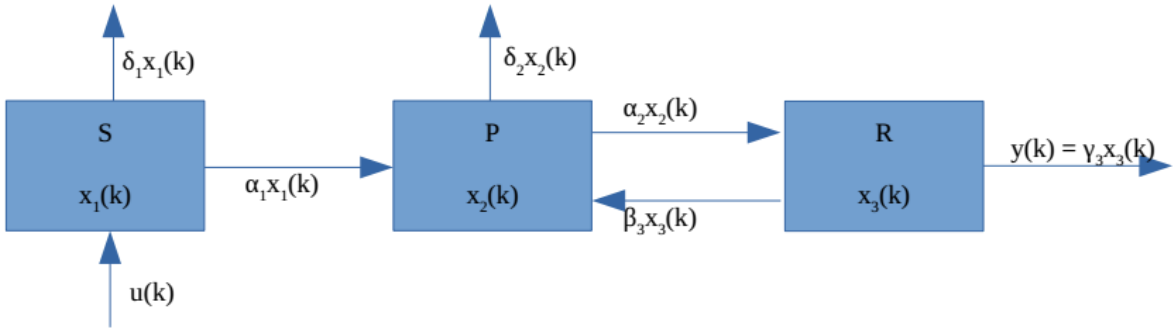
Sensors and Control for Mechatronics Systems
Tutorial 7

Question 1 : Discrete time system

Consider the supply chain system.



1.1 : Model the system in MATLAB and obtain the monthly sales for a period of 1 year ***(k= 0 to k = 12)***

δ_1 = 0.1
δ_2 = 0.1
α_1 = 0.6
α_2 = 0.6
β_3 = 0.05
γ_3 = 0.8
$x_1(0)$ = 300
$x_2(0)$ = 100
$x_3(0)$ = 80
u(k) = 120 for all k.

1.2 : Plot x_1, x_2, x_3 and y against k and interpret the system stability.

1.3 : Obtain the eigenvalues of the matrix A and corroborate your answer to 1.2.

1.4 : Test the controllability of the system using matrices A and B

1.5 : MATLAB state-space model (ss) can be used to create discrete time system models.
Ref : https://au.mathworks.com/help/control/ref/ss.html
Use function ***ss*** to create a discrete time state-space model of the above system.

1.6 : Use functions ***isstable*** (https://au.mathworks.com/help/control/ref/isstable.html) and ***ctrb*** (https://au.mathworks.com/help/control/ref/ctrb.html) to test for stability and controllability of the system.

Question 2 : LQR control

2.1 : For the closed-loop system that follows the feedback control law ***u(k) = -Kx(k)*** design the state feedback control to minimize the performance index

$$J(U) = {}^{N-1}\Sigma_{T=0}(x_T{}^T Q x_T + u_T{}^T R u_T) + x_N{}^T Q x_N$$

where Q is an identity matrix of size 3 and R is scalar 1 by solving the algebraic Riccati equation.

$$A = \begin{matrix} 1.5 & 1.75 & 1.3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$$

$$B = \begin{matrix} 1 \\ 0 \\ 0 \end{matrix}$$

$$C = \begin{matrix} 0 \\ 0 \\ 0.75 \end{matrix}$$

$$X_0 = \begin{matrix} 1 \\ 1 \\ 1 \end{matrix}$$

The following functions are available in MATLAB.

dare : Solve discrete time Riccati equation (ref: https://au.mathworks.com/help/control/ref/dare.html)

lqr : Linear quadratic regulator for state space system (ref: https://au.mathworks.com/help/control/ref/lqr.html)

Plot the states and output for LQR optimal input for step k = 1 to k = 20

2.2 : Change the values for Q and R independently and observe the effect on the change of inputs and rate of convergence. Explain this behavior.