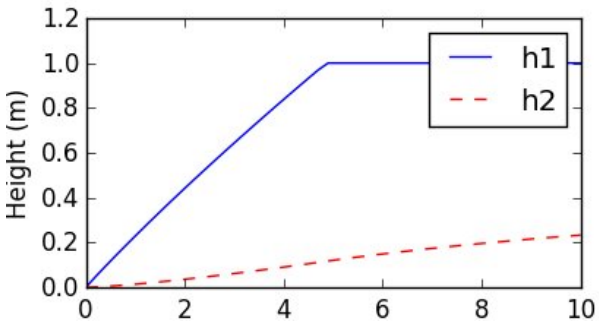
Solve Differential Equations in Python

Differential equations can be solved with different methods in Python. Below are examples that show how to solve differential equations with (1) **GEKKO Python**, (2) Euler's method, (3) the **ODEINT function from Scipy.Integrate**. Additional information is provided on using APM Python for parameter estimation with dynamic models and scale-up to large-scale problems.

- **Gravity Drained Tank Problem**



1. GEKKO Python

See **Introduction to GEKKO** for more information on solving differential equations in Python. GEKKO Python solves the differential equations with tank overflow conditions. When the first tank overflows, the liquid is lost and does not enter tank 2. The model is composed of variables and equations. The differential variables (h1 and h2) are solved with a mass balance on both tanks.

```
import numpy as np
import matplotlib.pyplot as plt
from gekko import GEKKO

m = GEKKO()

# integration time points
m.time = np.linspace(0,10)

# constants
c1 = 0.13
c2 = 0.20
Ac = 2        # m^2
# inflow
qin1 = 0.5    # m^3/hr

# variables
h1 = m.Var(value=0,lb=0,ub=1)
h2 = m.Var(value=0,lb=0,ub=1)
overflow1 = m.Var(value=0,lb=0)
overflow2 = m.Var(value=0,lb=0)

# outflow equations
qin2 = m.Intermediate(c1 * h1**0.5)
qout1 = m.Intermediate(qin2 + overflow1)
qout2 = m.Intermediate(c2 * h2**0.5 + overflow2)

# mass balance equations
m.Equation(Ac*h1.dt()==qin1-qout1)
m.Equation(Ac*h2.dt()==qin2-qout2)

# minimize overflow
m.Obj(overflow1+overflow2)

# set options
m.options.IMODE = 6 # dynamic optimization

# simulate differential equations
m.solve()

# plot results
plt.figure(1)
plt.plot(m.time,h1,'b-')
plt.plot(m.time,h2,'r--')
plt.xlabel('Time (hrs)')
plt.ylabel('Height (m)')
plt.legend(['height 1','height 2'])
plt.show()
```

[$[Get Code]]

2. Discretize with Euler's Method

Euler's method is used to solve a set of two differential equations in Excel and Python.

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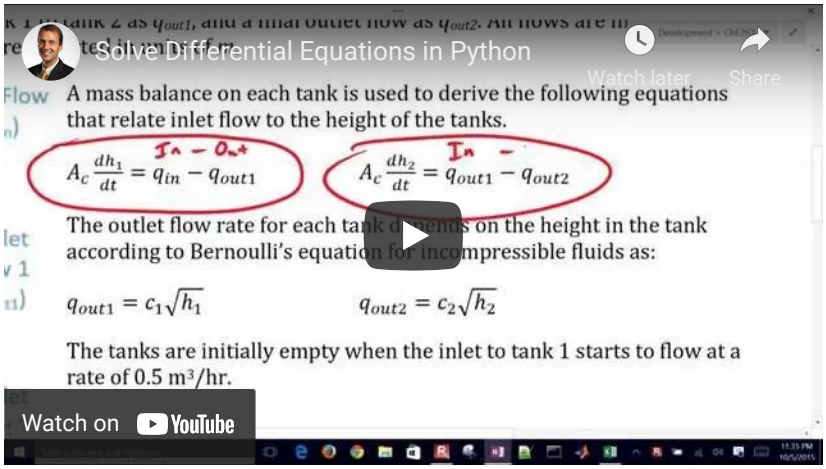
- **Gravity Drained Tank Files**




```
import numpy as np
import matplotlib.pyplot as plt

def tank(c1,c2):
  Ac = 2 # m^2
  qin = 0.5 # m^3/hr
  dt = 0.5 # hr
  tf = 10.0 # hr

  h1 = 0
  h2 = 0
  t = 0
  ts = np.empty(21)
  h1s = np.empty(21)
  h2s = np.empty(21)
  i = 0
  while t<=10.0:
     ts[i] = t
     h1s[i] = h1
     h2s[i] = h2

     qout1 = c1 * pow(h1,0.5)
     qout2 = c2 * pow(h2,0.5)
     h1 = (qin-qout1)*dt/Ac + h1
     if h1>1:
        h1 = 1
     h2 = (qout1-qout2)*dt/Ac + h2
     i = i + 1
     t = t + dt

  # plot data
  plt.figure(1)
  plt.plot(ts,h1s)
  plt.plot(ts,h2s)
  plt.xlabel("Time (hrs)")
  plt.ylabel("Height (m)")
  plt.show()

# call function
tank(0.13,0.20)
```

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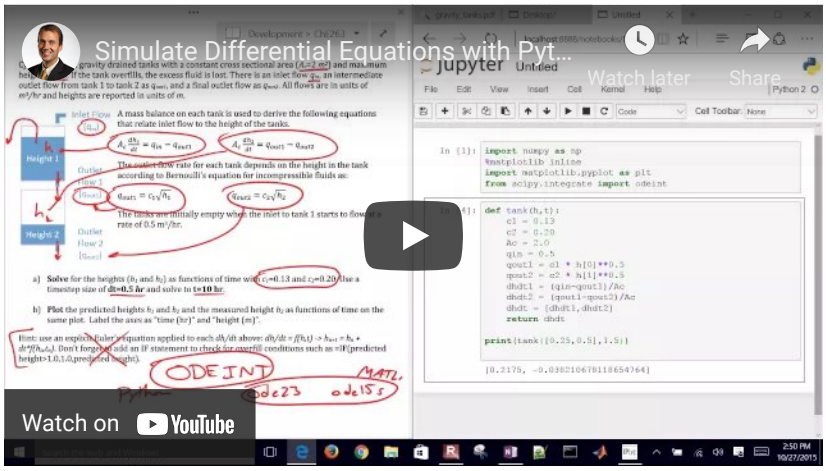
3. SciPy.Integrate ODEINT Function

See **Introduction to ODEINT** for more information on solving differential equations with SciPy.




```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint
```

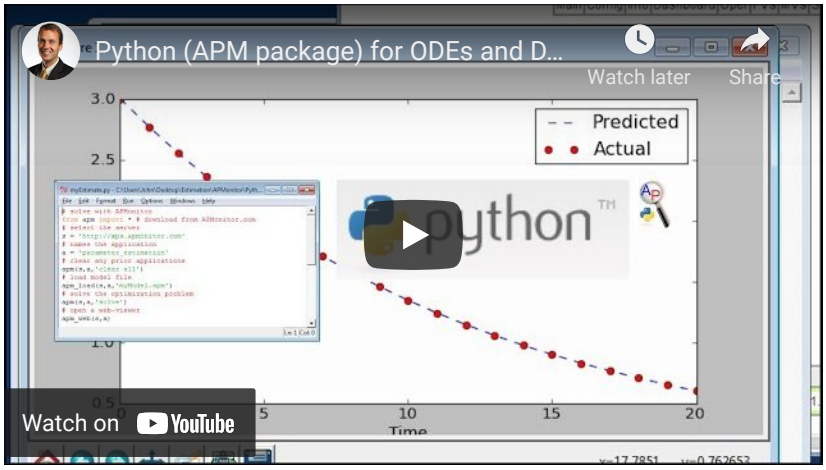
```
def tank(h,t):
    # constants
    c1 = 0.13
    c2 = 0.20
    Ac = 2       # m^2
    # inflow
    qin = 0.5    # m^3/hr
    # outflow
    qout1 = c1 * h[0]**0.5
    qout2 = c2 * h[1]**0.5
    # differential equations
    dhdt1 = (qin   - qout1) / Ac
    dhdt2 = (qout1 - qout2) / Ac
    # overflow conditions
    if h[0]>=1 and dhdt1>=0:
        dhdt1 = 0
    if h[1]>=1 and dhdt2>=0:
        dhdt2 = 0
    dhdt = [dhdt1,dhdt2]
    return dhdt

# integrate the equations
t = np.linspace(0,10) # times to report solution
h0 = [0,0]            # initial conditions for height
y = odeint(tank,h0,t) # integrate

# plot results
plt.figure(1)
plt.plot(t,y[:,0],'b-')
plt.plot(t,y[:,1],'r--')
plt.xlabel('Time (hrs)')
plt.ylabel('Height (m)')
plt.legend(['h1','h2'])
plt.show()
```

[$[Get Code]]

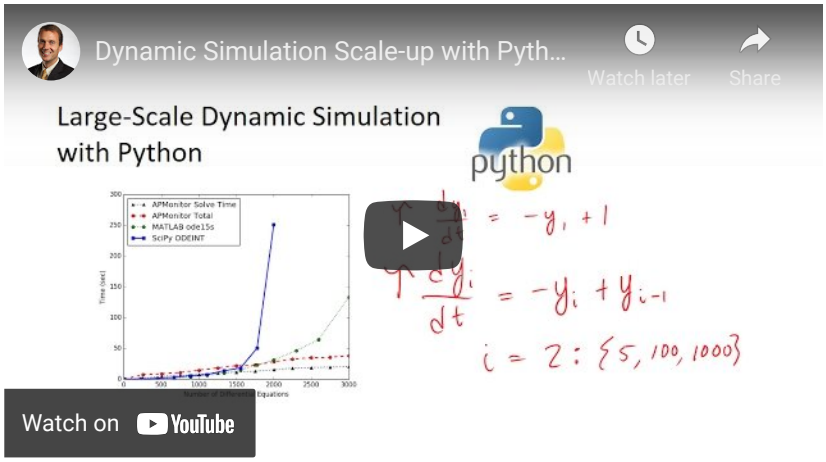
APM Python DAE Integrator and Optimizer

This tutorial gives step-by-step instructions on how to simulate dynamic systems. Dynamic systems may have differential and algebraic equations (DAEs) or just differential equations (ODEs) that cause a time evolution of the response. Below is an example of solving a first-order decay with the APM solver in Python. The objective is to fit the differential equation solution to data by adjusting unknown parameters until the model and measured values match.

- **Dynamic Estimation Files (dynamic_estimation.zip)**



Scale-up for Large Sets of Equations



Additional Material

This same example problem is also demonstrated with **Spreadsheet**

Programming and in the **Matlab programming language**. Another example problem demonstrates how to calculate the concentration of CO gas buildup in a room.

- **Case Study on CO Buildup in a Room**