Privilege separation in browser architectures

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ABSTRACT

In many software systems as modern web browsers the user and his sensitive data often interact with the untrusted outer world. This scenario can pose a serious threat to the user's private data and gives new relevance to an old story in computer science: providing controlled access to untrusted components, while preserving usability and ease of interaction. To address the threats of untrusted components, modern web browsers propose privilege-separated architectures, which isolate components that manage critical tasks and data from components which handle untrusted inputs. The former components are given strong permissions, possibly coinciding with the full set of permissions granted to the user, while the untrusted components are granted only limited privileges, to limit possible malicious behaviours: all the interactions between trusted and untrusted components is handled via message passing. In this thesis we introduce a formal semantics for privilege-separated architectures and we provide a general definition of privilege separation: we discuss how different privilege-separated architectures can be evaluated in our framework, identifying how different security threats can be avoided, mitigated or disregarded. Specifically, we evaluate in detail the existing Google Chrome Extension Architecture in our formal model and we discuss how its design can mitigate serious security risks, with only limited impact on the user experience.

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$$
\begin{array}{lcl}
\textit{Abstract cache} & \hat{C} & : & \mathcal{L} \to \hat{V} \\
\textit{Abstract variable environment} & \hat{\Gamma} & : & \mathcal{V} \to \hat{V} \\
\textit{Abstract memory} & \hat{\mu} & : & \mathcal{L} \times \mathcal{P} \to \hat{V} \\
\textit{Abstract permission cache} & \hat{P} & : & \mathcal{L} \to \mathcal{P}
\end{array}
$$

[*PE-Val*] $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} c : \hat{v} \text{ iff } \{d_c\} \subseteq \hat{v}$

[*PE-Var*] $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} x : \hat{v} \text{ iff } \hat{\Gamma}(x) \subseteq \hat{v}$

[*PE-Lambda*] $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \lambda x.e : \hat{v} \text{ iff } \{\lambda x.e\} \subseteq \hat{v}$

[*PE-Obj*]
$$\begin{aligned}
&(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \{\overrightarrow{str_i : e_i}\} : \hat{v} \gg \rho \text{ iff}\\
&\quad \forall i : (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_i : \hat{v}_i \gg \rho_i \wedge\\
&\qquad \{\overrightarrow{str_i : \hat{v}_i}\} \subseteq \hat{v} \wedge\\
&\qquad \rho_i \sqsubseteq \rho
\end{aligned}$$

[*PE-Let*]
$$\begin{aligned}
&(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \textbf{let } \overrightarrow{x_i = e_i} \textbf{ in } e' : \hat{v} \gg \rho \text{ iff}\\
&\quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e' : \hat{v} \gg \rho' \wedge\\
&\quad \rho' \sqsubseteq \rho \wedge\\
&\quad \forall i :\\
&\qquad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_i : \hat{v}_i \gg \rho_i \wedge\\
&\qquad \hat{v}_i \subseteq \hat{\Gamma}(x_i) \wedge\\
&\qquad \rho_i \sqsubseteq \rho
\end{aligned}$$

[*PE-App*]
$$\begin{aligned}
&(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1\, e_2 : \hat{v} \gg \rho \text{ iff}\\
&\quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge\\
&\quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge\\
&\quad \rho_1 \sqsubseteq \rho \wedge\\
&\quad \rho_2 \sqsubseteq \rho \wedge\\
&\quad \forall (\lambda x.e_0) \in \hat{v}_1 :\\
&\qquad \hat{v}_2 \subseteq \hat{\Gamma}(x) \wedge\\
&\qquad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_0 : \hat{v}_0 \gg \rho_0 \wedge\\
&\qquad \rho_0 \sqsubseteq \rho \wedge\\
&\qquad \hat{v}_0 \subseteq \hat{v}
\end{aligned}$$

[*PE-Op*]
$$\begin{aligned}
&(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} op(\overrightarrow{e_i}) : \hat{v} \gg \rho \text{ iff}\\
&\quad \forall i :\\
&\qquad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_i : \hat{v}_i \gg \rho_i \wedge\\
&\qquad \rho_i \sqsubseteq \rho \wedge\\
&\quad \widehat{op}(\overrightarrow{\hat{v}_i}) \subseteq \hat{v}
\end{aligned}$$

[*PE-Cond*]
$$\begin{aligned}
&(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \textbf{if } (e_0)\ \{\ e_1\ \} \textbf{ else } \{\ e_2\ \} : \hat{v} \gg \rho \text{ iff}\\
&\quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_0 : \hat{v}_0 \gg \rho_0 \wedge\\
&\quad \rho_0 \sqsubseteq \rho \wedge\\
&\quad \widehat{\textbf{true}} \in \hat{v}_0 \Rightarrow\\
&\qquad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge \hat{v}_1 \subseteq \hat{v} \wedge \rho_1 \sqsubseteq \rho \wedge\\
&\quad \widehat{\textbf{false}} \in \hat{v}_0 \Rightarrow\\
&\qquad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge \hat{v}_2 \subseteq \hat{v} \wedge \rho_2 \sqsubseteq \rho
\end{aligned}$$

[*PE-While*]
$$\begin{aligned}
&(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \textbf{while } (e_1)\ \{\ e_2\ \} : \hat{v} \gg \rho \text{ iff}\\
&\quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge\\
&\quad \rho_1 \sqsubseteq \rho \wedge\\
&\quad \widehat{\textbf{true}} \in \hat{v}_1 \Rightarrow\\
&\qquad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge \hat{v}_2 \subseteq \hat{v} \wedge \rho_2 \sqsubseteq \rho \wedge\\
&\quad \widehat{\textbf{false}} \in \hat{v}_1 \Rightarrow\\
&\qquad \widehat{\textbf{undefined}} \subseteq \hat{v}
\end{aligned}$$

[*PE-GetField*]
$$\begin{aligned}
&(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1[e_2] : \hat{v} \gg \rho \text{ iff}\\
&\quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge\\
&\quad \rho_1 \sqsubseteq \rho \wedge\\
&\quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge\\
&\quad \rho_2 \sqsubseteq \rho \wedge\\
&\quad \widehat{get}(\hat{v}_1, \hat{v}_2) \subseteq \hat{v}
\end{aligned}$$

$$
\begin{array}{ll}
[\textit{PE-SetField}] & (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_0[e_1] = e2 : \hat{v} \gg \rho \text{ iff} \\
& \quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_0 : \hat{v}_0 \gg \rho_0 \wedge \\
& \quad \rho_0 \sqsubseteq \rho \wedge \\
& \quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge \\
& \quad \rho_1 \sqsubseteq \rho \wedge \\
& \quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge \\
& \quad \rho_2 \sqsubseteq \rho \wedge \\
& \quad \widehat{set}(\hat{v}_0, \hat{v}_1, \hat{v}_2) \subseteq \hat{v} \\
[\textit{PE-DelField}] & (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \textbf{delete } e_1[e_2] : \hat{v} \gg \rho \text{ iff} \\
& \quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge \\
& \quad \rho_1 \sqsubseteq \rho \wedge \\
& \quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge \\
& \quad \rho_2 \sqsubseteq \rho \wedge \\
& \quad \widehat{del}(\hat{v}_1, \hat{v}_2) \subseteq \hat{v} \\
[\textit{PE-Ref}] & (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \textbf{ref}_{r,\rho_r}\ e : \{r\} \gg \rho \text{ iff} \\
& \quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e : \hat{v} \gg \rho \wedge \\
& \quad \rho_r \sqsubseteq \rho_s \Rightarrow \hat{v} \subseteq \hat{\mu}(r, \rho_r) \\
[\textit{PE-DeRef}] & (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \textbf{deref } e : \hat{v} \gg \rho \text{ iff} \\
& \quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e : \hat{v}_1 \gg \rho_1 \wedge \\
& \quad \rho_1 \sqsubseteq \rho \wedge \\
& \quad \forall r \in \hat{v}_1 : \forall \rho_r \sqsubseteq \rho_s : \hat{\mu}(r, \rho_r) \subseteq \hat{v} \\
[\textit{PE-SetRef}] & (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 = e_2 : \hat{v} \gg \rho \text{ iff} \\
& \quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e : \hat{v}_1 \gg \rho_1 \wedge \\
& \quad \rho_1 \sqsubseteq \rho \wedge \\
& \quad (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge \\
& \quad \rho_2 \sqsubseteq \rho \wedge \\
& \quad \forall r \in \hat{v}_1 : \forall \rho_r \sqsubseteq \rho_s : \\
& \quad\quad \hat{v}_2 \subseteq \hat{\mu}(r, \rho_r) \wedge \\
& \quad\quad \hat{v}_2 \subseteq \hat{v} \\
[\textit{PE-Send}] & \ldots \\
[\textit{PE-Err}] & \ldots \\
[\textit{PE-Exercise}] & \ldots
\end{array}
$$

3.4 Compositional Verbose

[*CV-Val*] $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (c)^{\ell}$ iff $\{d_c\} \subseteq \hat{C}(\ell)$

[*CV-Var*] $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (x)^{\ell}$ iff $\hat{\Gamma}(x) \subseteq \hat{C}(\ell)$

[*CV-Lambda*] $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\lambda x.{e_0}^{\ell_0})^{\ell}$ iff

$$
\begin{aligned}
& \{\lambda x.{e_0}^{\ell_0}\} \subseteq \hat{C}(\ell) \wedge \\
& (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e_0}^{\ell_0}
\end{aligned}
$$

[*CV-Obj*] $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\{\overrightarrow{str_i : {e_i}^{\ell_i}}\})^{\ell}$ iff

$$
\begin{aligned}
& \forall i : \\
& \quad (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e_i}^{\ell_i} \wedge \\
& \quad \hat{P}(\ell_i) \sqsubseteq \hat{P}(\ell) \wedge \\
& \{\overrightarrow{str_i : \hat{C}(\ell_i)}\} \subseteq \hat{C}_{\ell}
\end{aligned}
$$

[*CV-Let*] $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\textbf{let } \overrightarrow{x_i = {e_i}^{\ell_i}} \textbf{ in } {e'}^{\ell'})^{\ell}$ iff

$$
\begin{aligned}
& (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e'}^{\ell'} \wedge \\
& \hat{P}(\ell') \sqsubseteq \hat{P}(\ell) \wedge \\
& \hat{C}(\ell') \subseteq \hat{C}(\ell) \wedge \\
& \forall i : \\
& \quad (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e_i}^{\ell_i} \wedge \\
& \quad \hat{C}(\ell_i) \subseteq \hat{\Gamma}(x_i) \wedge \\
& \quad \hat{P}(\ell_i) \sqsubseteq \hat{P}(\ell)
\end{aligned}
$$

[*CV-App*] $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} ({e_1}^{\ell_1}\, {e_2}^{\ell_2})^{\ell}$ iff

$$
\begin{aligned}
& (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e_1}^{\ell_1} \wedge (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e_2}^{\ell_2} \wedge \\
& \hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge \hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell) \\
& \forall (\lambda x.{e_0}^{\ell_0}) \in \hat{C}(\ell_1) : \\
& \quad \hat{C}(\ell_2) \subseteq \hat{\Gamma}(x) \wedge \hat{C}(\ell_0) \subseteq \hat{C}(\ell) \wedge \\
& \quad \hat{P}(\ell_0) \sqsubseteq \hat{P}(\ell)
\end{aligned}
$$

[*CV-Op*] $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (op(\overrightarrow{{e_i}^{\ell_i}}))^{\ell}$ iff

$$
\begin{aligned}
& \forall i : \\
& \quad (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e_i}^{\ell_i} \wedge \\
& \quad \hat{P}(\ell_i) \sqsubseteq \hat{P}(\ell) \wedge \\
& \widehat{op}(\hat{C}(\ell_i)) \subseteq \hat{C}(\ell)
\end{aligned}
$$

[*CV-Cond*] $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\textbf{if } ({e_0}^{\ell_0}) \ \{\ {e_1}^{\ell_1}\ \} \textbf{ else } \{\ {e_2}^{\ell_2}\ \})^{\ell}$ iff

$$
\begin{aligned}
& (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e_0}^{\ell_0} \wedge \\
& \hat{P}(\ell_0) \sqsubseteq \hat{P}(\ell) \wedge \\
& \widehat{\mathbf{true}} \in \hat{C}(\ell_0) \Rightarrow \\
& \quad (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e_1}^{\ell_1} \wedge \hat{C}(\ell_1) \subseteq \hat{C}(\ell) \wedge \\
& \quad \hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge \\
& \widehat{\mathbf{false}} \in \hat{C}(\ell_0) \Rightarrow \\
& \quad (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e_2}^{\ell_2} \wedge \hat{C}(\ell_2) \subseteq \hat{C}(\ell) \wedge \\
& \quad \hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell)
\end{aligned}
$$

[*CV-While*] $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\textbf{while } ({e_1}^{\ell_1}) \ \{\ {e_2}^{\ell_2}\ \})^{\ell}$ iff

$$
\begin{aligned}
& (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e_1}^{\ell_1} \wedge \\
& \hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge \\
& \widehat{\mathbf{true}} \in \hat{C}(\ell_1) \Rightarrow \\
& \quad (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} {e_2}^{\ell_2} \wedge \hat{C}(\ell_2) \subseteq \hat{C}(\ell) \wedge \\
& \quad \hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell) \wedge \\
& \widehat{\mathbf{false}} \in \hat{C}(\ell_1) \Rightarrow \widehat{\mathbf{undefined}} \subseteq \hat{C}(\ell)
\end{aligned}
$$

$$
\begin{array}{ll}
[\textit{CV-GetField}] & (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} ({e_1}^{\ell_1}[{e_2}^{\ell_2}])^{\ell} \text{ iff} \\
& \quad (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} {e_1}^{\ell_1} \wedge \\
& \quad \hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge \\
& \quad (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} {e_2}^{\ell_2} \wedge \\
& \quad \hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell) \wedge \\
& \quad \widehat{get}(\hat{C}(\ell_1), \hat{C}(\ell_2)) \subseteq \hat{C}(\ell) \\
[\textit{CV-SetField}] & (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} ({e_0}^{\ell_0}[{e_1}^{\ell_1}] = {e_2}^{\ell_2})^{\ell} \text{ iff} \\
& \quad (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} {e_0}^{\ell_0} \wedge \\
& \quad \hat{P}(\ell_0) \sqsubseteq \hat{P}(\ell) \wedge \\
& \quad (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} {e_1}^{\ell_1} \wedge \\
& \quad \hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge \\
& \quad (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} {e_2}^{\ell_2} \wedge \\
& \quad \hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell) \wedge \\
& \quad \widehat{set}(\hat{C}(\ell_0), \hat{C}(\ell_1), \hat{C}(\ell_2)) \subseteq \hat{C}(\ell) \\
[\textit{CV-DelField}] & (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} (\mathbf{delete}\ {e_1}^{\ell_1}[{e_2}^{\ell_2}])^{\ell} \text{ iff} \\
& \quad (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} {e_1}^{\ell_1} \wedge \\
& \quad \hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge \\
& \quad (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} {e_2}^{\ell_2} \wedge \\
& \quad \hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell) \wedge \\
& \quad \widehat{del}(\hat{C}(\ell_1), \hat{C}(\ell_2)) \subseteq \hat{C}(\ell) \\
[\textit{CV-Ref}] & (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} (\mathbf{ref}_{r,\rho_r}\ {e_1}^{\ell_1})^{\ell} \text{ iff} \\
& \quad (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} {e_1}^{\ell_1} \wedge \\
& \quad \{r\} \subseteq \hat{C}(\ell) \wedge \\
& \quad \hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge \\
& \quad \rho_r \sqsubseteq \rho_s \Rightarrow \hat{C}(\ell_1) \subseteq \hat{\mu}(r, \rho_r) \\
[\textit{CV-DeRef}] & (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} (\mathbf{deref}\ {e_1}^{\ell_1})^{\ell} \text{ iff} \\
& \quad (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} {e_1}^{\ell_1} \wedge \\
& \quad \hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge \\
& \quad \forall r \in \hat{C}(\ell_1) : \forall \rho_r \sqsubseteq \rho_s : \\
& \quad\quad \hat{\mu}(r, \rho_r) \subseteq \hat{C}(\ell) \\
[\textit{CV-SetRef}] & (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} ({e_1}^{\ell_1} = {e_2}^{\ell_2})^{\ell} \text{ iff} \\
& \quad (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} {e_1}^{\ell_1} \wedge \\
& \quad \hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge \\
& \quad (\hat{C},\hat{\Gamma},\hat{\mu},\hat{P}) \models_{c\rho_s} {e_2}^{\ell_2} \wedge \\
& \quad \hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell) \wedge \\
& \quad \forall r \in \hat{C}(\ell_1) : \forall \rho_r \sqsubseteq \rho_s : \\
& \quad\quad \hat{C}(\ell_2) \subseteq \hat{\mu}(r, \rho_r) \wedge \\
& \quad \hat{C}(\ell_2) \subseteq \hat{C}(\ell) \\
[\textit{PE-Send}] & \ldots \\
[\textit{PE-Err}] & \ldots \\
[\textit{PE-Exercise}] & \ldots
\end{array}
$$

3.5 Theorem

3.6 Requirements for correctness

4.1 Abstract domains choice

$R_1 = \{\overrightarrow{\widehat{str_i} : \widehat{v_i}}\} \sqsubseteq \{\overrightarrow{\widehat{str_j} : \widehat{v_j}}\} = R_2$ sse:

1. R_1 ha meno campi di R_2
2. ogni campo di R_1 e' piu' preciso del **corrispondente** campo di R_2

$\forall i, \exists j : \widehat{str_i} \sqsubseteq \widehat{str_j}$
$\forall i, \exists j : \widehat{str_i} \sqsubseteq \widehat{str_j} \Rightarrow \widehat{v_i} \sqsubseteq \widehat{v_j}$
Set:

- Exact
  - $\exists \rightarrow Union$
  - $\nexists \rightarrow addinprefix$
- Prefix
  - aggiungo in $*$

$\hat{v} \sqsubseteq \hat{v}'$ iff $\forall \widehat{u}_i \in \hat{v}, \exists \widehat{u}_j \in \hat{v}' : \widehat{u}_i \sqsubseteq \widehat{u}_j$.
If Galois connection then
$\hat{v} \sqsubseteq \hat{v}'$ iff $\gamma(\hat{v}) \subseteq \gamma(\hat{v}')$
where $\gamma : \widehat{V} \rightarrow P(V)$ is the concretisation function.
$\gamma_p : \widehat{PV} \rightarrow P(V)$
$\gamma(\hat{v}) = \bigcup_{\widehat{u}_i \in \hat{v}} \gamma_p(\widehat{u}_i)$

$\widehat{pre_{bool}} = \widehat{true} | \widehat{false}$

$\widehat{u_{bool}} = \{\overrightarrow{\widehat{pre_{bool}}}\}$ with $\sqsubseteq = \subseteq$

$\widehat{pre_{int}} = \oplus | 0 | \ominus$

$\widehat{u_{int}} = \{\overrightarrow{\widehat{pre_{int}}}\}$ with $\sqsubseteq = \subseteq$

$\widehat{pre_{string}} = s | s*$

$\widehat{u_{string}} = \{\overrightarrow{\widehat{pre_{string}}}\}$ with $\sqsubseteq = \subseteq$

— Giulia's spec. is more tricky than $\subseteq$

$\widehat{pre_{ref}} = r$

$\widehat{u_{ref}} = \{\overrightarrow{\widehat{pre_{ref}}}\}$ with $\sqsubseteq = \subseteq$

$\widehat{pre_{\lambda}} = \lambda$

$\widehat{u_{\lambda}} = \{\overrightarrow{\widehat{pre_{\lambda}}}\}$ with $\sqsubseteq = \subseteq$

$\widehat{pre_{rec}} = \{\overrightarrow{\widehat{str}_i : \hat{v}_i}\}$

$\widehat{u_{rec}} = \widehat{pre_{rec}}$ with $\sqsubseteq = \widehat{u_{rec}}_{\sqsubseteq}$

$\hat{v} = (\widehat{u_{bool}}, \widehat{u_{int}}, \widehat{u_{string}}, \widehat{u_{ref}}, \widehat{u_{\lambda}}, \widehat{u_{rec}}, \{\widehat{Null}\}, \{\widehat{Undef}\})$

with $\hat{v} \sqsubseteq \hat{v}'$ iff

$\widehat{u_{bool}} \sqsubseteq \widehat{u_{bool}}' \wedge$
$\widehat{u_{int}} \sqsubseteq \widehat{u_{int}}' \wedge$
$\widehat{u_{string}} \sqsubseteq \widehat{u_{string}}' \wedge$
$\widehat{u_{ref}} \sqsubseteq \widehat{u_{ref}}' \wedge$
$\widehat{u_{\lambda}} \sqsubseteq \widehat{u_{\lambda}}' \wedge$
$\widehat{u_{rec}} \sqsubseteq \widehat{u_{rec}}' \wedge$
$\widehat{Null} \notin \hat{v}' \vee \widehat{Null} \in \hat{v} \wedge \widehat{Null} \in \hat{v}' \wedge$
$\widehat{Undef} \notin \hat{v}' \vee \widehat{Undef} \in \hat{v} \wedge \widehat{Undef} \in \hat{v}'$

4.2 Abstract operations

4.3 Requirements verification

5. IMPLEMENTATION

5.1 Constraint generation

Constraint elements: E.

$$
\begin{array}{lll}
\textit{Cache element} & \mathsf{C}(\ell) & : \ \mathcal{L} \to \hat{V} \\
\textit{Var element} & \Gamma(x) & : \ \mathcal{V} \to \hat{V} \\
\textit{State element} & \mathsf{M}(\mathcal{P}, ref) & : \ \mathcal{L} \times \mathcal{P} \to \hat{V}
\end{array}
$$

Permission Element: $\mathsf{P}(\ell) : \mathcal{L} \to \mathcal{P}$

Constraint form.

$$
\begin{array}{lrcl}
\textit{Term inclusion} & \{\hat{v}\} & \subseteq & \mathsf{E} \\
\textit{Element inclusion} & \mathsf{E} & \subseteq & \mathsf{E} \\
\textit{Permission inclusion} & \mathsf{P}(\ell) & \sqsubseteq & \mathsf{P}(\ell') \\
\textit{Operation} & \widehat{Op}(\overrightarrow{\mathsf{E}_i}) & \subseteq & \mathsf{E} \\
\textit{Implication} & \{\hat{v}\} \subseteq \mathsf{E} & \Rightarrow & \mathsf{E} \subseteq \mathsf{E}
\end{array}
$$

Misc:

r_*	is the set of all references of the program;
$lambda_*$	is the set of all lambdas of the program;

$$
\begin{array}{ll}
[\mathit{CG\text{-}Val}] & \mathcal{C}_{*\rho_s}[\![(c)^\ell]\!] = \{d_c\} \subseteq \mathsf{C}(\ell) \\
[\mathit{CG\text{-}Var}] & \mathcal{C}_{*\rho_s}[\![(x)^\ell]\!] = \mathsf{\Gamma}(x) \subseteq \mathsf{C}(\ell) \\
[\mathit{CG\text{-}Lambda}] & \mathcal{C}_{*\rho_s}[\![(\lambda x.{e_0}^{\ell_0})^\ell]\!] = \\
& \quad \{\{\lambda x.{e_0}^{\ell_0}\} \subseteq \mathsf{C}(\ell)\} \cup \\
& \quad \mathcal{C}_{*\rho_s}[\![({e_0}^{\ell_0})]\!] \\
[\mathit{CG\text{-}Obj}] & \mathcal{C}_{*\rho_s}[\![(\{\overrightarrow{str_i : {e_i}^{\ell_i}}\})^\ell]\!] = \\
& \quad \bigcup_i (\mathcal{C}_{*\rho_s}[\![({e_i}^{\ell_i})]\!] \cup \\
& \quad\quad \{\mathsf{P}(\ell_i) \sqsubseteq \mathsf{P}(\ell)\}) \cup \\
& \quad \{\{\overrightarrow{str_i : \mathsf{C}(\ell_i)}\} \subseteq \mathsf{C}(\ell)\} \\
[\mathit{CG\text{-}Let}] & \mathcal{C}_{*\rho_s}[\![(\mathbf{let}\ \overrightarrow{x_i = {e_i}^{\ell_i}}\ \mathbf{in}\ {e'}^{\ell'})^\ell]\!] = \\
& \quad \bigcup_i (\mathcal{C}_{*\rho_s}[\![({e_i}^{\ell_i})]\!] \cup \\
& \quad\quad \{\mathsf{C}(\ell_i) \subseteq \mathsf{\Gamma}(x_i)\} \cup \\
& \quad\quad \{\mathsf{P}(\ell_i) \subseteq \mathsf{P}(\ell)\}) \cup \\
& \quad \mathcal{C}_{*\rho_s}[\![({e'}^{\ell'})]\!] \cup \\
& \quad \{\mathsf{P}(\ell') \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{\mathsf{C}(\ell') \subseteq \mathsf{C}(\ell)\} \\
[\mathit{CG\text{-}App}] & \mathcal{C}_{*\rho_s}[\![({e_1}^{\ell_1}\, {e_2}^{\ell_2})^\ell]\!] = \\
& \quad \mathcal{C}_{*\rho_s}[\![({e_1}^{\ell_1})]\!] \cup \mathcal{C}_{*\rho_s}[\![({e_2}^{\ell_2})]\!] \cup \\
& \quad \{\mathsf{P}(\ell_1) \sqsubseteq \mathsf{P}(\ell)\} \cup \{\mathsf{P}(\ell_2) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{\{t\} \subseteq \mathsf{C}(\ell_1) \Rightarrow \mathsf{C}(\ell_2) \subseteq \mathsf{\Gamma}(x) \\
& \quad\quad | t = (\lambda x.{e_0}^{\ell_0}) \in \mathit{lambda}_*\} \cup \\
& \quad \{\{t\} \subseteq \mathsf{C}(\ell_1) \Rightarrow \mathsf{C}(\ell_0) \subseteq \mathsf{C}(\ell) \\
& \quad\quad | t = (\lambda x.{e_0}^{\ell_0}) \in \mathit{lambda}_*\} \cup \\
& \quad \{\{t\} \subseteq \mathsf{C}(\ell_1) \Rightarrow \mathsf{P}(\ell_0) \sqsubseteq \mathsf{P}(\ell) \\
& \quad\quad | t = (\lambda x.{e_0}^{\ell_0}) \in \mathit{lambda}_*\} \cup \\
[\mathit{CG\text{-}Op}] & \mathcal{C}_{*\rho_s}[\![(op(\overrightarrow{{e_i}^{\ell_i}}))^\ell]\!] = \\
& \quad \bigcup_i (\mathcal{C}_{*\rho_s}[\![({e_i}^{\ell_i})]\!] \cup \{\mathsf{P}(\ell_i) \sqsubseteq \mathsf{P}(\ell)\}) \cup \\
& \quad \{\widehat{op}(\mathsf{C}(\ell_i)) \subseteq \mathsf{C}(\ell)\} \\
[\mathit{CG\text{-}Cond}] & \mathcal{C}_{*\rho_s}[\![(\mathbf{if}\ ({e_0}^{\ell_0})\ \{\ {e_1}^{\ell_1}\ \}\ \mathbf{else}\ \{\ {e_2}^{\ell_2}\ \})^\ell]\!] = \\
& \quad \mathcal{C}_{*\rho_s}[\![({e_0}^{\ell_0})]\!] \cup \mathcal{C}_{*\rho_s}[\![({e_1}^{\ell_1})]\!] \cup \mathcal{C}_{*\rho_s}[\![({e_2}^{\ell_2})]\!] \cup \\
& \quad \{\hat{P}(\ell_0) \sqsubseteq \hat{P}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{true}} \in \mathsf{C}(\ell_0) \Rightarrow \mathsf{C}(\ell_1) \subseteq \mathsf{C}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{true}} \in \mathsf{C}(\ell_0) \Rightarrow \mathsf{P}(\ell_1) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{false}} \in \mathsf{C}(\ell_0) \Rightarrow \mathsf{C}(\ell_2) \subseteq \mathsf{C}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{false}} \in \mathsf{C}(\ell_0) \Rightarrow \mathsf{P}(\ell_2) \sqsubseteq \mathsf{P}(\ell)\} \\
[\mathit{CG\text{-}While}] & \mathcal{C}_{*\rho_s}[\![(\mathbf{while}\ ({e_1}^{\ell_1})\ \{\ {e_2}^{\ell_2}\ \})^\ell]\!] = \\
& \quad \mathcal{C}_{*\rho_s}[\![({e_1}^{\ell_1})]\!] \cup \mathcal{C}_{*\rho_s}[\![({e_2}^{\ell_2})]\!] \cup \\
& \quad \{\mathsf{P}(\ell_1) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{true}} \in \mathsf{C}(\ell_1) \Rightarrow \mathsf{C}(\ell_2) \subseteq \mathsf{C}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{true}} \in \mathsf{C}(\ell_1) \Rightarrow \mathsf{P}(\ell_2) \subseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{false}} \in \mathsf{C}(\ell_1) \Rightarrow \widehat{\mathbf{undefined}} \subseteq \mathsf{C}(\ell)\}
\end{array}
$$

$$
\begin{array}{ll}
[\mathit{CG\text{-}GetField}] & \mathcal{C}_{*\rho_s}[\![(e_1{}^{\ell_1}[e_2{}^{\ell_2}])^{\ell}]\!] = \\
& \quad \mathcal{C}_{*\rho_s}[\![(e_1{}^{\ell_1})]\!] \cup \mathcal{C}_{*\rho_s}[\![(e_2{}^{\ell_2})]\!] \cup \\
& \quad \{\mathsf{P}(\ell_1) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{\mathsf{P}(\ell_2) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \widehat{get}(\mathsf{C}(\ell_1), \mathsf{C}(\ell_2)) \subseteq \mathsf{C}(\ell) \\
[\mathit{CG\text{-}SetField}] & \mathcal{C}_{*\rho_s}[\![(e_0{}^{\ell_0}[e_1{}^{\ell_1}] = e_2{}^{\ell_2})]\!] = \\
& \quad \mathcal{C}_{*\rho_s}[\![(e_0{}^{\ell_0})]\!] \cup \mathcal{C}_{*\rho_s}[\![(e_1{}^{\ell_1})^{\ell}]\!] \cup \mathcal{C}_{*\rho_s}[\![(e_2{}^{\ell_2})]\!] \cup \\
& \quad \{\mathsf{P}(\ell_1) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{\mathsf{P}(\ell_2) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{\mathsf{P}(\ell_3) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \widehat{set}(\mathsf{C}(\ell_1), \mathsf{C}(\ell_2), \mathsf{C}(\ell_2)) \subseteq \mathsf{C}(\ell) \\
[\mathit{CG\text{-}DelField}] & \mathcal{C}_{*\rho_s}[\![(\mathbf{delete}\ e_1{}^{\ell_1}[e_2{}^{\ell_2}])^{\ell}]\!] = \\
& \quad \mathcal{C}_{*\rho_s}[\![(e_1{}^{\ell_1})]\!] \cup \mathcal{C}_{*\rho_s}[\![(e_2{}^{\ell_2})]\!] \cup \\
& \quad \{\mathsf{P}(\ell_1) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{\mathsf{P}(\ell_2) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \widehat{del}(\mathsf{C}(\ell_1), \mathsf{C}(\ell_2)) \subseteq \mathsf{C}(\ell) \\
[\mathit{CG\text{-}Ref}] & \mathcal{C}_{*\rho_s}[\![(\mathbf{ref}_{r,\rho_r}\ e_1{}^{\ell_1})^{\ell}]\!] = \\
& \quad \mathcal{C}_{*\rho_s}[\![(e_1{}^{\ell_1})]\!] \cup \\
& \quad \{\{r\} \subseteq \mathsf{C}(\ell)\} \cup \\
& \quad \{\mathsf{P}(\ell_1) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{\rho_r \sqsubseteq \rho_s \Rightarrow \mathsf{C}(\ell_1) \subseteq \mathsf{M}(r, \rho_r)\} \\
[\mathit{CG\text{-}DeRef}] & \mathcal{C}_{*\rho_s}[\![(\mathbf{deref}\ e_1{}^{\ell_1})^{\ell}]\!] = \\
& \quad \mathcal{C}_{*\rho_s}[\![(e_1{}^{\ell_1})]\!] \cup \\
& \quad \{\mathsf{P}(\ell_1) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{r \in \mathsf{C}(\ell_1) \Rightarrow \mathsf{M}(r, \rho_r) \subseteq \mathsf{C}(\ell) \\
& \qquad \mid r \in r_*, \rho_r \sqsubseteq \rho_s\} \\
[\mathit{CG\text{-}SetRef}] & \mathcal{C}_{*\rho_s}[\![(e_1{}^{\ell_1} = e_2{}^{\ell_2})^{\ell}]\!] = \\
& \quad \mathcal{C}_{*\rho_s}[\![(e_1{}^{\ell_1})]\!] \cup \mathcal{C}_{*\rho_s}[\![(e_2{}^{\ell_2})]\!] \cup \\
& \quad \{\mathsf{P}(\ell_1) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{\mathsf{P}(\ell_2) \sqsubseteq \mathsf{P}(\ell)\} \cup \\
& \quad \{r \in \mathsf{C}(\ell_1) \Rightarrow \mathsf{C}(\ell_2) \subseteq \mathsf{M}(r, \rho_r) \\
& \qquad \mid r \in r_*, \rho_r \sqsubseteq \rho_s\} \cup \\
& \quad \{\mathsf{C}(\ell_2) \subseteq \mathsf{C}(\ell)\} \\
[\mathit{PE\text{-}Send}] & \ldots \\
[\mathit{PE\text{-}Err}] & \ldots \\
[\mathit{PE\text{-}Exercise}] & \ldots
\end{array}
$$

5.2 Constraint solving

5.3 Implementation-specific details

6. EXPERIMENTS

6.1 Findings

6.2 Performance

SLOW... Very SLOW!!!

7. CONCLUSION

7.1 Conclusions

7.2 Future works (unbundling)