PSY9511: Seminar 4

The basics of regression and classification

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K-Nearest Neighbours

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K-Nearest Neighbours

Linear regression:

$$\hat{f}(X) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p$$

K-Nearest Neighbours:

$$\hat{f}(X) = \frac{1}{K} \sum_{x_i \in \mathcal{N}} y_i$$

K-Nearest Neighbours

Linear regression:

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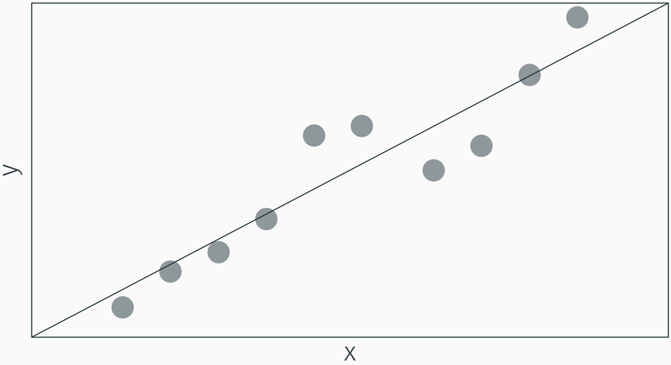
K-Nearest Neighbours:

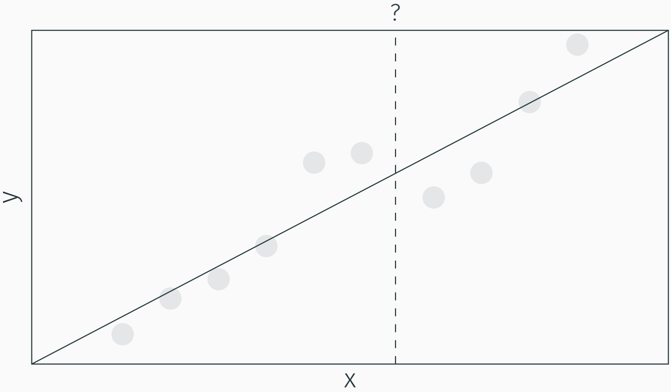
$$\hat{f}(X) = \frac{1}{K} \sum_{x_i \in \mathcal{N}} y_i$$

Blackboard!

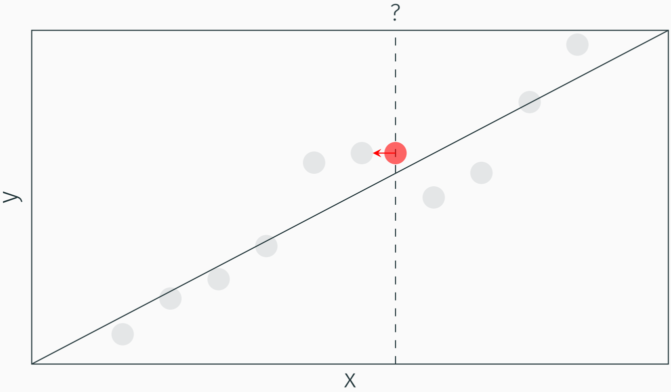
K-Nearest Neighbours



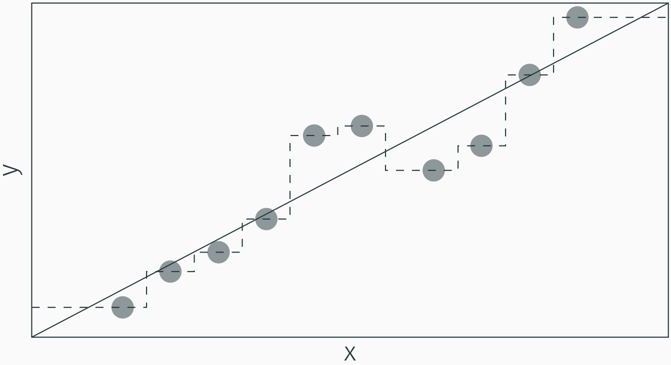
K-Nearest Neighbours



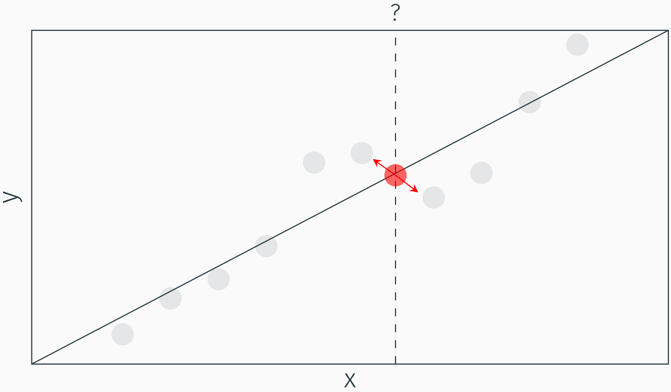
K-Nearest Neighbours



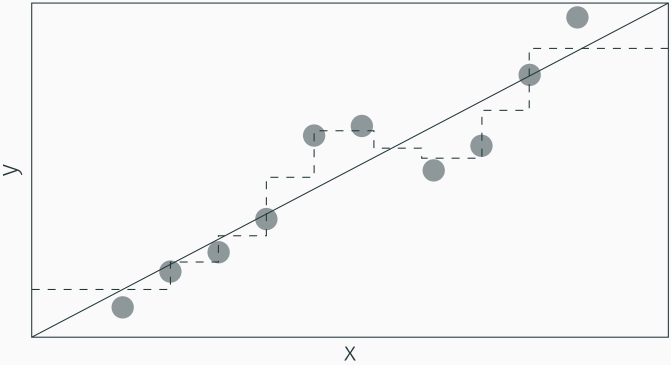
K-Nearest Neighbours



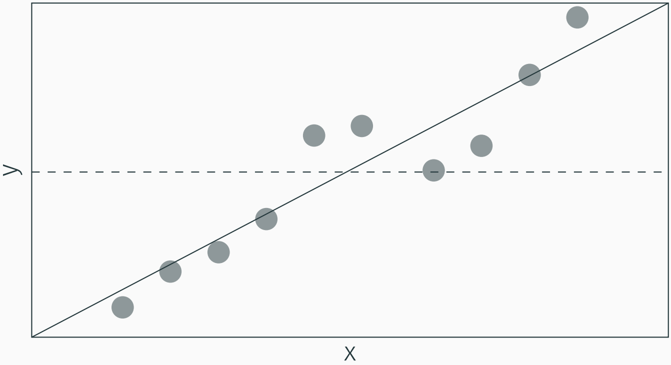
K-Nearest Neighbours



K-Nearest Neighbours



K-Nearest Neighbours

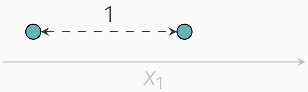


K-Nearest Neighbours

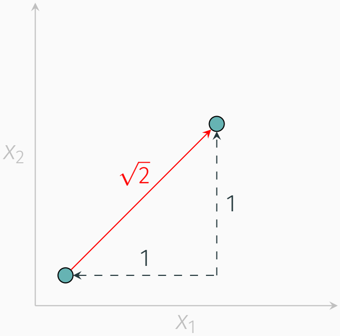
Blackboard exercise:

How does the bias-variance trade-off relate to the choice of K?

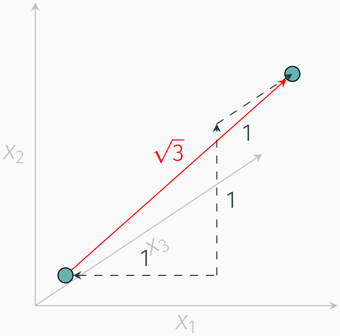
K-Nearest Neighbours



K-Nearest Neighbours



K-Nearest Neighbours



K-Nearest Neighbours

K-Nearest Neighbours: An intuitive model relying on similar datapoints to make a prediction

- No assumptions about the functional relationship between inputs and outputs
- Directly trades off bias and variance through the choice of K
- Does not work well in high dimensions as the neighbourhoods get sparse

Logistic regression

Logistic regression

mpg	manufacturer	chevrolet
36	Chevrolet	1
15	Ford	0
25	Chevrolet	1
26	Chevrolet	1
17	Ford	0
15	Ford	0
32	Chevrolet	1
14	Ford	0
14	Ford	0
28	Chevrolet	1

$$\widehat{\text{mpg}} = \beta_0 + \beta_1 \times \text{chevrolet}$$

Logistic regression

mpg	manufacturer	chevrolet
36	Chevrolet	1
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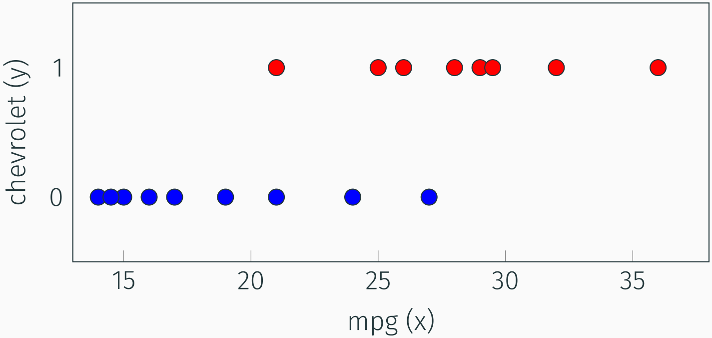
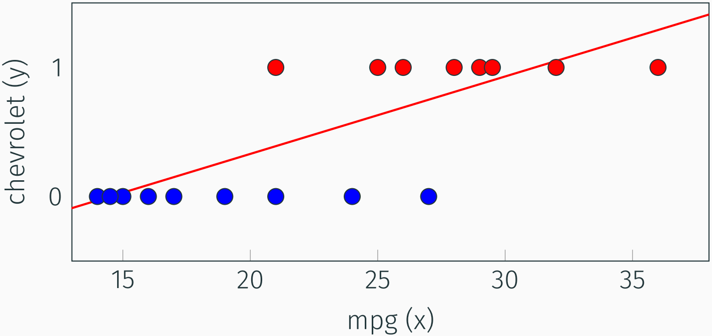
$$\widehat{\text{chevrolet}} = \beta_0 + \beta_1 \times \text{mpg}$$

Logistic regression

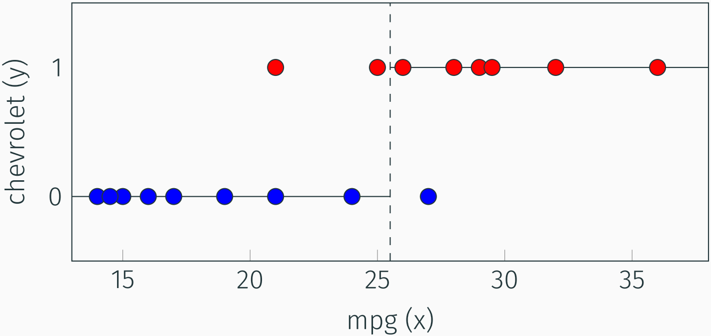


Logistic regression

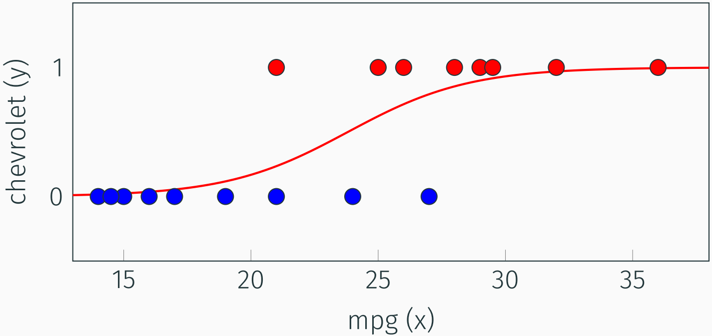


$$\widehat{\text{chevrolet}} = -0.87 + 0.06 \times \text{mpg}$$

Logistic regression



Logistic regression



$$\widehat{\text{chevrolet}} = \frac{e^{-10.22+0.42\times \text{mpg}}}{1+e^{-10.22+0.42\times \text{mpg}}}$$

Logistic regression

$$\hat{y} = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$

Logistic regression

$$\hat{y} = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$

$$e^{\beta_0 + \beta_1 x} \rightarrow 0$$

Logistic regression

$$\hat{y} = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

$$e^{\beta_0+\beta_1 x} \to 0 \implies \hat{y} = 0$$

Logistic regression

$$\hat{y} = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

$$e^{\beta_0+\beta_1 x} \to 0 \implies \hat{y} = 0$$

$$e^{\beta_0+\beta_1 x} \to \infty$$

Logistic regression

$$\hat{y} = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

$$\begin{aligned} e^{\beta_0+\beta_1 x} \to 0 \quad &\implies \hat{y} = 0 \\ e^{\beta_0+\beta_1 x} \to \infty \quad &\implies \hat{y} = 1 \end{aligned}$$

Logistic regression

□

$$\hat{y} = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

$$\begin{aligned} e^{\beta_0+\beta_1 x} \to 0 &\implies \hat{y} = 0 \\ e^{\beta_0+\beta_1 x} \to \infty &\implies \hat{y} = 1 \end{aligned}$$

$$0 \leq \hat{y} \leq 1$$

□

Logistic regression

$$\hat{y} = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

$$\begin{aligned} e^{\beta_0+\beta_1 x} \to 0 \quad &\implies \hat{y} = 0 \\ e^{\beta_0+\beta_1 x} \to \infty \quad &\implies \hat{y} = 1 \end{aligned}$$

$$0 \leq \hat{y} \leq 1 \quad \implies \hat{y} = Pr(Y = 1|X)$$

Logistic regression

$$\hat{y} = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

Logistic regression

$$\hat{y} = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

$$log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X$$

Logistic regression

$$\hat{y} = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

$$log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 x$$

"... *counterintuitive and challenging to interpret.*" - James Jaccard

Logistic regression

$$\hat{y} = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

Logistic regression

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More than one class?

Logistic regression

$$\hat{y} = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

More than one class?

$$Pr(Y = k|X = x) = \frac{e^{\beta_{0k}+\beta_{1k}x}}{\sum_{l=1}^{K} e^{\beta_{0l}+\beta_{1l}x}}$$

Logistic regression

$$\hat{y} = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

More than one class?

Blackboard!

$$Pr(Y = k|X = x) = \frac{e^{\beta_{0k}+\beta_{1k}x}}{\sum_{l=1}^{K} e^{\beta_{0l}+\beta_{1l}x}}$$

Generative models