

HMM expected counts

Emily Kellison-Linn, Tim O'Donnell, Elias Stengel-Eskin

1 Expected counts

Say we would like to compute the expected count of a particular HMM state given model parameters θ and some data x .

We can write this as

$$\mathbb{E}_{z \sim p(z|x, \theta)}[f_i(z)]$$

where $f_i(z)$ is a function over state sequences z which returns the number of occurrences of state i in a state sequence.

We can define $f_i(z)$ as

$$f_i(z) = \sum_{t=1}^T \delta_{i,t}(z)$$

where $\delta_{i,t}(z)$ is a delta function that returns 1 if there is an occurrence of state i at time t in z and 0 otherwise.

The full expected value then becomes

$$\begin{aligned} \mathbb{E}_{z \sim p(z|x, \theta)}[f_i(z)] &= \sum_z [p(z|x, \theta) f_i(z)] \\ &= \sum_z [p(z|x, \theta) \sum_{t=1}^T \delta_{i,t}(z)] \\ &= \sum_{t=1}^T \sum_z p(z|x, \theta) \delta_{i,t}(z) \\ &= \sum_{t=1}^T \sum_z \frac{p(z, x|\theta)}{p(x|\theta)} \delta_{i,t}(z) \end{aligned}$$