Notes of Electromagnetic Fields

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Contents

Chapter 1

Maxwell's equations

Table 1.1:

	Integral form	Differential form
Gauss's law		
Magnetic field of the magnetic flux		

Chapter 2

Transformation Coordinates

Assume that you have the follow joint of fields whose functions are in different coordenate systems

$$
\begin{aligned}
F &= f(x)a_x + f(y)a_y + f(z)a_z \\
F &= f(\rho)a_\rho + f(\phi)a_\phi + f(z)a_z \\
F &= f(r)a_r + f(\theta)a_\theta + f(\phi)a_\phi
\end{aligned}
$$

Now, How I can do for to change between the different systems? You can review all the time the tables for change of a system to other, but in the university that is not the sense. For that, is better use the follow method that is described by the Equation (2.1).

$$
\left[\begin{array}{l}
F = f(x)a_x + f(y)a_y + f(z)a_z \\
F = f(\rho)a_\rho + f(\phi)a_\phi + f(z)a_z \\
F = f(r)a_r + f(\theta)a_\theta + f(\phi)a_\phi
\end{array}\right]
\left[\begin{array}{ccccccc}
\cdot a_x & \cdot a_y & \cdots & \cdot a_\rho & \cdots & \cdot a_r & \cdots \\
\cdot a_x & \cdot a_y & \cdots & \cdot a_\rho & \cdots & \cdot a_r & \cdots \\
\cdot a_x & \cdot a_y & \cdots & \cdot a_\rho & \cdots & \cdot a_r & \cdots
\end{array}\right] =
$$

$$
\left[\begin{array}{llllll}
\text{In cartesian system, component x} & & \text{In cilindrical system, component } \phi & & \text{In spherical system,component r} & \\
F = f(x) & \cdots & F = A_2 f(x) + B_2 f(y) + C_2 f(z) & \cdots & F = A_3 f(x) + B_3 f(y) + C_3 f(z) & \cdots \\
F = A_1 f(\rho) + B_1 f(\phi) + C_1 f(z) & \cdots & F = f(\rho) & \cdots & F = D_3 f(\rho) + E_3 f(\phi) + F_3 f(z) & \cdots \\
F = D_1 f(r) + E_1 f(\theta) + F_1 f(\phi) & \cdots & F = D_2 f(r) + E_2 f(\theta) + F_2 f(\phi) & \cdots & F = f(\rho) & \cdots
\end{array}\right] \tag{2.1}
$$

Important: If for example I need change from cylindrical coordinates to cartesian coordenates all the variables have that be how the last system. The constants $A_i, B_i, C_i, D_i, E_i, F_i$ are find according to the Figures 2.1 and 2.2

To find what are the relation between the different vectors the Figure 2.1 and 2.2 are useful

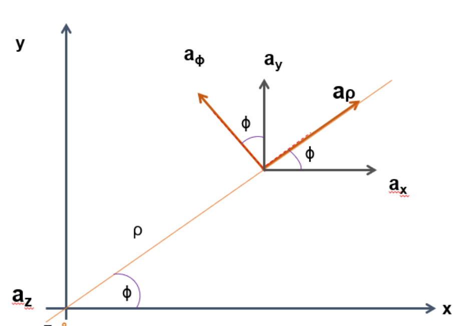

Figure 2.1: Relationship between cartesian and

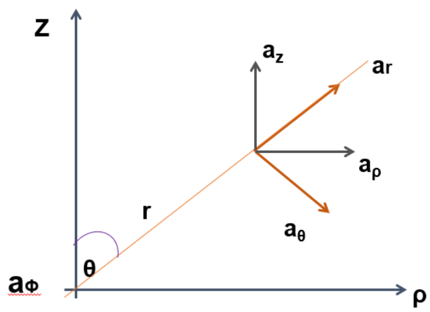

Figure 2.2: Relationship between cylindrical and spherical coordinates [1]

Example 2.0.1 *Find the matrix that change from cartesian coordinates to spherical coordinates and vice versa.*
Using the Equation (2.1) we have:

$$F \cdot a_x = f(x)a_x \cdot a_x + f(y)a_y \cdot a_x + f(z)a_z \cdot a_x$$
$$F \cdot a_x = f(\rho)a_\rho \cdot a_x + f(\phi)a_\phi \cdot a_x + f(z)a_z \cdot a_x$$

Thus, we obtain:

$$F \cdot a_x = f(x)$$
$$F \cdot a_x = f(\rho)cos(\phi) + f(\phi)cos\left(\phi + \frac{\pi}{2}\right) + f(z)0 \rightarrow$$
$$f(x) = f(\rho)cos(\phi) + f(\phi)cos\left(\phi + \frac{\pi}{2}\right) + f(z)0$$

Then, the component y es find how,

$$F \cdot a_x = f(x)a_x \cdot a_y + f(y)a_y \cdot a_y + f(z)a_z \cdot a_y$$
$$F \cdot a_x = f(\rho)a_\rho \cdot a_y + f(\phi)a_\phi \cdot a_y + f(z)a_z \cdot a_y$$

Thus, we obtain:

$$F \cdot a_x = f(y)$$
$$F \cdot a_x = f(\rho)cos\left(\frac{\pi}{2} - \phi\right) + f(\phi)cos\left(\phi\right)) + f(z)0 \rightarrow$$
$$f(y) = f(\rho)cos\left(\frac{\pi}{2} - \phi\right) + f(\phi)cos\left(\phi\right)) + f(z)0$$

In matrix system we have:

$$\begin{bmatrix} f(x) \\ f(y) \\ f(z) \end{bmatrix} = \begin{bmatrix} cos(\phi) & cos\left(\phi + \frac{\pi}{2}\right) & 0 \\ cos\left(\frac{\pi}{2} - \phi\right) & cos\left(\phi\right)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f(\rho) \\ f(\phi) \\ f(z) \end{bmatrix}$$

Now when a similar development we have:

$$\begin{bmatrix} f(\rho) \\ f(\phi) \\ f(z) \end{bmatrix} = \begin{bmatrix} cos(\phi) & cos\left(\phi - \frac{\pi}{2}\right) & 0 \\ cos\left(\frac{\pi}{2} + \phi\right) & cos\left(\phi\right)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f(x) \\ f(y) \\ f(z) \end{bmatrix}$$

Chapter 3

Space Charge Distribution

There are four ways of represent the Space Charge Distribution the Figure 3.1. All the quantities are scalars if for example were vectors will be electric flux density.

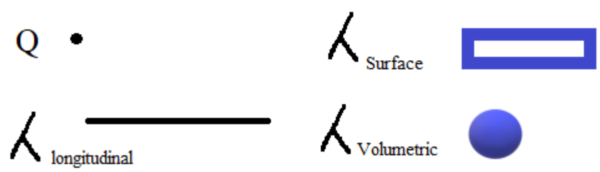

Figure 3.1: Space Charge Distribution

This are some observations

- Point load: When we do a comparison with other thing and we determinate that is despicable.
- $\lambda_{longitudinal}$: when in a bar the diameter in very small with respect to the length.
- $\lambda_{surfaces}$: the thickness is despicable with respect to total area.
- $\lambda_{Volumetric}$: All the physical dimensions are important.

Now the questions to answers is : How is the electric field E and D?

Coulomb's Law

$$F = \frac{kQ_0q}{R^2}a_r\ [N] \tag{3.1}$$

Where $k = \frac{1}{4\pi\epsilon_0}\left[\frac{m^2N}{C^2}\right]$

Its means is:

- F is directly proportional to the product between the source and the prove charge.

- The net force appears in the action line.
- The signs determine if the strength is of attraction or repulsion.
- The electric force is inversely proportional to the square of the distance.
- Is directly proportional to the factor K.

3.1 Point charge Distribution

The electric field intensity (E) is

$$E = \frac{F}{q} a_r = \frac{kQ_0}{R^2} a_r \left[\frac{N}{C} \right] \tag{3.2}$$

Respect to the electric field intensity we can say that:

- We can do a measure of the electric force.
- The electric field is a distortion [1] of the space due to the presence of a charge.

Ways of the represent a electric field

1. By vectors in direction to the electric force if a charge appear in the electric field.
2. Force lines, the length in proportional to the magnitude of the intensity.

3.2 Longitudinal Charge Longitudinal

According with the Figure 3.2 we have that $dq = \lambda_l dl$ and the intensity of the electric field is of the way $dE = \frac{Kdq}{R^2}$

[1] Is the presence of a force fields in this case is of attraction or repulsion; on other hand, the gravity field only is of attraction

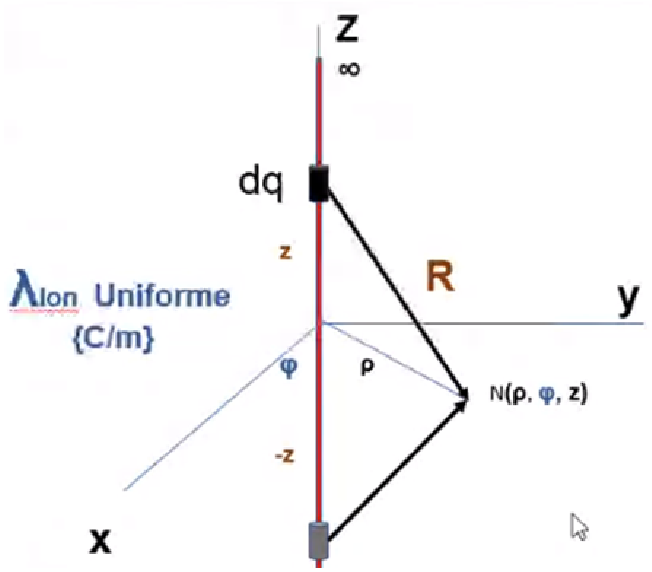

Figure 3.2: Longitudinal distribution

The Figure 3.3 show us a way to find the total electric field.

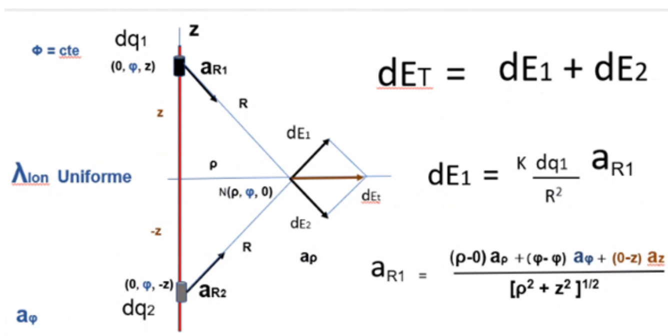

Figure 3.3: Longitudinal distribution with symmetry

We have the follow Equations

$$dE_T = dE_1 + dE_2$$

$$\begin{aligned}
dE_1 &= \frac{Kdq_1}{R_1^2} a_{R1} \\
a_{R1} &= \frac{(\rho - 0)a_\rho + (\phi - \phi)a_\phi + (0 - z)a_z}{\sqrt{\rho^2 + z^2}} \\
dE_1 &= \frac{Kdq_1}{(\rho^2 + z^2)^{\frac{3}{2}}}(\rho a_\rho - z a_z)
\end{aligned}$$

$$dE_2 = \frac{Kdq_2}{R_2^2} a_{R2}$$
$$a_{R2} = \frac{(\rho - 0)a_\rho + (\phi - \phi)a_\phi + (0 - (-z))a_z}{\sqrt{\rho^2 + z^2}}$$
$$dE_2 = \frac{Kdq_2}{(\rho^2 + z^2)^{\frac{3}{2}}}(\rho a_\rho + z a_z)$$

The value of E_T is:

$$E_T = \int_{-\infty}^{\infty} \frac{k\lambda_L \rho dz}{(\rho^2 + z^2)^{\frac{3}{2}}} a_\rho = k\lambda_L \rho \int_{-\infty}^{\infty} \frac{dz}{(\rho^2 + z^2)^{\frac{3}{2}}} a_\rho \tag{3.3}$$

Or

$$E_T = \int_{0}^{\infty} \frac{2k\lambda_L \rho dz}{(\rho^2 + z^2)^{\frac{3}{2}}} a_\rho = 2k\lambda_L \rho \int_{0}^{\infty} \frac{dz}{(\rho^2 + z^2)^{\frac{3}{2}}} a_\rho \tag{3.4}$$

And Finally

$$E_T = \frac{\lambda_L a_\rho}{2\pi\epsilon_0 \rho} \tag{3.5}$$

Chapter 4

Modeling

Table 4.1: The equivalent components of circuits

Capacitance	Resistance
$C = \frac{Q_e}{V} = \frac{\int D \cdot ds}{-\int E \cdot dL}$	$R = \frac{V}{I} = \frac{-\int E \cdot dL}{\int j \cdot dS}$
Inductance	Conductance
$L = \frac{\phi_{mag}}{I} = \frac{\int B \cdot dS}{\oint H \cdot dL}$	$G = \frac{I}{V} = \frac{\int j \cdot dS}{-\int E \cdot dL}$

Comments

- $V = \int j \cdot ds$: Gauss's law for the current.
- $V = -\int E \cdot dL$: Negative sign is because a external force do the work.
- $I = \oint H \cdot dL$: If you see the Maxwell equations, you can note that here be missing a term. According to the teacher, it is because only import the magnetic field.

Chapter 5

Efecto de los campos en los materiales

Pregunta 5.0.1 *¿Por qué se dañan los componentes eléctricos o electrónicos comúnmente?*
Por el desgaste de los aislantes. Por ejemplo, un transformador se daña porque se hace un corto circuito al desgastarsen los aislantes.

Pregunta 5.0.2 *¿Cuando un aislante cambia sus propiedades?*
Ocurre cuando sus cargas ligadas se convierten en cargas libres. En un aislante hay pocas cargas libres.

5.1 Polarización

El potencial se un dipolo en un punto N se puede plantear como:

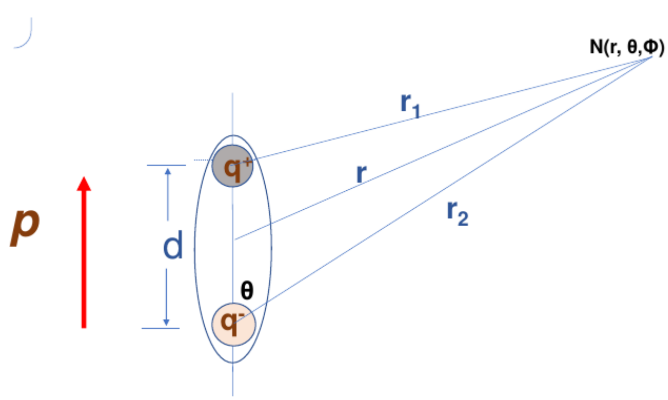

Figure 5.1:

El resultado es:

$$V_T = \left(\frac{q\ d}{4\pi\varepsilon_0\ r^2} \right)$$

Figure 5.2:

El *momento dipolar (p)* se define como

Momento dipolar

$$p = qd$$

Figure 5.3:

Es la capacidad que tiene una carga en orientarse según el campo eléctrico. La *(polarización P)* se puede plantear como:

Cuando no hay $\vec{E}$, las cargas están desorganizadas

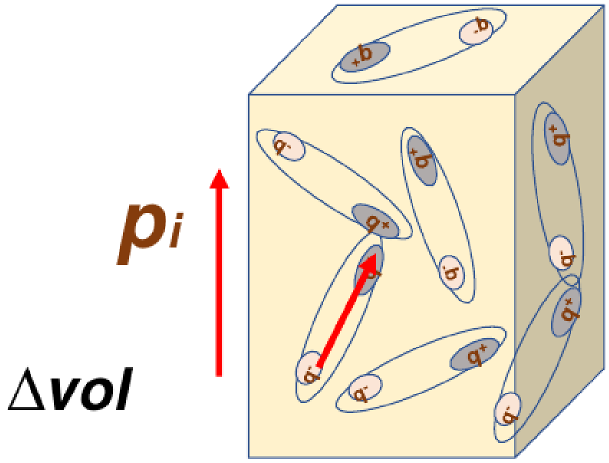

Figure 5.4:

En presencia de $\vec{E}$ Las cargas se organizan

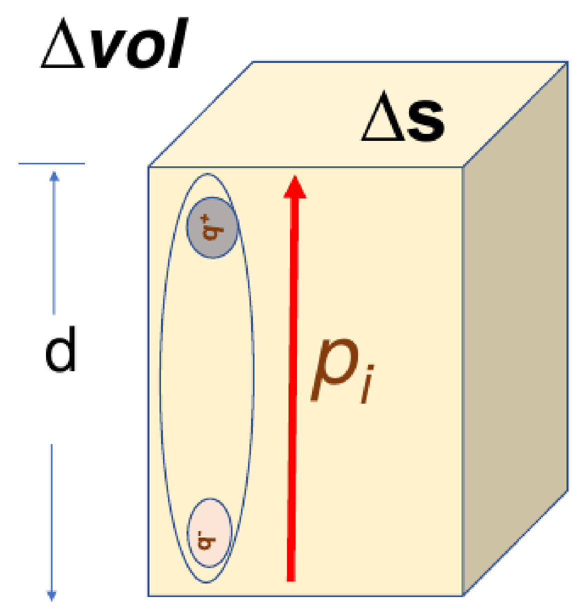

Figure 5.5:

Supongamos que hay una redistribución uniforme

$$\mathbf{P} = \sum_{i=1}^{N} \frac{p_i}{\Delta vol}$$

Figure 5.6:

P: La capacidad de generar polos por unidad de volumen

¿Sabias que...? 5.1.1 *La polarización (P) en el vació es cero, porque allí no hay ningún material*

5.2 Carga Libre

$$\mathbf{P} = \sum_{i=1}^{N} \frac{p_i}{\Delta vol} \quad [C/m^2]$$

$$Q_{Total} = Q_{libre} + Q_{ligada}$$

$$\Delta Q_{Total} = \Delta Q_{libre} + \Delta Q_{ligada}$$

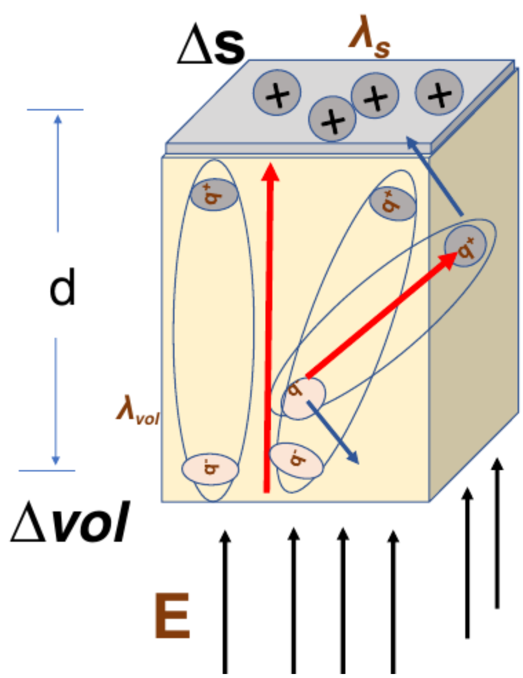

Figure 5.7:

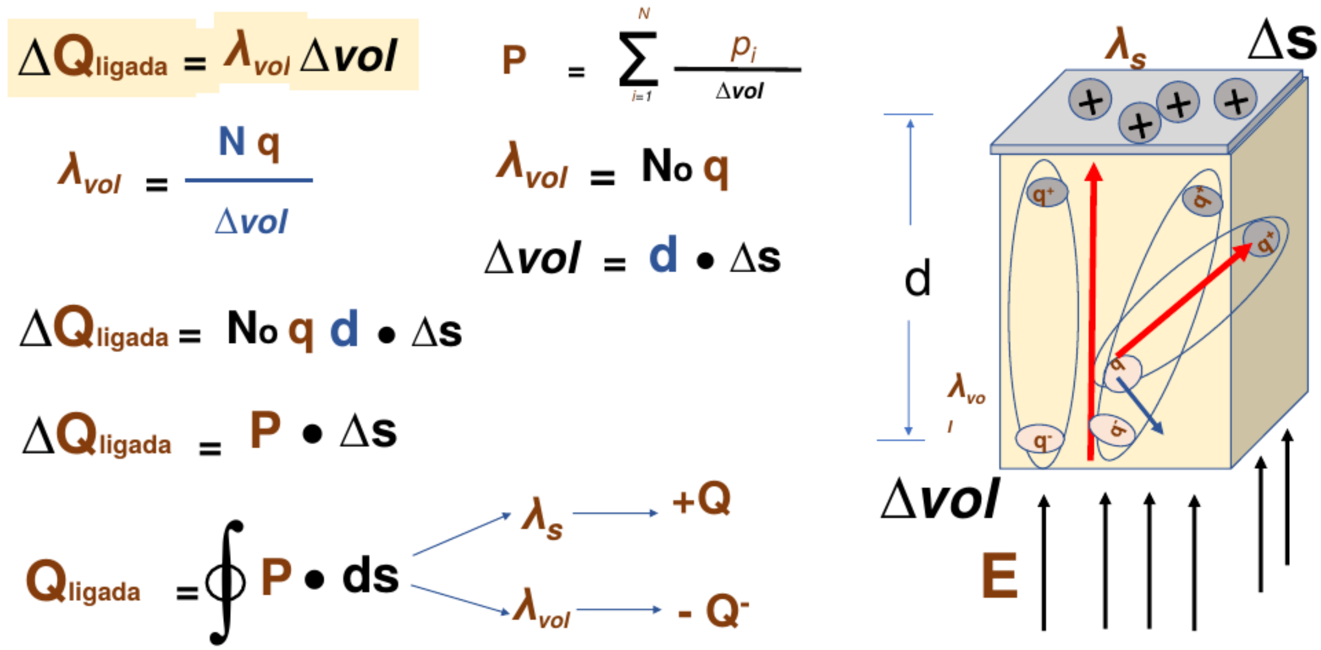

Figure 5.8:

Para continuar con el procedimiento se elige la carga negativa, porque se quiere trabajar con diferenciales de volumen.

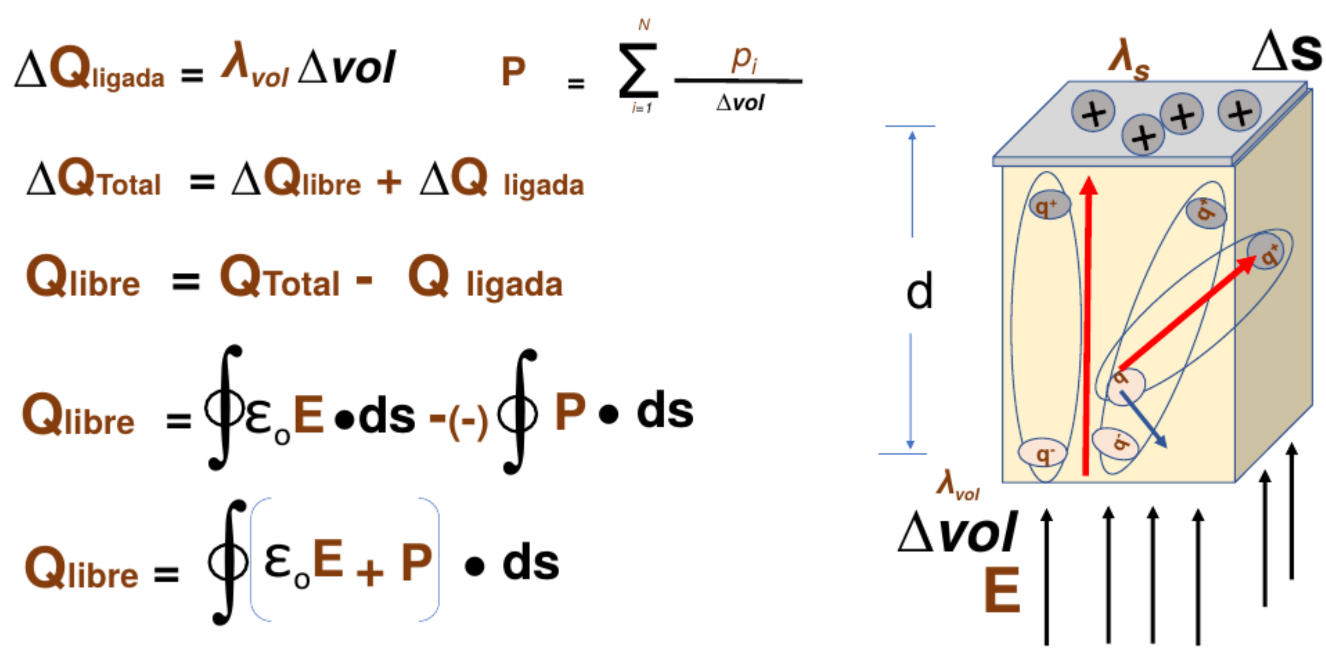

Figure 5.9:

Pregunta 5.2.1 *¿Por qué se elige a ϵ_o para la carga total si estamos en un material?*

Porque la mayor parte de la estructura se puede considerar vacío, por ejemplo en las estructuras cristalinas hay espacios intersticiales y en un núcleo atómico la mayor parte la compone el vacío.

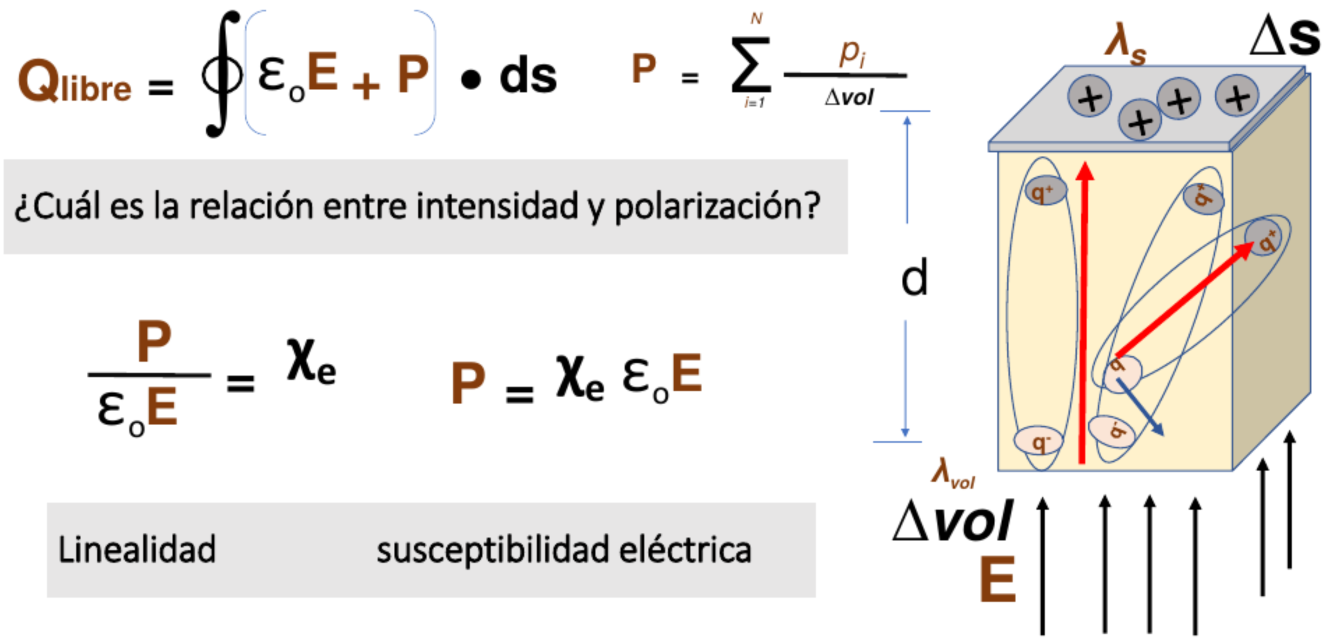

Figure 5.10:

Donde

$$P : \text{Polarización}$$
$$\chi_e : \text{suceptividad eléctrica}$$
$$\epsilon_0 : \text{Permitividad}$$

En caso magnético [1]

$$P : \text{Magnetización}$$
$$\chi_e : \text{Suceptividad Magnética}$$
$$\epsilon_0 : \text{Permeabilidad}$$

[1]Es redundante decir Permeabilidad Magnética

$$D = \left(\varepsilon_o + \chi_e \varepsilon_o\right) E \qquad \frac{D}{E} = \left(\varepsilon_o (1 + \chi_e)\right) = \varepsilon_m$$

Figure 5.11:

ϵ_m: Capacidad de producir flujo por unidad de area al aplicar una campo eléctrico

¿Sabias que...? 5.2.1 *En en el vacío $\chi = 0$ porque no hay dipolos y $\epsilon_r = 1$. En un material polar al aplicar $\vec{E}$ se polariza, en cambio en una dipolar la nube aumenta al tener las mismas condiciones.*

Entonces en un materiar polar al aplicar un $\vec{E}$ se genera un campo secundario P que refuerza a este primero.

Producir dipolos
Producir polarización (campo secundario)
Producir desplazamiento de carga ligada

Figure 5.12:

5.3 Disrupsión

De $E_0 - E_2$: linealidad, de $E_2 - E_c$: Ionización (punto antes de la disrupción) y de E_c-: Disrupción

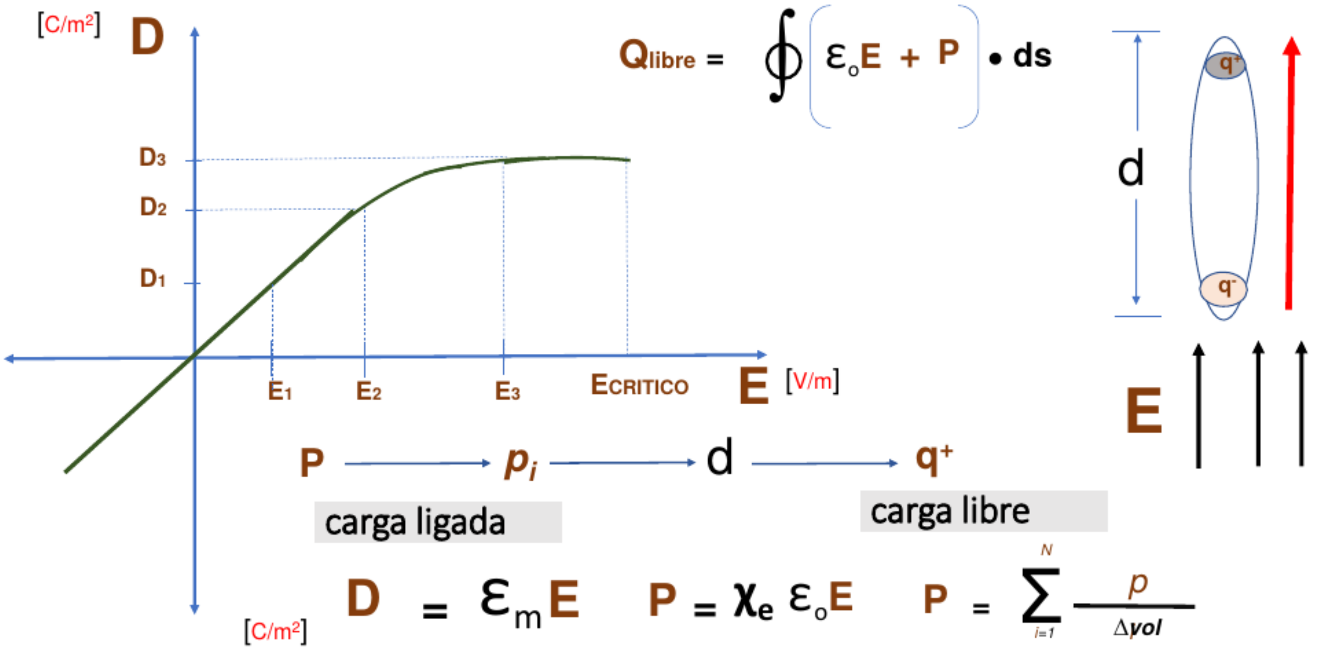

Figure 5.13:

Definition 1 (Disrupción)

Ejercicio 5.3.1 (Caucho) *Imagine que estire un caucho...*

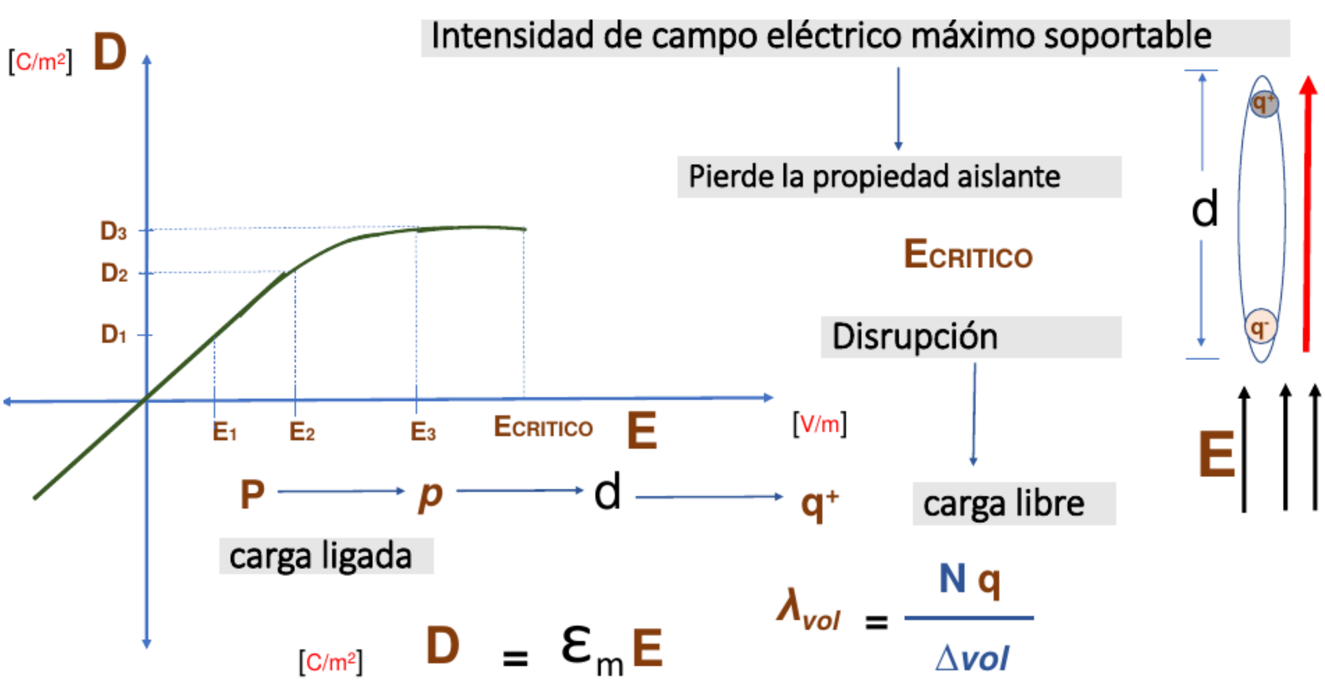

Figure 5.14:

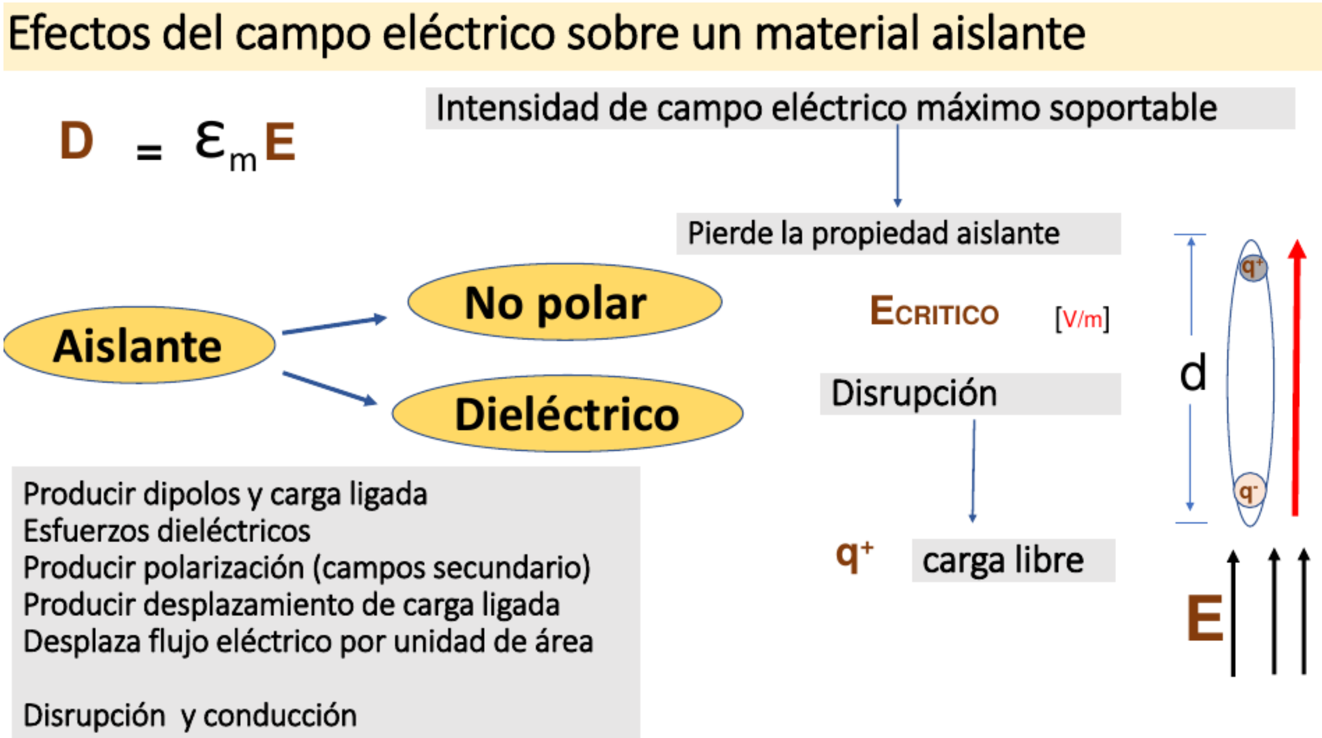
Efectos del campo eléctrico sobre un material aislante
D = εm E
Intensidad de campo eléctrico máximo soportable
Pierde la propiedad aislante
Aislante
No polar
Dieléctrico
ECRITICO
[V/m]
Disrupción
q+
carga libre
d
q+
q-
E
Producir dipolos y carga ligada
Esfuerzos dieléctricos
Producir polarización (campos secundario)
Producir desplazamiento de carga ligada
Desplaza flujo eléctrico por unidad de área
Disrupción y conducción

Figure 5.15:

Bibliography

[1] Fernando Augusto Herrera León. *Class's slides Electromagnetic Fields*. 2020.