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4.7 Applications of Quadratic Functions

Last Modified: Jun 01, 2017

A toy rocket is fired into the air from the top of a barn. Its height (h) above the ground in yards after t seconds is given by the function $h(t)=-5t^2+10t+20$.

1. What was the maximum height of the rocket?
2. How long was the rocket in the air before hitting the ground?
3. At what time(s) will the rocket be at a height of 22 yd?

Applications of Quadratic Functions

There are many real-world situations that deal with quadratics and parabolas. Throwing a ball, shooting a cannon, diving from a platform and hitting a golf ball are all examples of situations that can be modeled by quadratic functions.

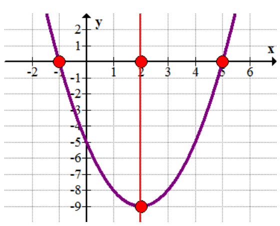
In many of these situations you will want to know the highest or lowest point of the parabola, which is known as the vertex. For example, consider that when you throw a football, the path it takes through the air is a parabola. Natural questions to ask are:

- "When does the football reach its maximum height?"
- "How high does the football get?"

If you know the equation for the function that models the situation, you can find the vertex. If the function is $f(x)=ax^2+bx+c$, the x-coordinate of the vertex will be $-\frac{b}{2a}$. The y-coordinate of the vertex can be found by substituting the x-coordinate into the function. In the case of the football:

- The x-coordinate of the vertex will give you the time when the football is at its maximum height.
- The y-coordinate will give you the maximum height.

One way to understand where the $-\frac{b}{2a}$ comes from is to consider where the vertex is on a parabola.

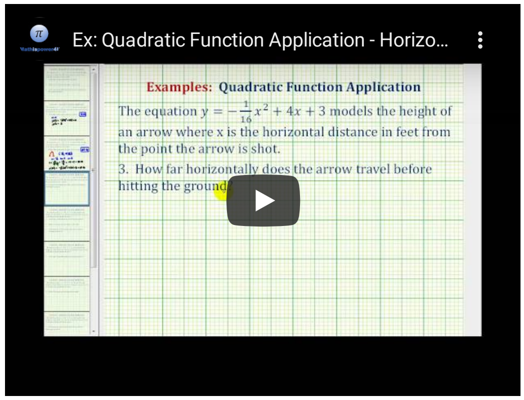


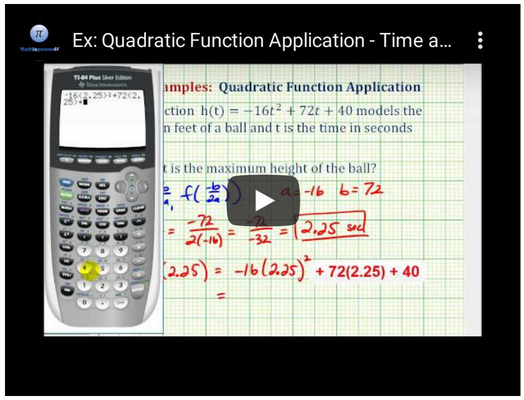
Due to the symmetry of parabolas, the x-coordinate of the vertex is directly between the two x-intercepts. The two x-intercepts are, according to the quadratic formula:

$$x=\frac{-b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}$$

So, $x=-\frac{b}{2a}$ is in the middle. One x-intercept is $\frac{\sqrt{b^2-4ac}}{2a}$ to the right and the other x-intercept is $\frac{\sqrt{b^2-4ac}}{2a}$ to the left.





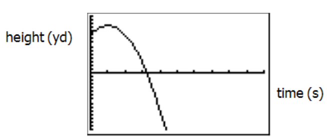
Let's solve the following problems using the quadratic formula:

1. A toy rocket is fired into the air from the top of a barn. Its height (h) above the ground in yards after t seconds is given by the function $h(t) = -5t^2 + 10t + 20$.

   a. What was the initial height of the rocket?

   b. When did the rocket reach its maximum height?

Sketch a graph of the function. Your graphing calculator can be used to produce the graph.



1. The initial height of the rocket is the height from which it was fired. The time is zero.

$$\begin{aligned} h(t) &= -5t^2 + 10t + 20 \\ h(t) &= -5(0)^2 + 10(0) + 20 \\ &\boxed{h(t) = 20\ yd} \end{aligned}$$

The initial height of the toy rocket is 20 yards. This is the y-intercept of the graph. The y-intercept of a quadratic function written in general form is the value of 'c'.

2. The time at which the rocket reaches its maximum height is the x-coordinate of the vertex.

$$\begin{aligned} t &= -\frac{b}{2a} \\ t &= -\frac{10}{2(-5)} \\ &\boxed{t = 1\ sec} \end{aligned}$$

It takes the toy rocket 1 second to reach its maximum height.

2. The sum of a number and its square is 272. Find the number.

Let n represent the number. Write an equation to represent the problem.

$n^2 + n = 272$

You can solve this equation using a few different methods. Here, solve by completing the square.

$$n^2 + n + \frac{1}{4} = 272 + \frac{1}{4}$$

$$n^2 + n + \frac{1}{4} = \frac{1089}{4}$$

$$\left(n + \frac{1}{2}\right)^2 = \frac{1089}{4}$$

$$\sqrt{\left(n + \frac{1}{2}\right)^2} = \sqrt{\frac{1089}{4}}$$
$$n + \frac{1}{2} = \pm\frac{33}{2}$$

$$n = \frac{32}{2} \ or \ n = -\frac{34}{2}$$
$$n = 16 \ or \ n = -17$$

These are both solutions to the problem. There are no restrictions listed in the problem regarding the solution.

3. The product of two consecutive positive odd integers is 195. Find the integers.

Let n represent the first positive odd integer. Let $n + 2$ represent the second positive odd integer. Write an equation to represent the problem.

$$n(n + 2) = 195$$
$$n^2 + 2n = 195$$

You can solve this equation with a few different methods. Here, use the quadratic formula.

$$n^2 + 2n - 195 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-195)}}{2(1)}$$

$$n = \frac{-2 \pm \sqrt{784}}{2}$$
$$n = \frac{-2 \pm 28}{2}$$

$$n = \frac{-2 + 28}{2} \ or \ n = \frac{-2 - 28}{2}$$

$$n = 13 \; or \; n = -15$$

There was a restriction on the solution presented in the problem. The solution must be an odd positive integer. Therefore, 13 is the solution you can use. The two positive odd integers are 13 and 15.

Examples

Example 1

Earlier, you were told about a toy rocket fired into the air from the top of a barn. Its height (h) above the ground in yards after t >seconds is given by the function $h(t) = -5t^2 + 10t + 20$.

1. What was the maximum height of the rocket?
2. How long was the rocket in the air before hitting the ground?
3. At what time(s) will the rocket be at a height of 22 yd?

1. The maximum height was reached by the rocket at one second as you found in part *b* of problem 1 from above.

$$\begin{aligned} &h(t) = -5t^2 + 10t + 20 \\ &h(t) = -5(1)^2 + 10(1) + 20 \\ &\boxed{h(t) = 25 \; yd} \end{aligned}$$

The maximum height reached by the rocket was 25 yd.

2. To find how long the rocket was in the air before hitting the ground, note that when the rocket hits the ground, its height will be zero.

$$\begin{aligned} h(t) &= -5t^2 + 10t + 20 \\ 0 &= -5t^2 + 10t + 20 \end{aligned}$$

Use the quadratic formula to solve for 't'. You have $a = -5, b = 10, c = 20$.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(10) \pm \sqrt{(10)^2 - 4(-5)(20)}}{2(-5)}$$

$$\begin{aligned} &t = \frac{-10 \pm 10\sqrt{5}}{-10} \\ &t = 1 \pm \sqrt{5} \end{aligned}$$

$$t = 1 + \sqrt{5} \; or \; t = 1 - \sqrt{5}$$

$$t = 3.24 \; s \; or \; t = -1.24 \; s$$

$$\boxed{t = 3.24 \; s}$$

Accept this solution

$$\boxed{t = -1.24 \; s}$$

Reject this solution. Time cannot be a negative quantity.

Thus, the toy rocket stayed in the air for approximately 3.24 seconds.

3. Now, to find the time that the rocket reached a height of 22 yd, remember that the rocket reached a maximum height of 25 yd at a time of 1 second. The rocket must reach a height of 22yd before and after one second. Remember the old saying: “What goes up must come down.”

   Use the quadratic formula to determine these times.

$$\begin{aligned} h(t) &= -5t^2 + 10t + 20 \\ 22 &= -5t^2 + 10t + 20 \end{aligned}$$

$$0 = -5t^2 + 10t - 2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(10) \pm \sqrt{(10)^2 - 4(-5)(-2)}}{2(-5)}$$

$$t = \frac{5 + \sqrt{15}}{5} \; or \; t = \frac{5 - \sqrt{15}}{5}$$

$$t = 1.77 \; s \; or \; t = 0.23 \; s$$

$$\boxed{t = 1.77 \; s}$$

   Accept this solution

$$\boxed{t = 0.23 \; s}$$

   Accept this solution.

   The rocket reached a height of 22 yd at 0.23 seconds on its way up and again at 1.77 seconds on its way down.

Example 2

A rectangular piece of cardboard measuring 40 in. by 30 in. is to be made into an open box with a base (bottom) of $900 \; in^2$ by cutting equal squares from the four corners and then bending up the sides. Find, to the nearest tenth of an inch, the length of the side of the square that must be cut from each corner.

Sketch a diagram to represent the problem.

Let the variable x represent the side length of the square.

- $L = 40 - 2x$
- $W = 30 - 2x$

The area of a rectangle is the product of its length and its width. The area of the base of the rectangle must be $900 \; in^2$, after the squares have been removed.

$$\begin{aligned} L \cdot W &= Area \\ (40 - 2x)(30 - 2x) &= 900 \end{aligned}$$

$$\begin{aligned} 1200 - 80x - 60x + 4x^2 &= 900 \\ 4x^2 - 140x + 1200 &= 900 \end{aligned}$$

You can now solve using the quadratic formula.

$$x = 32.7 \ or \ x = 2.3$$

The solution of 32.7 in must be rejected since it would cause the length and the width of the rectangle to result in negative values. The length of the side of the square that was cut from the cardboard was 2.3 inches.

Example 3

The local park has a rectangular flower bed that measures 10 feet by 15 feet. The caretaker plans on doubling its area by adding a strip of uniform width around the flower bed. Determine the width of the strip.

Sketch a diagram to represent the problem.

Let the variable x, represent the side length of the uniform strip.

$$\begin{array}{rr} L = 15 + 2x & L \cdot W = Area \\ W = 10 + 2x & (15)(10) = 150 \ ft^2 \end{array}$$

The area of a rectangle is the product of its length and its width. The area of the original flower bed is $150 \ ft^2$. The new flower bed must be twice this area which is $300 \ ft^2$.

$$\begin{aligned} L \cdot W &= Area \\ (15 + 2x)(10 + 2x) &= 300 \end{aligned}$$

$$\begin{aligned} 150 + 30x + 20x + 4x^2 &= 300 \\ 4x^2 + 50x + 150 &= 300 \end{aligned}$$

You can now solve using the quadratic formula.

$$x = 2.5 \ ft \ and \ x = -15 \ ft$$

$$\boxed{x = 2.5 \ ft}$$

Accept this solution.

$$\boxed{x = -15 \ ft}$$

Reject this solution. The width of the strip cannot be a negative value.

The width of the strip that is to be added to the flower bed is 2.5 feet.

Example 4

$h(t) = -4.9t^2 + 8t + 5$ represents Jeremiah's height (h) in meters above the water t seconds after he leaves the diving board.

1. What is the initial height of the diving board?
2. At what time did Jeremiah reach his maximum height?
3. What was Jeremiah's maximum height?
4. How long was Jeremiah in the air?

Sketch a graph of the function.

1. The initial height of the diving board is when the time is zero.

$$h(t) = -4.9t^2 + 8t + 5$$
$$h(t) = -4.9(0)^2 + 8(0) + 5$$
$$h(t) = 0 = 0 + 5$$
$$\boxed{h(t) = 5\ m}$$

The initial height of the diving board is 5 m.

2. The time at which Jeremiah reaches his maximum height is the x-coordinate of the vertex.

$$t = -\frac{b}{2a}$$
$$a = -4.9$$
$$b = 8$$
$$t = -\frac{8}{2(-4.9)}$$
$$t = \frac{-8}{-9.8}$$
$$\boxed{t = 0.82\ sec}$$

It took Jeremiah 0.82 seconds to reach his maximum height.

3. The maximum height was reached Jeremiah at 0.82 seconds.

$$h(t) = -4.9t^2 + 8t + 5$$
$$h(t) = -4.9(0.82)^2 + 8(0.82) + 5$$
$$h(t) = -3.29 + 6.56 + 5$$
$$\boxed{h(t) = 8.27\ m}$$

The maximum height reached by Jeremiah was 8.27 m.

4. When Jeremiah hits the water, his height will be zero.

$$h(t) = -4.9t^2 + 8t + 5$$
$$0 = -4.9t^2 + 8t + 5$$

Use the quadratic formula to solve for 't'.

$$t = -0.48\ s\ or\ t = 2.12\ s$$

$$\boxed{t = 2.12\ s}$$

Accept this solution

$$\boxed{t = -0.48\ s}$$

Reject this solution. Time cannot be a negative quantity.

Jeremiah stayed in the air for approximately 2.12 seconds.

Review

Solve the following problems using your knowledge of quadratic functions.

1. The product of two consecutive even integers is 224. Find the integers.
2. The hypotenuse of a right triangle is 26 inches. The sum of the legs is 34 inches. Find the length of the legs of the triangle.
3. The product of two consecutive integers is 812. What are the integers?
4. The width of a rectangle is 3 inches longer than the length. The area of the rectangle is 674.7904 square inches. What are the dimensions of the rectangle?
5. The product of two consecutive odd integers is 3135. What are the integers?

6. Josie wants to landscape her rectangular back garden by planting shrubs and flowers along a border of uniform width as shown in the diagram. Determine the width of the border if the outside fence has dimensions of 28 yd by 25 yd and the remaining garden is to be $\frac{3}{4}$ of the original size.
7. Gregory ran the 1800 yard race last year but knows that if he could run 0.5 yd/s faster, he could reduce his time by 30 seconds. What was Gregory's time when he ran the race last year?

During a high school baseball tournament, Lexie hits a pitch and the baseball stays in the air for 4.42 seconds. The function describes the height over time, where h is its height, in yards, and t is the time, in seconds, from the instant the ball is hit.

$h = -5t^2 + 22t + 0.5$

8. Algebraically determine the maximum height the ball reaches.
9. When will the ball reach its maximum height?
10. How long will the ball be at a height of less than 20 meters while it is in the air?

A rock is thrown off a 75 meter high cliff into some water. The height of the rock relative to the cliff after t seconds is given by $h(t) = -5t^2 + 20t$.

11. Where will the rock be after five seconds?
12. How long before the rock reaches its maximum height?
13. When will the rock hit the water?

You jump off the end of a ski jump. Your height in meters relative to the height of the ski jump after t seconds is given by $h(t) = -5t^2 + 12t$.

14. How high will you be after 2 seconds? At this point are you going up or coming down?
15. If you spend 6.1 seconds in the air, how far below the end of the ski jump do you land? (What is the vertical distance?)

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.4.

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