Travail sur le docking de l'hexarotor

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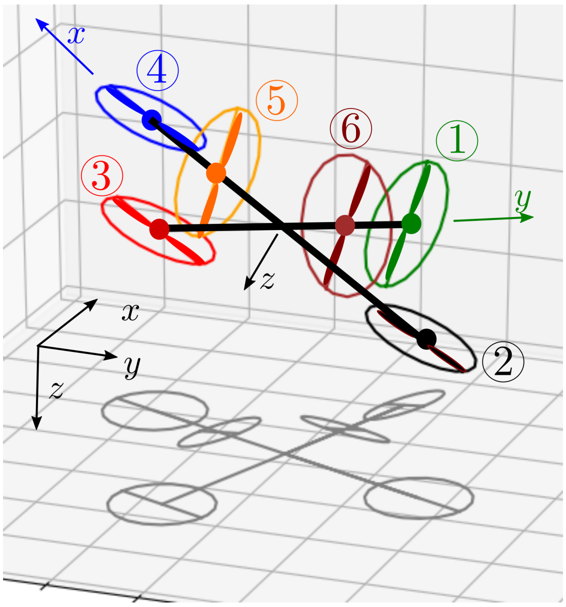
Consider a hexarotor with 6 rotors. Each rotor is at position $\mathbf{q}_i$ with a direction $\mathbf{d}_i$ and yields a force f_i. In the body frame, we have

i	1	2	3	4	5	6
$\mathbf{q}(i)$	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ \frac{1}{2} \\ 0 \end{pmatrix}$
$\mathbf{d}(i)$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

as illustrated by the figure. Each force f_i contributes to the total force on the robot as $\mathbf{d}(i) \cdot f_i$ and to the torque as $\mathbf{q}(i) \wedge \mathbf{d}(i) \cdot f_i$.



1) Find the state equations of the hexarotor.

2) Find a controller that makes the hexarotor able to dock on a platform at position $\mathbf{p}_d$ with an orientation $\mathbf{R}_d$.

Positioner. The *positioner* is a controller which computes the acceleration $\dot{\mathbf{v}}_r$ to apply to the robot to reach a desired target $\mathbf{p}_d(t)$. We define the error

$$\mathbf{e} = \mathbf{p}_d - \mathbf{p}.$$

To cancel this error, by choosing the right acceleration, we take here a proportional and derivative controller. For this, we want the error to satisfy

$$\ddot{\mathbf{e}} + \alpha_1 \dot{\mathbf{e}} + \alpha_0 \mathbf{e} = \mathbf{0}.$$

where α_0, α_1 are chosen to generate a stable characteristic polynomial. We have

$$\begin{array}{rl} & -\ddot{\mathbf{e}} = +\alpha_1 \dot{\mathbf{e}} + \alpha_0 \mathbf{e} \\ \Leftrightarrow & \ddot{\mathbf{p}} = \ddot{\mathbf{p}}_d + \alpha_1 (\dot{\mathbf{p}}_d - \dot{\mathbf{p}}) + \alpha_0 (\mathbf{p}_d - \mathbf{p}) \\ \Leftrightarrow & \dot{\mathbf{R}} \mathbf{v}_r + \mathbf{R} \dot{\mathbf{v}}_r = \ddot{\mathbf{p}}_d + \alpha_1 (\dot{\mathbf{p}}_d - \mathbf{R} \cdot \mathbf{v}_r) + \alpha_0 (\mathbf{p}_d - \mathbf{p}) \end{array}$$

Isolating $\dot{\mathbf{v}}_r$ in this expression gives us the expression of the positioner:

$$\dot{\mathbf{v}}_r = \mathbf{R}^{\mathrm{T}} \ddot{\mathbf{p}}_d + \alpha_1 (\mathbf{R}^{\mathrm{T}} \dot{\mathbf{p}}_d - \mathbf{v}_r) + \alpha_0 \mathbf{R}^{\mathrm{T}} (\mathbf{p}_d - \mathbf{p}) - \boldsymbol{\omega}_r \wedge \mathbf{v}_r.$$

Orientator. The *orientator* is a controller which tells us how to change the rotation vector in order to follow a desired orientation $\mathbf{R}_d(t)$.

We need to recall that

$$\frac{d}{dt} \log(\mathbf{R}(t)) = \mathbf{R}^{\mathrm{T}}(t) \cdot \dot{\mathbf{R}}(t). \tag{1}$$

Indeed,

$$\begin{array}{rl} & \frac{d}{dt} \exp(\log(\mathbf{R}(t)) = \frac{d}{dt} \mathbf{R}(t) \\ \Rightarrow & \exp(\log(\mathbf{R}(t)) \cdot \frac{d}{dt} \log(\mathbf{R}(t)) = \dot{\mathbf{R}}(t) \\ \Rightarrow & \frac{d}{dt} \log(\mathbf{R}(t)) = \mathbf{R}^{\mathrm{T}}(t) \cdot \dot{\mathbf{R}}(t) \end{array}$$

We want the orientation $\mathbf{R}_d(t)$. In SO(3), the error of our robot can be represented by $\mathbf{R}_d \cdot \mathbf{R}^{\mathrm{T}}$. Now, this quantity cannot be qualified as an error since SO(3) has no the addition. In the corresponding Lie algebra, we define the error as

$$\mathbf{e} = \wedge^{-1} \log(\mathbf{R}_d \mathbf{R}^{\mathrm{T}})$$

where $\mathbf{e}$ corresponds to the rotation vector we have to follow for 1sec to go from $\mathbf{R}$ to $\mathbf{R}_d$. To cancel thus error, by choosing the right torque $\boldsymbol{\tau}_r$, we can consider a proportional and derivative controller. For this, we want the error to satisfy

$$\ddot{\mathbf{e}} + \alpha_1 \dot{\mathbf{e}} + \alpha_0 \mathbf{e} = \mathbf{0},$$

where α_0, α_1 are chosen to generate a stable characteristic polynomial. Now,

$$\begin{array}{rcl} \mathbf{e} & = & \wedge^{-1} \log(\mathbf{R}_d \mathbf{R}^{\mathrm{T}}) \\ \dot{\mathbf{e}} & = & \wedge^{-1} \frac{d}{dt} \log(\mathbf{R}_d \mathbf{R}^{\mathrm{T}}) \\ & = & \wedge^{-1} \left(\mathbf{R} \mathbf{R}_d^{\mathrm{T}} \cdot \frac{d}{dt} (\mathbf{R}_d \mathbf{R}^{\mathrm{T}}) \right) \\ & = & \wedge^{-1} \left(\mathbf{R} \mathbf{R}_d^{\mathrm{T}} \cdot (\mathbf{R}_d \dot{\mathbf{R}}^{\mathrm{T}} + \dot{\mathbf{R}}_d \mathbf{R}^{\mathrm{T}}) \right) \\ & = & \wedge^{-1} \left(\mathbf{R} \mathbf{R}_d^{\mathrm{T}} \mathbf{R}_d \dot{\mathbf{R}}^{\mathrm{T}} + \mathbf{R} (\mathbf{R}_d^{\mathrm{T}} \dot{\mathbf{R}}_d) \mathbf{R}^{\mathrm{T}} \right) \\ & = & \wedge^{-1} \left(\mathbf{R} \dot{\mathbf{R}}^{\mathrm{T}} + \mathbf{R} \cdot ((\mathbf{R}_d^{\mathrm{T}} \boldsymbol{\omega}_d) \wedge) \cdot \mathbf{R}^{\mathrm{T}} \right) \\ & = & \wedge^{-1} \left(-(\boldsymbol{\omega} \wedge) + ((\mathbf{R} \cdot \mathbf{R}_d^{\mathrm{T}} \boldsymbol{\omega}_d) \wedge) \right) \\ & = & -\boldsymbol{\omega} + \mathbf{R} \mathbf{R}_d^{\mathrm{T}} \boldsymbol{\omega}_d \\ \ddot{\mathbf{e}} & = & -\dot{\boldsymbol{\omega}} + \dot{\mathbf{R}} \mathbf{R}_d^{\mathrm{T}} \boldsymbol{\omega}_d + \mathbf{R} \dot{\mathbf{R}}_d^{\mathrm{T}} \boldsymbol{\omega}_d + \mathbf{R} \mathbf{R}_d^{\mathrm{T}} \dot{\boldsymbol{\omega}}_d \end{array}$$

The differential equation for the error yields

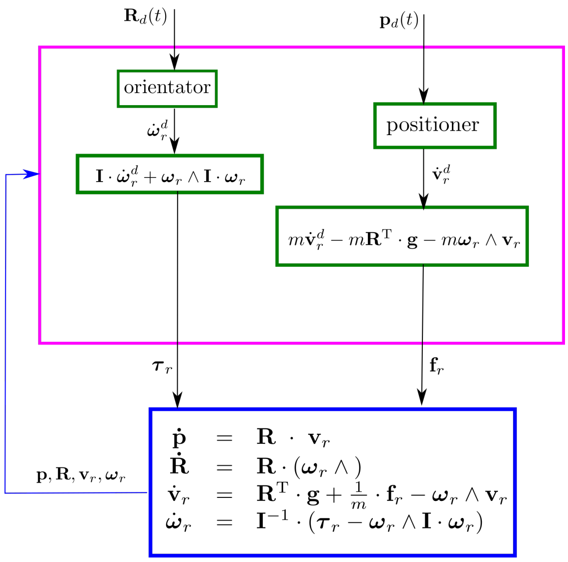
$$
\begin{aligned}
& -\ddot{\mathbf{e}} = \alpha_1\dot{\mathbf{e}} + \alpha_0\mathbf{e} \\
\Leftrightarrow\quad & \dot{\boldsymbol{\omega}} = \dot{\mathbf{R}}\mathbf{R}_d^{\mathrm{T}}\boldsymbol{\omega}_d + \mathbf{R}\dot{\mathbf{R}}_d^{\mathrm{T}}\boldsymbol{\omega}_d + \mathbf{R}\mathbf{R}_d^{\mathrm{T}}\dot{\boldsymbol{\omega}}_d + \alpha_1\dot{\mathbf{e}} + \alpha_0\mathbf{e} \\
\Leftrightarrow\quad & \dot{\boldsymbol{\omega}}_r = \mathbf{R}^{\mathrm{T}}\dot{\mathbf{R}}\mathbf{R}_d^{\mathrm{T}}\boldsymbol{\omega}_d + \dot{\mathbf{R}}_d^{\mathrm{T}}\boldsymbol{\omega}_d + \mathbf{R}_d^{\mathrm{T}}\dot{\boldsymbol{\omega}}_d + \alpha_1\mathbf{R}^{\mathrm{T}}\dot{\mathbf{e}} + \alpha_0\mathbf{R}^{\mathrm{T}}\mathbf{e} \\
\Leftrightarrow\quad & \dot{\boldsymbol{\omega}}_r = (\boldsymbol{\omega}_r \wedge)\mathbf{R}_d^{\mathrm{T}}\boldsymbol{\omega}_d + \dot{\mathbf{R}}_d^{\mathrm{T}}\boldsymbol{\omega}_d + \mathbf{R}_d^{\mathrm{T}}\dot{\boldsymbol{\omega}}_d + \alpha_1\mathbf{R}^{\mathrm{T}}\dot{\mathbf{e}} + \alpha_0\mathbf{R}^{\mathrm{T}}\mathbf{e}
\end{aligned}
$$

We can take as a characteristic polynomial for the error which is stable as for instance: $(s+1) = s^2+2s+1$. In this case, $\alpha_0 = 1$ and $\alpha_1 = 2$. As a consequence, to provide the robot an orientation $\mathbf{R}_d(t)$ we can take the following controller

	Orientator (in : $\mathbf{R}_d, \dot{\mathbf{R}}_d, \ddot{\mathbf{R}}_d, \mathbf{R}, \boldsymbol{\omega}_r$; out: $\dot{\boldsymbol{\omega}}_r^d$)
1	$\boldsymbol{\omega}_d = \wedge^{-1}(\dot{\mathbf{R}}_d \cdot \mathbf{R}_d^{\mathrm{T}})$
2	$\dot{\boldsymbol{\omega}}_d = \wedge^{-1}(\dot{\mathbf{R}}_d \cdot \dot{\mathbf{R}}_d^{\mathrm{T}} + \ddot{\mathbf{R}}_d \cdot \mathbf{R}_d^{\mathrm{T}})$
3	$\mathbf{e}_r = \mathbf{R}^{\mathrm{T}} \cdot \wedge^{-1}\log(\mathbf{R}_d\mathbf{R}^{\mathrm{T}})$
4	$\dot{\mathbf{e}}_r = -\boldsymbol{\omega}_r + \mathbf{R}_d^{\mathrm{T}}\boldsymbol{\omega}_d$
5	$\dot{\boldsymbol{\omega}}_r^d = \left((\boldsymbol{\omega}_r \wedge)\mathbf{R}_d^{\mathrm{T}} + \dot{\mathbf{R}}_d^{\mathrm{T}}\right) \cdot \boldsymbol{\omega}_d + \mathbf{R}_d^{\mathrm{T}}\dot{\boldsymbol{\omega}}_d - \alpha_1\dot{\mathbf{e}}_r - \alpha_0\mathbf{e}_r$

As illustrated by the figure, we are now able to generate the right total force $\mathbf{f}_r$ and the right torque $\boldsymbol{\tau}_r$ in order to track a time dependent position $\mathbf{p}_d(t)$ with the right time dependent orientation $\mathbf{R}_d(t)$.

To get generate the torque $\boldsymbol{\tau}_r$, we can either use the external forces such as propellers or use inertial disks inside the robot.



Concentrator. Each propeller yields a force f_i. As a consequence, the resulting force in the body frame is

$$
\begin{aligned}
\mathbf{f}_r &= \begin{pmatrix} \mathbf{d}(1) & \mathbf{d}(2) & \mathbf{d}(3) & \mathbf{d}(4) & \mathbf{d}(5) & \mathbf{d}(6) \end{pmatrix} \begin{pmatrix} f_1 \\ \vdots \\ f_6 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} f_1 \\ \vdots \\ f_6 \end{pmatrix}
\end{aligned}
$$

and

$$\boldsymbol{\tau}_r = \begin{pmatrix} \mathbf{q}(1)\wedge\mathbf{d}(1) & \cdots & \mathbf{q}(6)\wedge\mathbf{d}(6) \end{pmatrix} \begin{pmatrix} f_1 \\ \vdots \\ f_6 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} f_1 \\ \vdots \\ f_6 \end{pmatrix}$$

Therefore,

$$\begin{pmatrix} \mathbf{f}_r \\ \boldsymbol{\tau}_r \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}}_{\mathbf{C}} \begin{pmatrix} f_1 \\ \vdots \\ f_6 \end{pmatrix}$$

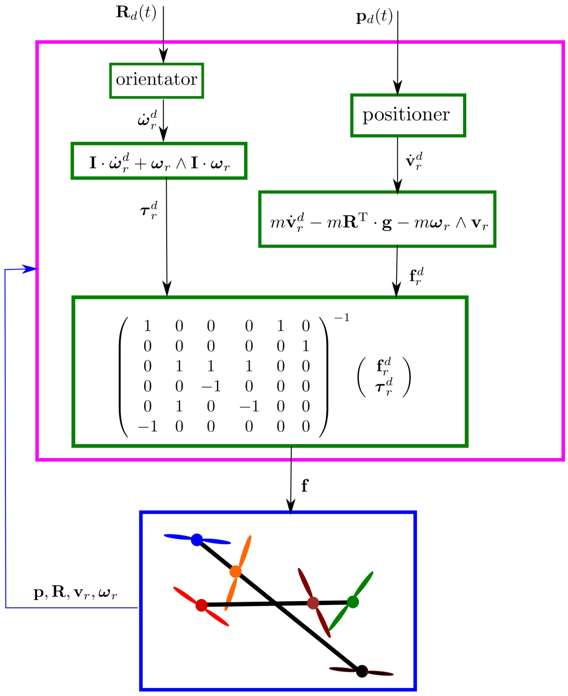
The matrix $\mathbf{C}$ is called the *concentrator*. The state equations for the hexarotor are given by

$$\begin{cases} \dot{\mathbf{p}} = \mathbf{R} \cdot \mathbf{v}_r \\ \dot{\mathbf{R}} = \mathbf{R} \cdot (\boldsymbol{\omega}_r \wedge) \\ \dot{\mathbf{v}}_r = \mathbf{R}^{\mathrm{T}} \cdot \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} + \frac{1}{m} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} f_1 \\ \vdots \\ f_6 \end{pmatrix} - \boldsymbol{\omega}_r \wedge \mathbf{v}_r \\ \dot{\boldsymbol{\omega}}_r = \mathbf{I}^{-1} \cdot \left(\begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} f_1 \\ \vdots \\ f_6 \end{pmatrix} - \boldsymbol{\omega}_r \wedge (\mathbf{I} \cdot \boldsymbol{\omega}_r) \right) \end{cases}$$

If we want the force $\mathbf{f}_r$ and the torque $\boldsymbol{\tau}_r$ we have to apply the forces

$$\begin{pmatrix} f_1 \\ \vdots \\ f_6 \end{pmatrix} = \mathbf{C}^{-1} \cdot \begin{pmatrix} \mathbf{f}_r \\ \boldsymbol{\tau}_r \end{pmatrix}$$

We thus get the controller of the figure below.



$\mathbf{R}_d(t)$
$\mathbf{p}_d(t)$
orientator
$\dot{\boldsymbol{\omega}}_r^d$
$\mathbf{I}\cdot\dot{\boldsymbol{\omega}}_r^d+\boldsymbol{\omega}_r\wedge\mathbf{I}\cdot\boldsymbol{\omega}_r$
$\boldsymbol{\tau}_r^d$
positioner
$\dot{\mathbf{v}}_r^d$
$m\dot{\mathbf{v}}_r^d-m\mathbf{R}^{\mathrm{T}}\cdot\mathbf{g}-m\boldsymbol{\omega}_r\wedge\mathbf{v}_r$
$\mathbf{f}_r^d$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{f}_r^d \\ \boldsymbol{\tau}_r^d \end{pmatrix}$
$\mathbf{f}$
$\mathbf{p},\mathbf{R},\mathbf{v}_r,\boldsymbol{\omega}_r$