

Privilege separation in browser architectures

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ABSTRACT

In many software systems as modern web browsers the user and his sensitive data often interact with the untrusted outer world. This scenario can pose a serious threat to the user's private data and gives new relevance to an old story in computer science: providing controlled access to untrusted components, while preserving usability and ease of interaction. To address the threats of untrusted components, modern web browsers propose privilege-separated architectures, which isolate components that manage critical tasks and data from components which handle untrusted inputs. The former components are given strong permissions, possibly coinciding with the full set of permissions granted to the user, while the untrusted components are granted only limited privileges, to limit possible malicious behaviours: all the interactions between trusted and untrusted components is handled via message passing. In this thesis we introduce a formal semantics for privilege-separated architectures and we provide a general definition of privilege separation: we discuss how different privilege-separated architectures can be evaluated in our framework, identifying how different security threats can be avoided, mitigated or disregarded. Specifically, we evaluate in detail the existing Google Chrome Extension Architecture in our formal model and we discuss how its design can mitigate serious security risks, with only limited impact on the user experience.

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1. MOTIVATION

1.1 Privilege escalation attacks

1.2 Chrome extension architecture overview

1.3 Chrome extension architecture weaknesses

1.4 Proposal

2. BACKGROUND

2.1 Chrome extension architecture details

2.2 Flow logic

3. FORMALIZATION

3.1 Calculus

3.2 Safety properties

3.3 Analysis specification

3.3.1 Abstract succinct

<i>Abstract cache</i>	$\hat{C} : \mathcal{L} \rightarrow \hat{V}$
<i>Abstract variable environment</i>	$\hat{\Gamma} : \mathcal{V} \rightarrow \hat{V}$
<i>Abstract memory</i>	$\hat{\mu} : \mathcal{L} \times \mathcal{P} \rightarrow \hat{V}$
<i>Abstract permission cache</i>	$\hat{P} : \mathcal{L} \rightarrow \mathcal{P}$

$[PE-Val]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} c : \hat{v} \text{ iff } \{d_c\} \subseteq \hat{v}$
$[PE-Var]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} x : \hat{v} \text{ iff } \hat{\Gamma}(x) \subseteq \hat{v}$
$[PE-Lambda]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \lambda x.e : \hat{v} \text{ iff } \{\lambda x.e\} \subseteq \hat{v}$
$[PE-Obj]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \overrightarrow{\{str_i : e_i\}} : \hat{v} \gg \rho \text{ iff}$ $\forall i : (\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_i : \hat{v}_i \gg \rho_i \wedge$ $\overrightarrow{\{str_i : \hat{v}_i\}} \subseteq \hat{v} \wedge$ $\rho_i \sqsubseteq \rho$
$[PE-Let]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \mathbf{let} \overrightarrow{x_i = e_i} \mathbf{in} e' : \hat{v} \gg \rho \text{ iff}$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e' : \hat{v} \gg \rho' \wedge$ $\rho' \sqsubseteq \rho \wedge$ $\forall i :$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_i : \hat{v}_i \gg \rho_i \wedge$ $\hat{v}_i \subseteq \hat{\Gamma}(x_i) \wedge$ $\rho_i \sqsubseteq \rho$
$[PE-App]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 e_2 : \hat{v} \gg \rho \text{ iff}$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge$ $\rho_1 \sqsubseteq \rho \wedge$ $\rho_2 \sqsubseteq \rho \wedge$ $\forall (\lambda x.e_0) \in \hat{v}_1 :$ $\hat{v}_2 \subseteq \hat{\Gamma}(x) \wedge$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_0 : \hat{v}_0 \gg \rho_0 \wedge$ $\rho_0 \sqsubseteq \rho \wedge$ $\hat{v}_0 \subseteq \hat{v}$
$[PE-Op]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} op(\overrightarrow{e_i}) : \hat{v} \gg \rho \text{ iff}$ $\forall i :$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_i : \hat{v}_i \gg \rho_i \wedge$ $\rho_i \sqsubseteq \rho \wedge$ $\widehat{op(\overrightarrow{v_i})} \subseteq \hat{v}$
$[PE-Cond]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \mathbf{if} (e_0) \{ e_1 \} \mathbf{else} \{ e_2 \} : \hat{v} \gg \rho \text{ iff}$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_0 : \hat{v}_0 \gg \rho_0 \wedge$ $\rho_0 \sqsubseteq \rho \wedge$ $\widehat{\mathbf{true}} \in \hat{v}_0 \Rightarrow$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge \hat{v}_1 \subseteq \hat{v} \wedge \rho_1 \sqsubseteq \rho \wedge$ $\widehat{\mathbf{false}} \in \hat{v}_0 \Rightarrow$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge \hat{v}_2 \subseteq \hat{v} \wedge \rho_2 \sqsubseteq \rho$
$[PE-While]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \mathbf{while} (e_1) \{ e_2 \} : \hat{v} \gg \rho \text{ iff}$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge$ $\rho_1 \sqsubseteq \rho \wedge$ $\widehat{\mathbf{true}} \in \hat{v}_1 \Rightarrow$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge \hat{v}_2 \subseteq \hat{v} \wedge \rho_2 \sqsubseteq \rho \wedge$ $\widehat{\mathbf{false}} \in \hat{v}_1 \Rightarrow$ $\widehat{\mathbf{undefined}} \subseteq \hat{v}$
$[PE-GetField]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1[e_2] : \hat{v} \gg \rho \text{ iff}$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge$ $\rho_1 \sqsubseteq \rho \wedge$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge$ $\rho_2 \sqsubseteq \rho \wedge$ $\widehat{get(\hat{v}_1, \hat{v}_2)} \subseteq \hat{v}$

$[PE-SetField]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_0[e_1] = e_2 : \hat{v} \gg \rho$ iff $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_0 : \hat{v}_0 \gg \rho_0 \wedge$ $\rho_0 \sqsubseteq \rho \wedge$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge$ $\rho_1 \sqsubseteq \rho \wedge$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge$ $\rho_2 \sqsubseteq \rho \wedge$ $\widehat{set}(\hat{v}_0, \hat{v}_1, \hat{v}_2) \subseteq \hat{v}$
$[PE-DelField]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \mathbf{delete} e_1[e_2] : \hat{v} \gg \rho$ iff $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 : \hat{v}_1 \gg \rho_1 \wedge$ $\rho_1 \sqsubseteq \rho \wedge$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge$ $\rho_2 \sqsubseteq \rho \wedge$ $\widehat{del}(\hat{v}_1, \hat{v}_2) \subseteq \hat{v}$
$[PE-Ref]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \mathbf{ref}_{r, \rho_r} e : \{r\} \gg \rho$ iff $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e : \hat{v} \gg \rho \wedge$ $\rho_r \sqsubseteq \rho_s \Rightarrow \hat{v} \subseteq \hat{\mu}(r, \rho_r)$
$[PE-DeRef]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} \mathbf{deref} e : \hat{v} \gg \rho$ iff $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e : \hat{v}_1 \gg \rho_1 \wedge$ $\rho_1 \sqsubseteq \rho \wedge$ $\forall r \in \hat{v}_1 : \forall \rho_r \sqsubseteq \rho_s : \hat{\mu}(r, \rho_r) \subseteq \hat{v}$
$[PE-SetRef]$	$(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_1 = e_2 : \hat{v} \gg \rho$ iff $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e : \hat{v}_1 \gg \rho_1 \wedge$ $\rho_1 \sqsubseteq \rho \wedge$ $(\hat{\Gamma}, \hat{\mu}) \models_{\rho_s} e_2 : \hat{v}_2 \gg \rho_2 \wedge$ $\rho_2 \sqsubseteq \rho \wedge$ $\forall r \in \hat{v}_1 : \forall \rho_r \sqsubseteq \rho_s :$ $\hat{v}_2 \subseteq \hat{\mu}(r, \rho_r) \wedge$ $\hat{v}_2 \subseteq \hat{v}$
$[PE-Send]$...
$[PE-Err]$...
$[PE-Exercise]$...

3.3.2 Compositional Verbose

[CV-Val]	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (c)^\ell$ iff $\{d_c\} \subseteq \hat{C}(\ell)$
[CV-Var]	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (x)^\ell$ iff $\hat{\Gamma}(x) \subseteq \hat{C}(\ell)$
[CV-Lambda]	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\lambda x. e_0^{\ell_0})^\ell$ iff $\{\lambda x. e_0^{\ell_0}\} \subseteq \hat{C}(\ell) \wedge$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_0^{\ell_0}$
[CV-Obj]	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\overrightarrow{\{str_i : e_i^{\ell_i}\}})^\ell$ iff $\forall i :$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_i^{\ell_i} \wedge$ $\hat{P}(\ell_i) \sqsubseteq \hat{P}(\ell) \wedge$ $\overrightarrow{\{str_i : \hat{C}(\ell_i)\}} \subseteq \hat{C}(\ell)$
[CV-Let]	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\mathbf{let} \ x_i = e_i^{\ell_i} \ \mathbf{in} \ e'^{\ell'})^\ell$ iff $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e'^{\ell'} \wedge$ $\hat{P}(\ell') \sqsubseteq \hat{P}(\ell) \wedge$ $\hat{C}(\ell') \subseteq \hat{C}(\ell) \wedge$ $\forall i :$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_i^{\ell_i} \wedge$ $\hat{C}(\ell_i) \subseteq \hat{\Gamma}(x_i) \wedge$ $\hat{P}(\ell_i) \sqsubseteq \hat{P}(\ell)$
[CV-App]	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (e_1^{\ell_1} e_2^{\ell_2})^\ell$ iff $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_1^{\ell_1} \wedge (\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_2^{\ell_2} \wedge$ $\hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge \hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell)$ $\forall (\lambda x. e_0^{\ell_0}) \in \hat{C}(\ell_1) :$ $\hat{C}(\ell_2) \subseteq \hat{\Gamma}(x) \wedge \hat{C}(\ell_0) \subseteq \hat{C}(\ell) \wedge$ $\hat{P}(\ell_0) \sqsubseteq \hat{P}(\ell)$
[CV-Op]	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (op(\overrightarrow{e_i^{\ell_i}}))^\ell$ iff $\forall i :$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_i^{\ell_i} \wedge$ $\hat{P}(\ell_i) \sqsubseteq \hat{P}(\ell) \wedge$ $\widehat{op}(\hat{C}(\ell_i)) \subseteq \hat{C}(\ell)$
[CV-Cond]	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\mathbf{if} \ (e_0^{\ell_0}) \ \{ e_1^{\ell_1} \} \ \mathbf{else} \ \{ e_2^{\ell_2} \})^\ell$ iff $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_0^{\ell_0} \wedge$ $\hat{P}(\ell_0) \sqsubseteq \hat{P}(\ell) \wedge$ $\widehat{\mathbf{true}} \in \hat{C}(\ell_0) \Rightarrow$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_1^{\ell_1} \wedge \hat{C}(\ell_1) \subseteq \hat{C}(\ell) \wedge$ $\hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge$ $\widehat{\mathbf{false}} \in \hat{C}(\ell_0) \Rightarrow$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_2^{\ell_2} \wedge \hat{C}(\ell_2) \subseteq \hat{C}(\ell) \wedge$ $\hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell)$
[CV-While]	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\mathbf{while} \ (e_1^{\ell_1}) \ \{ e_2^{\ell_2} \})^\ell$ iff $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_1^{\ell_1} \wedge$ $\hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge$ $\widehat{\mathbf{true}} \in \hat{C}(\ell_1) \Rightarrow$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_2^{\ell_2} \wedge \hat{C}(\ell_2) \subseteq \hat{C}(\ell) \wedge$ $\hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell) \wedge$ $\widehat{\mathbf{false}} \in \hat{C}(\ell_1) \Rightarrow \widehat{\mathbf{undefined}} \subseteq \hat{C}(\ell)$

$[CV-GetField]$	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (e_1^{\ell_1} [e_2^{\ell_2}])^\ell$ iff $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_1^{\ell_1} \wedge$ $\hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_2^{\ell_2} \wedge$ $\hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell) \wedge$ $\widehat{get}(\hat{C}(\ell_1), \hat{C}(\ell_2)) \subseteq \hat{C}(\ell)$
$[CV-SetField]$	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (e_0^{\ell_0} [e_1^{\ell_1}] = e_2^{\ell_2})^\ell$ iff $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_0^{\ell_0} \wedge$ $\hat{P}(\ell_0) \sqsubseteq \hat{P}(\ell) \wedge$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_1^{\ell_1} \wedge$ $\hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_2^{\ell_2} \wedge$ $\hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell) \wedge$ $\widehat{set}(\hat{C}(\ell_0), \hat{C}(\ell_1), \hat{C}(\ell_2)) \subseteq \hat{C}(\ell)$
$[CV-DelField]$	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\mathbf{delete} \ e_1^{\ell_1} [e_2^{\ell_2}])^\ell$ iff $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_1^{\ell_1} \wedge$ $\hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_2^{\ell_2} \wedge$ $\hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell) \wedge$ $\widehat{del}(\hat{C}(\ell_1), \hat{C}(\ell_2)) \subseteq \hat{C}(\ell)$
$[CV-Ref]$	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\mathbf{ref}_{r, \rho_r} \ e_1^{\ell_1})^\ell$ iff $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_1^{\ell_1} \wedge$ $\{r\} \subseteq \hat{C}(\ell) \wedge$ $\hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge$ $\rho_r \sqsubseteq \rho_s \Rightarrow \hat{C}(\ell_1) \subseteq \hat{\mu}(r, \rho_r)$
$[CV-DeRef]$	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (\mathbf{deref} \ e_1^{\ell_1})^\ell$ iff $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_1^{\ell_1} \wedge$ $\hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge$ $\forall r \in \hat{C}(\ell_1) : \forall \rho_r \sqsubseteq \rho_s :$ $\hat{\mu}(r, \rho_r) \subseteq \hat{C}(\ell)$
$[CV-SetRef]$	$(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} (e_1^{\ell_1} = e_2^{\ell_2})^\ell$ iff $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_1^{\ell_1} \wedge$ $\hat{P}(\ell_1) \sqsubseteq \hat{P}(\ell) \wedge$ $(\hat{C}, \hat{\Gamma}, \hat{\mu}, \hat{P}) \models_{c\rho_s} e_2^{\ell_2} \wedge$ $\hat{P}(\ell_2) \sqsubseteq \hat{P}(\ell) \wedge$ $\forall r \in \hat{C}(\ell_1) : \forall \rho_r \sqsubseteq \rho_s :$ $\hat{C}(\ell_2) \subseteq \hat{\mu}(r, \rho_r) \wedge$ $\hat{C}(\ell_2) \subseteq \hat{C}(\ell)$
$[PE-Send]$...
$[PE-Err]$...
$[PE-Exercise]$...

3.4 *Theorem*

3.5 *Requirements for correctness*

4. ABSTRACT DOMAINS

4.1 Abstract domains choice

$$R_1 = \{\widehat{\overrightarrow{str_i : \hat{v}_i}}\} \sqsubseteq \{\widehat{\overrightarrow{str_j : \hat{v}_j}}\} = R_2 \text{ sse:}$$

1. R_1 ha meno campi di R_2

2. ogni campo di R_1 e' piu' preciso del **corrispondente** campo di R_2

$$\begin{aligned} & \forall i, \exists j : \widehat{str_i} \sqsubseteq \widehat{str_j} \\ \forall i, \exists j : \widehat{str_i} \sqsubseteq \widehat{str_j} \Rightarrow \hat{v}_i \sqsubseteq \hat{v}_j \\ & \text{Set:} \end{aligned}$$

- Exact

- $\exists \rightarrow Union$
- $\nexists \rightarrow addinprefix$

- Prefix

- aggiungo in *

$$\hat{v} \sqsubseteq \hat{v}' \text{ iff } \forall \hat{u}_i \in \hat{v}, \exists \hat{u}_j \in \hat{v}' : \hat{u}_i \sqsubseteq \hat{u}_j.$$

If Galois connection then

$$\hat{v} \sqsubseteq \hat{v}' \text{ iff } \gamma(\hat{v}) \subseteq \gamma(\hat{v}')$$

where $\gamma : \widehat{V} \rightarrow P(V)$ is the concretisation function.

$$\gamma_p : \widehat{PV} \rightarrow P(V)$$

$$\gamma(\hat{v}) = \bigcup_{\hat{u}_i \in \hat{v}} \gamma_p(\hat{u}_i)$$

$$\widehat{pre_{bool}} = \widehat{true|false}$$

$$\widehat{u_{bool}} = \{\widehat{pre_{bool}}\}$$

$$\widehat{pre_{int}} = \widehat{\oplus|0|\ominus}$$

$$\widehat{u_{int}} = \{\widehat{pre_{int}}\}$$

$$\widehat{pre_{string}} = \widehat{s|s*}$$

$$\widehat{u_{string}} = \{\widehat{pre_{string}}\}$$

$$\widehat{pre_{ref}} = r$$

$$\widehat{u_{ref}} = \{\widehat{pre_{ref}}\}$$

$$\widehat{pre_{\lambda}} = \lambda$$

$$\widehat{u_{\lambda}} = \{\widehat{pre_{\lambda}}\}$$

$$\widehat{pre_{rec}} = \widehat{\{str_i : \hat{v}_i\}}$$

$$\widehat{u_{rec}} = \widehat{pre_{rec}}$$

$$\hat{v} = (\widehat{u_{bool}}, \widehat{u_{int}}, \widehat{u_{string}}, \widehat{u_{ref}}, \widehat{u_{\lambda}}, \widehat{u_{rec}}, \{\widehat{Null}\}, \{\widehat{Undef}\})$$

with $\sqsubseteq = \sqsubseteq$

with $\sqsubseteq = \sqsubseteq$

with $\sqsubseteq = \sqsubseteq$

— Giulia's spec. is more tricky than \sqsubseteq

with $\sqsubseteq = \sqsubseteq$

with $\sqsubseteq = \sqsubseteq$

with $\sqsubseteq = \widehat{u_{rec}} \sqsubseteq$

with $\hat{v} \sqsubseteq \hat{v}'$ iff

$$\widehat{u_{bool}} \sqsubseteq \widehat{u_{bool}'} \wedge$$

$$\widehat{u_{int}} \sqsubseteq \widehat{u_{int}'} \wedge$$

$$\widehat{u_{string}} \sqsubseteq \widehat{u_{string}'} \wedge$$

$$\widehat{u_{ref}} \sqsubseteq \widehat{u_{ref}'} \wedge$$

$$\widehat{u_{\lambda}} \sqsubseteq \widehat{u_{\lambda}'} \wedge$$

$$\widehat{u_{rec}} \sqsubseteq \widehat{u_{rec}'} \wedge$$

$$\widehat{Null} \notin \hat{v}' \vee \widehat{Null} \in \hat{v} \wedge \widehat{Null} \in \hat{v}' \wedge$$

$$\widehat{Undef} \notin \hat{v}' \vee \widehat{Undef} \in \hat{v} \wedge \widehat{Undef} \in \hat{v}'$$

4.2 Abstract operations

4.3 Requirements verification

5. IMPLEMENTATION

5.1 Constraint generation

Constraint elements: E .

$$\begin{array}{lll} \text{Cache element} & C(\ell) & : \mathcal{L} \rightarrow \hat{V} \\ \text{Var element} & \Gamma(x) & : \mathcal{V} \rightarrow \hat{V} \\ \text{State element} & M(\mathcal{P}, ref) & : \mathcal{L} \times \mathcal{P} \rightarrow \hat{V} \end{array}$$

Permission Element: $P(\ell) : \mathcal{L} \rightarrow \mathcal{P}$

Constraint form.

$$\begin{array}{lll} \text{Term inclusion} & \{\hat{v}\} & \subseteq E \\ \text{Element inclusion} & E & \subseteq E \\ \text{Permission inclusion} & P(\ell) & \sqsubseteq P(\ell') \\ \text{Operation} & \widehat{Op}(\vec{E}_i) & \subseteq E \\ \text{Implication} & \{\hat{v}\} \subseteq E & \Rightarrow E \subseteq E \end{array}$$

Misc:

r_* is the set of all references of the program;
 $lambda_*$ is the set of all lambdas of the program;

$$\begin{array}{ll}
[CG-Val] & \mathcal{C}_{*\rho_s} \llbracket (e)^\ell \rrbracket = \{d_c\} \subseteq \mathbf{C}(\ell) \\
[CG-Var] & \mathcal{C}_{*\rho_s} \llbracket (x)^\ell \rrbracket = \Gamma(x) \subseteq \mathbf{C}(\ell) \\
[CG-Lambda] & \mathcal{C}_{*\rho_s} \llbracket (\lambda x. e_0^{\ell_0})^\ell \rrbracket = \\
& \quad \{\{\lambda x. e_0^{\ell_0}\} \subseteq \mathbf{C}(\ell)\} \cup \\
& \quad \mathcal{C}_{*\rho_s} \llbracket (e_0^{\ell_0}) \rrbracket \\
[CG-Obj] & \mathcal{C}_{*\rho_s} \llbracket (\overrightarrow{\{str_i : e_i^{\ell_i}\}})^\ell \rrbracket = \\
& \quad \bigcup_i (\mathcal{C}_{*\rho_s} \llbracket (e_i^{\ell_i}) \rrbracket \cup \\
& \quad \quad \{\mathbf{P}(\ell_i) \sqsubseteq \mathbf{P}(\ell)\}) \cup \\
& \quad \overrightarrow{\{\{str_i : \mathbf{C}(\ell_i)\} \subseteq \mathbf{C}(\ell)\}} \\
[CG-Let] & \mathcal{C}_{*\rho_s} \llbracket (\text{let } x_i = e_i^{\ell_i} \text{ in } e'^{\ell'})^\ell \rrbracket = \\
& \quad \bigcup_i (\mathcal{C}_{*\rho_s} \llbracket (e_i^{\ell_i}) \rrbracket \cup \\
& \quad \quad \{\mathbf{C}(\ell_i) \subseteq \Gamma(x_i)\} \cup \\
& \quad \quad \{\mathbf{P}(\ell_i) \subseteq \mathbf{P}(\ell)\}) \cup \\
& \quad \mathcal{C}_{*\rho_s} \llbracket (e'^{\ell'}) \rrbracket \cup \\
& \quad \{\mathbf{P}(\ell') \sqsubseteq \mathbf{P}(\ell)\} \cup \\
& \quad \{\mathbf{C}(\ell') \subseteq \mathbf{C}(\ell)\} \\
[CG-App] & \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1} e_2^{\ell_2})^\ell \rrbracket = \\
& \quad \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1}) \rrbracket \cup \mathcal{C}_{*\rho_s} \llbracket (e_2^{\ell_2}) \rrbracket \cup \\
& \quad \{\mathbf{P}(\ell_1) \sqsubseteq \mathbf{P}(\ell)\} \cup \{\mathbf{P}(\ell_2) \sqsubseteq \mathbf{P}(\ell)\} \cup \\
& \quad \{\{t\} \subseteq \mathbf{C}(\ell_1) \Rightarrow \mathbf{C}(\ell_2) \subseteq \Gamma(x) \\
& \quad \quad | t = (\lambda x. e_0^{\ell_0}) \in \text{lambda}_*\} \cup \\
& \quad \{\{t\} \subseteq \mathbf{C}(\ell_1) \Rightarrow \mathbf{C}(\ell_0) \subseteq \mathbf{C}(\ell) \\
& \quad \quad | t = (\lambda x. e_0^{\ell_0}) \in \text{lambda}_*\} \cup \\
& \quad \{\{t\} \subseteq \mathbf{C}(\ell_1) \Rightarrow \mathbf{P}(\ell_0) \sqsubseteq \mathbf{P}(\ell) \\
& \quad \quad | t = (\lambda x. e_0^{\ell_0}) \in \text{lambda}_*\} \cup \\
& \quad \overrightarrow{\{t = (\lambda x. e_0^{\ell_0}) \in \text{lambda}_*\}} \\
[CG-Op] & \mathcal{C}_{*\rho_s} \llbracket (\overrightarrow{op}(e_i^{\ell_i}))^\ell \rrbracket = \\
& \quad \bigcup_i (\mathcal{C}_{*\rho_s} \llbracket (e_i^{\ell_i}) \rrbracket \cup \{\mathbf{P}(\ell_i) \sqsubseteq \mathbf{P}(\ell)\}) \cup \\
& \quad \{\widehat{op}(\mathbf{C}(\ell_i)) \subseteq \mathbf{C}(\ell)\} \\
[CG-Cond] & \mathcal{C}_{*\rho_s} \llbracket (\text{if } (e_0^{\ell_0}) \{ e_1^{\ell_1} \} \text{ else } \{ e_2^{\ell_2} \})^\ell \rrbracket = \\
& \quad \mathcal{C}_{*\rho_s} \llbracket (e_0^{\ell_0}) \rrbracket \cup \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1}) \rrbracket \cup \mathcal{C}_{*\rho_s} \llbracket (e_2^{\ell_2}) \rrbracket \cup \\
& \quad \{\widehat{\mathbf{P}}(\ell_0) \sqsubseteq \widehat{\mathbf{P}}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{true}} \in \mathbf{C}(\ell_0) \Rightarrow \mathbf{C}(\ell_1) \subseteq \mathbf{C}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{true}} \in \mathbf{C}(\ell_0) \Rightarrow \mathbf{P}(\ell_1) \sqsubseteq \mathbf{P}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{false}} \in \mathbf{C}(\ell_0) \Rightarrow \mathbf{C}(\ell_2) \subseteq \mathbf{C}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{false}} \in \mathbf{C}(\ell_0) \Rightarrow \mathbf{P}(\ell_2) \sqsubseteq \mathbf{P}(\ell)\} \\
[CG-While] & \mathcal{C}_{*\rho_s} \llbracket (\text{while } (e_1^{\ell_1}) \{ e_2^{\ell_2} \})^\ell \rrbracket = \\
& \quad \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1}) \rrbracket \cup \mathcal{C}_{*\rho_s} \llbracket (e_2^{\ell_2}) \rrbracket \cup \\
& \quad \{\mathbf{P}(\ell_1) \sqsubseteq \mathbf{P}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{true}} \in \mathbf{C}(\ell_1) \Rightarrow \mathbf{C}(\ell_2) \subseteq \mathbf{C}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{true}} \in \mathbf{C}(\ell_1) \Rightarrow \mathbf{P}(\ell_2) \subseteq \mathbf{P}(\ell)\} \cup \\
& \quad \{\widehat{\mathbf{false}} \in \mathbf{C}(\ell_1) \Rightarrow \widehat{\mathbf{undefined}} \subseteq \mathbf{C}(\ell)\}
\end{array}$$

$[CG-GetField]$	$ \begin{aligned} \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1} [e_2^{\ell_2}])^\ell \rrbracket = & \\ & \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1}) \rrbracket \cup \mathcal{C}_{*\rho_s} \llbracket (e_2^{\ell_2}) \rrbracket \cup \\ & \{P(\ell_1) \sqsubseteq P(\ell)\} \cup \\ & \{P(\ell_2) \sqsubseteq P(\ell)\} \cup \\ & \widehat{get}(C(\ell_1), C(\ell_2)) \subseteq C(\ell) \end{aligned} $
$[CG-SetField]$	$ \begin{aligned} \mathcal{C}_{*\rho_s} \llbracket (e_0^{\ell_0} [e_1^{\ell_1}] = e_2^{\ell_2}) \rrbracket = & \\ & \mathcal{C}_{*\rho_s} \llbracket (e_0^{\ell_0}) \rrbracket \cup \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1})^\ell \rrbracket \cup \mathcal{C}_{*\rho_s} \llbracket (e_2^{\ell_2}) \rrbracket \cup \\ & \{P(\ell_1) \sqsubseteq P(\ell)\} \cup \\ & \{P(\ell_2) \sqsubseteq P(\ell)\} \cup \\ & \{P(\ell_3) \sqsubseteq P(\ell)\} \cup \\ & \widehat{set}(C(\ell_1), C(\ell_2), C(\ell_2)) \subseteq C(\ell) \end{aligned} $
$[CG-DelField]$	$ \begin{aligned} \mathcal{C}_{*\rho_s} \llbracket (\mathbf{delete} \ e_1^{\ell_1} [e_2^{\ell_2}])^\ell \rrbracket = & \\ & \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1}) \rrbracket \cup \mathcal{C}_{*\rho_s} \llbracket (e_2^{\ell_2}) \rrbracket \cup \\ & \{P(\ell_1) \sqsubseteq P(\ell)\} \cup \\ & \{P(\ell_2) \sqsubseteq P(\ell)\} \cup \\ & \widehat{del}(C(\ell_1), C(\ell_2)) \subseteq C(\ell) \end{aligned} $
$[CG-Ref]$	$ \begin{aligned} \mathcal{C}_{*\rho_s} \llbracket (\mathbf{ref}_{r, \rho_r} \ e_1^{\ell_1})^\ell \rrbracket = & \\ & \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1}) \rrbracket \cup \\ & \{\{r\} \subseteq C(\ell)\} \cup \\ & \{P(\ell_1) \sqsubseteq P(\ell)\} \cup \\ & \{\rho_r \sqsubseteq \rho_s \Rightarrow C(\ell_1) \subseteq M(r, \rho_r)\} \end{aligned} $
$[CG-DeRef]$	$ \begin{aligned} \mathcal{C}_{*\rho_s} \llbracket (\mathbf{deref} \ e_1^{\ell_1})^\ell \rrbracket = & \\ & \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1}) \rrbracket \cup \\ & \{P(\ell_1) \sqsubseteq P(\ell)\} \cup \\ & \{r \in C(\ell_1) \Rightarrow M(r, \rho_r) \subseteq C(\ell) \\ & \quad \ r \in r_*, \rho_r \sqsubseteq \rho_s\} \end{aligned} $
$[CG-SetRef]$	$ \begin{aligned} \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1} = e_2^{\ell_2})^\ell \rrbracket = & \\ & \mathcal{C}_{*\rho_s} \llbracket (e_1^{\ell_1}) \rrbracket \cup \mathcal{C}_{*\rho_s} \llbracket (e_2^{\ell_2}) \rrbracket \cup \\ & \{P(\ell_1) \sqsubseteq P(\ell)\} \cup \\ & \{P(\ell_2) \sqsubseteq P(\ell)\} \cup \\ & \{r \in C(\ell_1) \Rightarrow C(\ell_2) \subseteq M(r, \rho_r) \\ & \quad \ r \in r_*, \rho_r \sqsubseteq \rho_s\} \cup \\ & \{C(\ell_2) \subseteq C(\ell)\} \end{aligned} $
$[PE-Send]$...
$[PE-Err]$...
$[PE-Exercise]$...

5.2 Constraint solving

5.3 Implementation-specific details

6. EXPERIMENTS

6.1 Findings

6.2 Performance

SLOW... Very SLOW!!!

7. CONCLUSION

7.1 *Conclusions*

7.2 *Future works (unbundling)*