PSY9511: Seminar 7

Deep learning for computer vision tasks

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07.11.24

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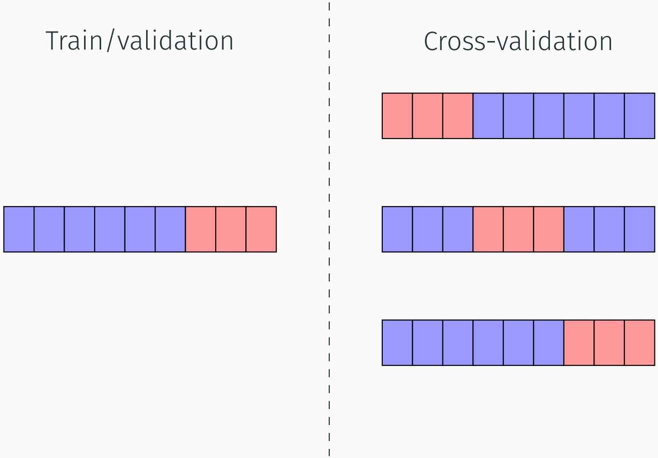
Outline

1. Exercise 4
2. Deep learning
    - Motivation
    - (Deep) neural networks
    - Training procedure
3. Convolutional neural networks for computer vision

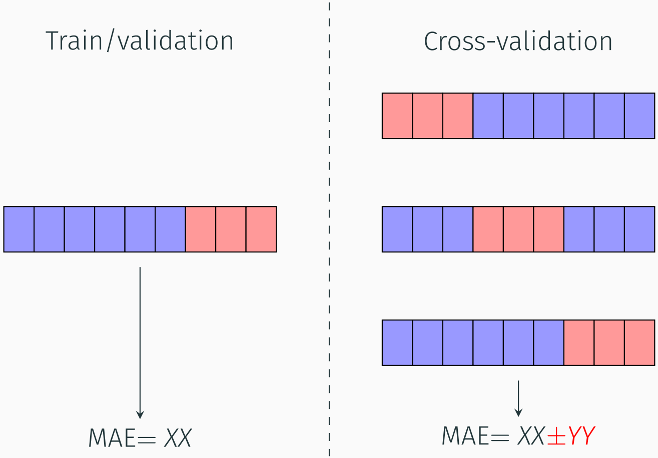
Weekly exercises

- The weekly exercises are **mandatory**
- The deadlines are **strict**

Validation procedures



Validation procedures



Deep learning

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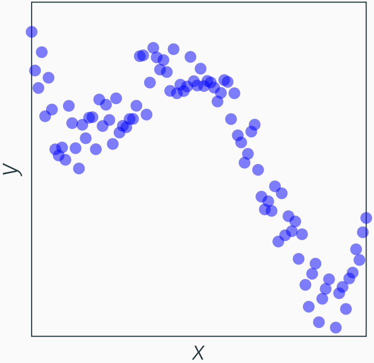
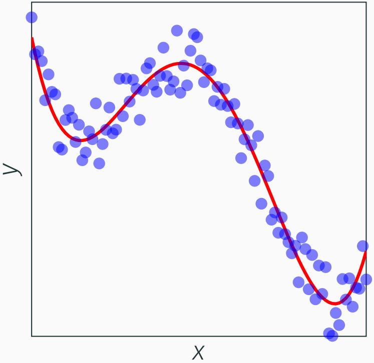
Deep learning: Motivation

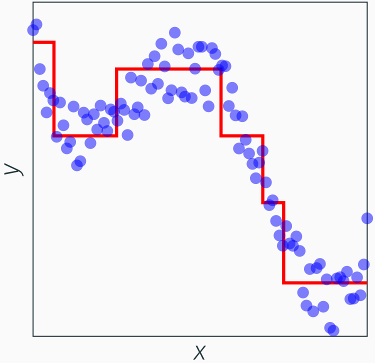


Deep learning: Motivation



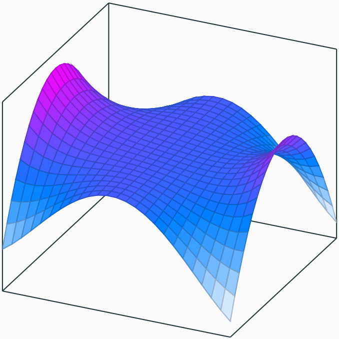
$$\hat{y} = s(x)$$

Deep learning: Motivation

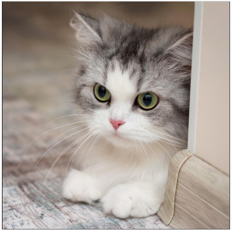


$$\hat{y} = \begin{cases} 4 & \ldots \\ 3 & 0.2 \leq x < 0.6 \\ 1.5 & \ldots \end{cases}$$

Deep learning: Motivation

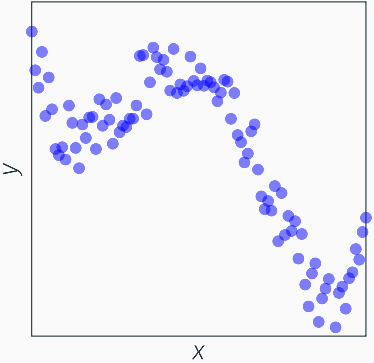


Deep learning: Motivation

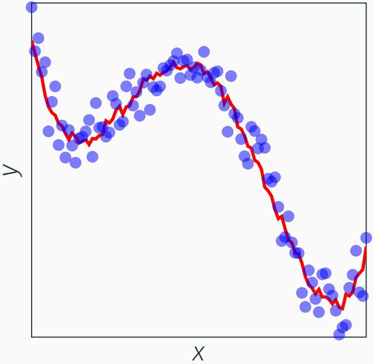


The cat wagged its tail

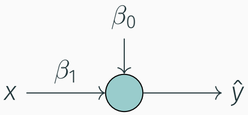
Deep learning: Motivation



Deep learning: Motivation

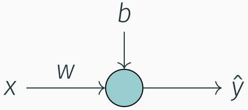


Deep learning: Artificial neural networks



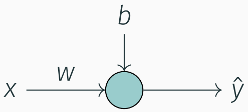
$$\hat{y} = \beta_0 + \beta_1 x$$

Deep learning: Artificial neural networks

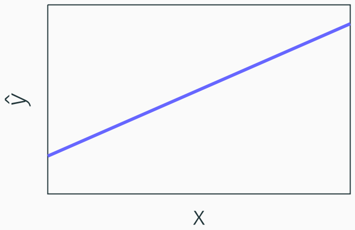


$$\hat{y} = wx + b$$

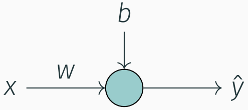
Deep learning: Artificial neural networks



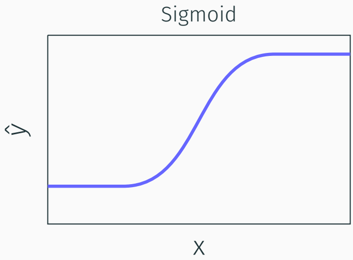
$$\hat{y} = wx + b$$



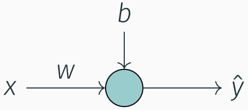
Deep learning: Artificial neural networks



$$\hat{y} = \frac{e^{wx+b}}{1 + e^{wx+b}}$$

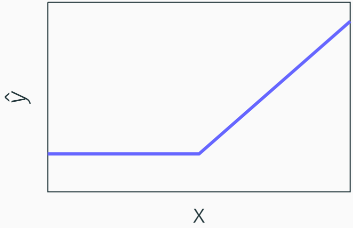


Deep learning: Artificial neural networks

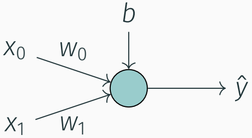


$$\hat{y} = max(0, wx + b)$$

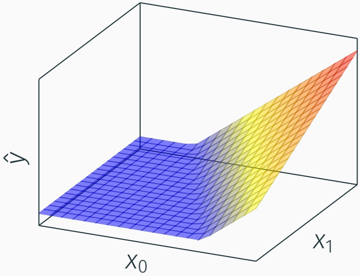
Rectified Linear Unit (ReLU)

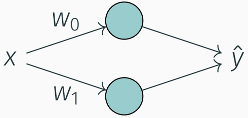


Deep learning: Artificial neural networks

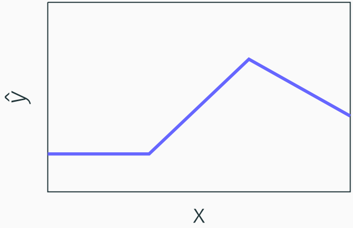


$$\hat{y} = max(0, w_0x_0 + w_1x_1 + b)$$

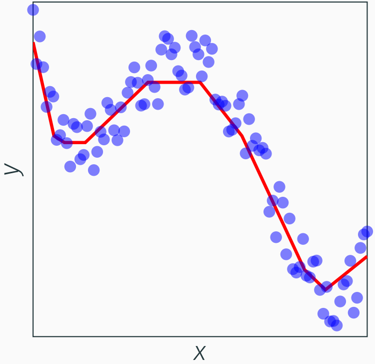




$$\hat{y} = max(0, w_0 x + b_0) + max(0, w_1 x + b_1)$$



Deep learning: Artificial neural networks

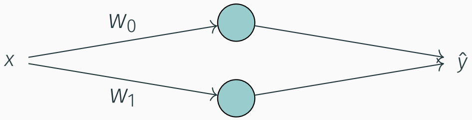


Piecewise linear function

Universal approximation theorem:

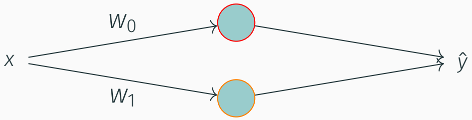
"Any relationship that can be described with a polynomial function can be approximated by a neural network with a single hidden layer."

Deep learning: Artificial neural networks



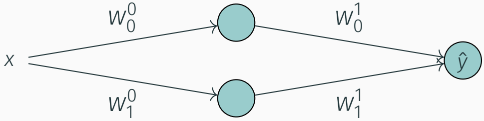
$$\hat{y} = max(0, w_0 x + b_0) + max(0, w_1 x + b_1)$$

Deep learning: Artificial neural networks



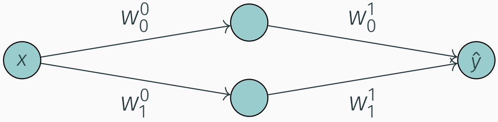
$$\hat{y} = \boxed{max(0, w_0x + b_0)} + \boxed{max(0, w_1x + b_1)}$$

Deep learning: Artificial neural networks



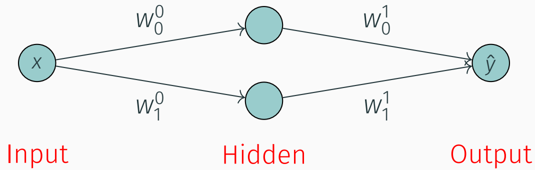
$$\hat{y} = max(0, w_0^1 * max(0, w_0^0 * x + b_0^0) + w_1^1 * max(0, w_1^0 * x + b_1^0) + b_0^1)$$

Deep learning: Artificial neural networks



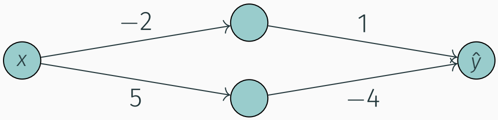
$$\hat{y} = max(0, w_0^1 * max(0, w_0^0 * x + b_0^0) + w_1^1 * max(0, w_1^0 * x + b_1^0) + b_0^1)$$

Deep learning: Artificial neural networks

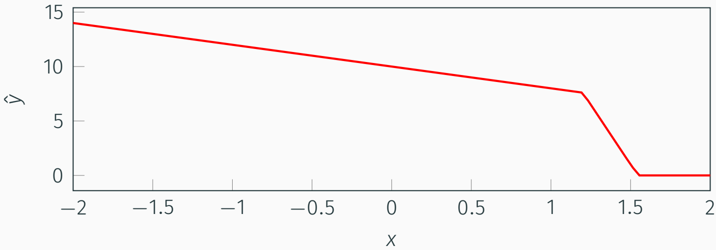


$$\hat{y} = max(0, w_0^1 * max(0, w_0^0 * x + b_0^0) + w_1^1 * max(0, w_1^0 * x + b_1^0) + b_0^1)$$

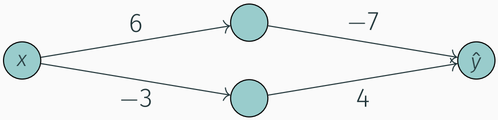
Deep learning: Artificial neural networks



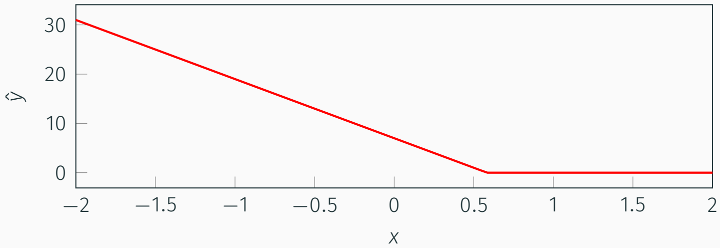
$$\hat{y} = max(0, 1 * max(0, (-2) * x + 3) + (-4) * max(0, 5 * x + (-6)) + 7)$$



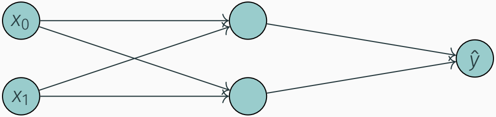
Deep learning: Artificial neural networks



$$\hat{y} = max(0, -7 * max(0, 6 * x + (-5)) + 4 * max(0, (-3) * x + 2) - 1)$$

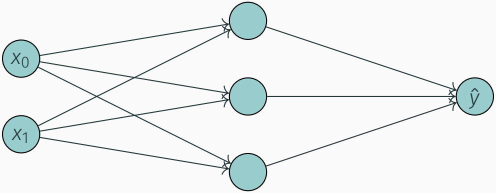


Deep learning: Artificial neural networks



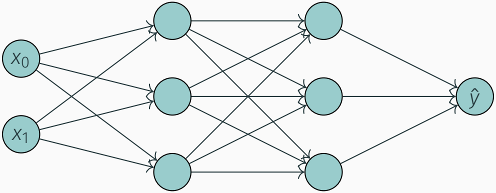
$$\begin{aligned}\hat{y} = max(0, &w^1_{0,0} * max(0, w^0_{0,0} * x_0 + w^0_{1,0} * x_1 + b_{0,0}) + \\ &w^1_{1,0} * max(0, w^0_{0,1} * x_0 + w^0_{1,1} * x_1 + b_{0,1}) + \\ &b_1)\end{aligned}$$

Deep learning: Artificial neural networks



$$\hat{y} = max(0, w^1_{0,0} * max(0, w^0_{0,0} * x_0 + w^0_{1,0} * x_1 + b_{0,0}) + w^1_{1,0} * max(0, w^0_{0,1} * x_0 + w^0_{1,1} * x_1 + b_{0,1}) + w^1_{2,0} * max(0, w^0_{0,2} * x_0 + w^0_{1,2} * x_1 + b_{0,2}) + b_1)$$

Deep learning: Artificial neural networks



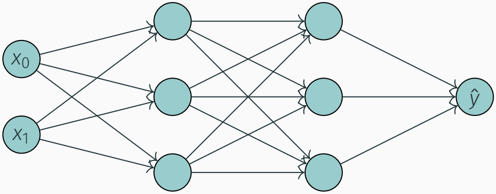
$$
\begin{aligned}
\hat{y} = max(0, & w^2_{0,0} * max(0, w^1_{0,0} * max(0, w^0_{0,0} * x_0 + w^0_{1,0} * x_1 + b_{0,0}) + \\
& \qquad w^1_{1,0} * max(0, w^0_{0,1} * x_0 + w^0_{1,1} * x_1 + b_{0,1}) + \\
& \qquad w^1_{2,0} * max(0, w^0_{0,2} * x_0 + w^0_{1,2} * x_1 + b_{0,2}) + \\
& \qquad b_{1,0}) + \\
& w^2_{1,0} * max(0, w^1_{0,1} * max(0, w^0_{0,0} * x_0 + w^0_{1,0} * x_1 + b_{0,0}) + \\
& \qquad w^1_{1,1} * max(0, w^0_{0,1} * x_0 + w^0_{1,1} * x_1 + b_{0,1}) + \\
& \qquad w^1_{2,1} * max(0, w^0_{0,2} * x_0 + w^0_{1,2} * x_1 + b_{0,2}) + \\
& \qquad b_{1,1}) + \\
& w^2_{2,0} * max(0, w^1_{0,2} * max(0, w^0_{0,0} * x_0 + w^0_{1,0} * x_1 + b_{0,0}) + \\
& \qquad w^1_{1,2} * max(0, w^0_{0,1} * x_0 + w^0_{1,1} * x_1 + b_{0,1}) + \\
& \qquad w^1_{2,2} * max(0, w^0_{0,2} * x_0 + w^0_{1,2} * x_1 + b_{0,2}) + \\
& \qquad b_{1,2}) + \\
& b_2)
\end{aligned}
$$

Deep learning: Artificial neural networks

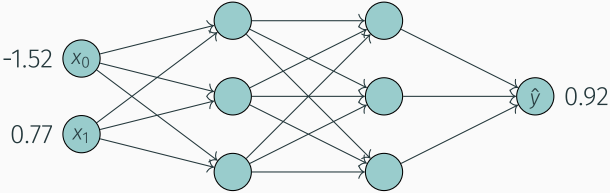
Artificial neural networks: Combines artificial neurons, simple computational units that compute a non-linear function of their inputs, in a computational graph

- Can approximate arbitrarily complex polynomial functions (given enough neurons)
- Organized in layers. We can expand a model in width (e.g. more neurons per layer) or depth (e.g. more layers)

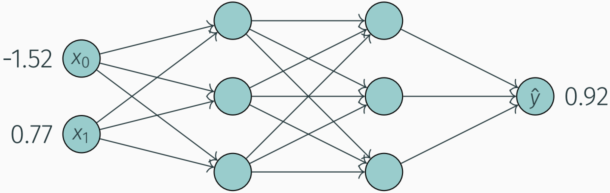
Deep learning: Loss functions



Deep learning: Loss functions

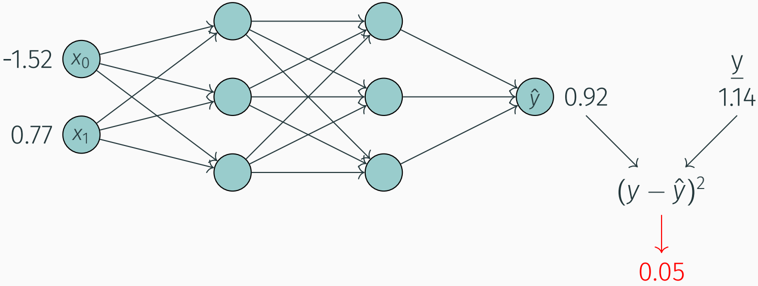


Deep learning: Loss functions

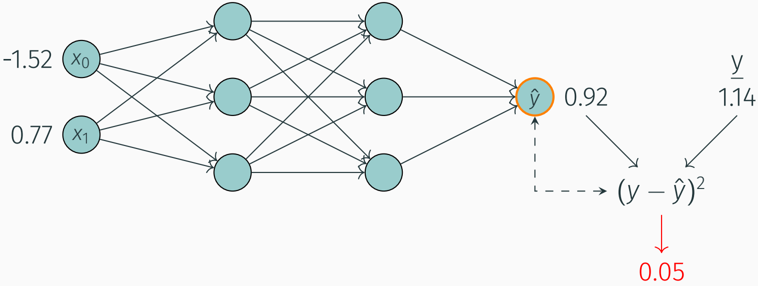


$\underline{y}$
1.14

Deep learning: Loss functions



Deep learning: Loss functions



Deep learning: Loss functions

In[1]:
```
from tensorflow.keras.layers import Input, Dense
from tensorflow.keras import Model

inputs = Input(shape=(2,))
hidden1 = Dense(units=3, activation='relu')(inputs)
hidden2 = Dense(units=3, activation='relu')(hidden1)
outputs = Dense(units=1, activation=?)(hidden2)

model = Model(inputs, outputs)
model.compile(loss=?)
```

In[1]:
```
from tensorflow.keras.layers import Input, Dense
from tensorflow.keras import Model

inputs = Input(shape=(2,))
hidden1 = Dense(units=3, activation='relu')(inputs)
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outputs = Dense(units=1, activation=?)(hidden2)

model = Model(inputs, outputs)
model.compile(loss=?)
```
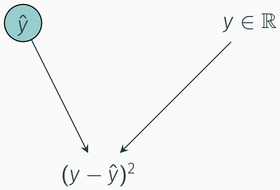

Deep learning: Loss functions

Regression



In[2]:
```
...
outputs = Dense(units=1, activation=None)(...)
...
model.compile(loss='mean_squared_error')
```

Deep learning: Loss functions

Binary classification

$\hat{y}$

$y \in \{0, 1\}$

$$-(y * log(\hat{y}) + (1 - y) * log(1 - \hat{y}))$$

In[3]:

```
...
outputs = Dense(units=1, activation='sigmoid')(...)
...
model.compile(loss='binary_crossentropy')
```

Multiclass classification

$\hat{y}$

$y \in \{cat, dog, bat\}$

Deep learning: Loss functions

Multiclass classification

$\hat{y}$

x_0	y
	cat
	dog
	bat
	dog

Deep learning: Loss functions

Multiclass classification

$\hat{y}$

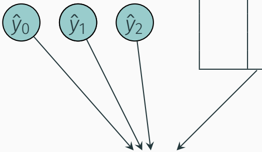
x_0	cat	dog	bat
	1	0	1
	0	1	0
	0	0	1
	0	1	0

Deep learning: Loss functions

Multiclass classification

$\hat{y}_0$ $\hat{y}_1$ $\hat{y}_2$

x_0	cat	dog	bat
	1	0	1
	0	1	0
	0	0	1
	0	1	0



x_0	cat	dog	bat
	1	0	1
	0	1	0
	0	0	1
	0	1	0

$$-\sum_{i=0}^{N} y_i * log(\hat{y}_i)$$

In[3]:

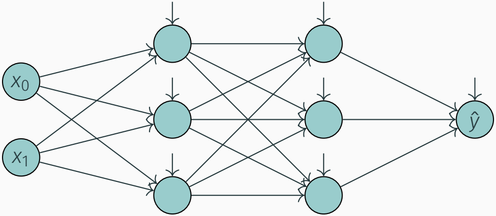
```
...
outputs = Dense(units=1, activation='softmax')(...)
...
model.compile(loss='categorical_crossentropy')
```

Deep learning: Loss functions

Loss functions: Behaves for neural networks as any other statistical learning model. However, important to **configure the final layer** correctly

- Regression: Mean squared error
  - No activation
- Binary classification: Binary cross-entropy
  - Sigmoid activation
- Multiclass classification: Categorical cross-entropy
  - Softmax activation

Deep learning: Training



$$
\begin{aligned}
\hat{y} = max(0, w^2_{0,0} * max(0, & w^1_{0,0} * max(0, w^0_{0,0} * x_0 + w^0_{1,0} * x_1 + b_{0,0}) + \\
& w^1_{1,0} * max(0, w^0_{0,1} * x_0 + w^0_{1,1} * x_1 + b_{0,1}) + \\
& w^1_{2,0} * max(0, w^0_{0,2} * x_0 + w^0_{1,2} * x_1 + b_{0,2}) + \\
& b_{1,0}) + \\
w^2_{1,0} * max(0, & w^1_{0,1} * max(0, w^0_{0,0} * x_0 + w^0_{1,0} * x_1 + b_{0,0}) + \\
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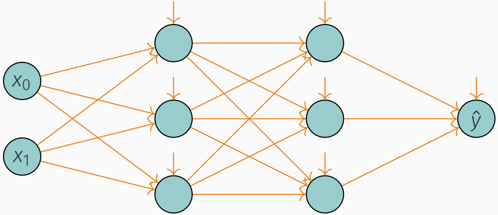
& w^1_{2,2} * max(0, w^0_{0,2} * x_0 + w^0_{1,2} * x_1 + b_{0,2}) + \\
& b_{1,2}) + \\
b_2)
\end{aligned}
$$

Deep learning: Training



$$
\begin{aligned}
\hat{y} = max(0, & w^2_{0,0} * max(0, w^1_{0,0} * max(0, w^0_{0,0} * x_0 + w^0_{1,0} * x_1 + b_{0,0}) + \\
& \qquad w^1_{1,0} * max(0, w^0_{0,1} * x_0 + w^0_{1,1} * x_1 + b_{0,1}) + \\
& \qquad w^1_{2,0} * max(0, w^0_{0,2} * x_0 + w^0_{1,2} * x_1 + b_{0,2}) + \\
& \qquad b_{1,0}) + \\
& w^2_{1,0} * max(0, w^1_{0,1} * max(0, w^0_{0,0} * x_0 + w^0_{1,0} * x_1 + b_{0,0}) + \\
& \qquad w^1_{1,1} * max(0, w^0_{0,1} * x_0 + w^0_{1,1} * x_1 + b_{0,1}) + \\
& \qquad w^1_{2,1} * max(0, w^0_{0,2} * x_0 + w^0_{1,2} * x_1 + b_{0,2}) + \\
& \qquad b_{1,1}) + \\
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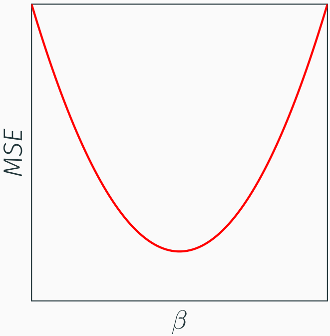
& \qquad b_{1,2}) + \\
& b_2)
\end{aligned}
$$

Deep learning: Training



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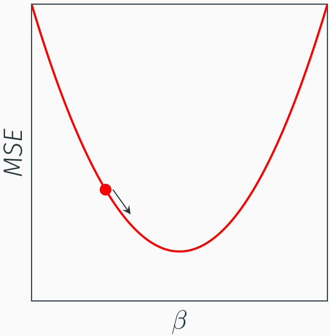

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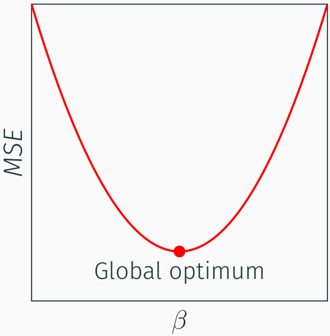

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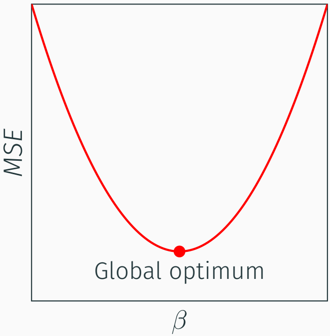

- $|\beta| \rightarrow 10^6 - 10^{12}$
- $\beta_x \implies \beta_y$

Learning representations by back-propagating errors

David E. Rumelhart*, Geoffrey E. Hinton† & Ronald J. Williams*

* Institute for Cognitive Science, C-015, University of California, San Diego, La Jolla, California 92093, USA
† Department of Computer Science, Carnegie-Mellon University, Pittsburgh, Philadelphia 15213, USA

We describe a new learning procedure, back-propagation, for networks of neurone-like units. The procedure repeatedly adjusts the weights of the connections in the network so as to minimize a measure of the difference between the actual output vector of the net and the desired output vector. As a result of the weight adjustments, internal 'hidden' units which are not part of the input or output come to represent important features of the task domain, and the regularities in the task are captured by the interactions of these units. The ability to create useful new features distinguishes back-propagation from earlier, simpler methods such as the perceptron-convergence procedure[1].

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Deep learning: Training

$$\Delta R(\theta^m) = \left.\frac{\partial R(\theta)}{\partial \theta}\right|_{\theta=\theta^m}$$

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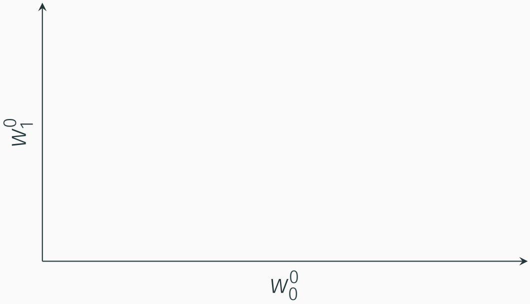

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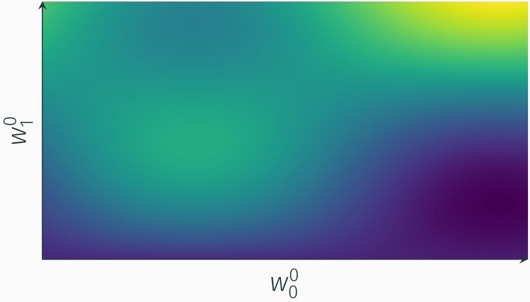
- θ are the parameters of the model



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- $R(\theta)$ is the loss as a function of the parameters

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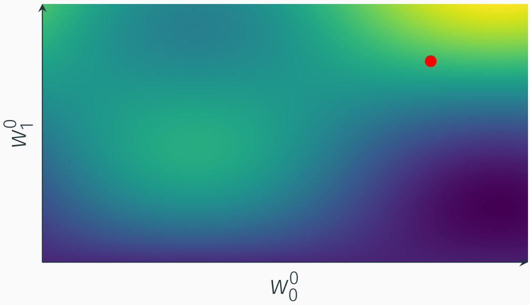

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- $R(\theta)$ is the loss as a function of the parameters
- θ^m is a specific configuration of parameters

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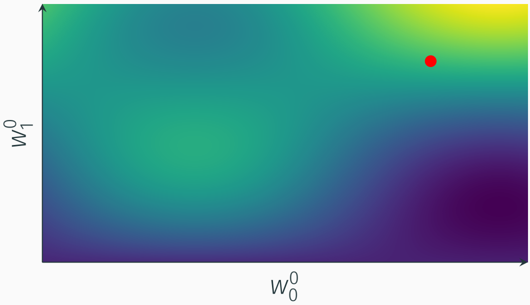

$$\frac{\partial R_i(\theta)}{\partial w_{jk}} = \frac{\partial R(\theta)}{\partial f_\theta(x_i)} \cdot \frac{\partial f_\theta(x_i)}{\partial g(z_{ik})} \cdot \frac{\partial g(z_{ik})}{\partial z_{ik}} \cdot \frac{z_{ik}}{w_{kj}}$$

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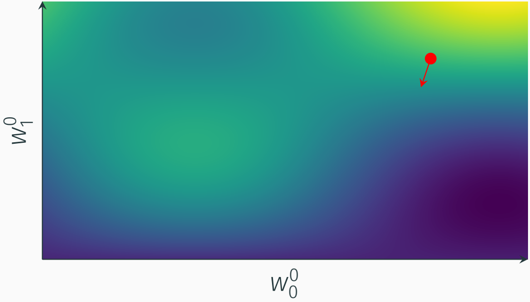

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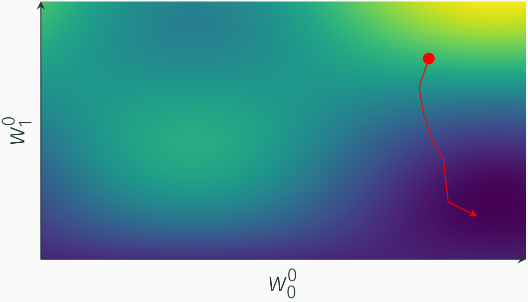

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Backpropagation: Uses gradient descent to iteratively determine how the model weights should be updated to minimize the loss function

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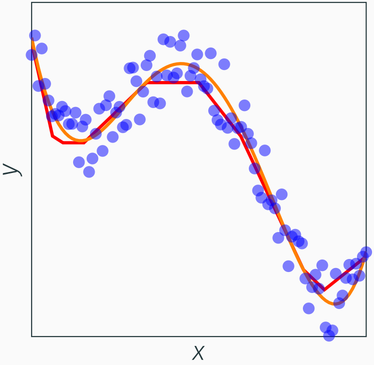
- **Gradient descent**: Calculate gradient based on all data points
- **Stochastic gradient descent**: Calculate gradient based on a batch of data points

Deep learning: The conundrum

Splines: A smooth curve implemented via piecewise polynomial functions

Neural networks: A piecewise linear function implemented as a hierarchy of artificial neurons



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- Overparameterization 🥳

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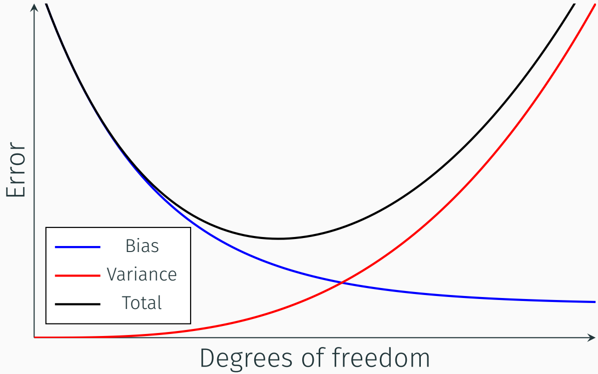

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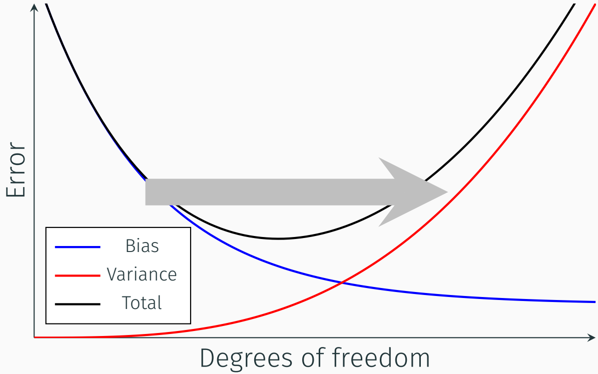

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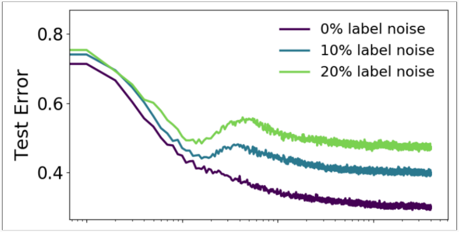

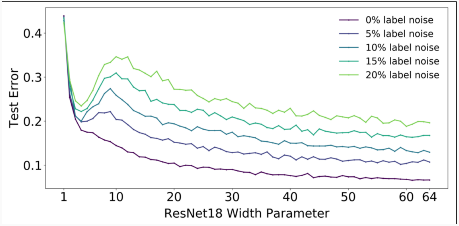

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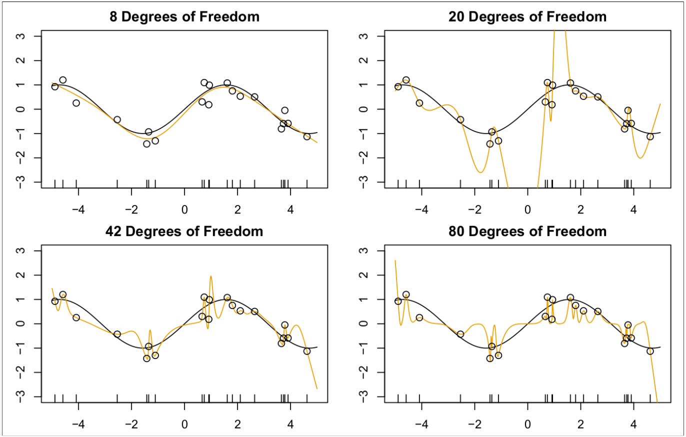

Deep learning: The conundrum

Overparameterization: Deep artificial neural networks generally have far more parameters than necessary (and often more than the number of data points)

- At face value, it is surprising that this does not yield severe overfitting
- However, it can be shown that neural networks, after perfectly fitting their training data, generally become more well-behaved and less wild

Deep learning: Regularization

- Weight decay: Applies an ℓ_2-penalty to the weights, similarly to ridge regression

  $R(\theta; \lambda) = R(\theta) + \lambda \sum \theta^2$

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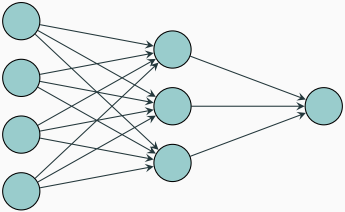

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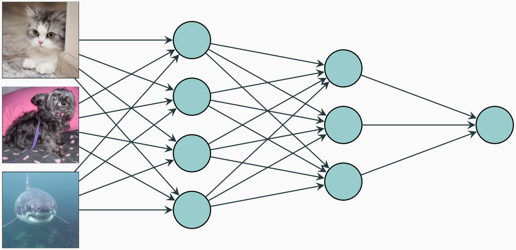

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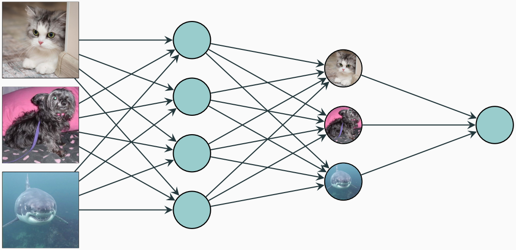

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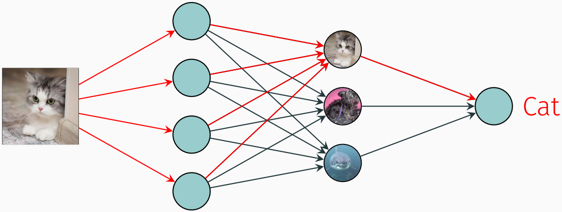

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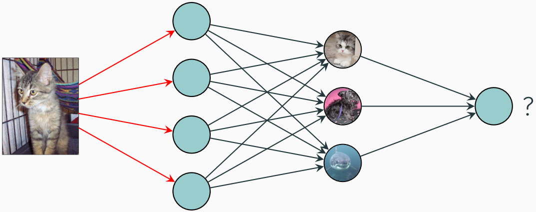

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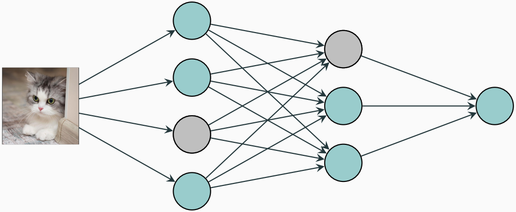

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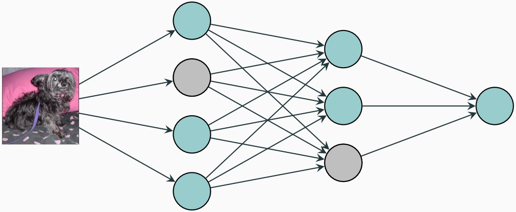

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- Data augmentation: Attempts to generate more data