Nombre: Julián Emiliano Diaz Buenrostro   EXAMEN UNIDAD II

Fecha: 31/05/2021   Hora: 16:00 hrs   Especialidad: Mecatrónica

1 Halle la solución general por el método de coeficientes indeterminados

a) $y'' - 3y' - 4y = -16x$

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda_{1,2} = \frac{3 \pm \sqrt{9 - 4(1)(-4)}}{2}$ $y = C_1 e^{4x} + C_2 e^{-x}$

$\lambda_{1,2} = \frac{3 \pm \sqrt{9+16}}{2}$ $g(x) = -16x \longrightarrow y_p = Ax + B$

$y'_p = A$

$\lambda_{1,2} = \frac{3 \pm \sqrt{25}}{2}$ $y''_p = 0$

$\lambda_{1,2} = \frac{3 \pm 5}{2}$

$0 - 3(A) - 4(Ax + B) = -16x$

$\lambda_1 = \frac{3+5}{2}$ $\lambda_2 = \frac{3-5}{2}$

$-3A - 4Ax - 4B = -16x$

$\lambda_1 = \frac{8}{2} = 4$ $\lambda_2 = \frac{-2}{2} = -1$

$-4Ax - 3A - 4B = -16x$

raíces reales y diferentes

$-4A = -16$ $-3\cancel{A}^{4} - 4B = 0$

$A = \frac{-16}{-4} = 4$ $-12 - 4B = 0$

$y = C_1 e^{4x} + C_2 e^{-x} + 4x - 3$ //

$-4B = 12$

$y_p = 4x - 3$ $B = \frac{12}{-4} = -3$

b) $y'' + 36y = 2\sin(6x) - 3\cos(6x)$

$\lambda^2 + 36 = 0$

$\sqrt{\lambda^2} = \sqrt{-36}$ $y = e^0(C_1\cos(6x) + C_2\operatorname{sen}(6x))$ $\beta = 6$

$\lambda = \pm 6i$ $y = C_1\cos(6x) + C_2\operatorname{sen}(6x)$ $b = 6$ $\therefore \beta = b$

$\alpha = 0 \quad \beta = 6$ $z = 1$

raíces complejas $g(x) = 2\sin(6x) - 3\cos(6x)$

$y_p = x(A\operatorname{sen}(6x) + B\cos(6x))$

$y'_p = \left[x(6A\cos(6x) - 6B\operatorname{sen}(6x)) + (A\operatorname{sen}(6x) + B\cos(6x))(1)\right]$

$= 6Ax\cos(6x) - 6Bx\operatorname{sen}(6x) + A\operatorname{sen}(6x) + B\cos(6x)$

$= (6Ax + B)\cos(6x) + (-6Bx + A)\operatorname{sen}(6x)$

$y''_p = \left[-6(6Ax + B)\operatorname{sen}(6x) + (6A)\cos(6x)\right]$

$+ \left[6(-6Bx + A)\cos(6x) + (-6B)\operatorname{sen}(6x)\right]$

$y''_p = (-36Ax - 6B)\operatorname{sen}(6x) + 6A\cos(6x) + (-36Bx + 6A)\cos(6x)$

$+ (-6B)\operatorname{sen}(6x)$

$= (-36Ax - 6B - 6B)\operatorname{sen}(6x) + (6A - 36Bx + 6A)\cos(6x)$

$(-36Ax-12B)\,sen(6x) + (-36Bx+12A)\cos(6x)$

$(-36Ax-12B)\,sen(6x) + (-36Bx+12A)\cos(6x) +$

$36\left[Ax\,sen(6x) + Bx\cos(6x)\right] = 2\sin(6x) - 3\cos(6x)$

$(-3Ax-12B+36Ax)\,sen(6x) + (-36Bx+12A+36Bx)\cos(6x)$
$= 2\,sen(6x) - 3\cos(6x)$

$-12B\,sen(6x) + 12A\cos(6x) = 2sen(6x) - 3\cos(6x)$

$-12B = 2 \qquad 12A = -3$

$B = \frac{2}{-12} \qquad A = -\frac{3}{12}$

$B = -\frac{1}{6} \qquad A = -\frac{1}{4}$

$y = C_1\cos(6x) + C_2 sen(6x) + x\left[-\frac{1}{4}sen(6x) - \frac{1}{6}\cos(6x)\right]$

2. Encuentre la solución general por el método de variación de parámetros.

a) $y'' + y = 4x\cos(x)$

$\lambda^2 + 1 = 0 \qquad y = e^{0x}(C_1\cos(x) + C_2 sen(x))$

$\sqrt{\lambda^2} = \sqrt{-1} \qquad y_h = C_1\cos(x) + C_2 sen(x)$

$\lambda = \pm i$

$\alpha = 0 \quad \beta = 1 \qquad \underset{y_1}{\uparrow} \qquad \underset{y_2}{\uparrow}$

$$w(y_1, y_2) = \begin{vmatrix} \cos(x) & sen(x) \\ -sen(x) & \cos(x) \end{vmatrix} = \cos^2(x) + sen^2(x) = 1$$

$$u(x) = -\int \frac{4x\cos(x)(sen(x))}{1}dx = -4\int x\cos(x)\,sen(x)$$

$u = x \qquad dv = \cos(x)\,sen(x)$

$du = 1 \qquad v = \int \cos(x)\,sen(x)\,dx$

$v = \int u\cos(x)\frac{1}{\cos(x)}du \qquad u = sen(x)$

$du = \cos(x)\,dx$

$v = \int u\,du = \frac{u^2}{2} \qquad dx = \frac{1}{\cos(x)}du$

$= \frac{sen^2(x)}{2}$

$$= -4\left[\frac{x\,sen^2(x)}{2} - \int \frac{sen^2(x)}{2}\,1\right]$$

Escaneado con CamScanner

$$-\frac{1}{2}\int sen^2(x)$$

$$-\frac{1}{2}\int \frac{1-\cos(2x)}{2} = -\frac{1}{2}\left[\int \frac{1}{2}dx - \int \cos(2x)\,dx\right]$$

$$= -\frac{1}{2}\left[\frac{1}{2}\int dx - \frac{1}{2}\int \cos(2x)\,2\,dx\right]$$

$$= -\frac{1}{2}\left[\frac{1}{2}x - \frac{sen(2x)}{2}\right] = -\frac{x}{4} + \frac{sen(2x)}{4}$$

$$U(x) = -4\left[\frac{x\,sen^2(x)}{2} - \frac{x}{4} + \frac{sen(2x)}{4}\right]$$

$$U(x) = -2x\,sen^2(x) + x - sen(2x)$$

$$V(x) = \int \frac{4x\cos(x)(\cos(x)}{1}\,dx = 4\int x\cos^2(x)\,dx$$

$$= 4\int x\left[\frac{1+\cos(2x)}{2}\right]dx$$

$$= 4\int \frac{x}{2} + \frac{x\cos(2x)}{2}$$

$$= 4\left[\frac{1}{2}\int x\,dx + \frac{1}{2}\int x\cos(2x)\right] \quad \begin{matrix} u = x & dv = \cos(2x) \\ du = 1 & v = \frac{1}{2}\int \cos(2x)(2)\,dx = \frac{sen(2x)}{2} \end{matrix}$$

$$= 4\left[\frac{1}{2}\left[\frac{x^2}{2}\right] + \frac{1}{2}\left[\frac{x\,sen(2x)}{2} - \int \frac{sen(2x)}{2}(1)\right]\right]$$

$$= 4\left[\frac{x^2}{4} + \frac{1}{2}\left[\frac{x\,sen(2x)}{2} - \frac{1}{2}\left[\frac{1}{2}\right]\int sen(2x)(2)\,dx\right]\right]$$

$$= 4\left[\frac{x^2}{4} + \frac{1}{2}\left[\frac{x\,sen(2x)}{2} - \frac{1}{4}(-\cos(2x))\right]\right]$$

$$= 4\left[\frac{x^2}{4} + \frac{x\,sen(2x)}{4} + \frac{\cos(2x)}{4}\right]$$

$$V(x) = x^2 + x\,sen(2x) + \cos(2x)$$

$$y_p = [-2x\,sen^2(x) + x - sen(2x)]\cos(x) + [x^2 + x\,sen(2x) + \cos(2x)]\,sen(x)$$

$$y = C_1\cos(x) + C_2\,sen(x) + [-2x\,sen^2(x) + x - sen(2x)]\cos(x) + [x^2 + x\,sen(2x) + \cos(2x)]\,sen(x)$$

b) $x^2y'' - xy' + y = 4x^3$

$a_2 = 1 \quad a_1 = -1 \quad a_0 = 1$

$m^2 + (-1-1)m + 1 = 0$

$m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0$

$m = 1$

raices reales e iguales

$y_h = C_1 x + C_2 x \ln(x)$

$(x^2y'' - xy' + y = 4x^3)\frac{1}{x^2}$

$y'' - \frac{y'}{x} + \frac{y}{x^2} = 4x$

$$\omega(y_1, y_2) = \begin{vmatrix} x & x\ln(x) \\ 1 & x\left[\frac{1}{\cancel{x}}\right] + \ln(x) \end{vmatrix} = x + x\ln(x) - x\ln(x) = x$$

$$u(x) = -\int \frac{4\cancel{x}(x\ln(x))}{\cancel{x}}dx = \int 4x\ln(x)dx = 4\int x\ln(x)$$

$u = \ln(x) \quad dv = x$

$du = \frac{1}{x} \quad v = \frac{x^2}{2}$

$$= 4\left[\frac{\ln(x)x^2}{2} - \int \frac{x^2}{2}\left[\frac{1}{x}\right]\right]$$

$$= 4\left[\frac{\ln(x)x^2}{2} - \int \frac{x}{2}\right]$$

$$= 4\left[\frac{\ln(x)x^2}{2} - \frac{1}{2}\int x dx\right]$$

$$-\frac{1}{2}\left[\frac{x^2}{2}\right] \doteq 4\left[\frac{x^2\ln(x)}{2} - \frac{x^2}{4}\right]$$

$$= -2x^2\ln(x) + x^2$$

$$v(x) = \int \frac{4x(x)}{x}dx = 4\int x dx = 4\left[\frac{x^2}{2}\right] = 2x^2$$

$$y_p = \left[-2x^2\ln(x) + x^2\right]x + \left[2x^2\right]x\ln(x)$$

$$y = C_1x + C_2x\ln(x) + \left[-2x^2\ln(x) + x^2\right]x + \left[2x^2\right]x\ln(x)$$