Lecture 2

Decision Theory

Agenda

> Introduction to Decision Making
> Decision Making Without Probabilities
> Decision Making With Probabilities
> Expected Value of Perfect Information
> Next Class

Decision Analysis

> Techniques used to make decisions among a set of discrete alternatives in the face of uncertain future events
- investments
- new products
- facility expansion
- choice of professors/courses

Key Characteristics

> Important problem
> Unique
> Non-instantaneous decision required
> Complex
> Uncertainty

Terminology

> Decision Alternatives
- controllable; our choice

> Outcomes (States of nature)
- future events, uncontrollable
- define all (mutually exclusive, collectively exhaustive)

> Payoffs
- V(D,S) ⇐ Value of A Decision in a certain State

> Criteria, for evaluation of alternatives

Structuring Decision Problems

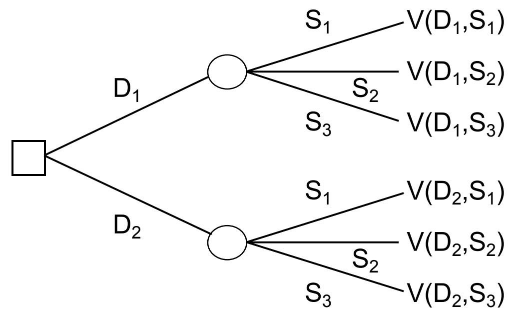
> Payoff Table
- matrix/table of actions vs outcomes, payoffs

> Decision Trees
- chronological representation of problem
- nodes and branches
- 2 types of nodes: square = decision, round = outcome
- payoffs at end of branches

Payoff Tables

		States of Nature		
		S_1	S_2	S_3
Decision	D_1	$V(D_1,S_1)$	$V(D_1,S_2)$	$V(D_1,S_3)$
	D_2	$V(D_2,S_1)$	$V(D_2,S_2)$	$V(D_2,S_3)$
	D_3	$V(D_3,S_1)$	$V(D_3,S_2)$	$V(D_3,S_3)$

Decision Trees



Types of Problems

- CERTAINTY - know which outcome (state of nature) will occur; hence select best alternative
- UNCERTAINTY - no probability information available
- RISK - probabilities of outcomes (states of nature) are available

Decisions Under Uncertainty

- When decision maker doesn't know which outcome will occur & has no probability estimates
- Four commonly used criteria :
  - equally likely (LAPLACE)
  - the optimistic approach (MAXIMAX)
  - the conservative approach (MAXIMIN)
  - the MINIMAX REGRET approach

Decisions Under Uncertainty

Payoff Table		States of Nature		
		S_1	S_2	S_3
Decision	D_1	10,000	6,500	-4,000
	D_2	8,000	6,000	1,000
	D_3	5,000	5,000	5,000

- What decision would you choose?
- Why?

Laplace Criterion

- Assume all states are equally likely

Payoff Table		States of Nature			Average Payoff
		S_1	S_2	S_3	
Decision	D_1	10,000	6,500	-4,000	4,167
	D_2	8,000	6,000	1,000	5,000
	D_3	5,000	5,000	5,000	5,000

Optimistic Approach

- Used by optimistic (aggressive?) decision maker
- Decision with the largest possible payoff is chosen
- Any concerns about this approach?

Maximax (Optimistic)

- Maximize the maximum payoff

Payoff Table		States of Nature			Maximum Payoff
		S_1	S_2	S_3	
Decision	D_1	10,000	6,500	-4,000	10,000
	D_2	8,000	6,000	1,000	8,000
	D_3	5,000	5,000	5,000	5,000

Conservative/Pessimistic Approach

- For conservatives or pessimists
- For each decision identify the minimum payoff; select the maximum of these minimum payoffs
- Any concerns about this approach?

Maximin (Pessimistic)

- Maximize the minimum payoff

Payoff Table		States of Nature			Minimum Payoff
		S_1	S_2	S_3	
Decision	D_1	10,000	6,500	-4,000	-4,000
	D_2	8,000	6,000	1,000	1,000
	D_3	5,000	5,000	5,000	5,000

Minimax Regret Approach

- Construct a regret (opportunity loss) table
- For each state of nature calculate the difference between each payoff and the largest payoff for that state of nature.
- Using regret table, the maximum regret for each possible decision is listed
- Chose decision which minimizes the maximum regrets

Minimax Regret

- Minimize the maximum regret

Payoff Table		States of Nature		
		S_1	S_2	S_3
Decision	D_1	10,000	6,500	-4,000
	D_2	8,000	6,000	1,000
	D_3	5,000	5,000	5,000

Regret Table		States of Nature			Maximum Regret
		S_1	S_2	S_3	
Decision	D_1	0	0	9,000	9,000
	D_2	2,000	500	4,000	4,000
	D_3	5,000	1,500	0	5,000

Minimization Problems

> These problems have been maximization problems - seeking the best payoff
> What is these had been minimization problems - seeking the best/lowest cost

Minimization Problems

> Minimin (Optimistic)
  - choose the global lowest cost option
> Minimax (Pessimistic)
  - identify maximum costs for each decision, then select the minimum of these
> Minimax regret
  - minimal of the maximum regrets

Minimization Objective

Payoff Table		States of Nature S_1	S_2	S_3	Minimum Cost	Maximum Cost
Decision	D_1	10,000	6,500	-4,000	-4,000	10,000
	D_2	8,000	6,000	1,000	1,000	8,000
	D_3	5,000	5,000	5,000	5,000	5,000
					Minimin	Minimax

Regret Table		States of Nature S_1	S_2	S_3	Maximum Regret	
Decision	D_1	5,000	1,500	0	5,000	Minimax Regret
	D_2	3,000	1,000	5,000	5,000	
	D_3	0	0	9,000	9,000	

Decisions Under Risk

> Probabilities known for Outcomes (States of Nature)
> Use Expected Value Approach
> The decision yielding the best expected return is chosen

Expected Value

> Expected value of a decision alternative is the sum of weighted payoffs for that alternative

Expected Value

> Expected value (EV) of decision alternative D_i is defined as:

$$EV(D_i) = \sum_{j=1}^{N} P(S_j) V(D_i, S_j)$$

> N = the number of states of nature
> $P(S_j)$ = the probability of state of nature s_j
> $V(D_i,S_j)$ = the payoff corresponding to decision alternative d_i and state of nature s_j

Expected Value

- That last slide will keep the Mathematicians and Statisticians happy
- A more practical approach, or view, of EV:

State	S_1	S_2	S_3	
P(S)	0.6	0.2	0.2	
Payoff	10,000	6,500	-4,000	EV
EV(Payoff)	6,000	1,300	-800	6,500

Sumproduct Function (Excel)

- The sumproduct function can speed up making up our spreadsheet, and add to its general applicability
- in cell e3: =sumproduct(b2..d2,b3..d3)

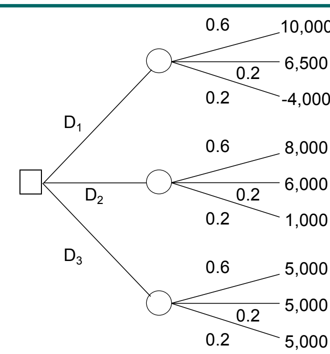
State	S_1	S_2	S_3	
P(S)	0.6	0.2	0.2	EV
Payoff	10,000	6,500	-4,000	6,500
EV(Payoff)	6,000	1,300	-800	6,500

Decisions Under Risk

- Expected value approach

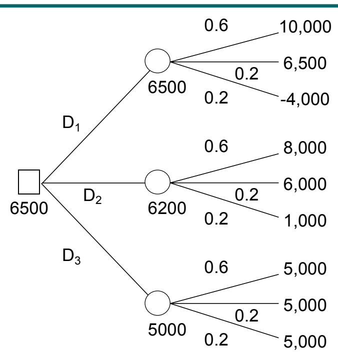
Payoff Table		States of Nature			$EV(D_i)$
		S_1	S_2	S_3	
P(S)		0.6	0.2	0.2	
Decision	D_1	10,000	6,500	-4,000	6,500
	D_2	8,000	6,000	1,000	6,200
	D_3	5,000	5,000	5,000	5,000

Decision Tree Approach



Roll back to a decision

Decision Tree Approach



Expected Value of Perfect Information

- Frequently information is available which can improve the probability estimates for the possible outcomes (states of nature)
- Expected value of perfect information (EVPI) is the increase in the expected profit that would result if one knew with certainty which state of nature would occur
- EVPI provides an upper bound on the expected value of any sample or survey information

EVPI Calculation

- Difference between the expected payoff under perfect information and the expected payoff of the optimal decision without perfect information

EVPI Calculation - I

Payoff Table		States of Nature			EV(D_i)
		S_1	S_2	S_3	
P(S)		0.6	0.2	0.2	
Decision	D_1	10,000	6,500	-4,000	6,500
	D_2	8,000	6,000	1,000	6,200
	D_3	5,000	5,000	5,000	5,000

Best	10,000	6,500	5,000	8,300
				-6,500
			EVPI	1,800

EVPI Calculation - II

- The EVPI can also be calculated as the expected regret under the optimal decision under uncertainty
- D_1 was the optimal decision under uncertainty

P(S)		0.6	0.2	0.2	E(Regret)
	D_1	0	0	9000	1,800

Next Class

- Practice Problems for
  - Linear Programming
  - Decision Theory