

Introduction to deep learning 1/?

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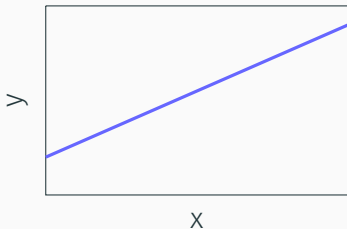
1. Building an artificial neural network (ANN)
2. Training the ANN
3. Transformation to a Convolutional Neural Network (CNN)

Building a neural network: Linear regression

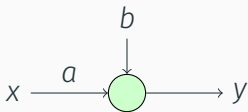
$$y = ax + b$$

Building a neural network: Linear regression

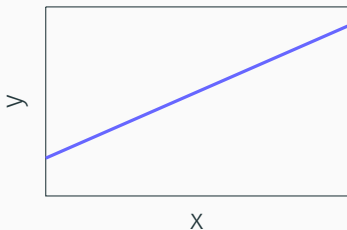
$$y = ax + b$$



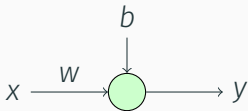
Building a neural network: Linear regression



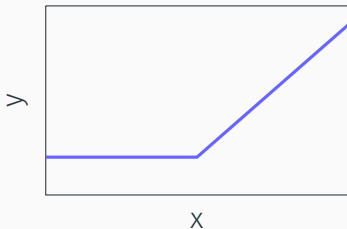
$$y = ax + b$$



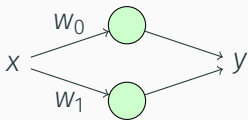
Building a neural network: Artificial neuron



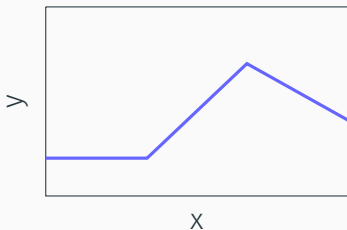
$$y = \max(0, wx + b)$$



Building a neural network: Artificial neural network (ANN)



$$y = \max(0, w_0x + b_0) + \max(0, w_1x + b_1)$$



Building a neural network: Universal approximation theorem

"Any relationship that can be described with a polynomial function can be approximated by a neural network with a single hidden layer."

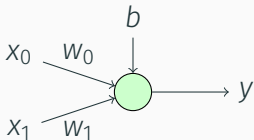
- Some guy in the 80s, probably

Building a neural network: Universal approximation theorem

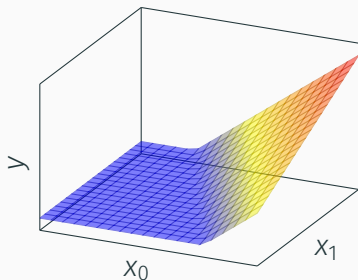
"Any relationship that can be described with a polynomial function can be approximated by a neural network with a single hidden layer."

- Some guy in the 80s, probably

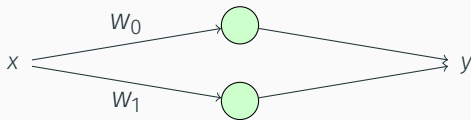
Building a neural network: Increasing dimensionality



$$y = \max(0, w_0x_0 + w_1x_1 + b)$$

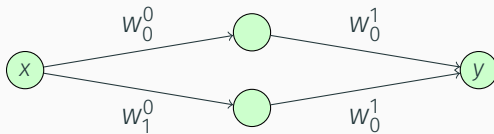


Building a neural network



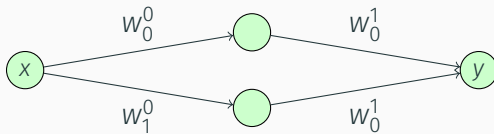
$$y = \max(0, w_0x + b_0) + \max(0, w_1x + b_1)$$

Building a neural network



$$y = \max(0, w_{0,0}^1 * \max(0, w_{0,0}^0 * x + b_{0,0}) + w_{1,0}^1 * \max(0, w_{0,1}^0 * x + b_{1,0}) + b_1)$$

Building a neural network



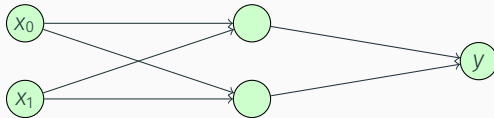
Input

Hidden

Output

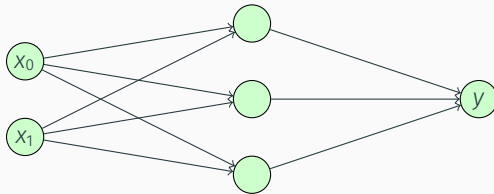
$$y = \max(0, w_{0,0}^1 * \max(0, w_{0,0}^0 * x + b_{0,0}) + w_{1,0}^1 * \max(0, w_{0,1}^0 * x + b_{1,0}) + b_1)$$

Building a neural network



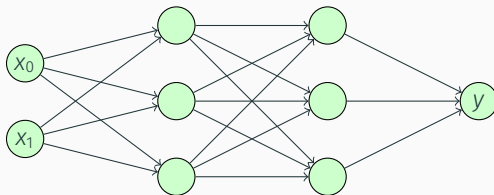
$$y = \max(0, w_{0,0}^1 * \max(0, w_{0,0}^0 * x_0 + w_{1,0}^0 * x_1 + b_{0,0}) + w_{1,0}^1 * \max(0, w_{0,1}^0 * x_0 + w_{1,1}^0 * x_1 + b_{0,1}) + b_1)$$

Building a neural network



$$y = \max(0, w_{0,0}^1 * \max(0, w_{0,0}^0 * x_0 + w_{1,0}^0 * x_1 + b_{0,0}) + w_{1,0}^1 * \max(0, w_{0,1}^0 * x_0 + w_{1,1}^0 * x_1 + b_{0,1}) + w_{2,0}^1 * \max(0, w_{0,2}^0 * x_0 + w_{1,2}^0 * x_1 + b_{0,2}) + b_1)$$

Building a neural network

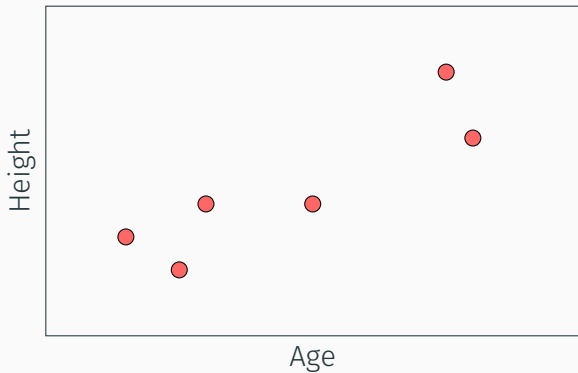


$$\begin{aligned}
 y = & \max(0, w_{0,0}^2 * \max(0, w_{0,0}^1 * \max(0, w_{0,0}^0 * x_0 + w_{1,0}^0 * x_1 + b_{0,0}) + \\
 & w_{1,0}^1 * \max(0, w_{0,1}^0 * x_0 + w_{1,1}^+ * w_1 + b_{0,1}) + \\
 & w_{2,0}^1 * \max(0, w_{0,2}^0 * x_0 + w_{1,2}^+ * w_1 + b_{0,2}) + \\
 & b_{1,0}) + \\
 & w_{1,0}^2 * \max(0, w_{0,1}^1 * \max(0, w_{0,0}^0 * x_0 + w_{1,0}^0 * x_1 + b_{0,0}) + \\
 & w_{1,1}^1 * \max(0, w_{0,1}^0 * x_0 + w_{1,1}^+ * w_1 + b_{0,1}) + \\
 & w_{2,1}^1 * \max(0, w_{0,2}^0 * x_0 + w_{1,2}^+ * w_1 + b_{0,2}) + \\
 & b_{1,1}) + \\
 & w_{2,0}^2 * \max(0, w_{0,2}^1 * \max(0, w_{0,0}^0 * x_0 + w_{1,0}^0 * x_1 + b_{0,0}) + \\
 & w_{1,2}^1 * \max(0, w_{0,1}^0 * x_0 + w_{1,1}^+ * w_1 + b_{0,1}) + \\
 & w_{2,2}^1 * \max(0, w_{0,2}^0 * x_0 + w_{1,2}^+ * w_1 + b_{0,2}) + \\
 & b_{1,2}) + \\
 & b_2)
 \end{aligned}$$

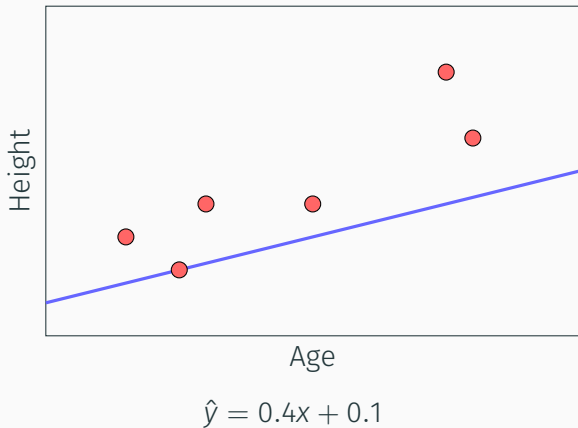
Building a neural network: Summary

- Artificial neurons are (linear) weighted sums wrapped in non-linear activation functions
- Multiple artificial neurons stacked together in a layerwise fashion comprise an artificial neural network
- Artificial neural networks allow us to model arbitrarily complex relationships between inputs and outputs

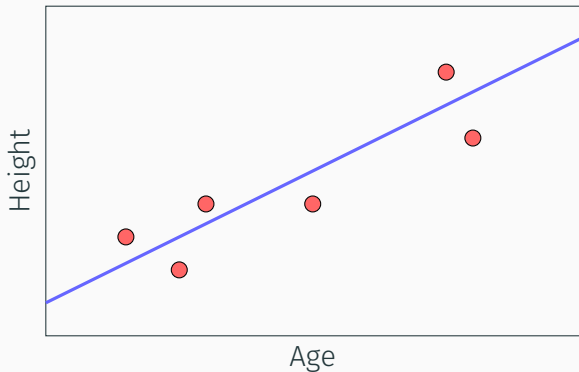
Training a neural network: Loss functions



Training a neural network: Loss functions

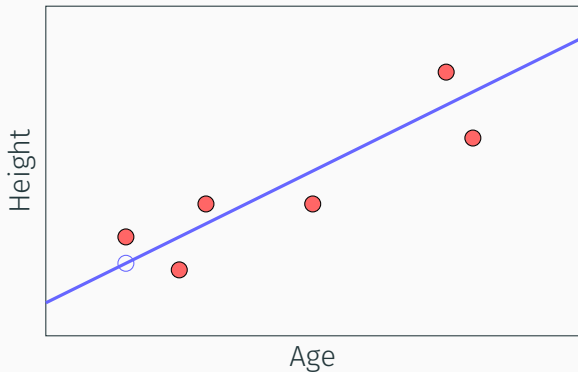


Training a neural network: Loss functions



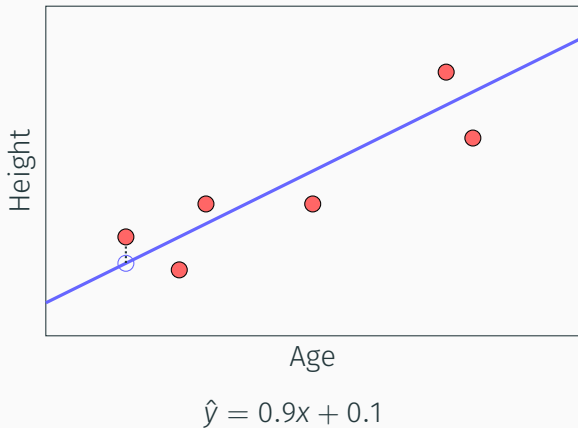
$$\hat{y} = 0.9x + 0.1$$

Training a neural network: Loss functions

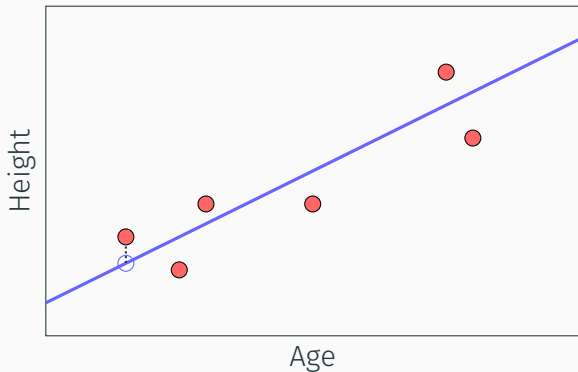


$$\hat{y} = 0.9x + 0.1$$

Training a neural network: Loss functions



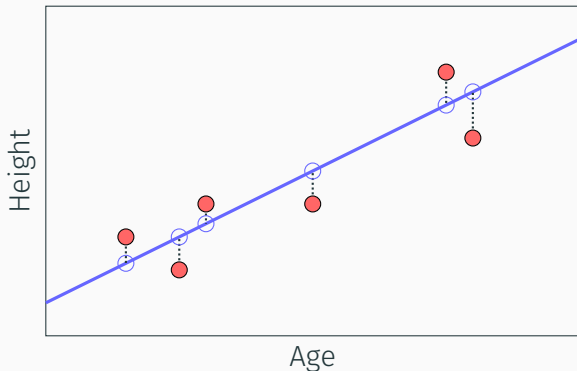
Training a neural network: Loss functions



$$\hat{y} = 0.9x + 0.1$$

$$error_i = |\hat{y} - y|$$

Training a neural network: Loss functions

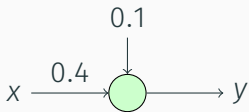


$$\hat{y} = 0.9x + 0.1$$

$$error_i = |\hat{y} - y|$$

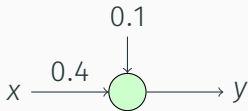
$$mse = \frac{1}{n} \sum_{i=1}^n (\hat{y}_i - y_i)^2$$

Training a neural network: Gradient descent



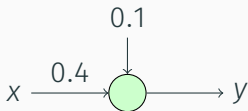
$$y = 0.4x + 0.1 \quad \text{loss} = (y - \hat{y})^2$$

Training a neural network: Gradient descent



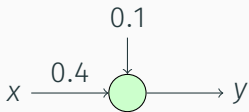
$$0.16 = 0.4 * 0.15 + 0.1 \quad 0.019 = (0.3 - 0.16)^2$$

Training a neural network: Gradient descent



$$(0.3 - (0.4 * 0.15 + 0.1))^2 = 0.019$$

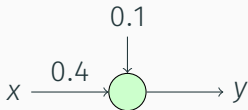
Training a neural network: Gradient descent



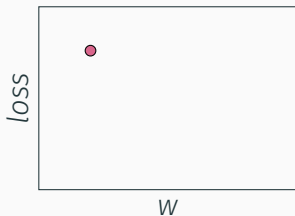
$$(0.3 - (0.4 * 0.15 + 0.1))^2 = 0.019$$

Diagram illustrating the calculation of the loss for a single neuron. The equation shows the difference between the target y (0.3) and the predicted value \hat{y} (0.15), weighted by w (0.4) and added to the bias b (0.1), squared to yield the loss (0.019). Red arrows indicate the mapping of variables: w points to 0.4, x points to 0.15, b points to 0.1, y points to 0.3, and $loss$ points to 0.019. A red underline is under the expression $(0.4 * 0.15 + 0.1)$, and a red arrow points up from \hat{y} to 0.15.

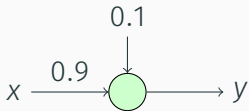
Training a neural network: Gradient descent



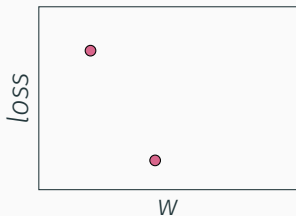
$$(0.3 - (0.4 * 0.15 + 0.1))^2 = 0.019$$



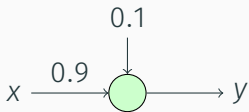
Training a neural network: Gradient descent



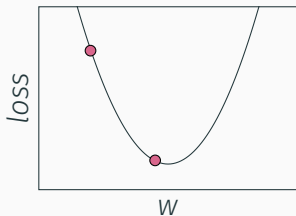
$$(0.3 - (0.9 * 0.15 + 0.1))^2 = 0.004$$



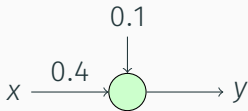
Training a neural network: Gradient descent



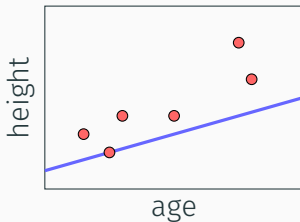
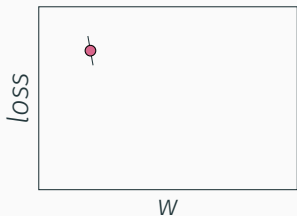
$$(0.3 - (0.9 * 0.15 + 0.1))^2 = 0.004$$



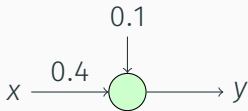
Training a neural network: Gradient descent



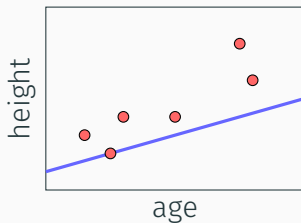
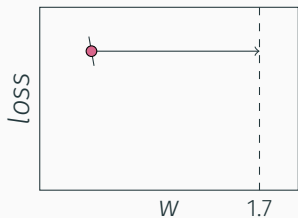
$$(0.3 - (0.4 * 0.15 + 0.1))^2 = 0.019$$



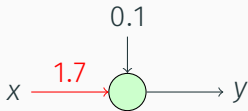
Training a neural network: Gradient descent



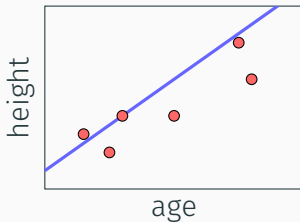
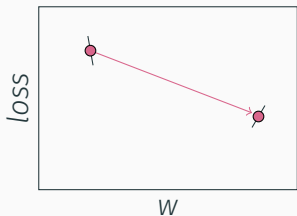
$$(0.3 - (0.4 * 0.15 + 0.1))^2 = 0.019$$



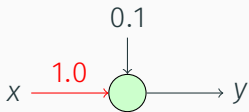
Training a neural network: Gradient descent



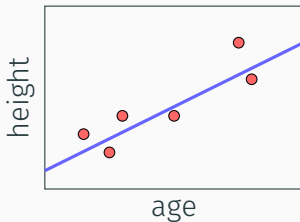
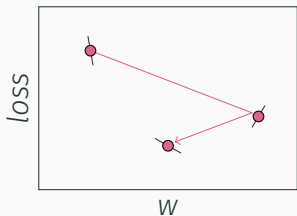
$$(0.3 - (1.7 * 0.15 + 0.1))^2 = 0.003$$



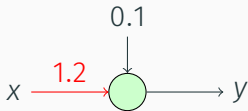
Training a neural network: Gradient descent



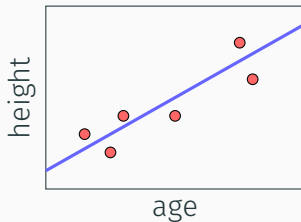
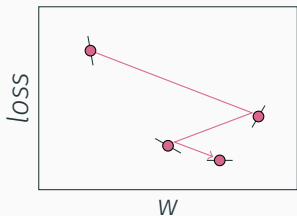
$$(0.3 - (1.0 * 0.15 + 0.1))^2 = 0.002$$



Training a neural network: Gradient descent



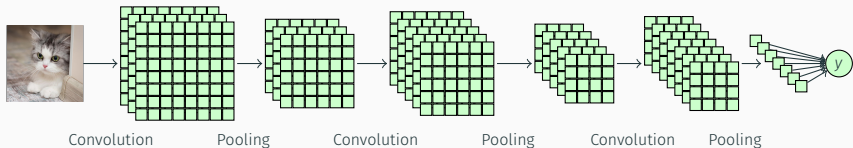
$$(0.3 - (1.2 * 0.15 + 0.1))^2 = 0.000$$



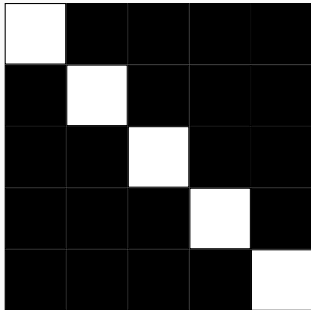
Training a neural network: Summary

- The loss function is a precise formalization of what we want the model to learn
- Calculating the gradient allows us to see how we can update the parameters of the model to decrease the loss
- Using gradient descent we can update the parameters sequentially until we have the perfect model with zero loss

Convolutional neural networks: Architecture

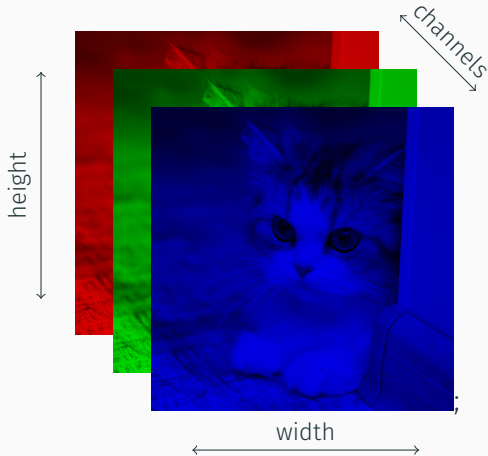


Convolutional neural networks: Images



1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Convolutional neural networks: Images



Convolutional neural networks: Convolution

0	0	1
0	1	0
1	0	0

 $*$

1	0	0
0	1	0
0	0	1

Convolutional neural networks: Convolution

0	0	1
0	1	0
1	0	0

 $*$

1	0	0
0	1	0
0	0	1

0*1

Convolutional neural networks: Convolution

0	0	1
0	1	0
1	0	0

 $*$

1	0	0
0	1	0
0	0	1

$$0*1+0*0$$

Convolutional neural networks: Convolution

0	0	1
0	1	0
1	0	0

 $*$

1	0	0
0	1	0
0	0	1

$$0*1+0*0+1*0$$

Convolutional neural networks: Convolution

0	0	1
0	1	0
1	0	0

 $*$

1	0	0
0	1	0
0	0	1

$$0*1+0*0+1*0+0*0$$

Convolutional neural networks: Convolution

0	0	1
0	1	0
1	0	0

 $*$

1	0	0
0	1	0
0	0	1

$$0*1+0*0+1*0+0*0+1*1+0*0+1*0+0*0+0*1=1$$

Convolutional neural networks: Convolution

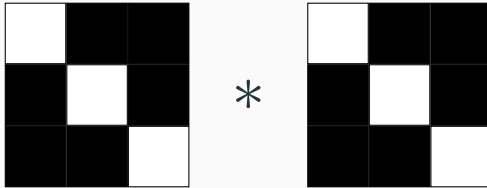
1	0	0
0	1	0
0	0	1

 $*$

1	0	0
0	1	0
0	0	1

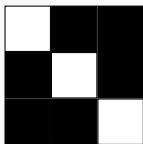
$$1*1+0*0+0*0+0*0+1*1+0*0+1*0+0*0+1*1=3$$

Convolutional neural networks: Convolution

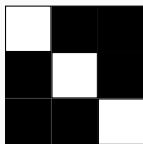


$$1*1+0*0+0*0+0*0+1*1+0*0+1*0+0*0+1*1=3$$

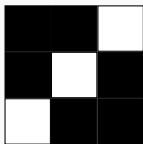
Convolutional neural networks: Convolutional layers



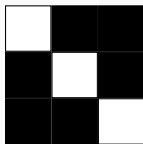
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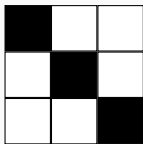
= 3



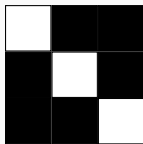
*



= 1



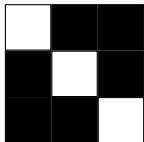
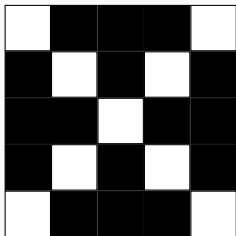
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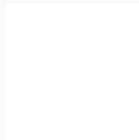
= 0

Convolutional neural networks: Convolutional layers

Image

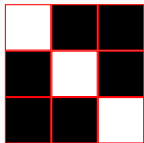
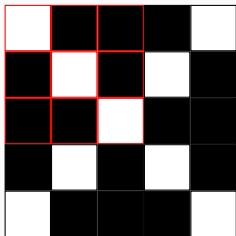


Pattern 1



Convolutional neural networks: Convolutional layers

Image

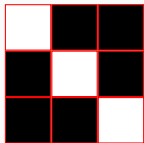
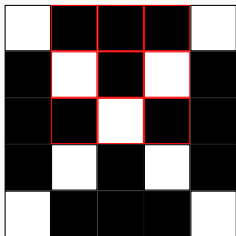


Pattern 1

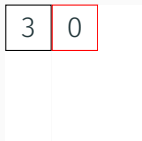
3

Convolutional neural networks: Convolutional layers

Image

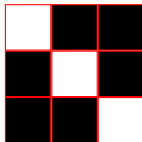
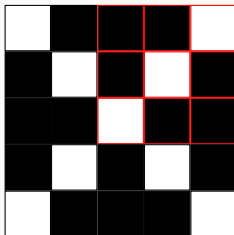


Pattern 1



Convolutional neural networks: Convolutional layers

Image

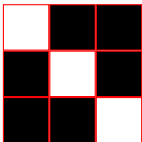
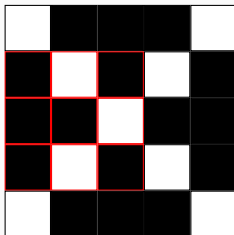


Pattern 1

3	0	1

Convolutional neural networks: Convolutional layers

Image

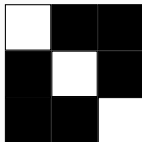
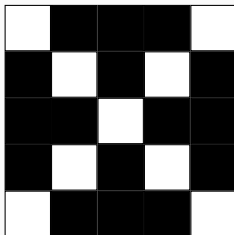


Pattern 1

3	0	1
0		

Convolutional neural networks: Convolutional layers

Image

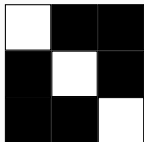
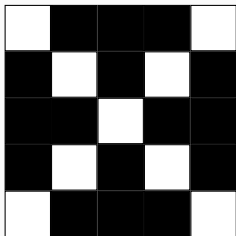


Pattern 1

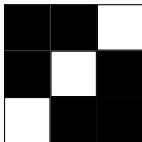
3	0	1
0	3	0
1	0	3

Convolutional neural networks: Convolutional layers

Image



Pattern 1

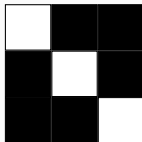
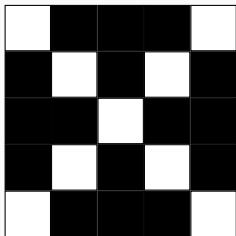


Pattern 2

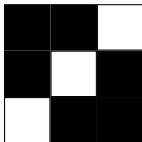
1	0	3
0	3	0
3	0	1

Convolutional neural networks: Convolutional layers

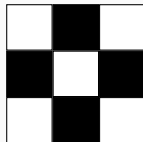
Image



Pattern 1



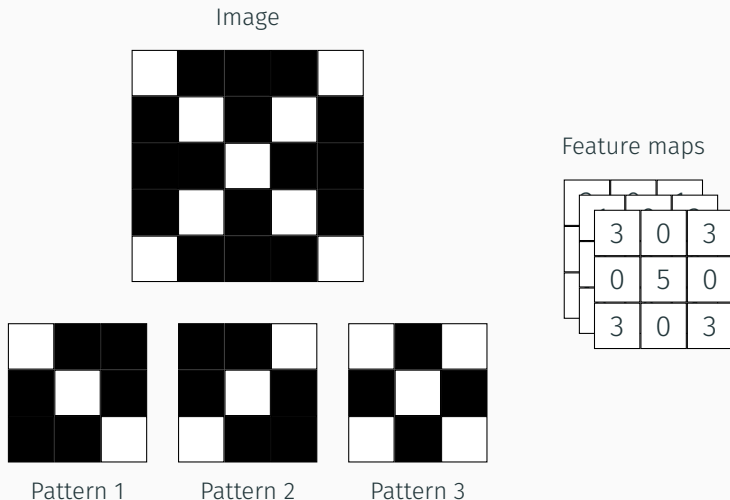
Pattern 2



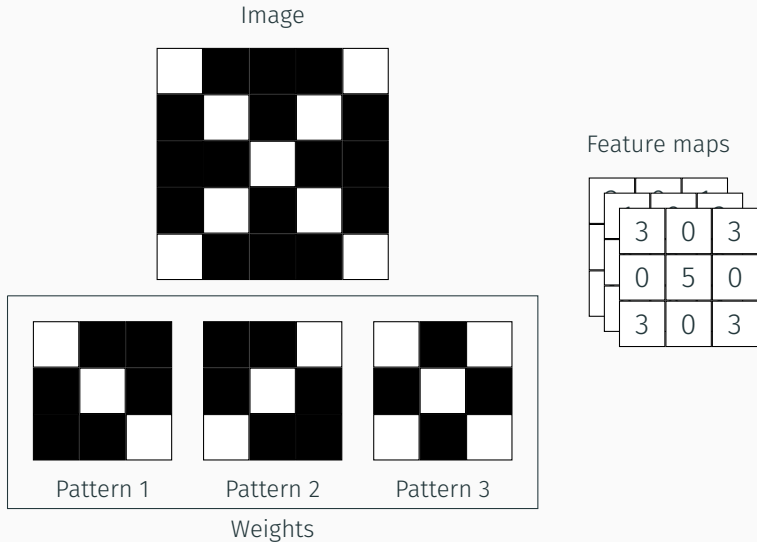
Pattern 3

3	0	3
0	5	0
3	0	3

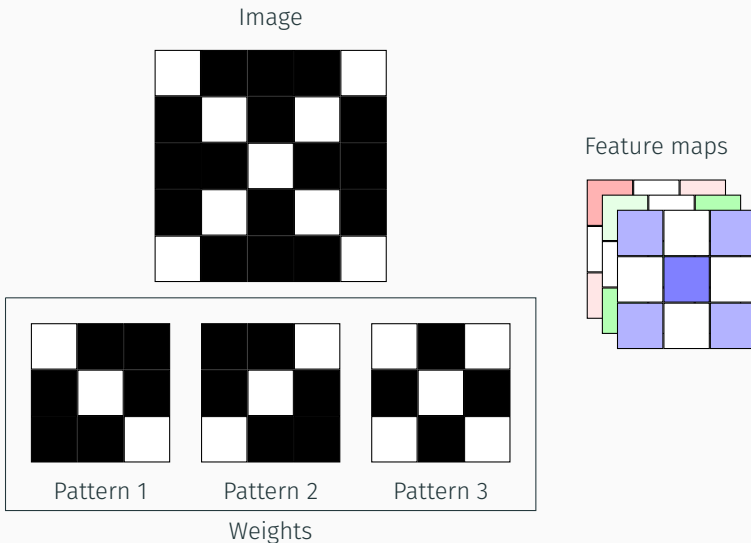
Convolutional neural networks: Convolutional layers



Convolutional neural networks: Convolutional layers



Convolutional neural networks: Convolutional layers



Convolutional neural networks: Pooling

Feature map

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

Convolutional neural networks: Pooling

Feature map

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

5

Convolutional neural networks: Pooling

Feature map

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

5	7
---	---

Convolutional neural networks: Pooling

Feature map

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

5	7
13	

Convolutional neural networks: Pooling

Feature map

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

5	7
13	15

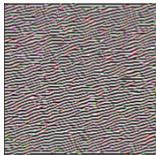
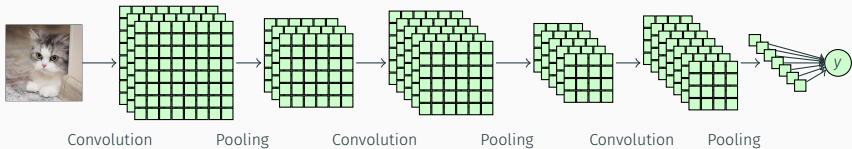
Convolutional neural networks: Pooling

Feature map

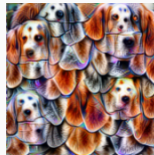
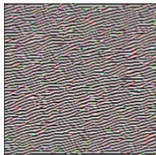
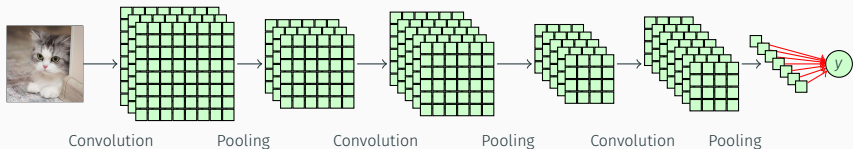
0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

5	7
13	15

Convolutional neural networks: Architecture



Convolutional neural networks: Architecture



Convolutional neural networks: Summary

- Images in computers are stored as matrices of numbers
- The convolution operation is a pattern matcher
- A convolutional layer is a battery of pattern matchers where the patterns are learned during training
- The pooling operation reduces spatial dimensions and distills information
- A convolutional neural network consists of alternating convolution and pooling, which means looking for more and more abstract patterns spanning larger and larger region of the images
- The final layer of the network makes a prediction based on the patterns it has found