Método del trapecio

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}(f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n))$$
$$x_i = x_0 + ih; \;\; i = 0, \ldots, n$$
$$h = \frac{x_n - x_0}{n}$$

Error de truncamiento

$$\epsilon = -\frac{(x_n - x_0)^3}{12n^2} f''(c)$$

n para un error dado

$$n = \text{ent}(\sqrt{\frac{(x_n - x_0)^3}{12\epsilon} f''(c)})$$
$$f''(c) = \max_{x_0 \leqslant x \leqslant x_n} |f''(x)|$$

Método de Simpson

$$\int_{x_o}^{x_n} f(x) = \frac{h}{3}(f(x_0) + 4\sum_{i=1}^{n/2} f(x_{2i-1}) + 2\sum_{i=1}^{n/2-1} f(x_{2i}) + f(x_n))$$
$$x_i = x_0 + ih; \;\; i = 0, \ldots, n$$
$$h = \frac{x_n - x_0}{n}$$

Error de truncamiento

$$\epsilon = -\frac{(x_n - x_0)^5}{180n^4} f^{(iv)}(c)$$

n para un error dado

$$n = \text{ent}(\sqrt[4]{\frac{(x_n - x_0)^5}{180\epsilon} f^{(iv)}(c)})$$
$$f^{(iv)}(c) = \max_{x_0 \leqslant x \leqslant x_n} |f^{(iv)}(x)|$$

Métodos de Runge-Kutta

Dado:

$$\frac{dy}{dx} = f(x, y)$$

Método de Euler

$$y_{i+1} = y_i + f(x_i, y_i)h$$
$$x_i = x_0 + ih$$

Método para series de Taylor

$$y_{i+1} = y_i + f(x_i, y_i)h + \frac{f'(x_i, y_i)}{2}h^2$$

$$f'(x_i, y_i) = \frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y}\frac{dx}{dy}$$

Método de Heun

$$y_{i+1} = y_i + \frac{h}{2}(f(x_i, y_i) + f(x_{i+1}, y_i) + hf(x_i, y_i)))$$
$$x_i = x_0 + ih$$

Método Runge-Kutta

$$y_{i+1} = y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x_i, y_i)$$
$$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h)$$
$$k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h)$$
$$k_4 = f(x_i + h, y_i + k_3 h)$$