HOMEWORK 1

Solve all of the following exercises. Please explain your work in a careful and logical way. **This assignment is due Friday, September 22nd 2023**.

Exercise 1

Prove that

- z is real if and only if $\bar{z} = z$;
- z is either real or pure imaginary if and only if $\bar{z}^2 = z^2$.

Exercise 2

Use de Moivre's formula to derive the following identities

- $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$;
- $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$.

Exercise 3

Find the four solutions of the equation $z^4 - 1 = 0$.

Exercise 4

Sketch the following sets

- $|z - 1 - i| < 4$, $|z - 4| \leq 3$;
- $Im(z) > 1$, $Im(z) = 1$.

Finally, which of those sets are open?

Exercise 5

Let S be the set consisting of all points z such that $|z| < 1$ or $|z-2| < 1$. Prove that S is not connected.

Exercise 6

Find the domain of definition for the following functions

- $f(z) = \frac{1}{z^2+1}$;
- $f(z) = z^6 + z^3 + 100$.

Exercise 7

In each case, write the function $f(z)$ in the form $f(z) = u(x, y) + iv(x, y)$:

- $f(z) = \frac{1}{z}$;
- $f(z) = z^3 + z + 1$.

Exercise 8

Solve Exercise number 1 page 54 in the textbook.

Exercise 9

Solve Exercise number 11 page 55 in the textbook.

Exercise 10

Solve Exercise number 2 page 61 in the textbook.