HOMEWORK 2

Solve all of the following exercises. Please explain your work in a careful and logical way. **This assignment is due Friday, November 3rd 2023**.

Exercise 1

Use the Cauchy-Riemann equations to show that $f(z) = \exp(\bar{z})$ is not analytic anywhere.

Exercise 2

Show that $Log(i^3) \neq 3Log(i)$. Here recall that $Log(z)$ is the principal branch of the logarithm.

Exercise 3

Show that

$$\sin(z) = \sin(x+iy) = \sin(x)\cosh(y) + i\cos(x)\sinh(y).$$

Use this identity to prove that

$$|\sin(z)|^2 = \sin(x)^2 + \sinh(y)^2,$$

and to find the zeros of $\sin(z)$ in $\mathbb{C}$.

Exercise 4

Use the Cauchy-Riemann equations to show that neither $\sin(\bar{z})$ nor $\cos(\bar{z})$ is an analytic function of z anywhere.

Exercise 5

Solve Exercise number 8 page 108 in the textbook.

Exercise 6

Solve Exercise number 9 page 108 in the textbook.

Exercise 7

Prove the following identities

$$-i\sinh(iz) = \sin(z), \quad -i\sin(iz) = \sinh(z), \quad \cos(iz) = \cosh(z).$$

Exercise 8

Prove the following identities

- $\sinh(z) = \sinh(x)\cos(y) + i\cosh(x)\sin(y)$;
- $\cosh(z) = \cosh(x)\cos(y) + i\sinh(x)\sin(y)$;
- $|\sinh(z)|^2 = \sinh^2(x) + \sin^2(y)$;
- $|\cosh(z)|^2 = \sinh^2(x) + \cos^2(y)$

Exercise 9

Solve Exercise number 2 page 119 in the textbook.

Exercise 10

Solve Exercise number 3 page 119 in the textbook.