

Real World Examples of Quadratic Equations

A **Quadratic Equation** looks like this:

$$5x^2 + 3x + 3 = 0$$

this makes it Quadratic (pointing to the x^2 term)

[Quadratic equations](#) pop up in many real world situations!

Here we have collected some examples for you, and solve each using different methods:

- [Factoring Quadratics](#)
- [Completing the Square](#)
- [Graphing Quadratic Equations](#)
- [The Quadratic Formula](#)
- [Online Quadratic Equation Solver](#)

Each example follows three general stages:

- Take the real world description and make some equations
- Solve!
- Use your common sense to interpret the results



Balls, Arrows, Missiles and Stones

When you throw a ball (or shoot an arrow, fire a missile or throw a stone) it goes up into the air, slowing as it travels, then comes down again faster and faster ...

... and a [Quadratic Equation](#) tells you its position at all times!

Example: Throwing a Ball

A ball is thrown straight up, from 3 m above the ground, with a velocity of 14 m/s. When does it hit the ground?

Ignoring air resistance, we can work out its height by adding up these three things:
(Note: **t** is time in seconds)

The height starts at 3 m:	3
It travels upwards at 14 meters per second (14 m/s):	14t
Gravity pulls it down, changing its position by <i>about</i> 5 m per second squared:	-5t ²
(Note for the enthusiastic: the -5t ² is simplified from -(1/2)at ² with a=9.8 m/s ²)	

Add them up and the height **h** at any time **t** is:

$$h = 3 + 14t - 5t^2$$

And the ball will hit the ground when the height is zero:

$$3 + 14t - 5t^2 = 0$$

Which is a [Quadratic Equation](#) !

In "Standard Form" it looks like:

$$-5t^2 + 14t + 3 = 0$$

It looks even better when we multiply all terms by -1 :

$$5t^2 - 14t - 3 = 0$$

Let us solve it ...

There are many ways to solve it, here we will factor it using the "Find two numbers that multiply to give $a \times c$, and add to give b " method in [Factoring Quadratics](#):

$$a \times c = -15, \text{ and } b = -14.$$

The factors of -15 are: $-15, -5, -3, -1, 1, 3, 5, 15$

By trying a few combinations we find that -15 and 1 work ($-15 \times 1 = -15$, and $-15 + 1 = -14$)

$$\text{Rewrite middle with } -15 \text{ and } 1: 5t^2 - 15t + t - 3 = 0$$

$$\text{Factor first two and last two: } 5t(t - 3) + 1(t - 3) = 0$$

$$\text{Common Factor is } (t - 3): (5t + 1)(t - 3) = 0$$

$$\text{And the two solutions are: } 5t + 1 = 0 \text{ or } t - 3 = 0$$

$$t = -0.2 \text{ or } t = 3$$

The " $t = -0.2$ " is a negative time, impossible in our case.

The " $t = 3$ " is the answer we want:

The ball hits the ground after 3 seconds!



Here is the graph of the [Parabola](#) $h = -5t^2 + 14t + 3$

It shows you the **height** of the ball vs **time**

Some interesting points:

$(0, 3)$ When $t=0$ (at the start) the ball is at 3 m

$(-0.2, 0)$ says that -0.2 seconds BEFORE we threw the ball it was at ground level. This never happened! So our common sense says to ignore it.

$(3, 0)$ says that at 3 seconds the ball is at ground level.

Also notice that the ball goes **nearly 13 meters** high.

Note: You can find exactly where the top point is!

The method is explained in [Graphing Quadratic Equations](#), and has two steps:

Find where (along the horizontal axis) the top occurs using $-b/2a$:

$$\bullet t = -b/2a = -(-14)/(2 \times 5) = 14/10 = \mathbf{1.4 \text{ seconds}}$$

Then find the height using that value (1.4)

$$\bullet h = -5t^2 + 14t + 3 = -5(1.4)^2 + 14 \times 1.4 + 3 = \mathbf{12.8 \text{ meters}}$$

So the ball reaches the highest point of 12.8 meters after 1.4 seconds.



Example: New Sports Bike

You have designed a new style of sports bicycle!

Now you want to make lots of them and sell them for profit.

Your **costs** are going to be:

- \$700,000 for manufacturing set-up costs, advertising, etc
- \$110 to make each bike

Based on similar bikes, you can expect **sales** to follow this "Demand Curve":

$$\bullet \text{Unit Sales} = 70,000 - 200P$$

Where " P " is the price.

For example, if you set the price:

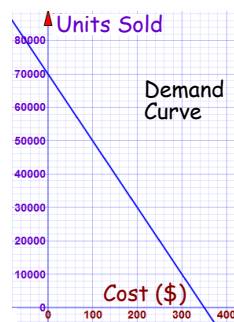
- at \$0, you just give away 70,000 bikes
- at \$350, you won't sell any bikes at all
- at \$300 you might sell $70,000 - 200 \times 300 = 10,000$ bikes

So ... what is the best price? And how many should you make?

Let us make some equations!

How many you sell depends on price, so use "P" for Price as the variable

- Unit Sales = $70,000 - 200P$
- Sales in Dollars = Units \times Price = $(70,000 - 200P) \times P = 70,000P - 200P^2$
- Costs = $700,000 + 110 \times (70,000 - 200P) = 700,000 + 7,700,000 - 22,000P = 8,400,000 - 22,000P$
- Profit = Sales - Costs = $70,000P - 200P^2 - (8,400,000 - 22,000P) = -200P^2 + 92,000P - 8,400,000$



$$\text{Profit} = -200P^2 + 92,000P - 8,400,000$$

Yes, a Quadratic Equation. Let us solve this one by [Completing the Square](#).

Solve: $-200P^2 + 92,000P - 8,400,000 = 0$

Step 1 Divide all terms by -200

$$P^2 - 460P + 42000 = 0$$

Step 2 Move the number term to the right side of the equation:

$$P^2 - 460P = -42000$$

Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation:

$$(b/2)^2 = (-460/2)^2 = (-230)^2 = 52900$$

$$P^2 - 460P + 52900 = -42000 + 52900$$

$$(P - 230)^2 = 10900$$

Step 4 Take the square root on both sides of the equation:

$$P - 230 = \pm\sqrt{10900} = \pm 104 \text{ (to nearest whole number)}$$

Step 5 Subtract (-230) from both sides (in other words, add 230):

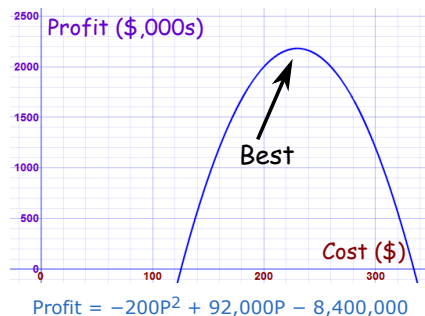
$$P = 230 \pm 104 = 126 \text{ or } 334$$

What does that tell us? It says that the profit is ZERO when the Price is \$126 or \$334

But we want to know the maximum profit, don't we?

It is exactly half way in-between! At \$230

And here is the graph:

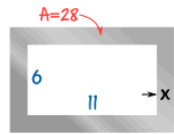


The best sale price is **\$230**, and you can expect:

- Unit Sales = $70,000 - 200 \times 230 = 24,000$
- Sales in Dollars = $\$230 \times 24,000 = \$5,520,000$
- Costs = $700,000 + \$110 \times 24,000 = \$3,340,000$
- Profit = $\$5,520,000 - \$3,340,000 = \$2,180,000$

A very profitable venture.

Example: Small Steel Frame



Your company is going to make frames as part of a new product they are launching.

The frame will be cut out of a piece of steel, and to keep the weight down, the final area should be **28 cm²**

The inside of the frame has to be **11 cm by 6 cm**

What should the width **x** of the metal be?

Area of steel before cutting:

$$\text{Area} = (11 + 2x) \times (6 + 2x) \text{ cm}^2$$

$$\text{Area} = 66 + 22x + 12x + 4x^2$$

$$\text{Area} = 4x^2 + 34x + 66$$

Area of steel after cutting out the 11 × 6 middle:

$$\text{Area} = 4x^2 + 34x + 66 - 66$$

$$\text{Area} = 4x^2 + 34x$$

Let us solve this one **graphically!**

Here is the graph of $4x^2 + 34x$:

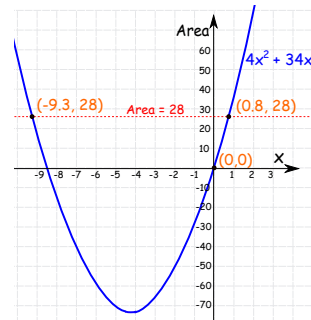
The desired area of **28** is shown as a horizontal line.

The area equals 28 cm² when:

$$x \text{ is about } -9.3 \text{ or } 0.8$$

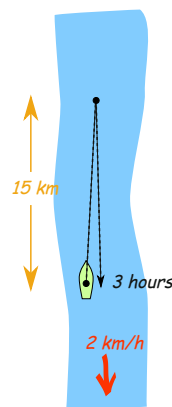
The negative value of **x** make no sense, so the answer is:

$$x = 0.8 \text{ cm (approx.)}$$



Example: River Cruise

A 3 hour river cruise goes 15 km upstream and then back again. The river has a current of 2 km an hour. What is the boat's speed and how long was the upstream journey?



There are two speeds to think about: the speed the boat makes in the water, and the speed relative to the land:

- Let **x** = the boat's speed in the water (km/h)
- Let **v** = the speed relative to the land (km/h)

Because the river flows downstream at 2 km/h:

- when going upstream, **v = x - 2** (its speed is reduced by 2 km/h)
- when going downstream, **v = x + 2** (its speed is increased by 2 km/h)

We can turn those speeds into times using:

$$\text{time} = \text{distance} / \text{speed}$$

(to travel 8 km at 4 km/h takes $8/4 = 2$ hours, right?)

And we know the total time is 3 hours:

$$\text{total time} = \text{time upstream} + \text{time downstream} = 3 \text{ hours}$$

Put all that together:

$$\text{total time} = 15/(x-2) + 15/(x+2) = 3 \text{ hours}$$

Now we use our algebra skills to solve for "x".

First, get rid of the fractions by multiplying through by **(x-2)(x+2)**:

$$3(x-2)(x+2) = 15(x+2) + 15(x-2)$$

Expand everything:

$$3(x^2-4) = 15x+30 + 15x-30$$

Bring everything to the left and simplify:

$$3x^2 - 30x - 12 = 0$$

It is a Quadratic Equation! Let us solve it using the [Quadratic Formula](#) :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where **a**, **b** and **c** are from the Quadratic Equation in "Standard Form": $ax^2 + bx + c = 0$

Solve $3x^2 - 30x - 12 = 0$

Coefficients are: $a = 3$, $b = -30$ and $c = -12$

Quadratic Formula: $x = [-b \pm \sqrt{(b^2-4ac)}] / 2a$

Put in a, b and c: $x = [-(-30) \pm \sqrt{((-30)^2-4 \times 3 \times (-12))}] / (2 \times 3)$

Solve: $x = [30 \pm \sqrt{(900+144)}] / 6$

$x = [30 \pm \sqrt{(1044)}] / 6$

$x = (30 \pm 32.31) / 6$

$x = -0.39$ **or** 10.39

Answer: $x = -0.39$ **or** 10.39 (to 2 decimal places)

$x = -0.39$ makes no sense for this real world question, but $x = 10.39$ is just perfect!

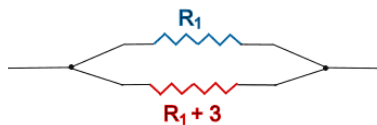
Answer: **Boat's Speed = 10.39 km/h** (to 2 decimal places)

And so the upstream journey = $15 / (10.39-2) = 1.79$ hours = **1 hour 47min**

And the downstream journey = $15 / (10.39+2) = 1.21$ hours = **1 hour 13min**

Example: Resistors In Parallel

Two resistors are in parallel, like in this diagram:



The total resistance has been measured at 2 Ohms, and one of the resistors is known to be 3 ohms more than the other.

What are the values of the two resistors?

The formula to work out total resistance " R_T " is:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

In this case, we have $R_T = 2$ and $R_2 = R_1 + 3$

$$\frac{1}{2} = \frac{1}{R_1} + \frac{1}{R_1+3}$$

To get rid of the fractions we can multiply all terms by $2R_1(R_1 + 3)$ and then simplify:

$$\text{Multiply all terms by } 2R_1(R_1 + 3): \frac{2R_1(R_1+3)}{2} = \frac{2R_1(R_1+3)}{R_1} + \frac{2R_1(R_1+3)}{R_1+3}$$

$$\text{Then simplify: } R_1(R_1 + 3) = 2(R_1 + 3) + 2R_1$$

$$\text{Expand: } R_1^2 + 3R_1 = 2R_1 + 6 + 2R_1$$

Bring all terms to the left: $R_1^2 + 3R_1 - 2R_1 - 6 - 2R_1 = 0$

Simplify: $R_1^2 - R_1 - 6 = 0$

Yes! A Quadratic Equation !

Let us solve it using our [Quadratic Equation Solver](#).

- Enter 1, -1 and -6
- And you should get the answers -2 and 3

R_1 cannot be negative, so **$R_1 = 3$ Ohms** is the answer.

The two resistors are 3 ohms and 6 ohms.

Others

Quadratic Equations are useful in many other areas:



For a parabolic mirror, a reflecting telescope or a satellite dish, the shape is defined by a quadratic equation.

Quadratic equations are also needed when studying lenses and curved mirrors.

And many questions involving time, distance and speed need quadratic equations.

here's
more:

- [Quadratic Equations](#)
- [Factoring Quadratics](#)
- [Completing the Square](#)
- [Graphing Quadratic Equations](#)
- [Quadratic Equation Solver](#)
- [Algebra Index](#)