

# Auction Games

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CS404 Agent-based Systems Coursework

## I. GAME 1 - FIRST TO BUY 3 OF ANY ARTIST WINS, THE HIGHEST BIDDER PAYS OWN BID, AND THE AUCTION ORDER KNOWN.

The first game goal is to collect 3 paintings from the same artist before to any other bidder, the value of the paintings is not being consider and the order is known; I set the utility as one in case of winning 3 paints of the same artists and zero otherwise. In order to decide my strategy for the game, it is necessary to calculate the probability of obtain 3 paintings of the same artist based on the order of appearance in the auction based on the assumption that everyone will bid the same for the paintings, then the strategy will look to maximise the expected utility. This assumption of everyone bid one third of their money for each piece in my strategy is based on the following reasoning; if someone bid more than one third of her money then her budget left for the following paintings will be not sufficient to win 2 more of them since all the other bidders will beat this proposition; also, if she bid less than one third of the money she is out of the game since everyone will bid more.

For example, reducing the game to a very simple one with only 2 players and both of them choose an artist to bid for, in this case since the budget is not an integer ( $1000/3 = 333.333$ ) then the option to bid one more at the start is 334 coins for the first piece and then 333 for the following 2; both of you are competing for the same artist then the first one will be bought for 334, then for the next paintings this player can bid only 333 splitting his money in 2 (if this player bid more than this then she will have a second painting but no money left for the third one), if the player one manage to acquire one more painting then the second one will bid 334 for the third one and then the probability of winning is again one half for this piece, the probability of winning for both players is always  $(1/2) * (1/2) = 1/4$ . Then deviate for this bid, putting the context of 2 players if one of them bid first 335 then the money left is 665 that is not sufficient to win 2 more paintings of the same artist and the other player will just bid one third of her money and win 3 paintings; if one of the players bid less than one third of the money it is out of the game to grab any pieces and at one time she will have to raise her money bid.

Therefore, since I am assuming that all the players will bid one third of their money for the first painting, one half for the second one and all their money for the last one, the strategy that maximise the expected utility is only driven

for the probability of winning these paintings based on the number of bidders and the order of the auction. Then, I am looking the probability of acquire the necessary pieces as a binomial negative distribution where the probability of 'success' is to buy a painting for one specific artist given the number of bidders going for that artist also.

The way of calculating the probability of success is determinate by the assumption that the majority or even all of the bidders will try to grab the paintings from the artist that will come first three times; then, other percentage of the bidders will go for the artists to come second three times and therefore. However, since it is a sequence of probabilities without replacement then it is necessary to calculate the numbers one by one; to start, I just determinate the artist that comes first, second, third and last three times given the order. For the first artist the probability of win that painting is one divided by the number of bidders going for the first, then to obtain the second painting the number of people going for the same artist will decrease (or at least stay the same if this artist is still the first one to come tree times) because some players who did not won the painting will recalibrate their chances and might go for other painter to win, but some players will maintain their decision in order compete against the one who has the first piece before she obtain a second one because then she will win with 334 budget at some point.

Since there are many scenarios depending the order of the paintings and the behaviour of all the players at each stage the exact probability of winning for each artist is difficult to calculate. My strategy is reduced to at each painting in the auction try to get the piece of art from the next first artist to become 3 times; also, bid one third of my money for the second one just if one of the paintings from this painter come before one of the first artist and there is more than 5 players in the auction; and, if there is 3 paintings of this artist (that is not the first one in order of appearance) coming in a subset of the auction based on 3 times the number of bidders and if the game has more than 5 bidders, I bid one third of my money minus one, so I will win if no one goes for it. For example, if the order is: [Picasso, Da Vinci, Da Vinci, Da Vinci, Picasso, Picasso, ...] then I would bid one third of my money for the Picasso even when Da Vinci is appearing first 3 times. When I win a piece then I strict to that artist with the half of my money, changing my choice would represent of do not have more budget to have 3 paintings of the same artist, I just change this in case there is no more than 2 painting for the artist that I already have one.

The decisions that I took are based on several strategies that I tried, and I put to compete against each other; however, there was not a single strategy that wins all the time and the simple final strategy that I choose in the end had good results.

## II. GAME 2 - FIRST TO BUY 3 OF ANY ARTIST WINS, THE HIGHEST BIDDER PAYS OWN BID, BUT THE AUCTION ORDER IS NOT KNOWN.

For the second game I am also assuming that the bids will be one third of the budget for the paints, and then again, the strategy that maximise the utility is driven by the selection of the artists to bid for. Since here we just have the information of how many paintings there is from each artists and not the order I am modelling the game for each artist as a random variable to appear in the auction as a negative binomial distribution of a sequence without replacement with the probability of success (that a piece of art from an artist is selected) is the number of paintings from that painter still on auction divided by the total number of items to be sell. I am taking this approach since we know how many pieces of art will are still to be sell and it can be selected independent randomly.

In order to select a strategy, in other words, how to select which artist I will bid one third of my budget, I am considering the expected value of the number of trial to have 3 successes for each artist. Given the number of paintings in the auction I am calculating the expected value of a negative binomial distribution for 3 successes. I am taking the bidding as a sequence of independent Bernoulli trials at each stage; each trial is determinate as a success if the piece of art for a given artist is out for selling and a failure otherwise where the probability of success is the number of paintings for that specific painter divided by the total number of paintings still to be sell in the auction and the random variable is the number of trials to collect three successes. The probability function is shown in the Equation 1 and the expected value of the random variable is given in Equation 2.

$$f(x; k, p) = \binom{x-1}{k-1} p^k (1-p)^{x-k} \quad (1)$$

$$E(x) = \frac{k(1-p)}{p} \quad (2)$$

where:

- 'k' is the number of success
- 'p' is the probability of success

Then, the artist that have minimum expected number of trial for three successes is selected to bid one third of my budget; the k number of successes is updated by the number of paintings that I have then from 3 is changing to 2 and to 1 as I am collecting the pieces of art for a given artist. However, I have also to consider the other players; again, most of the players are going to bid for the artist with the maximum

number of paintings and just selecting for which artist it is easy to check just going for the one with more paintings.

Then, my strategy consist in bid for the second one with less expected number of trial for three success if the number of bidders is more or equal to 5 and the difference between this one and the first one is less than 2 trials; or, when there is more than 10 bidders and the difference between the first artists and the third one in expected number of trials is less than 3 then bid for the third one; and bid for the first artists in less number of expected trial otherwise. I am considering the difference in 2 for going for the second artist because then I am expecting to compete with less people for 2 trials less than if I go for the first artists, same applies for the difference of 3 for going for the third artists.

In addition, I am bidding when I have no paintings yet one third of my money minus two in any case, this is only to grab a painting in case there is no other player going for it. Then, even when this artist is not appearing much in the auction the action to grabbing a piece of art for less money than one third ensure that the next two paintings from this artist I will be able to offer more than the other players, here also I check that there is more than 20 pieces from that artists still to be sell this to ensure that there is still paintings to try to grab before someone else wins.

## III. GAME 3 - HIGHEST TOTAL VALUE AT THE END WINS, THE HIGHEST BIDDER PAYS OWN BID, AND THE AUCTION ORDER KNOWN.

For the third game the goal is to maximise the total value at the end of the full auction, the final budget is not considered as a value and this point is important. For each artist a value is provided, and we know the order of the auction, we have the information of how many paintings remains to be sell. First, I am calculation the value that I give for each piece relatively with my budget, the number of paintings per artist and the value for each of them. I first calculate the maximum value that I can possibly achieve as the sum of all the values of the paintings to be sell (Equation 3); then, this maximum value is the denominator that distribute my budget, in other words, the percentage of my budget that must be dedicated to achieve this maximum value (Equation 3); finally, in order to calculate what is the value that I am giving to each pieces I multiply this percentage of my budget by the value of the painting for each artist to obtain this valuation (Equation 5).

$$MaxValue = \sum_{i=1} value(i) \quad (3)$$

where:

- 'i' are the paintings still to be sell in the auction.

$$\%Budget = \frac{money}{MaxValue} \quad (4)$$

$$bid_{artist} = value(artist) \times \%Budget \quad (5)$$

where:

the value artist is for each painting

This is my own valuation for each painting that is getting updated based on the auction order and what is my budget (then how many paintings I have); the valuation is getting bigger if I am not buying anything since the maximum possible value is getting smaller and my budget is the same; and also, it contemplates the real value for each piece of art. However, one important aspect of this game is to actually win some paintings since the final value is your own standings and the budget is not considered; then, at certain point of the game the maximum possible value will be small to be divided by all the players and at this stage of the game everyone will be eager to collect some piece of art.

Following the previous point, and since for each painting this is a common-value auction I need to consider others value in order to bid for the piece of art. This game is a sealed-bid auction then in order to create a full strategy it is necessary to try to estimate the highest bid of your competitors and then with this information elaborate your own bidding. In order to estimate this valuation, I am considering the last price paid by the winner and I am assuming that for the next bidding this will be the maximum value for all the bidders for that artist.

After estimate the maximum value for each artist then I need to compare this number with my own valuation and bid the value plus a certain amount in case this is lower than my own valuation and bid my own valuation minus a certain amount otherwise (in case the estimation is incorrect in order to have some profit); however, for this game this is more complicated since the utility is not strictly based on profitability but in term of value (the fact of having money at the end of the auction is useless, actually it would mean that I did not bid enough in order to catch a certain art work). Simplifying the game, if there is a single painting to be sell then the value of that painting is to win, every bidder would bid her full budget since win that painting represents to maximise the utility that can be modelled as 1 in case of winning the full game and zero otherwise; however, here we have the constrain if the budget, the number of pieces still on sale and the own valuation but winning one single painting is better than do not win any at all and keeping some of the budget in the end is not optimal.

After analysing the previous points, I adjust the valuation of the paintings bidding to actually win the highest valuable pieces, the art from Rembrandt and Da-Vinci, and my final strategy consist in the following points. First, if my budget is higher than 500 I bid against the estimated maximum valuation for the Rembrandt and Da-Vinci, I collect the winning bidding for these 2 artist and I bid the last price paid plus one, in order to win this piece; this is an aggressive strategy to start but the reason is, again, that I am evaluating

considering the fact that have a piece is better than do not have any of them and this can be achievable only going versus the aggressive players assuming we are many students in the class and the pieces are limited. Second, in case I am the actual winner of the painting then, the estimation of the maximum value of my opponents is incorrect; since I am winning then I am bidding more money than the necessary, or that I was selected randomly, anyway I am updating my bid if I am the winner by reducing 20% (money) in case I was the last winner of a Da-Vinci, and 15% (money) in case I was the last winner of a Rembrandt; in this way I am also avoiding to keep bidding too high in case there is very aggressive players at the beginning but I am assuming that there is no player with this strategy (mess others but not winning). Finally, in case I have less than 500 money budget I am bidding my actual valuation taking in consideration the maximum possible to win and the pieces still on sale (also I bid this amount for the Picasso and Van-Gogh all the time); this final point is because after winning some pieces in an aggressive way then I can wait for the end of the auction to reveal my actual value, not waiting till the end when there is not enough paintings to be distributed and do not spending my full budget in an aggressive way.

#### IV. GAME 4 - HIGHEST TOTAL VALUE AT THE END WINS, THE HIGHEST BIDDER PAYS THE SECOND HIGHEST BID, AND THE AUCTION ORDER KNOWN.

For the fourth and final game the goal is the same that the third one but with the difference of the winner pays the second highest bid. For this game I am maintaining the same valuation as in the previous one, calculating the maximum possible value and divide that number between my budget to bid the relative amount for the actual paint value. Therefore, the difference in this game is to estimate the others valuation and the difficulty of this is that you do not have the information of how much the winner bids, but the second one.

This is a Vickrey auction and we studied in class that this kind of auctions are strategy-proof, submitting your true valuation is always a dominant strategy. However, in this game submitting your true valuation is not a good strategy since your valuation is not linked to the actual profit but to the final game win; also, this is a sequence of auctions so the valuation is changing and the importance of the game is to discover the bidding of the others based on the price paid in order to win some paintings; an aggressive strategy results better because have even a single painting increase your expected utility than have your budget, and the actual risk of paying that money and loss a good percentage of your budget is decreased due the bidder is paying the second highest; your own valuation is getting bigger at each auction point as much as your budget allows you.

In order to estimate the evaluation of the winner, then I again consider the last price paid, this information is given by the principle of price discovery. For each artist at the last

painting I see the money that the winners pays, then I do a check if I am the winner or if I am not; if I am the winner and the amount that I am bidding is bigger than my valuation this represents that I am bidding more than the necessary amount in order to win and maybe some of the players are just pushing the biddings high in order to the aggressive first players pays more money that the need it and in a certain point their budget will be limited, then I adjust my bid to the price that I pay times 1.1; in this way I am still winning the painting (*ceteris paribus*) but also reducing the risk of loss my money easy at the beginning. If I am not the winner, then I adjust my bid for by 2 times the last winner price payed in order to win some paintings from Rembrandt and Da Vinci, again as in game 3, the utility is one if you are the winner and zero otherwise and the fact of not buying pieces. Also updating the valuation by 2 times the payed price will start to increase the money that the very aggressive players pay.

I play with this aggressive strategy only when my budget is less more than 500 money and for the high valuable artist, after I reach the half of my budget then I start to bid according to my valuation but being multiplied by 3 (also I bid this for all the Picassos and Van-Gogh) because being aggressive is a good strategy in this game because paying the money that you are bidding has a very low probability to occurs and the lack of high bids in the end leads to lose a lot of opportunities to collect paintings. After testing my code against other multiplier number and other kind of strategies then the number 3 for multiplication of my valuations gave me good results.