Algebraic Properties of Complex Numbers

- $z = x + iy$ • $z^{-1} = (\frac{x}{x^2+y^2}, \frac{-y}{x^2+y^2})$ • $(z * z^{-1} = 1)$
- $\frac{z_1}{z_2} = z_1 z_2^{-1}$ • $(z_1 + z_2)^n = \sum_{k=0}^{n} \binom{n}{k} z_1^{n-k} z_2^k$

Modulus Properties

- $|z| = \sqrt{x^2 + y^2}$ • $|z|^2 = (Re(z))^2 + (Im(z))^2$ • $Re(z) \leq |Re(z)| \leq |z|$
- $|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ • $|z_1 + z_2| \leq |z_1| + |z_2|$
- $|z_1 z_2|^2 = (|z_1||z_2|)^2$ • $|z^n| = |z|^n$

Complex Conjugate Properties

- $\bar{z} = x - iy$ • $|\bar{z}| = |z|$ • $\overline{z_1 + z_2} = \bar{z_1} + \bar{z_2}$

Exponential and Polar Form

- $z = r(cos\theta + i * sin\theta)$ • $-\pi < Arg(z) \leq \pi$ • $arg(z) = artctan(\frac{y}{x})$
- $arg(z) = Arg(z) + 2n\pi$ where $(n = 0, \pm 1, \pm 2, ...)$
- Eulers Formula: $e^{i\theta} = cos(\theta) + i * sin(\theta)$ • $e^{i\pi} = -1$ • $e^{i\pi/2} = i$
- Exponential form: $z = re^{i\theta}$ • $z^{-1} = \frac{1}{re^{i\theta}} = \frac{1}{r}(cos(\theta) - i * sin(\theta))$
- $e^{i\theta_1} e^{i\theta_2} = e^{i(\theta_1 + \theta_2)}$ • $z_1 z_2 = (r_1 r_2) e^{i(\theta_1 + \theta_2)}$ • $z^n = r^n e^{in\theta}$
- De Moivre's Formula: $(cos(\theta) + i * sin(\theta))^n = cos(n\theta) + i * sin(n\theta)$
- $arg(\frac{z_1}{z_2}) = arg(z_1) - arg(z_2)$
- Roots: $z^{1/n} = (x + iy)^{1/n} = r^{1/n} e^{i(\frac{\theta + 2\pi * k}{n})}$ where k= 0, 1, ... ,n-1

Complex Functions

- $f(z) = u(x, y) + iv(x, y)$ • $f(x + iy) = u + iv$
- $\lim_{z \to z_0} f(z) = \omega_0$ For each positive number ϵ, there is a positive number δ such that $|f(z) - \omega_0| < \epsilon$ whenever $0 < |z - z_0| < \delta$
- $\lim_{z \to z_0} f(z) = \infty$ if $\lim_{z \to z_0} \frac{1}{f(z)} = 0$
- $\lim_{z \to \infty} f(z) = \omega_0$ if $\lim_{z \to 0} f(\frac{1}{z}) = \omega_0$ • $\lim_{z \to \infty} f(z) = \infty$ if $\lim_{z \to 0} \frac{1}{f(1/z)} = 0$
- $\frac{d}{dz} c = 0$ • $\frac{d}{dz} z = 1$ • $\frac{d}{dz} z^n = nz^{n-1}$
- $\frac{d}{dz}[f(z) + g(z)] = f'(z) + g'(z)$ • $\frac{d}{dz}[f(z)g(z)] = f(z)g'(z) + f'(z)g(z)$

- $\frac{d}{dz}[\frac{f(z)}{g(z)} = \frac{f'(z)g(z)-f(z)g'(z)}{[g(z)]^2}$
- $F(z) = g[f(z)] \to F'(z) = g'[f(z)]f'(z)$

Terminology

- **Neighborhood**: "Closeness" of two points. $|z - z_0| < \epsilon$ which consists of all points z lying inside a circle centered at z_0 with a radius of ϵ
- **Deleted Neighborhood**: $0 < |z - z_0| < \epsilon$. Same idea as neighborhood but center is not contained in the set either
- **Interior Point**: A point z_0 is interior of a set S whenever there is some neighborhood of z_0 containing only points of S
- **Exterior Point**: A point is exterior when there is no neighborhood that contains any points of S
- **Boundary Point**: A point is a boundary point when every neighborhood consists of at least one exterior and interior point
- **Open Set**: Does not contain any of its boundary points and all points are interior
- **Closed Set**: Contains all of its boundary points
- **Closure**: The closure of a set is the closed set consisting of all points in set S together with the boundary of S.
- **Connected Set**: An open set is connected if each pair of points can be joined by a polygonal line consisting of finite number of line segments joined end to end