Chapter 2

Linear Programming: Applications and Model Formulation

"Whenever there is a hard job to be done I assign it to a lazy man; he is sure to find an easy way of doing it."

– Walter Chrysler

PREVIEW

Linear programming (LP) is a widely used mathematical modelling technique developed to help decision makers in planning and decision-making regarding optimal use of scarce resources. This chapter is devoted to illustrate the applications of LP programming in different functional areas of management and how LP models are formulated.

LEARNING OBJECTIVES

After reading this chapter you should be able to

- identify situations in which linear programming technique can be applied.
- understand fundamental concepts and general mathematical structure of a linear programming model.
- express objective function and resource constraints in LP model in terms of decision variables and parameters.
- appreciate the limitations and assumptions of linear programming technique with a view to interpret the solution.

CHAPTER OUTLINE

2.1 Introduction
2.2 Structure of Linear Programming Model
2.3 Advantages of Using Linear Programming
2.4 Limitations of Linear Programming
2.5 Application Areas of Linear Programming
2.6 General Mathematical Model of Linear Programming Problem
2.7 Guidelines on Linear Programming Model Formulation
2.8 Examples of LP Model Formulation
- Conceptual Questions
- Self Practice Problems
- Hints and Answers

❑ Chapter Summary
❑ Chapter Concepts Quiz
❑ Case Study

2.1 INTRODUCTION

The application of specific operations research techniques to determine the choice among several courses of action, so as to get an optimal value of the measures of effectiveness (objective or goal), requires to formulate (or construct) a mathematical model. Such a model helps to represent the essence of a system that is required for decision-analysis. The term *formulation* refers to the process of converting the verbal description and numerical data into mathematical expressions, which represents the relationship among relevant decision variables (or factors), objective and restrictions (constraints) on the use of scarce resources (such as labour, material, machine, time, warehouse space, capital, energy, etc.) to several competing activities (such as products, services, jobs, new equipment, projects, etc.) on the basis of a given criterion of optimality. The term *scarce resources* refers to resources that are not available in infinite quantity during the planning period. The criterion of optimality is generally either performance, return on investment, profit, cost, utility, time, distance and the like.

Linear Programming is a mathematical technique useful for allocation of 'scarce' or 'limited' resources, to several competing activities on the basis of a given criterion of optimality.

In 1947, during World War II, George B Dantzing while working with the US Air Force, developed LP model, primarily for solving military logistics problems. But now, it is extensively being used in all functional areas of management, airlines, agriculture, military operations, education, energy planning, pollution control, transportation planning and scheduling, research and development, health care systems, etc. Though these applications are diverse, all LP models have certain common properties and assumptions – that are essential for decision-makers to understand before their use.

Before discussing the basic concepts and applications of linear programming, it is important to understand the meaning of the words – *linear* and *programming.* The word *linear* refers to linear relationship among variables in a model. That is, a given change in one variable causes a proportional change in another variable. For example, doubling the investment on a certain project will also double the rate of return. The word *programming* refers to the mathematical modelling and solving of a problem that involves the use of limited resources, by choosing a particular *course of action* (or *strategy*) among the given courses of action (or strategies) in order to achieve the desired objective.

The usefulness of this technique is enhanced by the availability of several user-friendly computer software such as STORM, TORA, QSB_+, LINDO, etc. However, there is no computer software for building an LP model. Model building is an art that improves with practice. A variety of examples are given in this chapter to illustrate the formulation of an LP model.

2.2 STRUCTURE OF LINEAR PROGRAMMING MODEL

2.2.1 General Structure of an LP Model

The general structure of an LP model consists of following three basic components (or parts).

***Decision variables* (*activities*)** The evaluation of various courses of action (alternatives) and select the best to arrive at the optimal value of objective function, is guided by the nature of objective function and availability of resources. For this, certain activities (also called *decision variables*) usually denoted by $x_1, x_2, \ldots, x_n$ are conducted. The value of these variables (activities) represents the extent to which each of these is performed. For example, in a product-mix manufacturing problem, an LP model may be used to determine units of each of the products to be manufactured by using limited resources such as personnel, machinery, money, material, etc.

Four components of any LP model are: (i) decision variables, (ii) objective function, (iii) constraints, and (iv) non-negativity

The value of certain variables may or may not be under the decision-maker's control. If values are under the control of the decision-maker, then such variables are said to be *controllable*, otherwise they are said to be *uncontrollable*. These decision variables, usually interrelated in terms of consumption of resources, require simultaneous solutions. In an LP model all decision variables are continuous, controllable and non-negative. That is, $x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$.

The objective function The objective function of each LP problem is expressed in terms of decision variables to optimize the criterion of optimality (also called *measure-of-performance*) such as profit, cost, revenue, distance etc. In its general form, it is represented as:

$$\text{Optimize (Maximize or Minimize) } Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n,$$

where Z is the measure-of-performance variable, which is a function of $x_1, x_2, \ldots, x_n$. Quantities $c_1, c_2, \ldots, c_n$ are parameters that represent the contribution of a unit of the respective variable $x_1, x_2, \ldots, x_n$ to the measure-of-performance Z. The optimal value of the given objective function is obtained by the graphical method or simplex method.

The constraints There are always certain limitations (or constraints) on the use of resources, such as: labour, machine, raw material, space, money, etc., that limit the degree to which an objective can be achieved. Such constraints must be expressed as linear equalities or inequalities in terms of decision variables. The solution of an LP model must satisfy these constraints.

2.2.2 Assumptions of an LP Model

In all mathematical models, assumptions are made for reducing the complex real-world problems into a simplified form that can be more readily analyzed. The following are the major assumptions of an LP model:

Assumptions of an LP model are:
(i) certainty,
(ii) additivity,
(iii) proportionality, &
(iv) divisibility

1. ***Certainty:*** In LP models, it is assumed that all its parameters such as: availability of resources, profit (or cost) contribution per unit of decision variable and consumption of resources per unit of decision variable must be known and constant.
2. ***Additivity:*** The value of the objective function and the total amount of each resource used (or supplied), must be equal to the sum of the respective individual contribution (profit or cost) of the decision variables. For example, the total profit earned from the sale of two products A and B must be equal to the sum of the profits earned separately from A and B. Similarly, the amount of a resource consumed for producing A and B must be equal to the total sum of resources used for A and B individually.
3. ***Linearity (or proportionality):*** The amount of each resource used (or supplied) and its contribution to the profit (or cost) in objective function must be proportional to the value of each decision variable. For example, if production of one unit of a product uses 5 hours of a particular resource, then making 3 units of that product uses $3 \times 5 = 15$ hours of that resource.
4. ***Divisibility (or continuity):*** The solution values of decision variables are allowed to assume continuous values. For instance, it is possible to collect 6.254 thousand litres of milk by a milk dairy and such variables are divisible. But, it is not desirable to produce 2.5 machines and such variables are not divisible and therefore must be assigned integer values. Hence, if any of the variable can assume only integer values or are limited to discrete number of values, LP model is no longer applicable.

2.3 ADVANTAGES OF USING LINEAR PROGRAMMING

Following are certain advantages of using linear programming technique:

1. Linear programming technique helps decision-makers to use their productive resources effectively.
2. Linear programming technique improves the quality of decisions. The decision-making approach of the user of this technique becomes more objective and less subjective.
3. Linear programming technique helps to arrive at optimal solution of a decision problem by taking into account constraints on the use of resources. For example, saying that so many units of any product may be produced does not mean that all units can be sold.
4. Linear programming approach for solving decision problem highlight bottlenecks in the production processes. For example, when a bottleneck occurs, machine cannot produce sufficient number of units of a product to meet demand. Also, machines may remain idle.

2.4 LIMITATIONS OF LINEAR PROGRAMMING

In spite of having many advantages and wide areas of applications, there are some limitations associated with this technique. These are as follows:

1. Linear programming assumes linear relationships among decision variables. However, in real-life problems, decision variables, neither in the objective function nor in the constraints are linearly related.

2. While solving an LP model there is no guarantee that decision variables will get integer value. For example, how many men/machines would be required to perform a particular job, a non-integer valued solution will be meaningless. Rounding off the solution to the nearest integer will not yield an optimal solution.
3. The linear programming model does not take into consideration the effect of time and uncertainty.
4. Parameters in the model are assumed to be constant but in real-life situations, they are frequently neither known nor constant.
5. Linear programming deals with only single objective, whereas in real-life situations a decision problem may have conflicting and multiple objectives.

2.5 APPLICATION AREAS OF LINEAR PROGRAMMING

Linear programming is the most widely used technique of decision-making in business and industry and in various other fields. In this section, broad application areas of linear programming are discussed:

Applications in Agriculture

These applications fall into categories of farm economics and farm management. The former deals with inter-regional competition, optimum allocation of crop production, efficient production patterns under regional land resources and national demand constraints, while the latter is concerned with the problems of the individual farm such as allocation of limited resources such as acreage, labour, water supply, working capital, etc., so as to maximize the net revenue.

Applications in Military

Military applications include (i) selection of an air weapon system against the enemy, (ii) ensuring minimum use of aviation gasoline (iii) updating supply-chain to maximize the total tonnage of bombs dropped on a set of targets and takes care of the problem of community defence against disaster at the lowest possible cost.

Production Management

Product Mix To determine the quantity of several different products to be produced, knowing their per unit profit (cost) contribution and amount of limited production resources used. The objective is to maximize the total profit subject to all constraints.

- *Production Planning* This deals with the determination of minimum cost production plan over the planning period, of an item with a fluctuating demand, while considering the initial number of units in inventory, production capacity, constraints on production, manpower and all relevant cost factors. The objective is to minimize total operation costs.
- *Assembly-line Balancing* This problem is likely to arise when an item can be made by assembling different components. The process of assembling requires some specified sequence(s). The objective is to minimize the total elapse time.
- *Blending Problems* These problems arise when a product can be made from a variety of available raw materials, each of which has a particular composition and price. The objective here is to determine the minimum cost blend, subject to availability of the raw materials, and to minimum and maximum constraints on certain product constituents.
- *Trim Loss* When an item is made to a standard size (e.g. glass, paper sheet), the problem of determining which combination of requirements should be produced from standard materials in order to minimize the trim loss, arises.

Financial Management

- *Portfolio Selection* This deals with the selection of specific investment activity among several other activities. The objective here is to find the allocation which maximizes the total expected return or minimizes risk under certain limitations.
- *Profit Planning* This deals with the maximization of the profit margin from investment in plant facilities and equipment, cash in hand and inventory.

Marketing Management

- *Media Selection* The linear programming technique helps in determining the advertising media mix so as to maximize the effective exposure, subject to limitation of budget, specified exposure rates to different market segments, specified minimum and maximum number of advertisements in various media.
- *Travelling Salesman Problem* The salesman's problem is to find the shortest route from a given city to each of the specified cities and then returning to the original point of departure, provided no city would be visited twice during the tour. Such type of problems can be solved with the help of the modified assignment technique.
- *Physical Distribution* Linear programming determines the most economic and efficient manner of locating manufacturing plants and distribution centres for physical distribution.

Personnel Management

- *Staffing Problem* Linear programming is used to allocate optimum manpower to a particular job so as to minimize the total overtime cost or total manpower.
- *Determination of Equitable Salaries* Linear programming technique has been used in determining equitable salaries and sales incentives.
- *Job Evaluation and Selection* Selection of suitable person for a specified job and evaluation of job in organizations has been done with the help of the linear programming technique.

Other applications of linear programming lie in the area of administration, education, fleet utilization, awarding contracts, hospital administration, capital budgeting, etc.

2.6 GENERAL MATHEMATICAL MODEL OF LINEAR PROGRAMMING PROBLEM

The general linear programming problem (or model) with n decision variables and m constraints can be stated in the following form:

Optimize (Max. or Min.) $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

subject to the linear constraints,

$$\begin{aligned} a_{11}x_1 &+ a_{12}x_2 + \ldots + a_{1n}x_n \ (\leq, =, \geq)\ b_1 \\ a_{21}x_1 &+ a_{22}x_2 + \ldots + a_{2n}x_n \ (\leq, =, \geq)\ b_2 \\ \vdots \quad & \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ a_{m1}x_1 &+ a_{m2}x_2 + \ldots + a_{mn}x_n \ (\leq, =, \geq)\ b_m \end{aligned}$$

and $\quad x_1, x_2, \ldots, x_n \geq 0$

The above formulation can also be expressed in a compact form as follows.

Optimize (Max. or Min.) $Z = \sum_{j=1}^{n} c_j x_j$ (Objective function) (1)

subject to the linear constraints

$\sum_{j=1}^{n} a_{ij} x_j \ (\leq, =, \geq)\ b_i;\ i = 1, 2, \ldots, m$ (Constraints) (2)

and $x_j \geq 0;\ j = 1, 2, \ldots, n$ (Non-negativity conditions) (3)

where, the c_j s are coefficients representing the per unit profit (or cost) of decision variable x_j to the value of objective function. The a_{ij}'s are referred as *technological coefficients* (*or input-output coefficients*). These represent the amount of resource, say i consumed per unit of variable (activity) x_j. These coefficients can be positive, negative or zero. The b_i represents the *total availability of the ith resource.* The term resource is used in a very general sense to include any numerical value associated with the right-hand side of a constraint. It is assumed that $b_i \geq 0$ for all i. However, if any $b_i < 0$, then both sides of constraint i is multiplied by –1 to make $b_i > 0$ and reverse the inequality of the constraint.

In the general LP problem, the expression (≤, =, ≥) means that in any specific problem each constraint may take only one of the three possible forms:

(i) less than or equal to (≤)
(ii) equal to (=)
(iii) greater than or equal to (≥)

2.7 GUIDELINES ON LINEAR PROGRAMMING MODEL FORMULATION

The effective use and application requires, as a first step, the mathematical formulation of an LP model. Steps of LP model formulation are summarized as follows:

Step 1: Identify the decision variables

(a) Express each constraint in words. For this you should first see whether the constraint is of the form $\geq$ (at least as large as), of the form $\leq$ (no larger than) or of the form = (exactly equal to).
(b) Express verbally the objective function.
(c) Verbally identify the decision variables with the help of Step (a) and (b). For this you need to ask yourself the question – *What decisions must be made in order to optimize the objective function?*

LP model formulation requires: (i) identification of decision variables and input data, (ii) formulation of constraints, and (iii) objective function

Having followed Step 1(a) to (c) decide the symbolic notation for the decision variables and specify their units of measurement. Such specification of units of measurement would help in interpreting the final solution of the LP problem.

Step 2: Identify the problem data

To formulate an LP model, identify the problem data in terms of constants, and parameters associated with decision variables. It may be noted that the decision-maker can control values of the variables but cannot control values in the data set.

Step 3: Formulate the constraints

Convert the verbal expression of the constraints in terms of resource requirement and availability of each resource. Then express each of them as linear equality or inequality, in terms of the decision variables defined in Step 1.

Values of these decision variables in the optimal LP problem solution must satisfy these constraints in order to constitute an acceptable (feasible) solution. Wrong formulation can either lead to a solution that is not feasible or to the exclusion of a solution that is actually feasible and possibly optimal.

Step 4: Formulate the objective function

Identify whether the objective function is to be maximized or minimized. Then express it in the form of linear mathematical expression in terms of decision variables along with profit (cost) contributions associated with them.

After gaining enough experience in model building, readers may skip verbal description. The following are certain examples of LP model formulation that may be used to strengthen the ability to translate a real-life problem into a mathematical model.

2.8 EXAMPLES OF LP MODEL FORMULATION

In this section a number of illustrations have been presented on LP model formulation with the hope that readers may gain enough experience in model building.

2.8.1 Examples on production

Example 2.1 A manufacturing company is engaged in producing three types of products: A, B and C. The production department produces, each day, components sufficient to make 50 units of A, 25 units of B and 30 units of C. The management is confronted with the problem of optimizing the daily production of the products in the assembly department, where only 100 man-hours are available daily for assembling the products. The following additional information is available:

Type of Product	*Profit Contribution per Unit of Product (Rs)*	*Assembly Time per Product (hrs)*
A	12	0.8
B	20	1.7
C	45	2.5

The company has a daily order commitment for 20 units of products A and a total of 15 units of products B and C. Formulate this problem as an LP model so as to maximize the total profit.

LP model formulation The data of the problem is summarized as follows:

Resources/Constraints	*Product Type*			*Total*
	A	*B*	*C*	
Production capacity (units)	50	25	30	
Man-hours per unit	0.8	1.7	2.5	100
Order commitment (units)	20	15 (both for B and C)		
Profit contribution (Rs/unit)	12	20	45	

Decision variables Let x_1, x_2 and x_3 = number of units of products A, B and C to be produced, respectively.

The LP model

Maximize (total profit) $Z = 12x_1 + 20x_2 + 45x_3$

subject to the constraints

(i) Labour and materials

(a) $0.8x_1 + 1.7x_2 + 2.5x_3 \leq 100$, (b) $x_1 \leq 50$, (c) $x_2 \leq 25$, (d) $x_3 \leq 30$

(ii) Order commitment

(a) $x_1 \geq 20$; (b) $x_2 + x_3 \geq 15$

and $x_1, x_2, x_3 \geq 0$.

Example 2.2 A company has two plants, each of which produces and supplies two products: A and B. The plants can each work up to 16 hours a day. In plant 1, it takes three hours to prepare and pack 1,000 gallons of A and one hour to prepare and pack one quintal of B. In plant 2, it takes two hours to prepare and pack 1,000 gallons of A and 1.5 hours to prepare and pack a quintal of B. In plant 1, it costs Rs 15,000 to prepare and pack 1,000 gallons of A and Rs 28,000 to prepare and pack a quintal of B, whereas in plant 2 these costs are Rs 18,000 and Rs 26,000, respectively. The company is obliged to produce daily at least 10 thousand gallons of A and 8 quintals of B.

Formulate this problem as an LP model to find out as to how the company should organize its production so that the required amounts of the two products be obtained at the minimum cost.

LP model formulation The data of the problem is summarized as follows:

Resources/Constraints	*Product*		*Total Availability (hrs)*
	A	*B*	
Preparation time (hrs)	Plant 1: 3 hrs/thousand gallons	1 hr/quintal	16
	Plant 2: 2 hrs/thousand gallons	1.5 hr/quintal	16
Minimum daily production	10 thousand gallons	8 quintals	
Cost of production (Rs)	Plant 1: 15,000/thousand gallons	28,000/quintals	
	Plant 2: 18,000/thousand gallons	26,000/quintals	

Decision variables Let

x_1, x_2 = quantity of product A (in ’000 gallons) to be produced in plant 1 and 2, respectively.

x_3, x_4 = quantity of product B (in quintals) to be produced in plant 1 and 2, respectively.

The LP model

Minimize (total cost) $Z = 15{,}000x_1 + 18{,}000x_2 + 28{,}000x_3 + 26{,}000x_4$

subject to the constraints

(i) Preparation time

(a) $3x_1 + 2x_2 \leq 16$, (b) $x_3 + 1.5x_4 \leq 16$

(ii) Minimum daily production requirement

(a) $x_1 + x_2 \geq 10$, (b) $x_3 + x_4 \geq 8$

and $x_1, x_2, x_3, x_4 \geq 0$.

Example 2.3 An electronic company is engaged in the production of two components C_1 and C_2 that are used in radio sets. Each unit of C_1 costs the company Rs 5 in wages and Rs 5 in material, while each of C_2 costs the company Rs 25 in wages and Rs 15 in material. The company sells both products on one-period credit terms, but the company's labour and material expenses must be paid in cash. The selling price of C_1 is Rs 30 per unit and of C_2 it is Rs 70 per unit. Because of the company's strong monopoly in these components, it is assumed that the company can sell, at the prevailing prices, as many units as it produces. The company's production capacity is, however, limited by two considerations. First, at the beginning of period 1, the company has an initial balance of Rs 4,000 (cash plus bank credit plus collections from past credit sales). Second, the company has, in each period, 2,000 hours of machine time and 1,400 hours of assembly time. The production of each C_1 requires 3 hours of machine time and 2 hours of assembly time, whereas the production of each C_2 requires 2 hours of machine time and 3 hours of assembly time. Formulate this problem as an LP model so as to maximize the total profit to the company.

LP model formulation The data of the problem is summarized as follows:

Resources/Constraints	*Components*		*Total Availability*
	C_1	C_2	
Budget (Rs)	10/unit	40/unit	Rs 4,000
Machine time	3 hrs/unit	2 hrs/unit	2,000 hours
Assembly time	2 hrs/unit	3 hrs/unit	1,400 hours
Selling price	Rs 30	Rs 70	
Cost (wages + material) price	Rs 10	Rs 40	

Decision variables Let x_1 and x_2 = number of units of components C_1 and C_2 to be produced, respectively.

The LP model

Maximize (total profit) Z = Selling price – Cost price

$$= (30 - 10)\, x_1 + (70 - 40)\, x_2 = 20x_1 + 30x_2$$

subject to the constraints

(i) The total budget available

$$10x_1 + 40x_2 \le 4{,}000$$

(ii) Production time

(a) $3x_1 + 2x_2 \le 2{,}000$; (b) $2x_1 + 3x_2 \le 1{,}400$

and $x_1, x_2 \ge 0$.

Example 2.4 A company has two grades of inspectors 1 and 2, the members of which are to be assigned for a quality control inspection. It is required that at least 2,000 pieces be inspected per 8-hour day. Grade 1 inspectors can check pieces at the rate of 40 per hour, with an accuracy of 97 per cent. Grade 2 inspectors check at the rate of 30 pieces per hour with an accuracy of 95 per cent.

The wage rate of a Grade 1 inspector is Rs 5 per hour while that of a Grade 2 inspector is Rs 4 per hour. An error made by an inspector costs Rs 3 to the company. There are only nine Grade 1 inspectors and eleven Grade 2 inspectors available to the company. The company wishes to assign work to the available inspectors so as to minimize the total cost of the inspection. Formulate this problem as an LP model so as to minimize the daily inspection cost. [*Delhi Univ., MBA, 2004, 2006*]

LP model formulation The data of the problem is summarized as follows:

	Inspector	
	Grade 1	*Grade 2*
Number of inspectors	9	11
Rate of checking	40 pieces/hr	30 pieces/hr
Inaccuracy in checking	1 – 0.97 = 0.03	1 – 0.95 = 0.05
Cost of inaccuracy in checking	Rs 3/piece	Rs 3/piece
Wage rate/hour	Rs 5	Rs 4
Duration of inspection = 8 hrs per day		
Total pieces which must be inspected = 2,000		

Decision variables Let x_1 and x_2 = number of Grade 1 and 2 inspectors to be assigned for inspection, respectively.

The LP model

Hourly cost of each inspector of Grade 1 and 2 can be computed as follows:

Inspector Grade 1 : Rs $(5 + 3 \times 40 \times 0.03)$ = Rs 8.60

Inspector Grade 2 : Rs $(4 + 3 \times 30 \times 0.05)$ = Rs 8.50

Based on the given data, the LP model can be formulated as follows:

Minimize (daily inspection cost) $Z = 8\,(8.60x_1 + 8.50x_2) = 68.80x_1 + 68.00x_2$

subject to the constraints

(i) Total number of pieces that must be inspected in an 8-hour day

$$8 \times 40x_1 + 8 \times 30x_2 \geq 2000$$

(ii) Number of inspectors of Grade 1 and 2 available

(a) $x_1 \leq 9$, (b) $x_2 \leq 11$

and $x_1, x_2 \geq 0$.

Example 2.5 An electronic company produces three types of parts for automatic washing machines. It purchases casting of the parts from a local foundry and then finishes the part on drilling, shaping and polishing machines.

The selling prices of parts A, B and C are Rs 8, Rs 10 and Rs 14 respectively. All parts made can be sold. Castings for parts A, B and C, respectively cost Rs 5, Rs 6 and Rs 10.

The shop possesses only one of each type of casting machine. Costs per hour to run each of the three machines are Rs 20 for drilling, Rs 30 for shaping and Rs 30 for polishing. The capacities (parts per hour) for each part on each machine are shown in the table:

Machine	*Capacity per Hour*		
	Part A	*Part B*	*Part C*
Drilling	25	40	25
Shaping	25	20	20
Polishing	40	30	40

The management of the shop wants to know how many parts of each type it should produce per hour in order to maximize profit for an hour's run. Formulate this problem as an LP model so as to maximize total profit to the company. [*Delhi Univ., MBA, 2001, 2004, 2007*]

LP model formulation Let x_1, x_2 and x_3 = numbers of type A, B and C parts to be produced per hour, respectively.

Since 25 type A parts per hour can be run on the drilling machine at a cost of Rs 20, then Rs 20/25 = Re 0.80 is the drilling cost per type A part. Similar reasoning for shaping and polishing gives

$$\text{Profit per type A part} = (8 - 5) - \left(\frac{20}{25} + \frac{30}{25} + \frac{30}{40}\right) = 0.25$$

$$\text{Profit per type B part} = (10 - 6) - \left(\frac{20}{40} + \frac{30}{20} + \frac{30}{30}\right) = 1$$

$$\text{Profit per type C part} = (14 - 10) - \left(\frac{20}{25} + \frac{30}{20} + \frac{30}{40}\right) = 0.95$$

On the drilling machine, one type A part consumes 1/25th of the available hour, a type B part consumes 1/40th, and a type C part consumes 1/25th of an hour. Thus, the drilling machine constraint is

$$\frac{x_1}{25} + \frac{x_2}{40} + \frac{x_3}{25} \leq 1$$

Similarly, other constraints can be established.

The LP model

Maximize (total profit) $Z = 0.25\,x_1 + 1.00\,x_2 + 0.95\,x_3$

subject to the constraints

(i) Drilling machine: $\dfrac{x_1}{25} + \dfrac{x_2}{40} + \dfrac{x_3}{25} \leq 1$, (ii) Shaping machine: $\dfrac{x_1}{25} + \dfrac{x_2}{20} + \dfrac{x_3}{20} \leq 1$,

(iii) Polishing machine: $\frac{x_1}{40} + \frac{x_2}{30} + \frac{x_3}{40} \le 1,$

and $x_1, x_2, x_3 \ge 0.$

Example 2.6 A pharmaceutical company produces two pharmaceutical products: A and B. Production of both these products requires the same process – I and II. The production of B also results in a by-product C at no extra cost. The product A can be sold at a profit of Rs 3 per unit and B at a profit of Rs 8 per unit. Some quantity of this by-product can be sold at a unit profit of Rs 2, the remainder has to be destroyed and the destruction cost is Re 1 per unit. Forecasts show that only up to 5 units of C can be sold. The company gets 3 units of C for each unit of B produced. The manufacturing times are 3 hours per unit for A on process I and II, respectively, and 4 hours and 5 hours per unit for B on process I and II, respectively. Because the product C is a by product of B, no time is used in producing C. The available times are 18 and 21 hours of process I and II, respectively. Formulate this problem as an LP model to determine the quantity of A and B which should be produced, keeping C in mind, to make the highest total profit to the company. [*Delhi Univ., MBA* (*HCA*)*, 2001, 2008*]

LP model formulation The data of the problem is summarized as follows:

Constraints/Resources	*Time (hrs) Required by*			*Availability*
	A	*B*	*C*	
Process I	3	4	–	18 hrs
Process II	3	5	–	21 hrs
By-product ratio from B	–	1	3	5 units (max. units that
Profit per unit (Rs)	3	8	2	can be sold)

Decision variables Let

x_1, x_2 = units of product A and B to be produced, respectively

x_3, x_4 = units of product C to be produced and destroyed, respectively.

The LP model

Maximize (total profit) $Z = 3x_1 + 8x_2 + 2x_3 - x_4$

subject to the constraints

(i) Manufacturing constraints for product A and B

(a) $3x_1 + 4x_2 \le 18,$ (b) $3x_1 + 5x_2 \le 21$

(ii) Manufacturing constraints for by-product C

(a) $x_3 \le 5,$ (b) $-3x_2 + x_3 + x_4 = 0$

and $x_1, x_2, x_3, x_4 \ge 0.$

Example 2.7 A tape recorder company manufactures models A, B and C, which have profit contributions per unit of Rs 15, Rs 40 and Rs 60, respectively. The weekly minimum production requirements are 25 units for model A, 130 units for model B and 55 units for model C. Each type of recorder requires a certain amount of time for the manufacturing of the component parts for assembling and for packing. Specifically, a dozen units of model A require 4 hours for manufacturing, 3 hours for assembling and 1 hour for packaging. The corresponding figures for a dozen units of model B are 2.5, 4 and 2 and for a dozen units of model C are 6, 9 and 4. During the forthcoming week, the company has available 130 hours of manufacturing, 170 hours of assembling and 52 hours of packaging time. Formulate this problem as an LP model so as to maximize the total profit to the company.

LP model formulation The data of the problem is summarized as follows:

Resources/Constraints	*Models*			*Total Availability (hrs)*
	A	*B*	*C*	
Production requirement (units)	25	130	55	
Manufacturing time (per dozen)	4	2.5	6	130
Assembling time (per dozen)	3	4	9	170
Packaging time (per dozen)	1	2	4	52
Contribution per unit (Rs)	15	40	60	

Decision variables Let x_1, x_2 and x_3 = units of model A, B and C to be produced per week, respectively.

The LP model

Maximize (total profit) = $15x_1 + 40x_2 + 60x_3$

subject to the constraints

(i) Minimum production requirement:

(a) $x_1 \geq 25$, (b) $x_2 \geq 130$, (c) $x_3 \geq 55$

(ii) Manufacturing time : $\dfrac{4x_1}{12} + \dfrac{2.5x_2}{12} + \dfrac{6x_3}{12} \leq 130$

(iii) Assembling time : $\dfrac{3x_1}{12} + \dfrac{4x_2}{12} + \dfrac{9x_3}{12} \leq 170$

(iv) Packaging time : $\dfrac{x_1}{12} + \dfrac{2x_2}{12} + \dfrac{4x_3}{12} \leq 52$

and $x_1, x_2, x_3 \geq 0$.

Example 2.8 Consider the following problem faced by a production planner of a soft drink plant. He has two bottling machines A and B. A is designed for 8-ounce bottles and B for 16-ounce bottles. However, each can also be used for both types of bottles with some loss of efficiency. The manufacturing data is as follows:

Machine	*8-ounce Bottles*	*16-ounce Bottles*
A	100/minute	40/minute
B	60/minute	75/minute

The machines can be run for 8 hours per day, 5 days per week. The profit on an 8-ounce bottle is Rs 1.5 and on a 16-ounce bottle is Rs 2.5. Weekly production of the drink cannot exceed 3,00,000 bottles and the market can absorb 25,000, 8-ounce bottles and 7,000, 16-ounce bottles per week. The planner wishes to maximize his profit, subject of course, to all the production and marketing restrictions. Formulate this problem as an LP model to maximize total profit.

LP model formulation The data of the problem is summarized as follows:

Constraints	*Production*		*Availability*
	8-ounce Bottles	*16-ounce Bottles*	
Machine A time	100/minute	40/minute	8 × 5 × 60 = 2,400 minutes
Machine B time	60/minute	75/minute	8 × 5 × 60 = 2,400 minutes
Production	1	1	3,00,000 units/week
Marketing	1	–	25,000 units/week
	–	1	7,000 units/week
Profit/unit (Rs)	1.5	2.5	

Decision variables Let x_1 and x_2 = units of 8-ounce and 16-ounce bottles to be produced weekly, respectively

The LP model

Maximize (total profit) $Z = 1.5x_1 + 2.5x_2$

subject to the constraints

(i) Machine time : (a) $\dfrac{x_1}{100} + \dfrac{x_2}{40} \leq 2{,}400$ and (b) $\dfrac{x_1}{60} + \dfrac{x_2}{75} \leq 2{,}400$

(ii) Production : $x_1 + x_2 \leq 3{,}00{,}000$

(iii) Marketing : (a) $x_1 \leq 25{,}000$, (b) $x_2 \leq 7{,}000$

and $x_1, x_2 \geq 0$.

Example 2.9 A company engaged in producing tinned food has 300 trained employees on its rolls, each of whom can produce one can of food in a week. Due to the developing taste of public for this kind of food, the company plans to add to the existing labour force, by employing 150 people, in a phased manner, over the next five weeks. The newcomers would have to undergo a two-week training programme before being put to work. The training is to be given by employees from among the existing ones and it is a known fact that one employee can train three trainees. Assume that there would be no production from the trainers and the trainees during training period, as the training is off-the-job. However, the trainees would be remunerated at the rate of Rs 300 per week, the same rate would apply as for the trainers.

The company has booked the following orders to supply during the next five weeks:

Week	:	1	2	3	4	5
No. of cans	:	280	298	305	360	400

Assume that the production in any week would not be more than the number of cans ordered for, so that every delivery of the food would be 'fresh'.

Formulate this problem as an LP model to develop a training schedule that minimizes the labour cost over the five-week period. [*Delhi Univ., MBA, 2003, 2005*]

LP model formulation The data of the problem is summarized as given below:

(i) Cans supplied

Week	:	1	2	3	4	5
Number	:	280	298	305	360	400

(ii) Each trainee has to undergo a two-week training.
(iii) One employee is required to train three trainees.
(iv) Every trained worker produces one can/week but there would be no production from trainers and trainees during training.
(v) Number of employees to be employed = 150
(vi) The production in any week is not to exceed the cans required.
(vii) Number of weeks for which newcomers would be employed: 5, 4, 3, 2, 1.

Observations based on given data are as follows:

(a) Workers employed at the beginning of the first week would get salary for all the five weeks; those employed at the beginning of the second week would get salary for four weeks and so on.
(b) The value of the objective function would be obtained by multiplying it by 300 because each person would get a salary of Rs 300 per week.
(c) Inequalities have been used in the constraints because some workers might remain idle in some week(s).

Decision variables Let x_1, x_2, x_3, x_4 and x_5 = number of trainees appointed in the beginning of week 1, 2, 3, 4 and 5, respectively.

The LP model

Minimize (total labour force) $Z = 5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5$

subject to the constraints

(i) Capacity

(a) $300 - \dfrac{x_1}{3} \geq 280$, (b) $300 - \dfrac{x_1}{3} - \dfrac{x_2}{3} \geq 298$

(c) $300 + x_1 - \dfrac{x_2}{3} - \dfrac{x_3}{3} \geq 305$, (d) $300 + x_1 + x_2 - \dfrac{x_3}{3} - \dfrac{x_4}{3} \geq 360$

(e) $300 + x_1 + x_2 + x_3 - \dfrac{x_4}{3} - \dfrac{x_5}{3} \geq 400$

(ii) New recruitment

$$x_1 + x_2 + x_3 + x_4 + x_5 = 150$$

and $x_1, x_2, x_3, x_4, x_5 \geq 0$.

Example 2.10 XYZ company assembles and markets two types of mobiles – A and B. Presently 200 mobiles of each type are manufactured per week. You are advised to formulate the production schedule which will maximize the profits in the light of the following information:

Type	*Total Component Cost Per Mobile (Rs)*	*Man-Hours of Assembly Time Per Mobile*	*Average Man-Minutes of Inspection and Correction*	*Selling Price Per Mobile (Rs)*
A	2000	12	10	6000
B	1600	6	35	4800

The company employs 100 assemblers who are paid Rs 50 per hour actually worked and who will work up to a maximum of 48 hours per week. The inspectors, who are presently four, have agreed to a plan, whereby they average 40 hours of work per week each. However, the four inspectors have certain other administrative duties which have been found to take up an average of 8 hours per week between them. The inspectors are each paid a fixed wage of Rs 12000 per week.

Each mobile of either type requires one camera of same type. However, the company can obtain a maximum supply of 600 cameras per week. Their cost has been included in the component's cost, given for each mobile in the table above. The other cost incurred by the company are fixed overheads of Rs 20,000 per week.

LP model formulation Computation of contribution from radio types A and B is as follows:

		A	B
Component cost	:	2000	1600
Labour cost in assembly (Rs 50 per hour)	:	600	300
Labour cost for inspection			
$\frac{1200}{40}\times\frac{1}{60}=$ Re 0.50 per minute		5.00	17.5
Total variable cost		2605.00	1917.50
Selling price		6000	4800
Contribution (selling price – cost price)		3395	2882.50

Decision variables Let x_1 and x_2 = number of units of radio types A and B to be produced, respectively.

The LP model

Maximize (total contribution) $Z = 3395x_1 + 2882.5x_2 - 20{,}000$

subject to the constraints

(i) $12x_1 + 6x_2 \le 48 \times 100$, (ii) $10x_1 + 35x_2 \le 4 \times \left(40 - \frac{25}{3}\right) \times 60 = 7{,}600$

(iii) $x_1 + x_2 \le 600$

and $x_1, x_2 \ge 0$.

Example 2.11 A plastic products manufacturer has 1,200 boxes of transparent wrap in stock at one factory and another 1,200 boxes at its second factory. The manufacturer has orders for this product from three different retailers, in quantities of 1,000, 700 and 500 boxes, respectively. The unit shipping costs (in rupees per box) from the factories to the retailers are as follows:

	Retailer I	*Retailer II*	*Retailer III*
Factory A	14	11	13
Factory B	13	13	12

Determine a minimum cost shipping schedule for satisfying all demands from current inventory. Formulate this problem as an LP model.

LP model formulation Given that the total number of boxes available at factory A and B = total number of boxes required by retailers 1, 2 and 3.

Decision variables Let x_1, x_2 and x_3 = number of boxes to be sent from factory A to retailer 1; factory B to retailer 2 and factory C to retailer 3, respectively.

	Number of Boxes to be Sent		
	Retailer 1	*Retailer 2*	*Retailer 3*
Factory A	x_1	x_2	$1{,}200 - (x_1 + x_2)$
Factory B	$1{,}000 - x_1$	$700 - x_2$	$500 - [(1{,}000 - x_1) + (700 - x_2)]$

The LP model

$$\text{Minimize (total distance) } Z = 14x_1 + 13x_2 + 11\,(1{,}200 - x_1 - x_2) + 13\,(1{,}000 - x_1) + 13\,(700 - x_2) + 12\,(x_1 - x_2 - 700) = 2x_1 + x_2 + 26{,}900$$

subject to the constraints

(i) $x_1 + x_2 \leq 1{,}200$, (ii) $x_1 \leq 1{,}000$, (iii) $x_2 \leq 700$;

and $x_1, x_2 \geq 0$.

Example 2.12 A company produces two types of sauces: A and B. Both these sauces are made by blending two ingredients – X and Y. A certain level of flexibility is permitted in the formulae of these products. Indeed, the restrictions are that (i) B must contain no more than 75 per cent of X, and (ii) A must contain no less than 25 per cent of X, and no less than 50 per cent of Y. Up to 400 kg of X and 300 kg of Y could be purchased. The company can sell as much of these sauces as it produces at a price of Rs 18 for A and Rs 17 for B. The X and Y cost Rs 1.60 and 2.05 per kg, respectively.

The company wishes to maximize its net revenue from the sale of these sauces. Formulate this problem as an LP model.

LP model formulation Let x_1, x_2 = kg of sauces A and B to be produced, respectively.
y_1, y_2 = kg of ingredient X used to make sauces A and B, respectively.
y_3, y_4 = kg of ingredient Y used to make sauces A and B, respectively.

The LP model

Maximize $Z = 18x_1 + 17x_2 - 1.60\,(y_1 + y_2) - 2.05\,(y_3 + y_4)$

subject to the constraints

(i) $y_1 + y_3 - x_1 = 0$, (ii) $y_2 + y_4 - x_2 = 0$

(iii) $\left.\begin{array}{ll} y_1 + y_2 & \leq 400 \\ y_3 + y_4 & \leq 300 \end{array}\right\}$ (Purchase) (iv) $\left.\begin{array}{l} y_1 - 0.25\,x_1 \geq 0 \\ y_2 - 0.50\,x_2 \geq 0 \end{array}\right\}$ (Sauce A)

(v) $y_2 - 0.75x_2 \geq 0$ (Sauce B)

and $x_1, x_2, y_1, y_2, y_3, y_4 \geq 0$.

Example 2.13 A complete unit of a certain product consists of four units of component A and three units of component B. The two components (A and B) are manufactured from two different raw materials of which 100 units and 200 units, respectively, are available. Three departments are engaged in the production process with each department using a different method for manufacturing the components per production run and the resulting units of each component are given below:

Department	*Input of Raw Materials per Run (units)*		*Output of Components per Run (units)*	
	I	*II*	*A*	*B*
1	7	5	6	4
2	4	8	5	8
3	2	7	7	3

Formulate this problem as an LP model to determine the number of production runs for each department which will maximize the total number of complete units of the final product.

LP model formulation Let x_1, x_2 and x_3 = number of production runs for departments 1, 2 and 3, respectively.

Since each unit of the final product requires 4 units of component A and 3 units of component B, therefore maximum number of units of the final product cannot exceed the smaller value of

$$\left\{\frac{\text{Total number of units of A produced}}{4}\,;\,\frac{\text{Total number of units of B produced}}{3}\right\}$$

or
$$\left\{\frac{6x_1 + 5x_2 + 7x_3}{4} \text{ and } \frac{4x_1 + 8x_2 + 3x_3}{3}\right\}$$

Also if y is the number of component units of final product, then we obviously have

$$\frac{6x_1 + 5x_2 + 7x_3}{4} \geq y \text{ and } \frac{4x_1 + 8x_2 + 3x_3}{3} \geq y$$

The LP model

$$\text{Maximize } Z = \text{Min}\left\{\frac{6x_1 + 5x_2 + 7x_3}{4}\,;\,\frac{4x_1 + 8x_2 + 3x_3}{3}\right\}$$

subject to the constraints

(i) Raw material

(a) $7x_1 + 4x_2 + 2x_3 \leq 100$ (Material I), (b) $5x_1 + 8x_2 + 7x_3 \leq 200$ (Material II)

(ii) Number of component units of final product

(a) $6x_1 + 5x_2 + 7x_3 - 4y \geq 0$, (b) $4x_1 + 8x_2 + 3x_3 - 4y \geq 0$

and $x_1, x_2, x_3 \geq 0$.

Example 2.14 ABC company manufactures three grades of paint: Venus, Diana and Aurora. The plant operates on a three-shift basis and the following data is available from the production records:

Requirement of Resource	*Grade*			*Availability (capacity/month)*
	Venus	*Diana*	*Aurora*	
Special additive (kg/litre)	0.30	0.15	0.75	600 tonnes
Milling (kilolitres per machine shift)	2.00	3.00	5.00	100 machine shifts
Packing (kilolitres per shift)	12.00	12.00	12.00	80 shifts

There are no limitations on the other resources. The particulars of sales forecasts and the estimated contribution to overheads and profits are given below:

	Venus	*Diana*	*Aurora*
Maximum possible sales per month (kilolitres)	100	400	600
Contribution (Rs/kilolitre)	4,000	3,500	2,000

Due to the commitments already made, a minimum of 200 kilolitres per month, of Aurora, must be supplied the next year.

Just when the company was able to finalize the monthly production programme for the next 12 months, it received an offer from a nearby competitor for hiring 40 machine shifts per month of milling capacity for grinding Diana paint that could be spared for at least a year. However, due to additional handling at the competitor's facility, the contribution from Diana would be reduced by Re 1 per litre.

Formulate this problem as an LP model for determining the monthly production programme to maximize contribution. [*Delhi Univ., MBA, 2006*]

LP model formulation Let

x_1 = quantity of Venus (kilolitres) produced in the company
x_2 = quantity of Diana (kilolitres) produced in the company
x_3 = quantity of Diana (kilolitres) produced by hired facilities
x_4 = quantity of Aurora (kilolitres) produced in the company

The LP model

Maximize (total profit) $Z = 4{,}000x_1 + 3{,}500x_2 + (3{,}500 - 1{,}000)x_3 + 2{,}000x_4$

subject to the constraints

(i) Special additive : $0.30x_1 + 0.15x_2 + 0.15x_3 + 0.75x_4 \leq 600$

(ii) Own milling facility : $\dfrac{x_1}{2} + \dfrac{x_2}{3} + \dfrac{x_4}{5} \leq 100$

(iii) Hired milling facility : $\dfrac{x_3}{3} \leq 40$

(iv) Packing : $\dfrac{x_1}{12} + \dfrac{x_2 + x_3}{12} + \dfrac{x_4}{12} \leq 80$

(v) Marketing:

(i) $x_1 \leq 100$ (Venus); (ii) $x_2 + x_3 \leq 400$ (Diana); (iii) $200 \leq x_4 \leq 600$ (Aurora)

and $x_1, x_2, x_3, x_4 \geq 0$.

Example 2.15 Four products have to be processed through a particular plant, the quantities required for the next production period are:

Product 1 :	2,000 units	Product 2 :	3,000 units
Product 3 :	3,000 units	Product 4 :	6,000 units

There are three production lines on which the products could be processed. The rates of production in units per day and the total available capacity in days are given in the following table. The corresponding cost of using the lines is Rs 600, Rs 500 and Rs 400 per day, respectively.

Production Line (days)	*Product* 1	2	3	4	*Maximum Line*
1	150	100	500	400	20
2	200	100	760	400	20
3	160	80	890	600	18
Total	2,000	3,000	3,000	6,000	

Formulate this problem as an LP model to minimize the cost of operation.

LP model formulation Let x_{ij} = number of units of product i (i = 1, 2, 3, 4) produced on production line j (j = 1, 2, 3)

The LP model

$$\text{Minimize (total cost) } Z = 600 \sum_{i=1}^{4} x_{i1} + 500 \sum_{i=1}^{4} x_{i2} + 400 \sum_{i=1}^{4} x_{i3}$$

subject to the constraints

(i) Production: (a) $\sum_{i=1}^{3} x_{i1} = 2{,}000$, (b) $\sum_{i=1}^{3} x_{i2} = 3{,}000$

(c) $\sum_{i=1}^{3} x_{i3} = 3{,}000$, (d) $\sum_{i=1}^{3} x_{i4} = 6{,}000$

(ii) Line capacity

(a) $\dfrac{x_{11}}{150} + \dfrac{x_{12}}{100} + \dfrac{x_{13}}{500} + \dfrac{x_{14}}{400} \le 20$, (b) $\dfrac{x_{21}}{200} + \dfrac{x_{22}}{100} + \dfrac{x_{23}}{760} + \dfrac{x_{24}}{400} \le 20$

(c) $\dfrac{x_{31}}{160} + \dfrac{x_{32}}{80} + \dfrac{x_{33}}{890} + \dfrac{x_{34}}{600} \le 18$

and $x_{ij} \ge 0$ for all i and j.

Example 2.16 XYZ company produces a specific automobile spare part. A contract that the company has signed with a large truck manufacturer calls for the following 4-month shipping schedule.

Month	*Number of Parts to be Shipped*
January	3,000
February	4,000
March	5,000
April	5,000

The company can manufacture 3,000 parts per month on a regular time basis and 2,000 parts per month on an overtime basis. Its production cost is Rs 15,000 for a part produced during regular time and 25,000 for a part produced during overtime. Its monthly inventory holding cost is Rs 500. Formulate this problem as an LP model to minimize the overall cost.

LP model formulation Let x_{ijk} = number of units of automobile spare part manufactured in month i (i = 1, 2, 3, 4) using shift j (j = 1, 2) and shipped in month k (k = 1, 2, 3, 4)

The LP model

Minimize (total cost) Z = Regular time production cost + Overtime production cost + One-month inventory cost + Two-month inventory cost + Three-month inventory cost

$$= 15{,}000(x_{111} + x_{112} + x_{113} + x_{114} + x_{212} + x_{213} + x_{214} + x_{313} + x_{314} + x_{414})$$
$$+ 25{,}000(x_{121} + x_{122} + x_{123} + x_{124} + x_{222} + x_{223} + x_{224} + x_{323} + x_{324} + x_{424})$$
$$+ 500(x_{112} + x_{122} + x_{213} + x_{223} + x_{314} + x_{324}) + 1{,}000(x_{113} + x_{123} + x_{214}$$
$$+ x_{224}) + 1{,}500(x_{114} + x_{124})$$

subject to the constraints

(i) Monthly regular time production

(a) $x_{111} + x_{112} + x_{113} + x_{114} \leq 3{,}000$, (b) $x_{212} + x_{213} + x_{214} \leq 3{,}000$

(c) $x_{313} + x_{314} \leq 3{,}000$, (d) $x_{414} \leq 3{,}000$

(ii) Monthly overtime production constraints

(a) $x_{121} + x_{122} + x_{123} + x_{124} \leq 2{,}000$, (b) $x_{222} + x_{223} + x_{224} \leq 2{,}000$

(c) $x_{323} + x_{324} \leq 2{,}000$, (d) $x_{424} \leq 2{,}000$

(iii) Monthly demand constraints

(a) $x_{111} + x_{121} = 3{,}000$, (b) $x_{112} + x_{122} + x_{212} + x_{222} = 4{,}000$

(c) $x_{113} + x_{123} + x_{213} + x_{223} + x_{313} + x_{323} = 5{,}000$

(d) $x_{114} + x_{124} + x_{214} + x_{224} + x_{314} + x_{324} + x_{414} + x_{424} = 5{,}000$

and $x_{ijk} \geq 0$ for all *i, j* and *k*.

2.8.2 Examples on Marketing

Example 2.17 An advertising company wishes to plan an advertising campaign for three different media: television, radio and a magazine. The purpose of the advertising is to reach as many potential customers as possible. The following are the results of a market study:

	Television			
	Prime Day (*Rs*)	*Prime Time* (*Rs*)	*Radio* (*Rs*)	*Magazine* (*Rs*)
Cost of an advertising unit	40,000	75,000	30,000	15,000
Number of potential customers reached per unit	4,00,000	9,00,000	5,00,000	2,00,000
Number of women customers reached per unit	3,00,000	4,00,000	2,00,000	1,00,000

The company does not want to spend more than Rs 8,00,000 on advertising. It is further required that

(i) at least 2 million exposures take place amongst women,

(ii) the cost of advertising on television be limited to Rs 5,00,000,

(iii) at least 3 advertising units be bought on prime day and two units during prime time; and

(iv) the number of advertising units on the radio and the magazine should each be between 5 and 10.

Formulate this problem as an LP model to maximize potential customer reach.

LP model formulation Let x_1, x_2, x_3 and x_4 = number of advertising units bought in prime day and time on television, radio and magazine, respectively.

The LP model

Maximize (total potential customer reach) $Z = 4{,}00{,}000x_1 + 9{,}00{,}000x_2 + 5{,}00{,}000x_3 + 2{,}00{,}000x_4$

subject to the constraints

(i) Advertising budget: $40{,}000x_1 + 75{,}000x_2 + 30{,}000x_3 + 15{,}000x_4 \leq 8{,}00{,}000$

(ii) Number of women customers reached by the advertising campaign

$$3{,}00{,}000x_1 + 4{,}00{,}000x_2 + 2{,}00{,}000x_3 + 1{,}00{,}000x_4 \geq 20{,}00{,}000$$

(iii) Television advertising : (a) $40{,}000x_1 + 75{,}000x_2 \leq 5{,}00{,}000$; (b) $x_1 \geq 3$; (c) $x_2 \geq 2$

(iv) Radio and magazine advertising : (a) $5 \leq x_3 \leq 10$; (b) $5 \leq x_4 \leq 10$

and $x_1, x_2, x_3, x_4 \geq 0$.

Example 2.18 A businessman is opening a new restaurant and has budgeted Rs 8,00,000 for advertisement, for the coming month. He is considering four types of advertising:

(i) 30 second television commercials

(ii) 30 second radio commercials

(iii) Half-page advertisement in a newspaper

(iv) Full-page advertisement in a weekly magazine which will appear four times during the coming month.

The owner wishes to reach families (a) with income over Rs 50,000 and (b) with income under Rs 50,000. The amount of exposure of each media to families of type (a) and (b) and the cost of each media is shown below:

Media	*Cost of Advertisement (Rs) Rs 50,000 (a)*	*Exposure to Families with Annual Income Over Rs 50,000 (b)*	*Exposure to Families with Annual Income Under*
Television	40,000	2,00,000	3,00,000
Radio	20,000	5,00,000	7,00,000
Newspaper	15,000	3,00,000	1,50,000
Magazine	5,000	1,00,000	1,00.000

To have a balanced campaign, the owner has determined the following four restrictions:

(i) there should be no more than four television advertisements
(ii) there should be no more than four advertisements in the magazine
(iii) there should not be more than 60 per cent of all advertisements in newspaper and magazine put together
(iv) there must be at least 45,00,000 exposures to families with annual income of over Rs 50,000.

Formulate this problem as an LP model to determine the number of each type of advertisement to be given so as to maximize the total number of exposures.

LP model formulation Let x_1, x_2, x_3 and x_4 = number of television, radio, newspaper, magazine advertisements to be pursued, respectively.

The LP model

Maximize (total number of exposures of both groups) Z

$$= (2,00,000 + 3,00,000)\, x_1 + (5,00,000 + 7,00,000)\, x_2 + (3,00,000 + 1,50,000)\, x_3 + (1,00,000 + 1,00,000)\, x_4$$
$$= 5,00,000\, x_1 + 12,00,000\, x_2 + 4,50,000\, x_3 + 2,00,000\, x_4$$

subject to the constraints

(i) Available budget : $40,000x_1 + 20,000x_2 + 15,000x_3 + 5,000x_4 \le 8,00,000$
(ii) Maximum television advertisement : $x_1 \le 4$
(iii) Maximum magazine advertisement
$x_4 \le 4$ (because magazine will appear only four times in the next month)
(iv) Maximum newspaper and magazine advertisement

$$\frac{x_3 + x_4}{x_1 + x_2 + x_3 + x_4} \le 0.60 \quad \text{or} \quad -0.6x_1 - 0.6x_2 + 0.4x_3 + 0.4x_4 \le 0$$

(v) Exposure to families with income over Rs 50,000
$$2,00,000x_1 + 5,00,000x_2 + 3,00,000x_3 + 1,00,000x_4 \ge 45,00,000$$

and $x_1, x_2, x_3, x_4 \ge 0$.

Example 2.19 An advertising agency is preparing an advertising campaign for a group of agencies. These agencies have decided that different characteristics of their target customers should be given different importance (weightage). The following table gives the characteristics with their corresponding importance (weightage).

	Characteristics	*Weightage (%)*
Age	25–40 years	20
Annual income	Above Rs 60,000	30
Female	Married	50

The agency has carefully analyzed three media and has compiled the following data:

Data Item	*Media*		
	Women's Magazine (%)	*Radio (%)*	*Television (%)*
Reader characteristics			
(i) Age: 25–40 years	80	70	60
(ii) Annual income: Above Rs 60,000	60	50	45
(iii) Females/Married	40	35	25
Cost per advertisement (Rs)	9,500	25,000	1,00,000
Minimum number of advertisement allowed	10	5	5
Maximum number of advertisement allowed	20	10	10
Audience size (1000s)	750	1,000	1,500

The budget for launching the advertising campaign is Rs 5,00,000. Formulate this problem as an LP model for the agency to maximize the total expected effective exposure.

LP model formulation Let x_1, x_2 and x_3 = number of advertisements made using advertising media: women's magazines, radio and television, respectively.

The effectiveness coefficient corresponding to each of the advertising media is calculated as follows:

Media	*Effectiveness Coefficient*
Women's magazine	0.80 (0.20) + 0.60 (0.30) + 0.40 (0.50) = 0.54
Radio	0.70 (0.20) + 0.50 (0.30) + 0.35 (0.50) = 0.46
Television	0.60 (0.20) + 0.45 (0.30) + 0.25 (0.50) = 0.38

The coefficient of the objective function, i.e. effective exposure for all the three media employed, can be computed as follows:

Effective exposure = Effectiveness coefficient × Audience size

where effectiveness coefficient is a weighted average of audience characteristics. Thus, the effective exposure of each media is as follows:

$$\begin{aligned}\text{Women's magazine} &= 0.54 \times 7{,}50{,}000 = 4{,}05{,}000\\ \text{Radio} &= 0.46 \times 10{,}00{,}000 = 4{,}60{,}000\\ \text{Television} &= 0.38 \times 15{,}00{,}000 = 5{,}70{,}000\end{aligned}$$

The LP model

Maximize (effective exposure) $Z = 4{,}05{,}000x_1 + 4{,}60{,}000x_2 + 5{,}70{,}000x_3$

subject to the constraints

(i) Budget: $9{,}500x_1 + 25{,}000x_2 + 1{,}00{,}000x_3 \leq 5{,}00{,}000$

(ii) Minimum number of advertisements allowed

(a) $x_1 \geq 10$; (b) $x_2 \geq 5$; and (c) $x_3 \geq 5$

(iii) Maximum number of advertisements allowed constraints

(a) $x_1 \leq 20$; (b) $x_2 \leq 10$; and (c) $x_3 \leq 10$

and $x_1, x_2, x_3 \geq 0$.

2.8.3 Examples on Finance

Example 2.20 An engineering company planned to diversify its operations during the year 2005-06. The company allocated capital expenditure budget equal to Rs 5.15 crore in the year 2005 and Rs 6.50 crore in the year 2006. The company had to take five investment projects under consideration. The estimated net returns at that present value and the expected cash expenditures on each project in those two years are as follows.

Assume that the return from a particular project would be in direct proportion to the investment in it, so that, for example, if in a project, say A, 20% (of 120 in 2005 and of 320 in 2006) was invested, then the resulting net return in it would be 20% (of 240). This assumption also implies that individuality of the project should be ignored. Formulate this capital budgeting problem as an LP model to maximize the net return.

Project	*Estimated Net Returns (in '000 Rs)*	*Cash Expenditure (in '000 Rs)*	
		Year 2005	*Year 2006*
A	240	120	320
B	390	550	594
C	80	118	202
D	150	250	340
E	182	324	474

LP model formulation Let x_1, x_2, x_3, x_4 and x_5 = proportion of investment in projects A, B, C, D and E, respectively.

The LP model

Maximize (net return) $= 240x_1 + 390x_2 + 80x_3 + 150x_4 + 182x_5$

subject to the constraints

(i) Capital expenditure budget

(a) $120x_1 + 550x_2 + 118x_3 + 250x_4 + 324x_5 \leq 515$ [For year 2005]

(b) $320x_1 + 594x_2 + 202x_3 + 340x_4 + 474x_5 \leq 650$ [For year 2006]

(ii) 0-1 integer requirement constraints

(a) $x_1 \leq 1$, (b) $x_2 \leq 1$, (c) $x_3 \leq 1$, (d) $x_4 \leq 1$, (e) $x_5 \leq 1$

and $x_1, x_2, x_3, x_4, x_5 \geq 0$

Example 2.21 XYZ is an investment company. To aid in its investment decision, the company has developed the investment alternatives for a 10-year period, as given in the following table. The return on investment is expressed as an annual rate of return on the invested capital. The risk coefficient and growth potential are subjective estimates made by the portfolio manager of the company. The terms of investment is the average length of time period required to realize the return on investment as indicated.

Investment Alternative	*Length of Investment*	*Annual Rate of Return (Year)*	*Risk Coefficient*	*Growth Potential Return (%)*
A	4	3	1	0
B	7	12	5	18
C	8	9	4	10
D	6	20	8	32
E	10	15	6	20
F	3	6	3	7
Cash	0	0	0	0

The objective of the company is to maximize the return on its investments. The guidelines for selecting the portfolio are:

(i) The average length of the investment for the portfolio should not exceed 7 years.
(ii) The average risk for the portfolio should not exceed 5.
(iii) The average growth potential for the portfolio should be at least 10%.
(iv) At least 10% of all available funds must be retained in the form of cash, at all times.

Formulate this problem as an LP model to maximize total return.

LP model formulation Let x_j = proportion of funds to be invested in the jth investment alternative ($j = 1, 2, \ldots, 7$)

The LP model

Maximize (total return) $Z = 0.03x_1 + 0.12x_2 + 0.09x_3 + 0.20x_4 + 0.15x_5 + 0.06x_6 + 0.00x_7$

subject to the constraints

(i) Length of investment : $4x_1 + 7x_2 + 8x_3 + 6x_4 + 10x_5 + 3x_6 + 0x_7 \leq 7$
(ii) Risk level : $x_1 + 5x_2 + 4x_3 + 8x_4 + 6x_5 + 3x_6 + 0x_7 \leq 5$
(iii) Growth potential : $0x_1 + 0.18x_2 + 0.10x_3 + 0.32x_4 + 0.20x_5 + 0.07x_6 + 0x_7 \geq 0.10$
(iv) Cash requirement : $x_7 \geq 0.10$
(v) Proportion of funds : $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 1$

and $x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$.

Example 2.22 An investor has three investment opportunities available to him at the beginning of each years, for the next 5 years. He has a total of Rs 5,00,000 available for investment at the beginning of the first year. A summary of the financial characteristics of the three investment alternatives is presented in the following table:

Investment Alternative	*Allowable Size of Initial Investment (Rs)*	*Return (%)*	*Timing of Return*	*Immediate Reinvestment Possible?*
1	1,00,000	19	1 year later	yes
2	unlimited	16	2 years later	yes
3	50,000	20	3 years later	yes

The investor wishes to determine the investment plan that will maximize the amount of money which can be accumulated by the beginning of the 6th year in the future. Formulate this problem as an LP model to maximize total return. [*Delhi Univ., MBA, 2002, 2008*]

LP model formulation Let

x_{ij} = amount to be invested in investment alternative, i (i = 1, 2, 3) at the beginning of the year j (j =1, 2, . . ., 5)

y_j = amount not invested in any of the investment alternatives in period j

The LP model

Minimize (total return) $Z = 1.19x_{15} + 1.16x_{24} + 1.20x_{33} + y_5$

subject to the constraints

(i) Yearly cash flow

(a) $x_{11} + x_{21} + x_{31} + y_1 = 5{,}00{,}000$ (year 1)

(b) $-y_1 - 1.19x_{11} + x_{12} + x_{22} + x_{32} + y_2 = 0$ (year 2)

(c) $-y_2 - 1.16x_{21} - 1.19x_{12} + x_{23} + x_{23} + x_{33} + y_3 = 0$

(d) $-y_3 - 1.20x_{31} - 1.16x_{22} - 1.19x_{13} + x_{14} + x_{24} + x_{34} + y_4 = 0$ (year 4)

(e) $-y_4 - 1.20x_{32} - 1.16x_{23} - 1.19x_{14} + x_{15} + x_{25} + x_{35} + y_5 = 0$ (year 5)

(ii) Size of investment

$x_{11} \le 1{,}00{,}000$, $x_{12} \le 1{,}00{,}000$, $x_{13} \le 1{,}00{,}000$, $x_{14} \le 1{,}00{,}000$, $x_{15} \le 1{,}00{,}000$

$x_{31} \le 50{,}000$, $x_{32} \le 50{,}000$, $x_{33} \le 50{,}000$, $x_{34} \le 50{,}000$, $x_{35} \le 50{,}000$

and $x_{ij}, y_j \ge 0$ for all i and j.

Remark To formulate the first set of constraints of yearly cash flow, the following situation is adopted:

$$\frac{\text{Investment alternatives}}{x_{12} + x_{22} + x_{32} + y_2} = \frac{\text{Investment alternatives}}{y_1 + 1.19x_{11}}$$

or $$-y_1 - 1.19x_{11} + x_{12} + x_{22} + x_{32} + y_2 = 0.$$

Example 2.23 A leading CA is attempting to determine the 'best' investment portfolio and is considering six alternative investment proposals. The following table indicates point estimates for the price per share, the annual growth rate in the price per share, the annual dividend per share and a measure of the risk associated with each investment.

Portfolio Data

Shares Under Consideration	*A*	*B*	*C*	*D*	*E*	*F*
Current price per share (Rs)	80.00	100.00	160.00	120.00	150.00	200.00
Projected annual growth rate	0.08	0.07	0.10	0.12	0.09	0.15
Projected annual dividend per share (Rs)	4.00	4.50	7.50	5.50	5.75	0.00
Projected risk return	0.05	0.03	0.10	0.20	0.06	0.08

The total amount available for investment is Rs 25 lakh and the following conditions are required to be satisfied:

(i) The maximum rupee amount to be invested in alternative F is Rs 2,50,000.
(ii) No more than Rs 5,00,000 should be invested in alternatives A and B combined.
(iii) Total weighted risk should not be greater than 0.10, where

$$\text{Total weighted risk} = \frac{(\text{Amount invested in alternative } j)\,(\text{Risk of alternative } j)}{\text{Total amount invested in all the alternatives}}$$

(iv) For the sake of diversity, at least 100 shares of each stock should be purchased.
(v) At least 10 per cent of the total investment should be in alternatives A and B combined.
(vi) Dividends for the year should be at least 10,000.

Rupee return per share of stock is defined as the price per share one year hence, less current price per share plus dividend per share. If the objective is to maximize total rupee return, formulate this problem as an LP model for determining the optimal number of shares to be purchased in each of the shares under consideration. You may assume that the time horizon for the investment is one year.

LP model formulation Let x_1, x_2, x_3, x_4, x_5 and x_6 = number of shares to be purchased in each of the six investment proposals A, B, C, D, E and F, respectively.

Rupee return per share = Price per share one year hence – Current price per share + Dividend per share

= Current price per share × Projected annual growth rate (i.e. Projected growth each year + Dividend per share).

Thus, we compute the following data:

Investment Alternatives	:	*A*	*B*	*C*	*D*	*E*	*F*
No. of shares purchased	:	x_1	x_2	x_3	x_4	x_5	x_6
Projected growth for each share (Rs)	:	6.40	7.00	16.00	14.40	13.50	30.00
Projected annual dividend per share (Rs)	:	4.00	4.50	7.50	5.50	5.75	0.00
Return per share (Rs)	:	10.40	11.50	23.50	19.90	19.25	30.00

The LP model

Maximize (total return) $R = 10.40x_1 + 11.50x_2 + 23.50x_3 + 19.90x_4 + 19.25x_5 + 30.00x_6$

subject to the constraints

(i) $80x_1 + 100x_2 + 160x_3 + 120x_4 + 150x_5 + 200x_6 \leq 25{,}00{,}000$ (total fund available)

(ii) $200x_6 \leq 2{,}50{,}000$ [from condition (i)]

(iii) $80x_1 + 100x_2 \leq 5{,}00{,}000$ [from condition (ii)]

(iv) $$\frac{80x_1\,(0.05) + 100x_2\,(0.03) + 160x_3\,(0.10) + 120x_4\,(0.02) + 150x_5\,(0.06) + 200x_6\,(0.08)}{80x_1 + 100x_2 + 160x_3 + 120x_4 + 150x_5 + 200x_6} \leq 1$$

$$4x_1 + 3x_2 + 16x_3 + 24x_4 + 9x_5 + 16x_6 \leq 8x_1 + 10x_2 + 16x_3 + 12x_4 + 15x_5 + 20x_6$$

$$-4x_1 - 7x_1 + 0x_3 + 12x_4 - 6x_5 - 4x_6 \leq 0$$

(v) $x_1 \geq 100,\; x_2 \geq 100,\; x_3 \geq 100,\; x_4 \geq 100,\; x_5 \geq 100,\; x_6 \geq 100$ [from condition (iv)]

(vi) $80x_1 + 100x_2 \geq 0.10\,(80x_1 + 100x_2 + 160x_3 + 120x_4 + 150x_5 + 200x_6)$ [from condition (v)]

$$80x_1 + 100x_2 \geq 8x_1 + 10x_2 + 16x_3 + 12x_4 + 15x_5 + 20x_6$$

$$72x_1 + 90x_2 - 16x_3 - 12x_4 - 15x_5 - 20x_6 \geq 0$$

(vii) $4x_1 + 4.5x_2 + 7.5x_3 + 5.5x_4 + 5.75x_5 \geq 10{,}000$ [from condition (vi)]

and $x_j \geq 0;\ j = 1, 2, 3, 4, 5$ and 6.

Example 2.24 A company must produce two products over a period of three months. The company can pay for materials and labour from two sources: company funds and borrowed funds.

The firm has to take three decisions:

(a) How many units of product 1 should it produce?
(b) How many units of product 2 should it produce?
(c) How much money should it borrow to support the production of the products?

The firm must take these decisions in order to maximize the profit contribution, subject to the conditions stated below:

(i) Since the company's products enjoy a seller's market, the company can sell as many units as it can produce. The company would therefore like to produce as many units as possible, subject to its production capacity and financial constraints. The capacity constraints, together with cost and price data, are shown in the following table:

Capacity, Price and Cost Data

Product	*Selling Price (Rs per Unit)*	*Cost of Production (Rs per Unit)*	*Required Hours per Unit in Department*		
			A	*B*	*C*
1	14	10	0.5	0.3	0.2
2	11	8	0.3	0.4	0.1
Available hours per production period of three months :			500.00	400.00	200.00

(ii) The available company funds during the production period will be Rs 3 lakh.

(iii) A bank will give loans up to Rs 2 lakh per production period at an interest rate of 20 per cent per annum provided that company's acid (quick) test ratio is at 1 to 1 while the loan is outstanding. Take a simplified acid-test ratio given by

$$\frac{\text{Surplus cash on hand after production + Accounts receivable}}{\text{Bank borrowings + Interest occurred thereon}}$$

(iv) Also make sure that the needed funds are made available for meeting production costs. Formulate this problem as an LP model.

LP model formulation Let x_1, x_2 = number of units of products 1 and 2 produced, respectively.
x_3 = amount of money borrowed.

The LP model

Profit contribution per unit of each product = (Selling price – Variable cost of production)

Maximize Z = Total profit by producing two products – Cost of borrowed money

$= (14 - 10)x_1 + (11 - 8)x_2 - 0.05x_3 = 4x_1 + 3x_2 - 0.05x_3$

(since the interest rate is 20 per cent per annum, it will be 5 per cent for a period of three months)

subject to the constraints

(i) The production capacity constraints for each department

(a) $0.5x_1 + 0.3x_2 \leq 500$, (b) $0.3x_1 + 0.4x_2 \leq 400$, (c) $0.2x_1 + 0.1x_2 \leq 200$

(ii) The funds available for production are the sum of Rs 3,00,000 in cash that the firm has and borrowed funds maximum up to Rs 2,00,000. Consequently, production is limited to the extent that the funds are available to pay for production costs. Thus, we write the constraint as:

Funds required for production ≤ Funds available

$$10x_1 + 8x_2 \leq 3,00,000 + x_3$$
$$10x_1 + 8x_2 - x_3 \leq 3,00,000$$

(iii) Borrowed funds constraint [from condition (iii) of the problem]

$$x_1 \leq 2,00,000$$

(iv) Acid-test condition constraint

$$\frac{\text{Surplus cash on hand after production + Accounts receivable}}{\text{Bank borrowings + Interest accrued thereon}} \geq 1$$

$$\frac{(3,00,000 + x_3 - 10x_1 - 8x_2) + 14x_1 + 11x_2}{x_3 + 0.2x_3} \geq 1$$

$$3,00,000 + x_3 + 4x_1 + 3x_2 \geq x_3 + 0.2x_3$$

or $$-4x_1 - 3x_2 + 0.2x_3 \leq 3,00,000$$

and $$x_1, x_2, x_3 \geq 0.$$

Example 2.25 The most recent audited summarized balance sheet of Shop Financial Service is given below:

The company intends to enhance its investment in the lease portfolio by another Rs 1,000 lakh. For this purpose, it would like to raise a mix of debt and equity in such a way that the overall cost of raising additional funds is minimized. The following constraints apply to the way the funds can be mobilized:

(i) Total debt divided by net owned funds, cannot exceed 10.
(ii) Amount borrowed from financial institutions cannot exceed 25 per cent of the net worth.
(iii) Maximum amount of bank borrowings cannot exceed three times the net owned funds.

Balance Sheet as on 31 March 2008

Liabilities	(*Rs lakh*)	*Assets*	(*Rs lakh*)
Equity Share Capital	65	*Fixed Assets:*	
Reserves & Surplus	110	Assets on Lease	
		(Original Cost: Rs 550 lakhs)	375
Term Loan from IFCI	80	Other Fixed Assets	50
Public Deposits	150	Investments (on wholly owned subsidiaries)	20
Bank Borrowings	147	*Current Assets:*	
Other Current Liabilities	50	Stock on Hire	80
	602	Receivables	30
		Other Current Assets	35
		Miscellaneous Expenditure (not written off)	12
			602

(iv) The company would like to keep the total public deposit limited to 40 per cent of the total debt. The post-tax costs of the different sources of finance are as follows:

Equity	Term Loans	Public Deposits	Bank Borrowings
2.5%	8.5%	7%	10%

Formulate this problem as an LP model to minimize cost of funds raised.

Note: (a) Total Debt = Term loans from Financial Institutions + Public deposits + Bank borrowings
(b) Net worth = Equity share capital + Reserves and surplus
(c) Net owned funds = Net worth – Miscellaneous expenditures

LP model formulation Let x_1, x_2, x_3 and x_4 = quantity of additional funds (in lakh) raised on account of additional equity, term loans, public deposits, bank borrowings, respectively.

The LP model

Minimize (cost of additional funds raised) $Z = 0.025x_1 + 0.085x_2 + 0.07x_3 + 0.1x_4$

subject to the constraints

(i) $$\frac{\text{Total Debt}}{\text{Net owned funds}} \le 10 \quad \text{or} \quad \frac{\text{Existing debt+Additional total debt}}{\text{(Equity share capital + Reserve \& surplus + Additional equity} - \text{Misc. exp.)}} \le 10$$

$$\frac{80+150+147+x_2+x_3+x_4}{(65+110+x_1)-12} \le 10 \quad \text{or} \quad \frac{x_2+x_3+x_4+377}{x_1+163} \le 10$$

$$x_2 + x_3 + x_4 + 377 \le 10\,x_1 + 1{,}630 \quad \text{or} \quad -10x_1 + x_2 + x_3 + x_4 \le 1{,}253.$$

(ii) Amount borrowed (from financial institutions) ≤ 25% of net worth

or (Existing long-term loan from financial institutions + Additional loan) ≤ 25% (Existing equity capital + Reserve & surplus + Addl. equity capital)

$$80 + x_2 \le 0.25\,(175 + x_1)$$
$$320 + 4x_1 \le 175 + x_1$$
$$-x_1 + 4x_2 \le -145 \text{ or } x_1 - 4x_2 \ge 145.$$

(iii) Maximum bank borrowings ≤ 3 (Net owned funds)

or (Existing bank borrowings + Addl. bank borrowings ≤ 3 (Existing equity capital + Reserves & surplus + Addl. equity capital – Misc. exp.)

$$(147 + x_4) \le 3\,(65 + 110 + x_1 - 12)$$
$$x_4 - 3x_1 \le 525 - 36 - 147$$
$$-3x_1 + x_4 \le 342.$$

(iv) Total public deposit ≤ 40% of total debt.

or (Existing public deposits + Addl. public deposits) ≤ 0.40 (Existing total debt + Addl. total debt)

or $150 + x_3 \le 0.40\,(80 + 150 + 147 + x_2 + x_3 + x_4)$ or $150 + x_3 \le 0.40\,(x_2 + x_3 + x_4 + 377)$

$1{,}500 + 10x_3 \le 4x_2 + 4x_3 + 4x_4 + 1{,}508$ or $-4x_2 + 6x_3 - 4x_4 \le 8.$

(v) Addl. equity capital + Addl. term loan + Addl. public deposits + Addl. bank borrowings = 1,000 (since the company wants to enhance the investment by Rs 1,000 lakh)

or $x_1 + x_2 + x_3 + x_4 = 1{,}000$

and $x_1, x_2, x_3, x_4 \ge 0.$

Example 2.26 Renco-Foundries is in the process of drawing up a Capital Budget for the next three years. It has funds to the tune of Rs 1,00,000 that can be allocated among projects A, B, C, D and E. The net cash flows associated with an investment of Re 1 in each project are provided in the following table.

Cash Flow at Time

Investment in	*0*	*1*	*2*	*3*
A	– Re 1	+ Re 0.5	+ Re 1	Re 0
B	Re 0	– Re 1	+ Re 0.5	+ Re 1
C	– Re 1	+ Rs 1.2	Re 0	Re 0
D	– Re 1	Re 0	Re 0	Rs 1.9
E	Re 0	Re 0	– Re 1	Rs 1.5

Note: Time 0 = present, Time 1 = 1 year from now. Time 2 = 2 years from now. Time 3 = 3 years from now.

For example, Re 1 invested in investment B requires a Re 1 cash outflow at time 1 and returns Re 0.50 at time 2 and Re 1 at time 3.

To ensure that the firm remains reasonably diversified, the firm will not commit an investment exceeding Rs 75,000 for any project. The firm cannot borrow funds and therefore, the cash available for investment at any time is limited to the cash in hand. The firm will earn interest at 8 per cent per annum by parking the un-invested funds in money market investments. Assume that the returns from investments can be immediately re-invested. For example, the positive cash flow received from project C at time 1 can immediately be re-invested in project B. Formulate this problem as an LP model so as to maximize cash on hand at time 3. [*CA, 2000; Delhi Univ., MBA, 2007*]

LP model formulation Let x_1, x_2, x_3, x_4 and x_5 = Amount of rupees invested in investments A, B, C, D and E, respectively.

s_i = Money invested in money market instruments at time i (for i = 0, 1, 2).

Firm earns interest at 8 per cent per annum by parking the un-invested funds in money market instruments, hence Rs s_0, Rs s_1 and Rs s_2 which are invested in these instruments at times 0, 1 and 2 will become $1.08s_0$, $1.08s_1$ and $1.08s_2$ at times 1, 2 and 3, respectively.

Note: Cash available for investment in time t = cash on hand at time t.

From the given data, it can be computed that at time 3:

$$\begin{aligned}\text{Cash on hand} &= x_1 \times 0 + x_2 \times 1 + x_3 \times 0 + 1.9x_4 + 1.5x_5 + 1.08s_2\\ &= \text{Rs } (x_2 + 1.9x_4 + 1.5x_5 + 1.08s_2)\end{aligned}$$

The LP model

Maximize (Cash on hand at time 3) $Z = x_2 + 1.9x_4 + 1.5x_5 + 1.08s_2$

subject to the constraints

At time 0: Total fund of Rs 1,00,000 is available for investing on projects A, C and D. That is

$$x_1 + x_2 + x_3 + s_0 = 1,00,000$$

At time 1: Rs $0.5x_1$, Rs $1.2x_3$, and Rs $1.08s_0$ will be available as a result of investment made at time 0. Since Rs x_2 and s_1 are invested in project B and money market instruments, respectively at time 1, therefore we write

$$0.5x_1 + 1.2x_3 + 1.08s_0 = x_2 + s_1$$

At time 2: Rs x_1; Rs $0.5x_2$ and Rs 1.08 s_1 will be available for investment. As Rs x_5 and Rs s_2 are invested at time 2. Thus

$$x_1 + 0.5x_2 + 1.08s_1 = x_5 + s_2$$

Also, since the company will not commit an investment exceeding Rs 75,000 in any project, therefore the constraint becomes: $x_i \leq 75,000$ for i = 1, 2, 3, 4, 5.

and $x_1, x_2, x_3, x_4, x_5, s_0, s_1, s_2 \geq 0$.

2.8.4 Examples on Agriculture

Example 2.27 A cooperative farm owns 100 acres of land and has Rs 25,000 in funds available for investment. The farm members can produce a total of 3,500 man-hours worth of labour during September – May and 4,000 man-hours during June – August. If any of these man-hours are not needed, some members of the firm would use them to work on a neighbouring farm for Rs 2 per hour during September – May and Rs 3 per hour during June – August. Cash income can be obtained from the three main crops and two types of livestock: dairy cows and laying hens. No investment funds are needed for the crops. However, each cow will require an investment outlay of Rs 3,200 and each hen will require Rs 15.

In additon each cow will also require 15 acres of land, 100 man-hours during the summer. Each cow will produce a net annual cash income of Rs 3,500 for the farm. The corresponding figures for each hen are: no acreage, 0.6 man-hours during September – May; 0.4 man-hours during June – August, and an annual net cash income of Rs 200. The chicken house can accommodate a maximum of 4,000 hens and the size of the cattle-shed limits the members to a maximum of 32 cows.

Estimated man-hours and income per acre planted in each of the three crops are:

	Paddy	*Bajra*	*Jowar*
Man-hours			
September-May	40	20	25
June-August	50	35	40
Net annual cash income (Rs)	1,200	800	850

The cooperative farm wishes to determine how much acreage should be planted in each of the crops and how many cows and hens should be kept in order to maximize its net cash income. Formulate this problem as an LP model to maximize net annual cash income.

LP model formulation The data of the problem is summarized as follows:

Constraints	*Cows*	*Hens*	*Crop*			*Extra Hours*		*Total Availability*
			Paddy	*Bajra*	*Jowar*	*Sept–May*	*June–Aug*	
Man-hours								
Sept–May	100	0.6	40	20	25	1	–	3,500
June–Aug	50	0.4	50	35	40	–	1	4,000
Land	1.5	–	1	1	1	–	–	100
Cow	1	–	–	–	–	–	–	32
Hens	–	1	–	–	–	–	–	4,000
Net annual cash income (Rs)	3,500	200	1,200	800	850	2	3	

Decision variables Let

x_1 and x_2 = number of dairy cows and laying hens, respectively.

x_3, x_4 and x_5 = average of paddy crop, *bajra* crop and *jowar* crop, respectively.

x_6 = extra man-hours utilized in Sept–May.

x_7 = extra man-hours utilized in June–Aug.

The LP model

Maximize (net cash income) $Z = 3{,}500x_1 + 200x_2 + 1{,}200x_3 + 800x_4 + 850x_5 + 2x_6 + 3x_7$

subject to the constraints

(i) Man-hours: $100x_1 + 0.6x_2 + 40x_3 + 20x_4 + 25x_5 + x_6 = 3{,}500$ (Sept-May duration)

$50x_1 + 0.4x_2 + 50x_3 + 35x_4 + 40x_5 + x_7 = 4{,}000$ (June-Aug duration)

(ii) Land availability: $1.5x_1 + x_3 + x_4 + x_5 \leq 100$

(iii) Livestock: (a) $x_1 \leq 32$ (dairy cows), (b) $x_2 \leq 4{,}000$ (laying hens)

and $x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$.

Example 2.28 A certain farming organization operates three farms of comparable productivity. The output of each farm is limited both by the usable acreage and by the amount of water available for irrigation. The data for the upcoming season is as shown below:

Farm	*Usable Acreage*	*Water Available (in cubic feet)*
1	400	1,500
2	600	2,000
3	300	900

The organization is considering planting crops which differ primarily in their expected profit per acre and in their consumption of water. Furthermore, the total acreage that can be devoted to each of the crops is limited by the amount of appropriate harvesting equipment available.

Crop	*Maximum Acreage*	*Water Consumption (in cubic feet)*	*Expected Profit per Acre (Rs)*
A	700	5	4,000
B	800	4	3,000
C	300	3	1,000

In order to maintain a uniform workload among the three farms, it is the policy of the organization that the percentage of the usable acreage planted be the same for each farm. However, any combination of the crops may be grown at any of the farms. The organization wishes to know how much of each crop should be planted at the respective farms in order to maximize expected profit.

Formulate this problem as an LP model in order to maximize the total expected profit.

LP model formulation The data of the problem is summarized below:

Crop	*Farm*			*Crop Requirement (in acres)*	*Expected Profit per Acre (Rs)*
	1	*2*	*3*		
A	x_{11}	x_{12}	x_{13}	700	4,000
B	x_{21}	x_{22}	x_{23}	800	3,000
C	x_{31}	x_{32}	x_{33}	300	1,000
Usable acreage	400	600	300		
Water available per acre	1,500	2,000	900		

Decision variables Let x_{ij} = number of acres to be allocated to crop i (i = 1, 2, 3) to farm j (j = 1, 2)
The LP model

Maximize (net profit) $Z = 4{,}000(x_{11} + x_{12} + x_{13}) + 3{,}000(x_{21} + x_{22} + x_{23}) + 1{,}000(x_{31} + x_{32} + x_{33})$
subject to the constraints

(i) Crop requirement

(a) $x_{11} + x_{12} + x_{13} \le 700$, (b) $x_{21} + x_{22} + x_{23} \le 800$, (c) $x_{31} + x_{32} + x_{33} \le 300$

(ii) Available acreage

(a) $x_{11} + x_{21} + x_{31} \le 400$, (b) $x_{12} + x_{22} + x_{32} \le 600$, (c) $x_{13} + x_{23} + x_{33} \le 300$

(iii) Water available (in acre feet)

(a) $5x_{11} + 4x_{21} + 3x_{31} \le 1{,}500$, (b) $5x_{12} + 4x_{22} + 3x_{32} \le 2{,}000$, (c) $5x_{13} + 4x_{23} + 3x_{33} \le 900$

(iv) Social equality

(a) $\dfrac{x_{11}+x_{21}+x_{31}}{400} = \dfrac{x_{12}+x_{22}+x_{32}}{600}$, (b) $\dfrac{x_{12}+x_{22}+x_{32}}{600} = \dfrac{x_{13}+x_{23}+x_{33}}{300}$,

(c) $\dfrac{x_{13}+x_{23}+x_{33}}{300} = \dfrac{x_{11}+x_{21}+x_{31}}{400}$

and $x_{ij} \ge 0$ for all i and j.

2.8.5 Examples on Transportation

Example 2.29 ABC manufacturing company wishes to develop its monthly production schedule for the next three months. Depending upon the sales commitments, the company can either keep the production constant, allowing fluctuation in inventory; or its inventories can be maintained at a constant level, with fluctuating production. Fluctuating production makes overtime work necessary, the cost of which is estimated to be double the normal production cost of Rs 12 per unit. Fluctuating inventories result in an inventory carrying cost of Rs 2 per unit/month. If the company fails to fulfil its sales commitment, it incurs a shortage cost of Rs 4 per unit/month. The production capacities for the next three months are in the table:

Month	*Production Capacity (units)*		*Sales (units)*
	Regular	*Overtime*	
1	50	30	60
2	50	0	120
3	60	50	40

Formulate this problem as an LP model to minimize the total production cost.

[*Delhi Univ., MBA, 2008*]

LP model formulation The data of the problem is summarized as follows:

Month	*Production Capacity*		*Sales*
	Regular	*Overtime*	
1	50	30	60
2	50	0	120
3	60	50	40

Normal production cost : Rs 12 per unit Overtime cost : Rs 24 per unit
Carrying cost : Rs 2 per unit per month Shortage cost : Rs 4 per unit per month

Assume five sources of supply: three regular and two overtime (because the second months overtime production is zero) production capacities. The demand for the three months will be the sales during these months.

All supplies against the order have to be made and can be made in the subsequent month if it is not possible to make them during the month of order, with additional cost equivalent to shortage cost, i.e. in month 2. The cumulative production of months 1 and 2 in regular and overtime is 130 units while the orders are for 180 units. This balance can be supplied during month 3 at an additional production cost of Rs 4.

The given information can now be presented in matrix form as follows:

	M_1	M_2	M_3	Production (supply)
M_1	12	14	16	50
M_2	16	12	14	50
M_3	20	16	12	60
M_1(OT)	24	26	28	30
M_2(OT)	32	28	24	50
Sales (demand)	60	120	40	

Decision variables Let x_{ij} = amount of commodity sent from source of supply i (i = 1, 2, . . ., 5) to destination j (j = 1, 2, 3)

The LP model

Minimize (total cost) $Z = 12x_{11} + 14x_{12} + 16x_{13} + 16x_{21} + 12x_{22} + 14x_{23} + 20x_{31} + 16x_{32} + 12x_{33} + 24x_{41} + 26x_{42} + 28x_{43} + 32x_{51} + 28x_{52} + 24x_{53}$

subject to the constraints

(i) Production (supply) constraints

(a) $x_{11} + x_{12} + x_{13} = 50$, (b) $x_{21} + x_{22} + x_{23} = 50$, (c) $x_{31} + x_{32} + x_{33} = 60$,
(d) $x_{41} + x_{42} + x_{43} = 30$, (e) $x_{51} + x_{52} + x_{53} = 30$

(ii) Sales (demand) constraints

(a) $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 60$, (b) $x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 120$,
(c) $x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 40$

and $x_{ij} \geq 0$ for all i and j.

Example 2.30 A trucking firm has received an order to move 3,000 tonnes of industrial material to a destination 1,000 km away. The firm has available, at the moment, a fleet of 150 class-A 15-tonne trailer trucks and another fleet of 100 class-B 10-tonne trailer trucks. The operating costs of these trucks are Rs 3 and Rs 4 per tonne per km, respectively. Based on past experience, the firm has a policy of retaining at least one class-A truck with every two class-B trucks in reserve. It is desired to know how many of these two classes of vehicles should be despatched to move the material at minimal operating costs. Formulate this problem as an LP model.

LP model formulation Let x_1 and x_2 = number of class A and B trucks to be despatched, respectively.

The LP model

Minimize (total operating cost) $Z = 3x_1 + 4x_2$

subject to the constraints

$$15x_1 + 10x_2 \leq 3{,}000$$

$$\left.\begin{array}{l} x_1 \leq 149 \\ x_2 \leq 98 \end{array}\right\}$$ (due to the policy of retaining at least one class-A truck with every two class-B truck in reserve)

and $x_1, x_2 \geq 0$.

Example 2.31 A ship has three cargo loads – forward, after and centre. Their capacity limits are:

	Weight (kg)	*Volume (cu cm)*
Forward	2,000	1,00,000
Centre	3,000	1,35,000
After	1,500	30,000

The following cargos are offered to be carried in the ship. The ship owner may accept all or any part of each commodity:

Commodity	*Weight (kg)*	*Volume (cu cm)*	*Profit (in Rs) per kg*
A	6,000	60	60
B	4,000	50	80
C	2,000	25	50

In order to preserve the trim of the ship, the weight in each cargo must be proportional to the capacity in kg. The cargo is to be distributed in a way so as to maximize profit. Formulate this problem as an LP model.

LP model formulation x_{iA}, x_{iB} and x_{iC} = weight (in kg) of commodities A, B and C to be accommodated in the direction $i(i = 1, 2, 3$ – forward, centre and after), respectively.

The LP model

Maximize (total profit) $Z = 60\,(x_{1A} + x_{2A} + x_{3A}) + 80\,(x_{1B} + x_{2B} + x_{3B}) + 50\,(x_{1C} + x_{2C} + x_{3C})$
subject to the constraints

$$x_{1B} + x_{2B} + x_{3B} \le 4{,}000; \qquad x_{1B} + x_{2B} + x_{3B} \le 4{,}000;$$
$$x_{1B} + x_{2B} + x_{3B} \le 4{,}000; \qquad x_{1A} + x_{1B} + x_{1C} \le 2{,}000$$
$$x_{1A} + x_{2B} + x_{3C} \le 4{,}000; \qquad x_{3A} + x_{3B} + x_{3C} \le 1{,}500$$

$$60x_{1A} + 50x_{1B} + 25x_{1C} \le 1{,}00{,}000$$
$$60x_{2A} + 50x_{2B} + 25x_{2C} \le 1{,}35{,}000$$
$$60x_{3A} + 50x_{3B} + 25x_{3C} \le 30{,}000$$

and $\qquad x_{iA},\ x_{iB},\ x_{iC} \ge 0$, for all i.

2.8.6 Examples on Personnel

Example 2.32 Evening shift resident doctors in a government hospital work five consecutive days and have two consecutive days off. Their five days of work can start on any day of the week and their schedule rotates indefinitely. The hospital requires the following minimum number of doctors to work on the given days:

Sun	Mon	Tues	Wed	Thus	Fri	Sat
35	55	60	50	60	50	45

No more than 40 doctors can start their five working days on the same day. Formulate this problem as an LP model to minimize the number of doctors employed by the hospital.

[*Delhi Univ., MBA (HCA), 2006*]

LP model formulation Let x_j = number of doctors who start their duty on day j (j = 1, 2, . . ., 7) of the week.

The LP model

Minimize (total number of doctors) $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$
subject to the constraints

(i) $x_1 + x_4 + x_5 + x_6 + x_7 \ge 35$, $\qquad$ (ii) $x_2 + x_5 + x_6 + x_7 + x_1 \ge 55$
(iii) $x_3 + x_6 + x_7 + x_1 + x_2 \ge 60$, $\qquad$ (iv) $x_4 + x_7 + x_1 + x_2 + x_3 \ge 50$
(v) $x_5 + x_1 + x_2 + x_3 + x_4 \ge 60$, $\qquad$ (vi) $x_6 + x_2 + x_3 + x_4 + x_5 \ge 50$
(vii) $x_7 + x_3 + x_4 + x_5 + x_6 \ge 45$, $\qquad$ (viii) $x_j \le 40$

and $\qquad x_j \ge 0$ for all j.

Example 2.33 A machine tool company conducts on-the-job training programme for machinists. Trained machinists are used as teachers for the programme, in the ratio of one for every ten trainees. The training programme lasts for one month. From past experience it has been found that out of the ten trainees hired, only seven complete the programme successfully and the rest are released.

Trained machinists are also needed for machining. The company's requirement for machining for the next three months is as follows: January 100, February 150 and March 200. In addition, the company requires 250 machinists by April. There are 130 trained machinists available at the beginning of the year. Pays per month are:

Each trainee	:	Rs 4,400
Each trained machinist (machining and teaching)	:	Rs 4,900
Each trained machinist idle	:	Rs 4,700

Formulate this problem as an LP model to minimize the cost of hiring and training schedule and the company's requirements.

LP model formulation Let

x_1, x_2 = trained machinist teaching and idle in January, respectively
x_3, x_4 = trained machinist teaching and idle in February, respectively
x_5, x_6 = trained machinist teaching and idle in March, respectively

The LP model

Minimize (total cost) Z = Cost of training programme (teachers and trainees) + Cost of idle machinists + Cost of machinists doing machine work (constant)

$$= 4{,}400\,(10x_1 + 10x_3 + 10x_5) + 4{,}900\,(x_1 + x_3 + x_5) + 4{,}700\,(x_2 + x_4 + x_6)$$

subject to the constraints

(i) Total trained machinists available at the beginning of January = Number of machinists doing machining + Teaching + Idle

$$130 = 100 + x_1 + x_2 \quad \text{or} \quad x_1 + x_2 = 30$$

(ii) Total trained machinists available at the beginning of February = Number of machinists in January + Joining after training programme

$$130 + 7x_1 = 150 + x_3 + x_4 \quad \text{or} \quad 7x_1 - x_3 - x_4 = 20$$

In January there are $10x_1$ trainees in the programme and out of those only $7x_1$ will become trained machinists.

(iii) Total trained machinists available at the beginning of March = Number of machinists in January + Joining after training programme in January and February

$$130 + 7x_1 + 7x_3 = 200 + x_5 + x_6$$
$$7x_1 + 7x_3 - x_5 - x_6 = 70$$

(iv) Company requires 250 trained machinists by April

$$130 + 7x_1 + 7x_3 + 7x_5 = 250$$
$$7x_1 + 7x_3 + 7x_5 = 120$$

and $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$.

Example 2.34 The super bazzar in a city daily needs anything between 22 to 30 workers in the bazzar depending on the time of day. The rush hours are between noon and 2 pm. The table indicates the number of workers needed at various hours when the bazzar is open.

The super bazzar now employs 24 full-time workers, but also needs a few part-time workers. A part-time worker must put in exactly 4 hours per day, but can start any time between 9 am and 1 pm. Full- time workers work from 9 am to 5 pm but are allowed an hour for lunch (half of the full-timers eat at 12 noon, the other half at 1 am). Full-timers thus provide 35 hours per week of productive labour time.

Time Period	*Number of Workers Needed*
9 AM – 11 AM	22
11 AM – 1 PM	30
1 PM – 3 PM	25
3 PM – 5 PM	23

The management of the super bazzar limits part-time hours to a maximum of 50 per cent of the day's total requirement.

Part-timers earn Rs 28 per day on the average, while full-timers earn Rs 90 per day in salary and benefits on the average. The management wants to set a schedule that would minimize total manpower costs.

Formulate this problem as an LP model to minimize total daily manpower cost.

LP model formulation Let

y = full-time workers
x_j = part-time workers starting at 9 am, 11 am and 1 pm, respectively (j = 1, 2, 3)

The LP model

Minimize (total daily manpower cost) $Z = 90y + 28(x_1 + x_2 + x_3)$
subject to the constraints

(i) $y + x_1 \geq 22$ [9 am – 11 am], (ii) $\frac{1}{2}y + x_1 + x_2 \geq 30$ [11 am – 1 pm],

(iii) $\frac{1}{2}y + x_2 + x_3 \geq 25$ [1 pm – 3 pm], (iv) $y + x_3 \geq 23$ [3 pm – 5 pm],

(v) $y \leq 24$ [Full-timers available], (iv) $4(x_1 + x_2 + x_3) \leq 0.50\,(22 + 30 + 25 + 23)$

[Part-timers' hours cannot exceed 50% of total hours required each day which is the sum of the workers needed each hour]

and $y, x_j \geq 0$ for all j.

Example 2.35 The security and traffic force, on the eve of Republic Day, must satisfy the staffing requirements as shown in the table. Officers work 8-hour shifts starting at each of the 4-hour intervals as shown below. How many officers should report for duty at the beginning of each time period in order to minimize the total number of officers needed to satisfy the requirements?

Time	*Number of Officers Required*
0:01 – 4:00	5
4:01 – 8:00	7
8:01 – 12:00	15
12:01 – 16:00	7
16:01 – 20:00	12
20:01 – 24:00	9

Formulate this problem as an LP model so as to determine the minimum number of officers required on duty at beginning of each time period.

LP model formulation x_i = number of officers who start in shift i (i = 1, 2, 3, . . ., 6)

The LP model

Minimize (number of officers required on duty) $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$
subject to the constraints

(i) $x_1 + x_2 \geq 7$, (ii) $x_2 + x_3 \geq 15$, (iii) $x_3 + x_4 \geq 7$,

(iv) $x_4 + x_5 \geq 12$, (v) $x_5 + x_6 \geq 9$, (vi) $x_6 + x_1 \geq 5$

and $x_j \geq 0$, for all j.

CONCEPTUAL QUESTIONS

1. (a) What is linear programming? What are its major assumptions and limitations? [*Delhi Univ., MBA, Nov. 2005*]
   (b) Two of the major limitations of linear programming are: assumption of 'additivity' and 'single objective'. Elaborate by giving appropriate examples. [*Delhi Univ., MBA, Nov. 2009*]
2. Linear programming has no real-life applications'. Do you agree with this statement? Discuss. [*Delhi Univ., MBA, 2004*]
3. In relation to the LP problem, explain the implications of the following assumptions of the model:
   (i) Linearity of the objective function and constraints,
   (ii) Continuous variables,
   (iii) Certainty.
4. What is meant by a feasible solution of an LP problem?
5. 'Linear programming is one of the most frequently and successfully applied operations research technique to managerial decisions.' Elucidate this statement with some examples. [*Delhi Univ., MBA, 2008*]
6. (a) What are the advantages and limitations of LP models?
   (b) Discuss and describe the role of linear programming in managerial decision-making, bringing out limitations, if any. [*Delhi Univ., MBA, 2003*]
7. Regardless of the way one defines linear programming, certain basic requirements are necessary before this technique can be employed to business problems. What are these basic requirements in formulation? Explain briefly.
8. Discuss in brief linear programming as a technique for resource utilization. [*Delhi Univ., MBA* (*HCA*), *2004*]
9. What are the four major types of allocation problems that can be solved using the linear programming technique? Briefly explain each with an example.
10. Give the mathematical and economic structure of linear programming problems. What requirements should be met in order to apply linear programming?
11. Discuss and describe the role of linear programming in managerial decision-making bringing out limitations, if any. [*Delhi Univ., MBA, 2004, 2009*]