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Nonlinear Optimization Logistic Model in the Problem of Cargo Transportation

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Abstract

The paper is devoted to the construction of an algorithm for solving an optimization logistic problem with a quadratic objective function. This problem is a quadratic programming problem with constraints such as equalities and inequalities. One of the constraints contains a parameter. The solution of the problem needs to be obtained in general form for any parameter value. In the process of constructing the algorithm, the concept of corner points is introduced. The solution of the problem is reduced to finding a finite set of corner points. All other optimal solutions are represented as linear combinations of two corner points. The constructed algorithm is implemented on a test example.

Keywords: Logistics; cargo transportation; optimization; nonlinear programming; parameter.

1. Introduction

As you know, the main goal of logistics is to implement a set of management and production measures to organize the movement, storage and sale of products or resources of various characters with minimal costs. It is considered that the basis of transport problems and production optimization problems are linear optimization problems (the problems of linear programming). The methods of solving such problems are currently well known (Silva et al., 2005; Brodetsky, 2010; Baldin et al., 2018; Dantzig, 2002), and the development of modern IT allows solving transport problems of any dimension.

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Recently, nonlinear models are often used in modeling traffic flows, for example, models of nonlinear dynamics (the Verhulst logistic equation) (Nakicenovic, 1989; Semenychev and Kozhukhova, 2013), nonlinear optimization problems (Chertkov et al., 2014). This work will be devoted to nonlinear optimization problems in the field of cargo transportation.

The methods of choice for solving multidimensional nonlinear optimization problems are more often numerical gradient methods for finding the optimal solution (Schmedders, 2008; Gilli et al., 2019) (steepest descent methods, proximal methods, Frank-Wolf methods and others). However, like any other numerical methods, gradient methods have a number of disadvantages:

- when approaching the minimum point, the convergence rate decreases significantly;
- the accumulation of errors with an increase in the number of iterations.

In this paper, we will consider an optimization problem in which the objective cost function has a quadratic form and the constraints have the form of equalities and inequalities. Moreover, it can contain a parameter from the constraints, which can take any value from a given set. It is required to obtain in the form of a certain set the entire set of optimal solutions for each parameter value from its domain of definition. Let's consider the mathematical formulation of this problem.

2. Problem Statement

Some warehouse B makes a plan to deliver weight products (sugar, flour, sand, cement, etc.) in the volume of N conventional units to n points A_i, $i=\overline{1,n}$ with its subsequent sale. The expected income from the sale of one conventional unit in point A_i, is m_i, respectively. We assume that the total cost function of the transportation of goods has a quadratic form

$$f(x_1,...,x_n)=\sum_{i,j=1}^{n} c_{ij}x_i x_j \to \min, \tag{1}$$

where c_{ij} are known coefficients, x_i is the volume of products delivered to point A_i.

It is required to determine the optimal product transportation plan from warehouse B to the destination points A_i.

Weight products are considered here. Since it is already necessary to use integer programming methods for piece products.

We would like this task to be solved simultaneously in the context of minimizing the target function of costs and maximizing the total profit from the sale of the all products. However, it has been repeatedly shown (Afanasyeva et al., 2007) that the problem has no solution in such a formulation. Therefore, we will fix the level of income from the sale of all products, considering it parametrically set. Then we have the following problem

$$\begin{cases} C=\sum_{i,j=1}^{n} c_{ij}x_i x_j \to \min, \\ \sum_{i=1}^{n} m_i x_i = M, \\ \sum_{i=1}^{n} x_i = N,\ x_i \ge 0, \end{cases} \tag{2}$$

where M is the parameter.

It is required to solve the problem for each parameter value from its definition area.

It is quite difficult to get a solution of this problem in the form of an algebraic function, but it is possible to obtain a solution using angular elements that will allow you to get a graphical representation of the specified functional dependence.

We construct an algorithm for solving an optimization problem with parameter (2) and illustrate the constructed algorithm for the given numerical values of constants.

3. Corner Point Method

We introduce the concept of corner points for the problem (2). There are two points $X^1 = (x_1^1,...,x_n^1)$ and $X^2 = (x_1^2,...,x_n^2)$ that deliver a local minimum to the problem (2) at the values M_1 and M_2 of parameter M. If a point X^* that is a linear combination of points X^1 and X^2

$$X^* = \alpha X^1 + (1-\alpha)X^1, \forall\alpha \in [0,1] \tag{3}$$

also delivers a local minimum to task (1) for the parameter value

$$M^* = \sum_{i=1}^{n} m_i \left(\alpha x_i^1 + (1-\alpha)x_i^2\right), \tag{4}$$

then the points X^1 and X^2 are called “corner points”.

Thus, the values of the C and M expression for all corner points and their linear combinations can be obtained. To find the corner points, we will use the algorithm described by the author in (Mikishanina, 2018), which was devoted to solving the optimization problem of constructing an effective set of securities.

Here we briefly present some of the main points of the algorithm for solving a quadratic programming problem with a parameter.

To solve (2), it is necessary to proceed to the quadratic programming problem

$$\begin{cases} F = \sum_{i,j=1}^{n} c_{ij}x_i x_j - \lambda\sum_{i=1}^{n} m_i x_i \to \min, \\ \sum_{i=1}^{n} x_i - N = 0, \\ -x_i \le 0,\ i = \overline{1,n}. \end{cases}, \tag{5}$$

where λ is a parameter. The problem (5) is solved by the method of indefinite Lagrange multipliers with restrictions of the type of equalities and inequalities (Afanasyeva et al., 2007), where the Lagrange function will take the form

$$\Lambda = \sum_{i,j=1}^{n} c_{ij}x_i x_j - \lambda\cdot\sum_{i=1}^{n} x_i \cdot m_i + \gamma_0 \cdot (\sum_{i=1}^{n} x_i - N) - \sum_{i=1}^{n} \gamma_i \cdot x_i \ , \tag{6}$$

$\gamma_0, \gamma_1, ..., \gamma_n$ are the indefinite multipliers.

The solution of the problems (5) will be piecewise broken functions $x_i(\lambda)$, $\lambda \in [0,+\infty)$. The values of the parameter λ in which at least one of the functions $x_i(\lambda)$ suffers a break are angular, and the point corresponding to this value of parameter λ is an angular point $(x_1,...,x_n)$.

4. Numerical Example

It is assumed that the goods will be distributed from the warehouse to three points of sale in the amount of 100 units of goods. The vector of expected profits from sales is set

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 500 \\ 800 \\ 600 \end{bmatrix}, \tag{7}$$

The objective cost function is given and has the form

$$C = 4x_1^2 + 5x_2^2 + 8x_3^2 - 4x_2x_3 - 0.5x_1x_3 + 2x_1x_2 \ , \tag{8}$$

Let's determine in what volumes x_1, x_2, x_3 it is necessary to organize the supply of products to minimize costs for all possible values of the expected total profit M.

The task has the form

$$\begin{cases} 4x_1^2 + 5x_2^2 + 8x_3^2 - 4x_2x_3 - 0.5x_1x_3 + 2x_1x_2 - \lambda\left(500x_1 + 800x_2 + 600x_3\right) \to \min, \\ x_1 + x_2 + x_3 = 100, \\ -x_1, -x_2, -x_3 \le 0. \end{cases} , \tag{9}$$

Graphs of solutions are shown in Fig. 1.

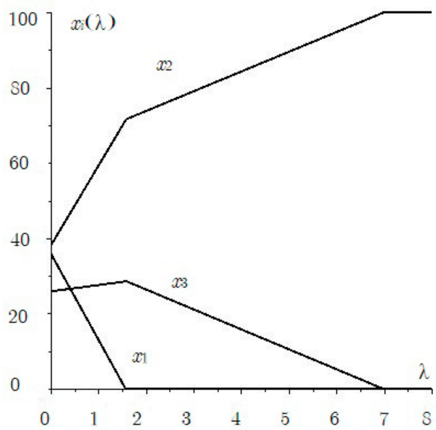

Fig. 1. Graphs of output volume functions.

The problem has three angular values of the parameter $\lambda = \{0, 1.57, 7\}$. They correspond to three angular distributions of products at points of sale

$$\left(35.8, 38.2, 26\right), \quad \left(0, 71.4, 28.6\right), \quad \left(0, 100, 0\right). \tag{10}$$

The set of solutions to problem (9) is shown in Fig. 2.

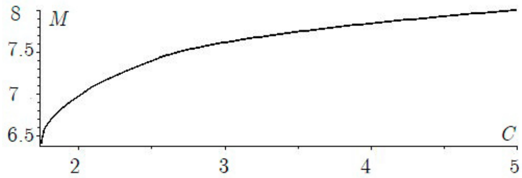

Fig. 2. Graphs of output volume functions.

5. Conclusion

The algorithm for solving optimization problems given in this paper can be applied to a wide class of problems in physics, logistics and economics. Optimization problems are usually solved when all the input numerical parameters of the problem are given in a single way. Here is an algorithm for solving an optimization problem when one of the input numerical parameters can take an integer set of values. The solution is obtained for the entire set of these values. The results obtained in the work can be useful for various research teams.

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