# PSY9511: Seminar 5

## Unsupervised learning

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24.10.24


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# Outline

1. Overview of unsupervised learning
2. Clustering
    - K-means
    - Hierarchical
3. Dimensionality reduction
    - Principal component analysis (PCA)
    - Independent component analysis (ICA)

# Unsupervised learning


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# Unsupervised learning: Motivation

Supervised learning: Find $\hat{y} = f(X)$

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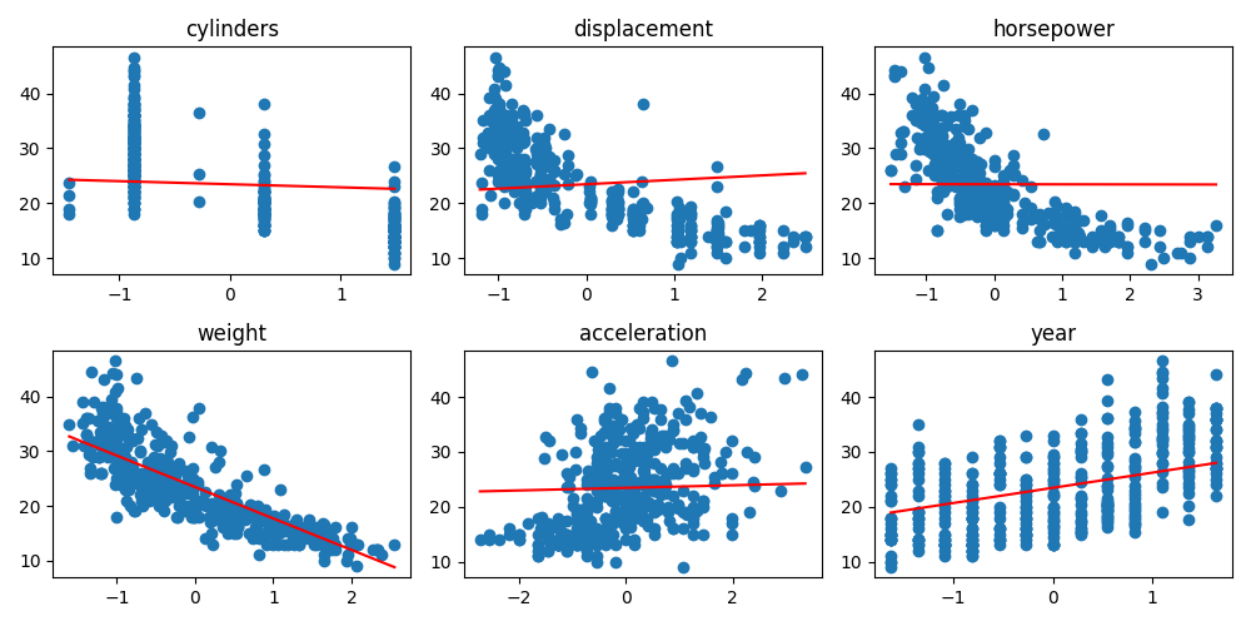

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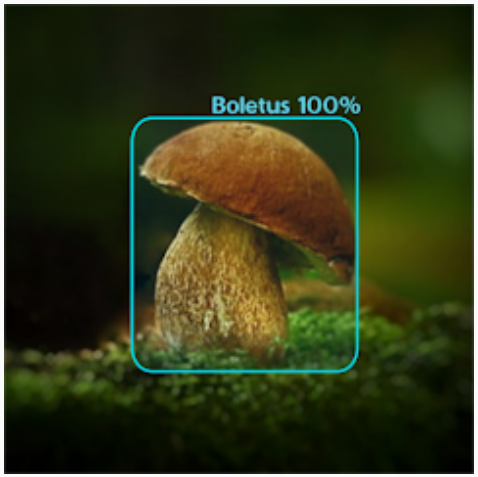

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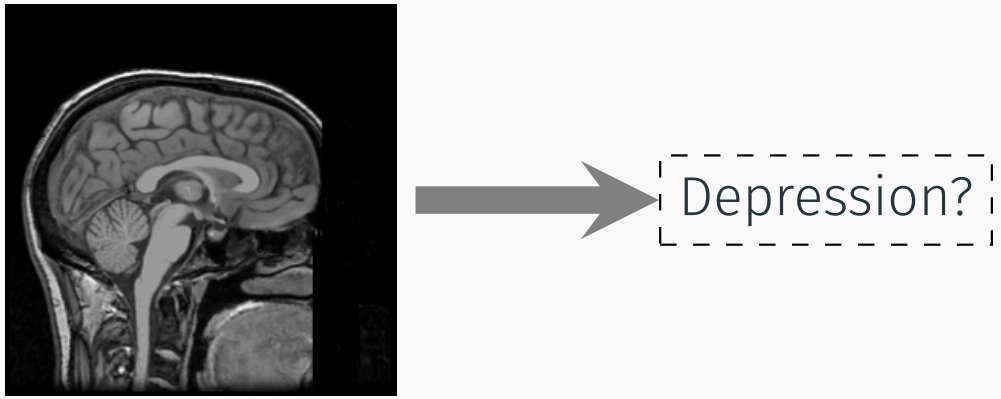

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**Unsupervised learning: Are there some interesting patterns in X?**

- Can we find subgroups or interesting axes of variability?
- Visualization

# Clustering


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|---|---|
| 0.20 | -0.26 |
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| 0.03 | 0.07 |
| -0.07 | -0.01 |
| -0.06 | 0.00 |
| 0.28 | -0.24 |
| 0.21 | -0.35 |
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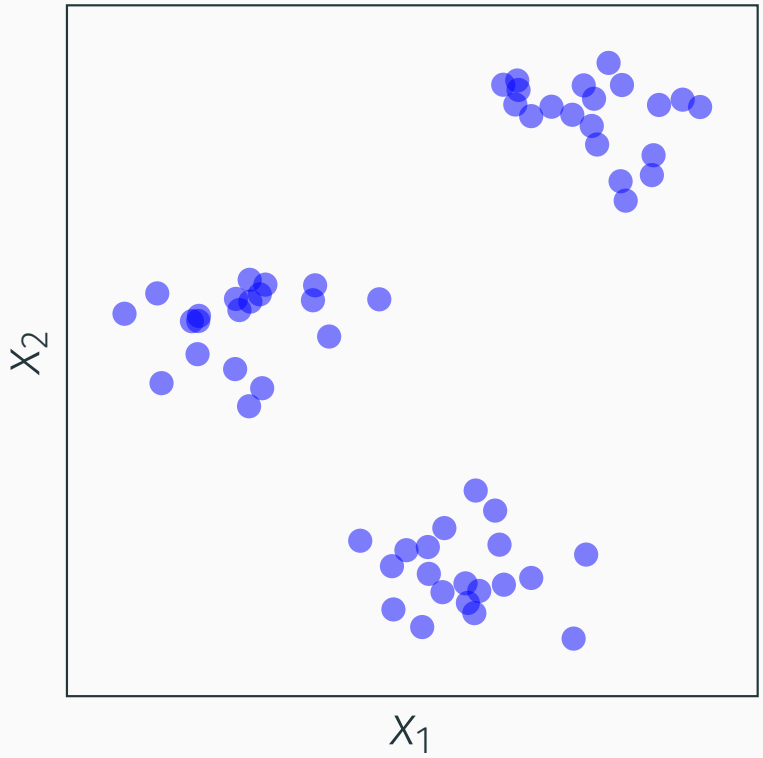

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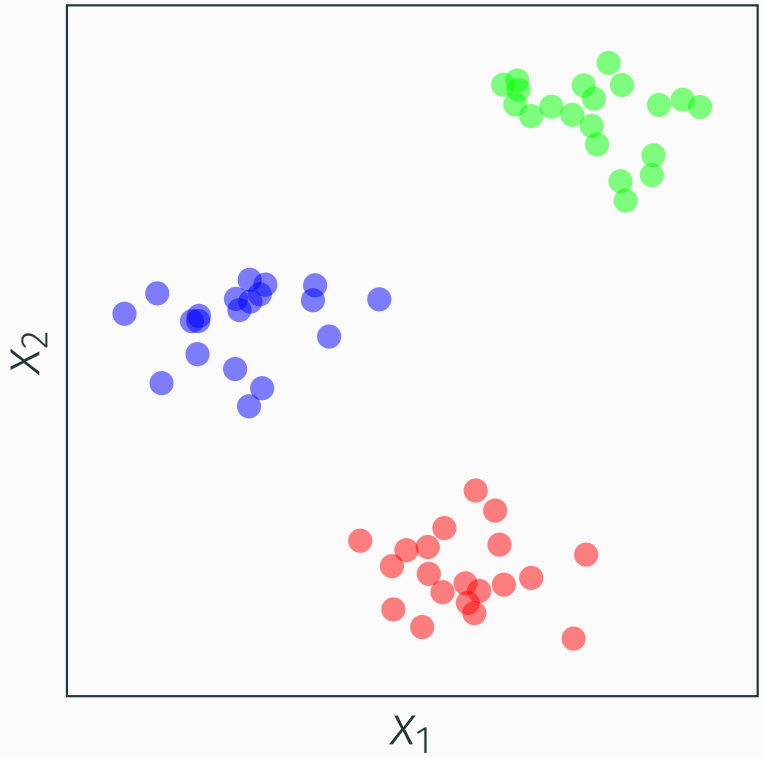

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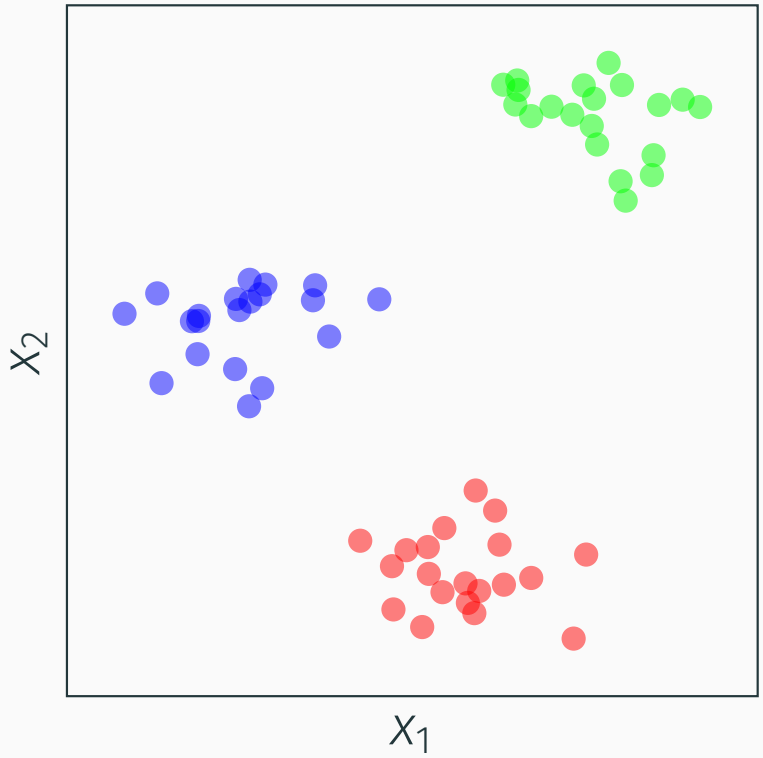

# K-means clustering

K-means clustering: Find $k$ clusters in the data to minimze the *within-cluster variance*:

$$\underset{C_1,\ldots,C_k}{\text{minimize}} \left( \sum_{k=1}^{K} \frac{1}{|C_k|} \sum_{i,i' \in C_k} \sum_{j=1}^{p} (x_{ij} - x_{i'j})^2 \right)$$

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|---|---|
| 1 | 1 |
| 1.5 | 1 |
| 1.25 | 1.25 |
| -1 | 1 |
| -1.5 | 1 |
| -1.25 | 1.25 |
| 0.25 | 0 |
| -0.25 | 0 |
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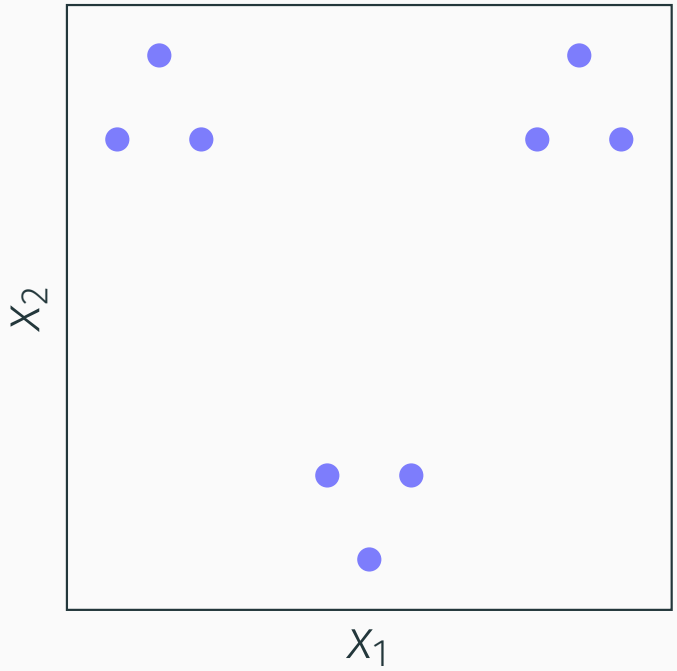


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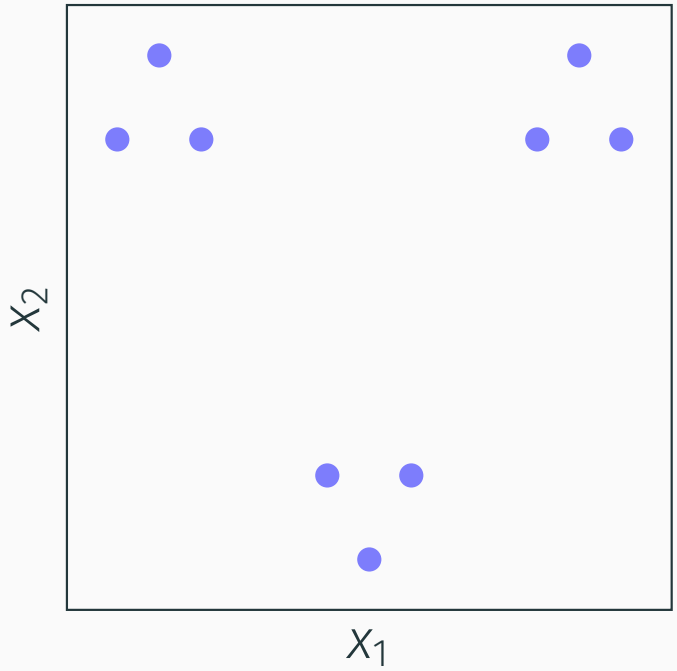


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| -1 | 1 | $C_2$ |
| -1.5 | 1 | $C_2$ |
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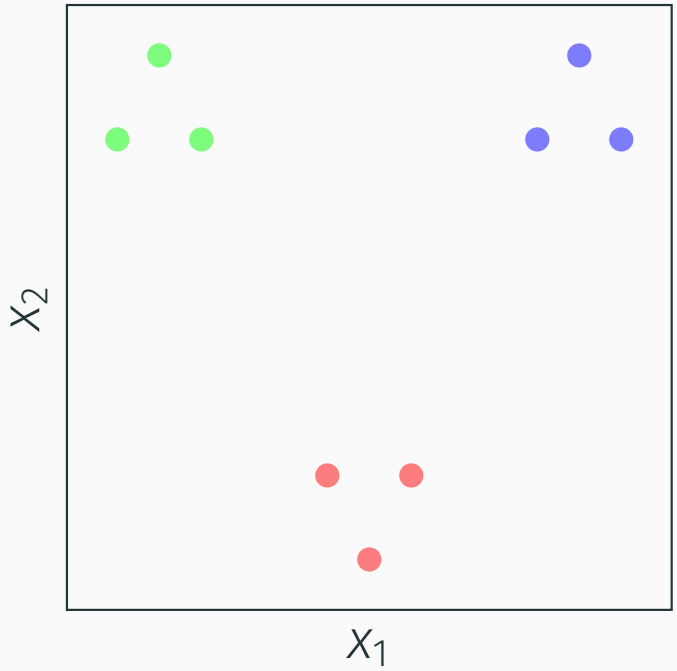


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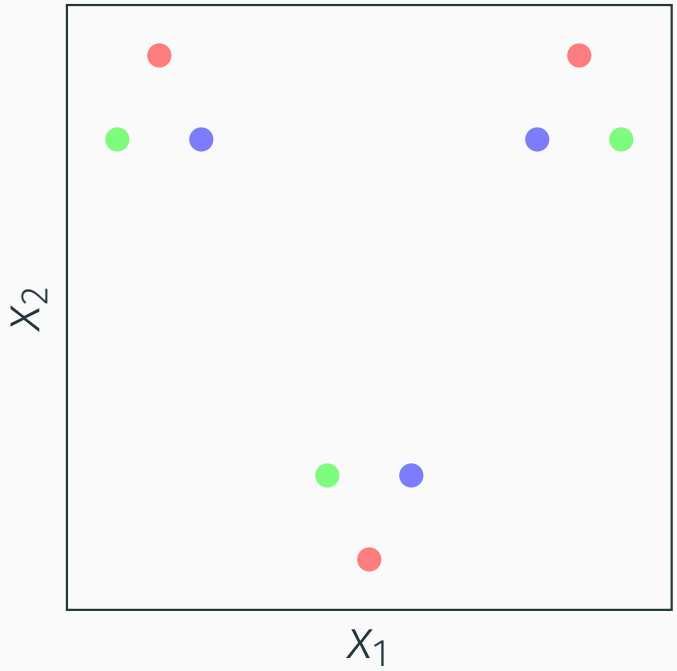


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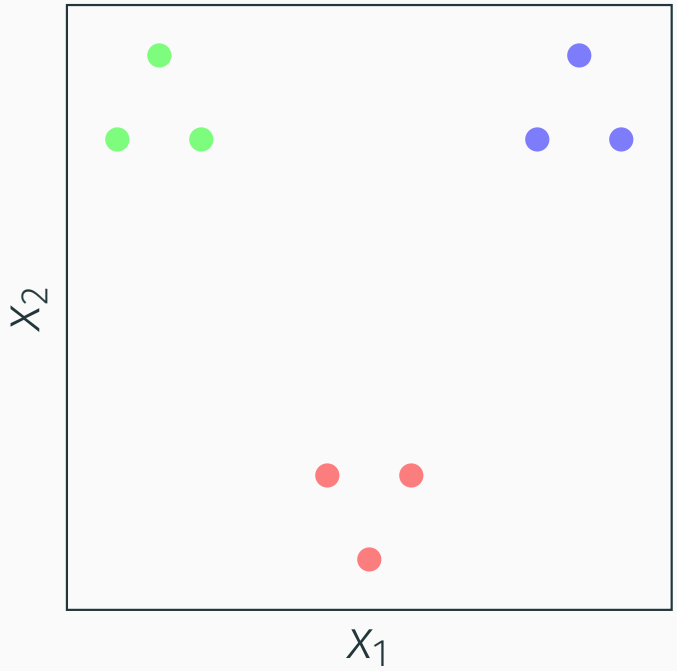


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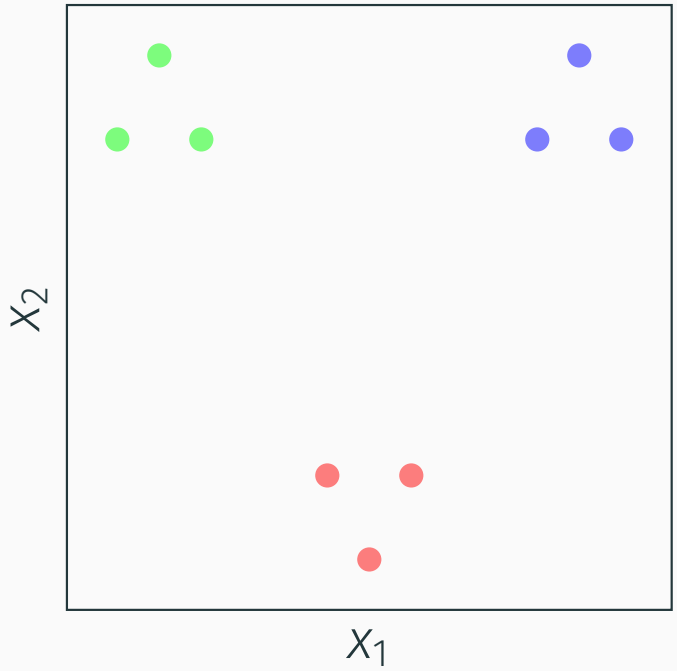


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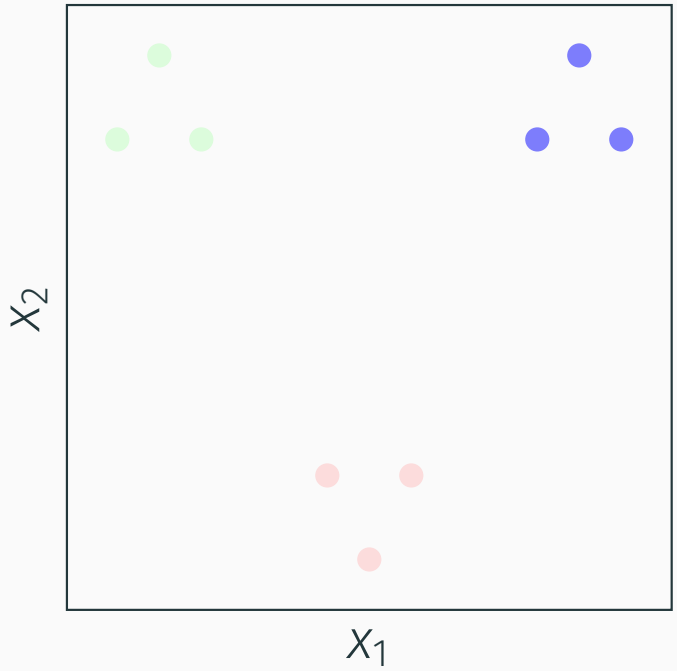


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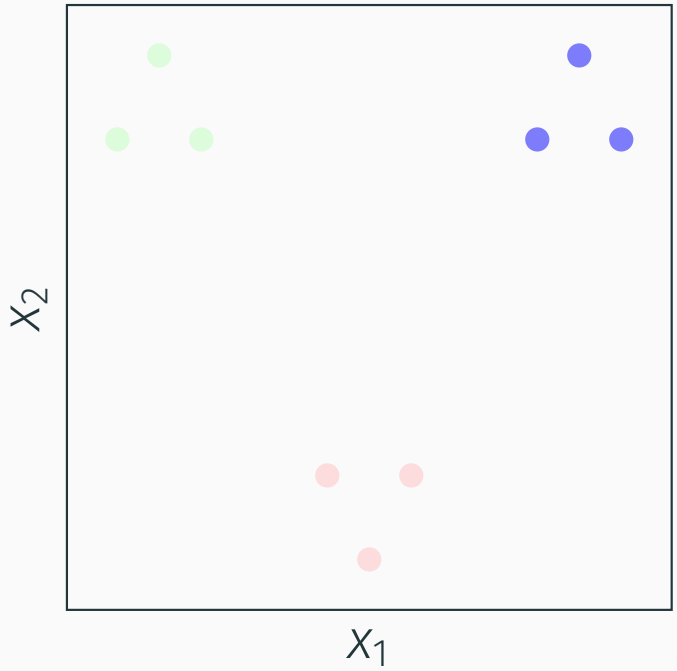


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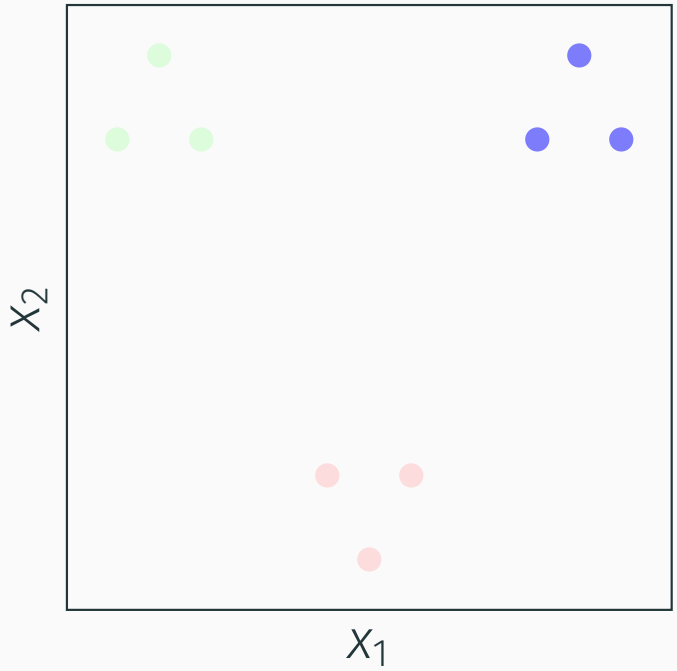


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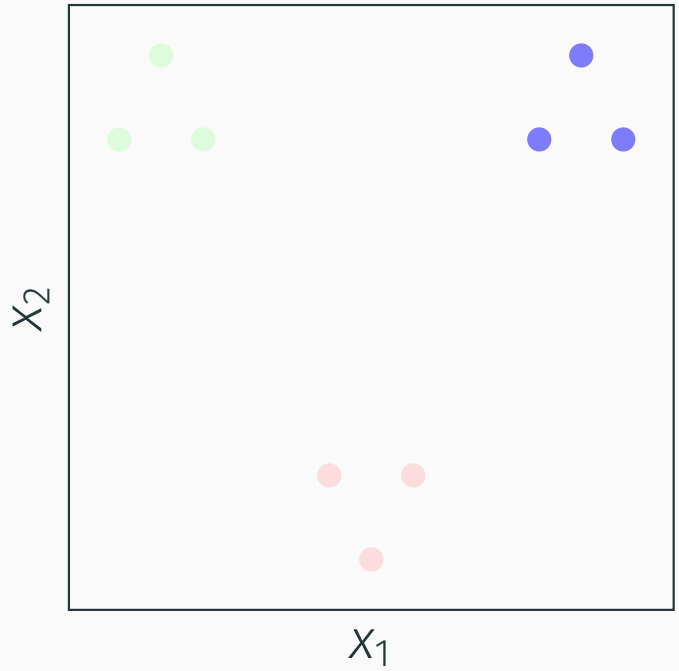


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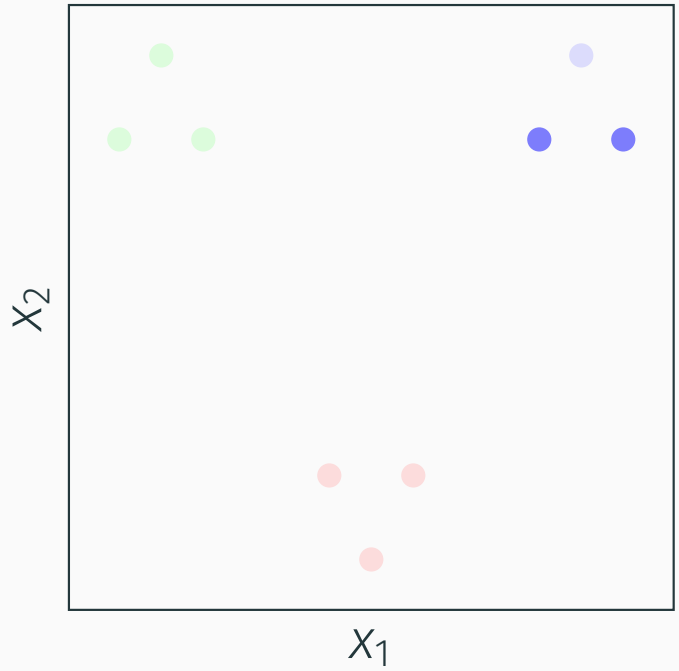


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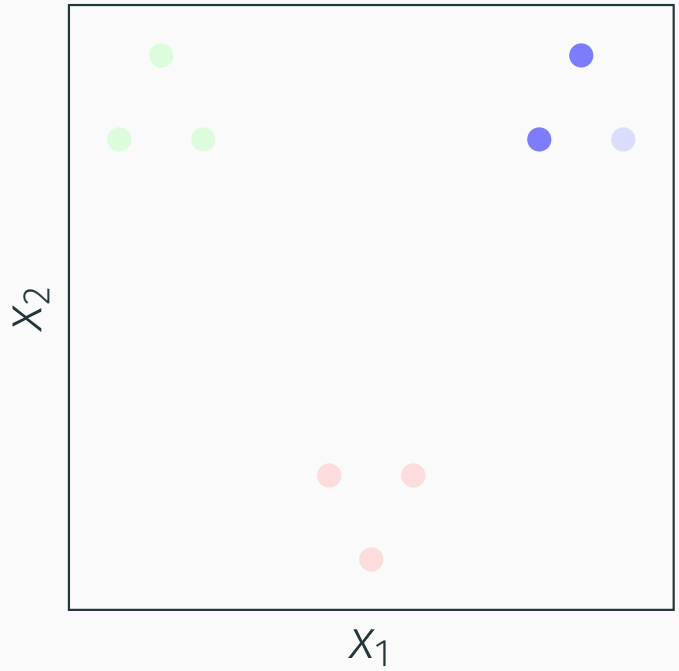


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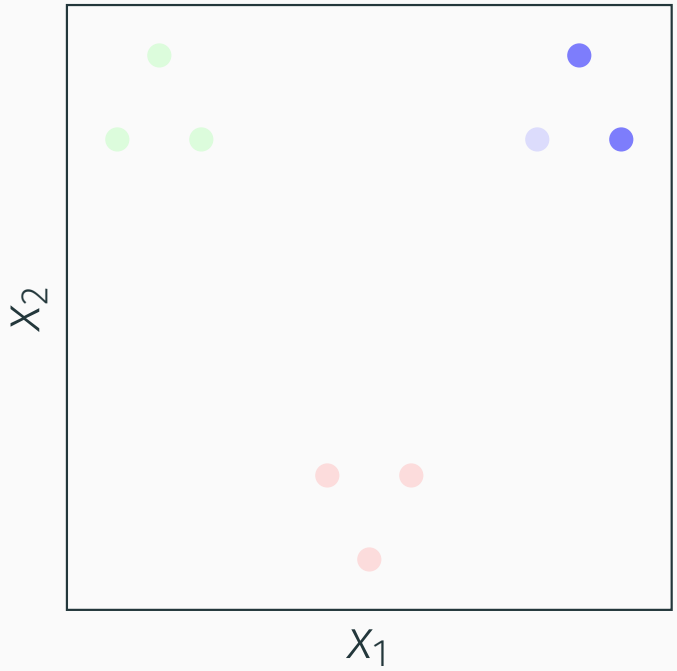


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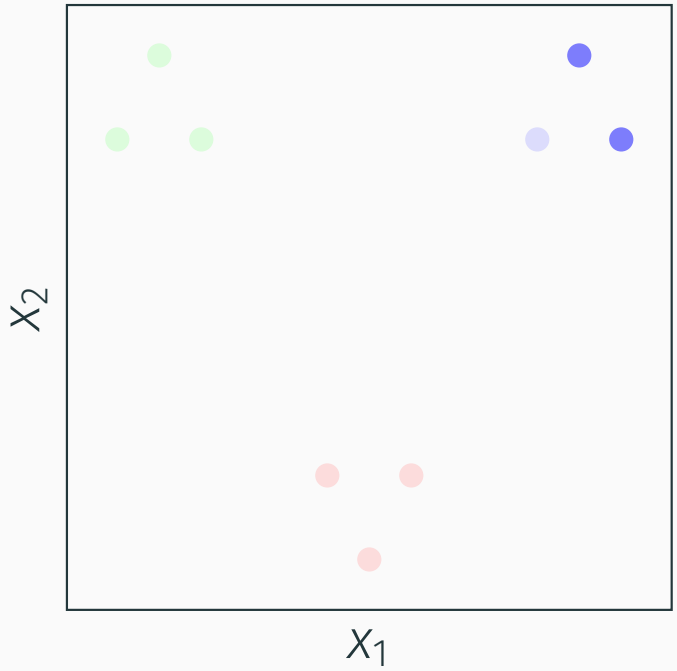


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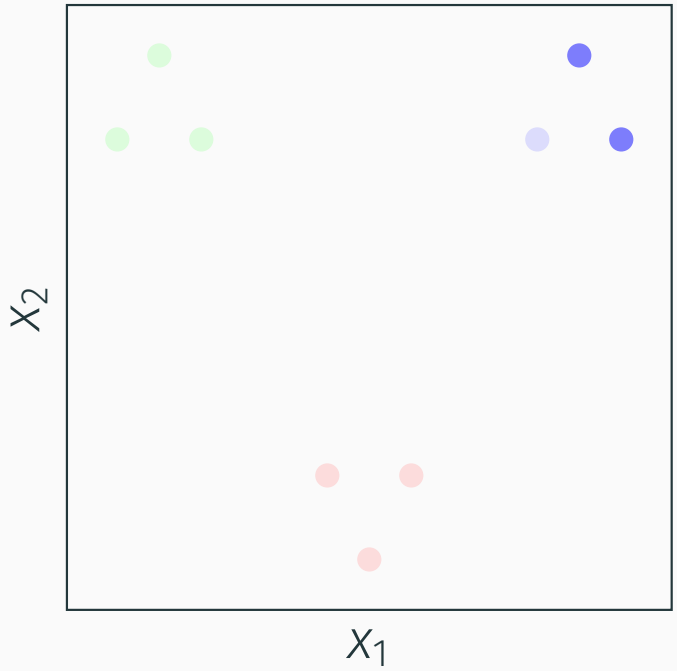
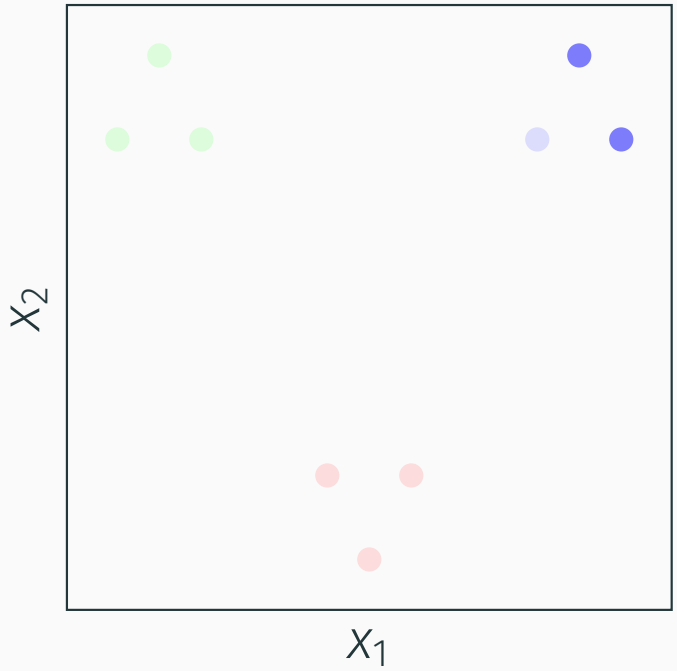
| -0.25 | 0 |
| 0 | -0.25 |




$$\underset{C_1,\ldots,C_k}{\text{minimize}} \left( \sum_{k=1}^{K} \frac{1}{|C_k|} \sum_{i,i' \in C_k} \sqrt{\sum_{j=1}^{p} (x_{ij} - x_{i'j})^2} \right)$$

# K-means clustering


| $x_1$ | $x_2$ |
|---|---|
| 1 | 1 |
| 1.5 | 1 |
| 1.25 | 1.25 |
| -1 | 1 |
| -1.5 | 1 |
| -1.25 | 1.25 |
| 0.25 | 0 |
| -0.25 | 0 |
| 0 | -0.25 |




$$\underset{C_1,\ldots,C_k}{\text{minimize}} \left( \sum_{k=1}^{K} \frac{1}{|C_k|} \sum_{i,i' \in C_k} \sqrt{\sum_{j=1}^{p} (x_{ij} - x_{i'j})^2} \right)$$

# K-means clustering

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|---|---|
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| 1.5 | 1 |
| 1.25 | 1.25 |
| -1 | 1 |
| -1.5 | 1 |
| -1.25 | 1.25 |
| 0.25 | 0 |
| -0.25 | 0 |
| 0 | -0.25 |

$$\underset{C_1,\ldots,C_k}{\text{minimize}} \left( \sum_{k=1}^{K} \frac{1}{|C_k|} \sum_{i,i' \in C_k} \sqrt{\sum_{j=1}^{p} (x_{ij} - x_{i'j})^2} \right)$$

# K-means clustering

| $x_1$ | $x_2$ |
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| 1 | 1 |
| 1.5 | 1 |
| 1.25 | 1.25 |
| -1 | 1 |
| -1.5 | 1 |
| -1.25 | 1.25 |
| 0.25 | 0 |
| -0.25 | 0 |
| 0 | -0.25 |

$$\underset{C_1,\ldots,C_k}{\text{minimize}} \left( \sum_{k=1}^{K} \frac{1}{|C_k|} \sum_{i,i' \in C_k} \sqrt{\sum_{j=1}^{p} (x_{ij} - x_{i'j})^2} \right)$$