1 Parsing-as-deduction rules for noisy channel

1.1 Item format

Each item has the following entries. In general, numerical indices are denoted by lowercase letters and other entries are denoted by uppercase letters.

1. frame index $(i, j, ...)$
2. PLU-internal HMM state ($s \in \{start, mid, end\}$)
3. PLU bottom type $(A, B, ...)$
4. PLU bottom index $(a, b, ...\ )$
5. edit operation type ($E \in \{NONE, IB, IT, SUB\}$)
6. PLU top index $(m, n, ...)$
7. the probability of the item (P or P')

We also assume the existence of the following functions:

- $TOP(m)$ returns the type of the PLU at position M in the top-level PLU sequence.
- $p_{hmm}(s_1 \rightarrow s_2)$ returns the probability of transitioning from PLU-internal HMM state s_1 to s_2. This is always 0.5 under the current implementation.
- $p_{op}(E)$ returns the probability of the given operation type ($\{IB, IT, SUB\}$).
- $p_{ib}(A)$ returns the probability of the insert bottom operation for PLU A, given that $E = IB$.
- $p_{it}(M)$ returns the probability of the insert top operation for PLU M, given that $E = IT$.
- $p_{sub}(M, A)$ returns the probability of the substitute operation that substitutes PLU A for PLU M, given that $E = SUB$.
- $lh(A, s, i)$ returns the likelihood of state s of PLU A at frame i (based on the audio input).

1.2 Moves in Levenshtein matrix (PLU transitions)

1.2.1 Insert Bottom

$$\frac{[i, end, A, a, E, m, P]}{\begin{array}{c}[i+1, start, B, a+1, IB, m,\\ P' = P \cdot p_{hmm}(end \rightarrow start) \cdot p_{op}(IB) \cdot p_{ib}(B) \cdot lh(B, start, i+1)]\end{array}}$$

1.2.2 Insert Top

$$\frac{[i, end, A, a, E, m, P]}{\begin{array}{c}[i, end, A, a, IT, m+1,\\ P' = P \cdot p_{op}(IT) \cdot p_{it}(TOP(m+1))]\end{array}}$$

1.2.3 Substitute

$$\frac{[i, end, A, a, E, m, P]}{\begin{array}{c}[i+1, start, B, a+1, SUB, m+1,\\ P' = P \cdot p_{hmm}(end \rightarrow start) \cdot p_{op}(SUB) \cdot p_{sub}(TOP(m+1), B) \cdot lh(B, start, i+1)]\end{array}}$$

1.3 PLU-internal transitions

1.3.1 HMM-state-internal transition

$$\frac{[i, s, A, a, E \in \{NONE, IB, SUB\}, m, P]}{\begin{array}{c}[i+1, s, A, a, NONE, m,\\ P' = P \cdot p_{hmm}(s \rightarrow s) \cdot lh(A, s, i+1)]\end{array}}$$

1.3.2 PLU-internal HMM state transition

$$\frac{[i, s \in \{start, mid\}, A, a, E \in \{NONE, IB, SUB\}, m, P]}{\begin{array}{c}[i+1, s+1, A, a, NONE, m,\\ P' = P \cdot p_{hmm}(s \rightarrow s+1) \cdot lh(A, s+1, i+1)]\end{array}}$$

1.4 Start items

The start items are as follows:

- $\{\ [\mathbf{i = 0}, start, A, \mathbf{a = 0}, IB, \mathbf{m = -1}, P = p_{op}(IB) \cdot p_{ib}(A) \cdot lh(A, start, 0)]\ \}$, $\forall A \in \{$bottom-level PLUs$\}$
- $\{\ [\mathbf{i = -1}, end, A, \mathbf{a = -1}, IT, \mathbf{m = 0}, P = p_{op}(IT) \cdot p_{it}(TOP(0))]\ \}$

- { $[\mathbf{i}=\mathbf{0}, start, A, \mathbf{a}=\mathbf{0}, SUB, \mathbf{m}=\mathbf{0}, P = p_{op}(SUB) \cdot p_{sub}(TOP(0), A) \cdot lh(A, start, 0)]$ },
  $\forall A \in$ {bottom-level PLUs}

1.5 Completion rules

The parse is complete when any item of the following is reached, where x is the number of frames in the audio input and y is the number of PLUs in the top-level sequence.

- { $[\mathbf{i}=\mathbf{n}, end, A, a, E, \mathbf{m}=\mathbf{y}, P]$ }, $\forall A \in$ {bottom-level PLUs}, $1 \leq a \leq max\_bottom$

2 Implementation details

2.1 Iteration order

Iterating through the items in a correct order, such that all items from which item x is reachable are completed before item x is entered, is nontrivial.

As a starting point for thinking about ordering, below is a list of all items from which item $x = [i, s, A, a, E, m, P]$ is reachable, and thus must be completed before x is entered, given certain conditions on the parameters.

Item	Conditions	Transition type
$[i-1, end, B, a-1, E', m, P']$	if $E = IB, s = start$	Insert-bottom operation
$[i, end, A, a, E', m-1, P']$	if $E = IT, s = end$	Insert-top operation
$[i-1, end, B, a-1, E', m-1, P']$	if $E = SUB, s = start$	Substitute operation
$[i-1, s, A, a, NONE, m, P']$	if $E \in \{NONE, IB, SUB\}$	PLU-internal HMM self-transiti
$[i-1, s-1, A, a, NONE, m, P']$	if $E \in \{NONE, IB, SUB\}, s \in \{mid, end\}$	PLU-internal HMM transition

2.2 Time complexity analysis

The unpruned chart, under the 3-state-HMM implementation, has the following dimensionality.

Dimension	Length	Typical value for a 15-second utterance
Number of frames	i	1500
HMM state	3 (constant)	3
Number of PLU bottom types	A	50
Number of PLU bottom indices	a	130
Number of edit operation types	4 (constant)	4
Number of PLU top indices	m	120

Full chart contains $n = O(i \times 3 \times A \times a \times 4 \times m) = O(iAam)$ items. However:

- The number of PLU top indices has an approximately linear relationship with the number of frames (and is upper bounded by it). We can express this as $m = O(i)$.
- Likewise, the number of PLU bottom indices also has an approximately linear relationship with i, so $a = O(i)$.
- Therefore, $n = O(Ai^3)$. In other words, the number of chart items grows as the cube of the number of frames and linearly as the number of PLU bottom types.

Full chart for a typical 15-second utterance contains approximately 1.4×10^{10} items.