Russell's Paradox

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Background

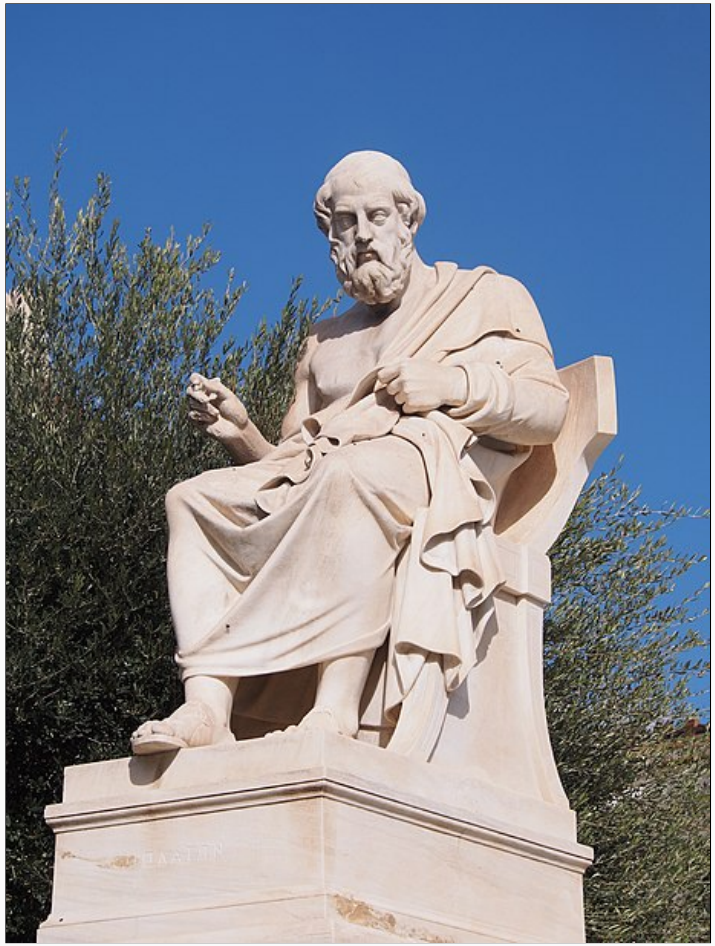
Background

$$\{S|S \notin S\}$$

Background

Source: A guy I met at a party once

Historical underpinnings



Historical underpinnings

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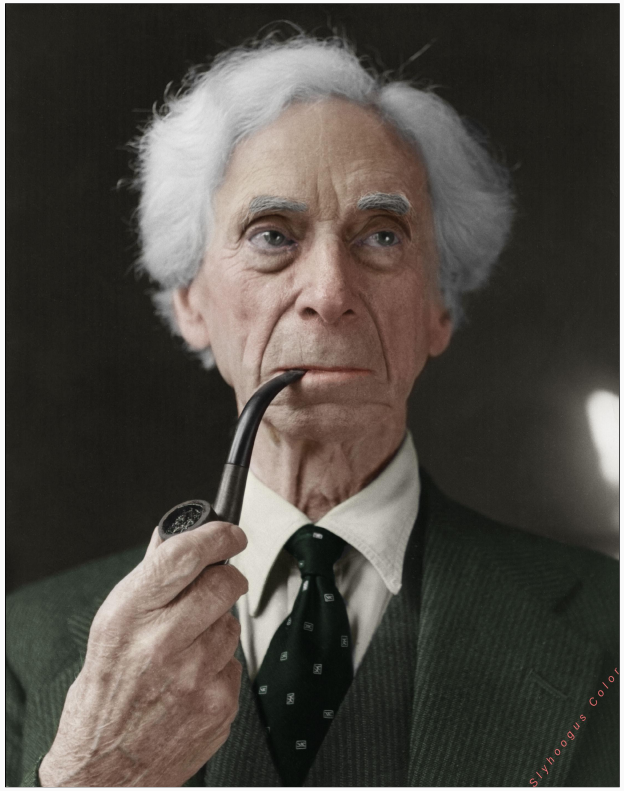


The project

$$0, 1, 2, 3, 4, \ldots$$

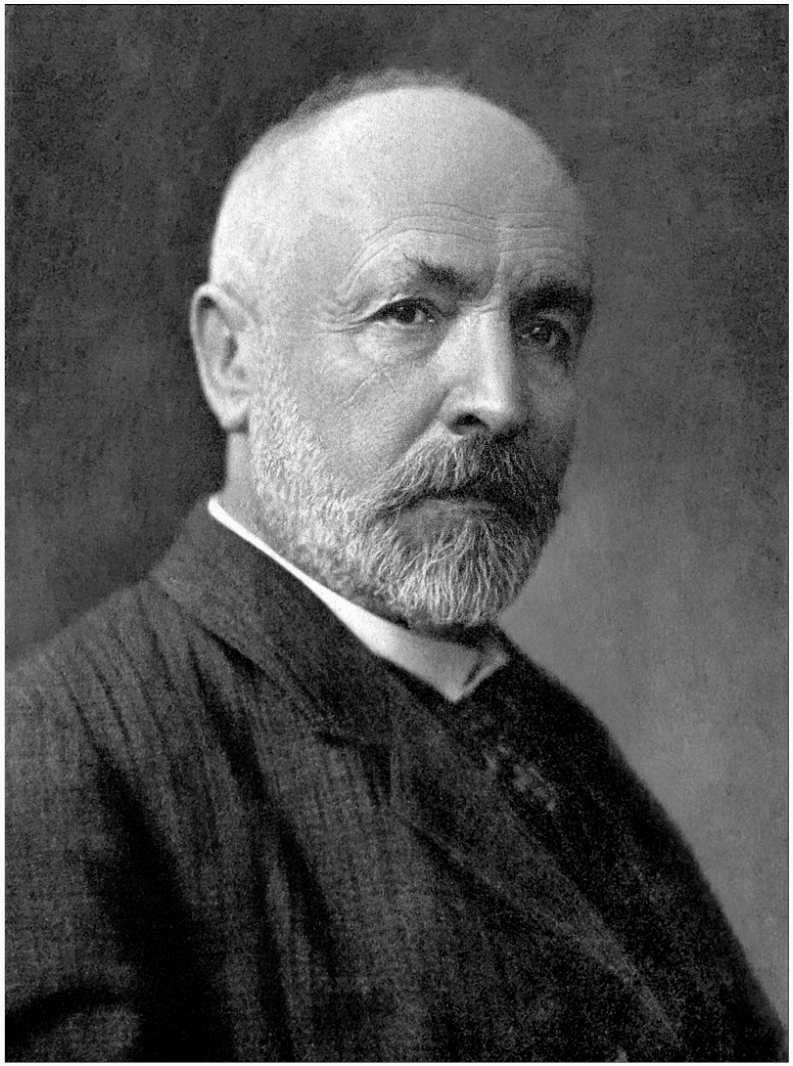
$$x + y = z$$
$$x - y = z$$
$$\ldots$$

The project

$0, 1, 2, 3, 4, \ldots \quad \Longrightarrow$ 🤔

$x + y = z$
$x - y = z \quad \Longrightarrow$ ✅
...

Set theory



Set theory

{🤓, 👩, 👧, 🐶}

{🤓, 😎, 🧐, 👩‍🔬, 👩‍💻, 👩‍🏫}

{monday, tuesday, wednesday, thursday,
friday, saturday, sunday}

{💻, 📱}

Set theory

Set theory

$$\{1, 3, 5\}$$

$$\{1, 10, 100\}$$

$$\{\}$$

Set theory

$$\{1, 2, 3, \ldots\}$$

Set theory

$$\{1, 2, 3, \ldots\}$$

$$\{x \mid x > 0\}$$

Set theory

$$\{1, 3, 5, \ldots\}$$

$$\{x \mid x \,\%\, 2 \neq 0\}$$

Set theory

{{🤓, 👱‍♀️, 👧, 🐶}, {🤓, 😎, 🧐, 👩‍🔬, 👩‍💻, 👩‍🏫},
{monday, tuesday, wednesday, thursday, friday, saturday, sunday},
{💻, 📱}, $\{1, 3, 5\}$, $\{1, 10, 100\}$, $\{\}$, $\{x \mid x > 0\}$, $\{x \mid x \,\%\, 2 \neq 0\}$

Set theory

{{🤓, 👧, 👧, 🐶}, {🤓, 😎, 🧐, 🤦‍♀️, 🤷‍♀️, 👩‍🏫},
{monday, tuesday, wednesday, thursday, friday, saturday, sunday},
{💻, 📱}, $\{1, 3, 5\}$, $\{1, 10, 100\}$, $\{\}$, $\{x \mid x > 0\}$, $\{x \mid x \,\%\, 2 \neq 0\}$}

$$\{\{\}, \{0\}, \{1\}, \{0, 1\}, \ldots\}$$

Set theory

{{🤓, 👱‍♀️, 👧, 🐶}, {🤓, 😎, 🧐, 👩‍🔬, 👩‍💻, 👩‍🏫},

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{💻, 📱}, $\{1, 3, 5\}$, $\{1, 10, 100\}$, $\{\}$, $\{x \mid x > 0\}$, $\{x \mid x \,\%\, 2 \neq 0\}$

$$\{\{\}, \{0\}, \{1\}, \{0, 1\}, \ldots\}$$

$$\{x \mid x \text{ is a set}\}$$

Set theory

{{🤓, 👧, 👧, 🐶}, {🤓, 😎, 🧐, 👩‍🔬, 👩‍💻, 👩‍🏫},

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{💻, 📱}, $\{1, 3, 5\}$, $\{1, 10, 100\}$, $\{\}$, $\{x \mid x > 0\}$, $\{x \mid x \,\%\, 2 \neq 0\}$

$$\{\{\}, \{0\}, \{1\}, \{0, 1\}, \ldots\}$$

$$S = \{x \mid x \text{ is a set}\}$$

Set theory

{{🤓, 👧, 👧, 🐶}, {🤓, 😎, 🧐, 🤦‍♀️, 👩‍💻, 👩‍🏫},
{monday, tuesday, wednesday, thursday, friday, saturday, sunday},
{💻, 📱}, $\{1, 3, 5\}$, $\{1, 10, 100\}$, $\{\}$, $\{x \mid x > 0\}$, $\{x \mid x \,\%\, 2 \neq 0\}$

$$\{\{\}, \{0\}, \{1\}, \{0, 1\}, \ldots\}$$

$$S = \{x \mid x \text{ is a set}\}$$

$$S \in S\,?$$

Set theory

$$S = \{\{\}, \{0\}, \{1\}, \{0, 1\}, \ldots, S, \ldots\}$$

$$S \in S$$

Set theory

$$S = \{\{\}, \{0\}, \{1\}, \{0, 1\}, \ldots, S, \ldots\} \qquad S = \{1, 3, 5\}$$

$$S \in S \qquad S \notin S$$

Set theory

$$S = \{\{\}, \{0\}, \{1\}, \{0, 1\}, \ldots, S, \ldots\} \qquad S = \{1, 3, 5\}$$

$$S \in S \qquad S \notin S$$

$$\{S \mid S \in S\}$$

Set theory

$$S = \{\{\}, \{0\}, \{1\}, \{0,1\}, \ldots, S, \ldots\} \qquad S = \{1,3,5\}$$

$$S \in S \qquad S \notin S$$

$$\{S \mid S \in S\}$$
$$\{S \mid S \notin S\}$$

Set theory

$$T = \{S \mid S \notin S\}$$

Set theory

$$T = \{S \mid S \notin S\}$$

$$T \in T \, ?$$

Set theory

$$T = \{S \mid S \notin S\}$$

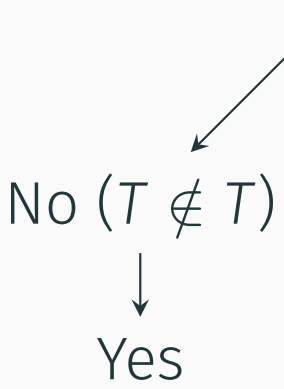
$$T \in T\ ?$$

$$\text{No } (T \notin T)$$

Set theory

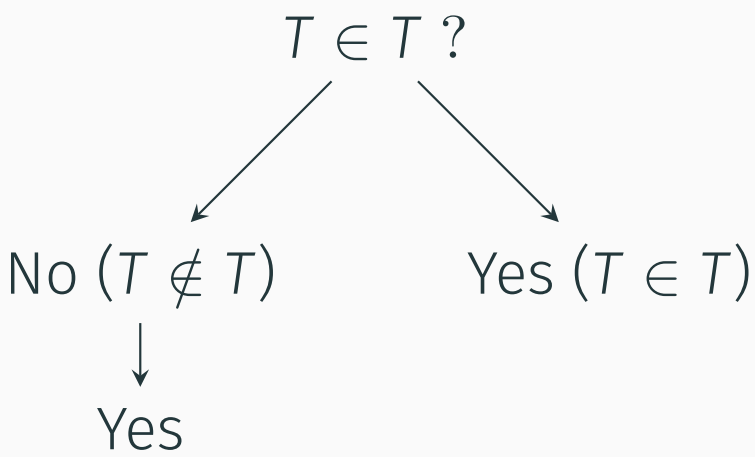
$$T = \{S \mid S \notin S\}$$

$$T \in T\ ?$$



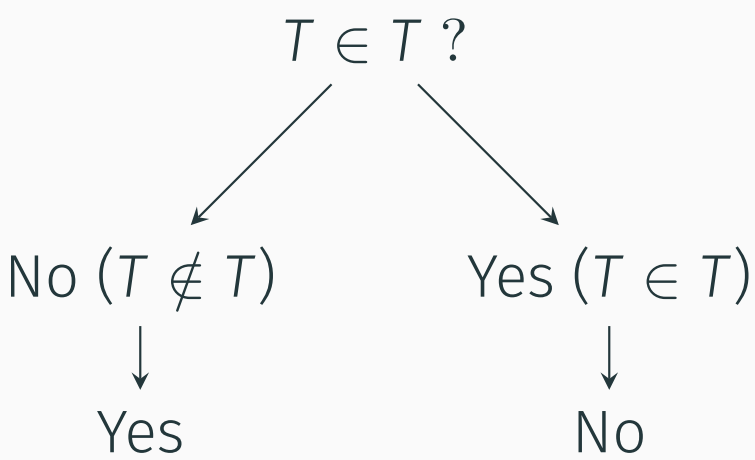
Set theory

$$T = \{S \mid S \notin S\}$$



Set theory

$$T = \{S \mid S \notin S\}$$



Set theory

Set theory

”Hardly anything more unfortunate can befall a scientific writer than to have one of the foundations of his edifice shaken after the work is finished. This was the position I was placed in by a letter of Mr. Bertrand Russell, just when the printing of this volume was nearing its completion.”

Later examples