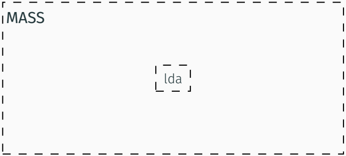
# PSY9511: Seminar 3

## Variable selection and regularization

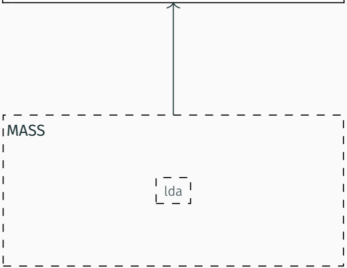
Esten H. Leonardsen
07.09.23

# Outline

```
```

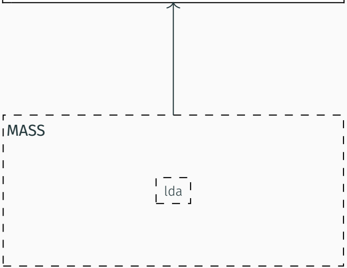

MASS
lda

```r
library(MASS)
```



# Python: Imports

```r
library(MASS)

lda_fit <- lda(display ~ age + fb,
               data = display)
```



# Python: Imports

```r
library(MASS)

lda_fit <- lda(display ~ age + fb,
               data = display)
```

In[1]:
```python
from sklearn import *

lda = discriminant_analysis.
       LinearDiscriminantAnalysis()
lda.fit(display[['age', 'fb']],
        display['display'])
```

```r
library(MASS)

lda_fit <- lda(display ~ age + fb,
               data = display)
```

In[1]:

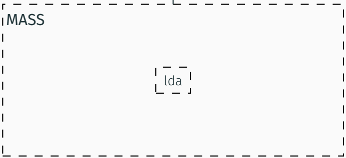
```python
import sklearn

lda = sklearn.discriminant_analysis.
       LinearDiscriminantAnalysis()
lda.fit(display[['age', 'fb']],
        display['display'])
```

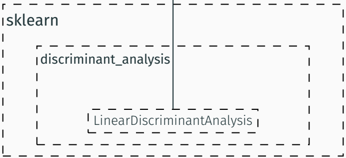
# Python: Imports

```r
library(MASS)

lda_fit <- lda(display ~ age + fb,
               data = display)
```

In[1]:

```python
from sklearn.discriminant_analysis \
    import LinearDiscriminantAnalysis

lda = LinearDiscriminantAnalysis()
lda.fit(display[['age', 'fb']],
        display['display'])
```



```r
path <- '/Users/esten/Downloads/Auto.csv'
df <- read.csv(path)
head(df, 10)
```

In[1]:

```python
import pandas as pd

path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)
df.head(10)
```

| | mpg | cylinders | displacement | horsepower |
|---|---|---|---|---|
| 1 | 18 | 8 | 307.0 | 130 |
| 2 | 15 | 8 | 350.0 | 165 |
| 3 | 18 | 8 | 318.0 | 150 |
| 4 | 16 | 8 | 304.0 | 150 |
| 5 | 17 | 8 | 302.0 | 140 |
| 6 | 15 | 8 | 429.0 | 198 |
| 7 | 14 | 8 | 454.0 | 220 |
| 8 | 14 | 8 | 440.0 | 215 |
| 9 | 14 | 8 | 455.0 | 225 |
| 10 | 15 | 8 | 390.0 | 190 |

Out[1]:

| | mpg | cylinders | displacement | horsepower |
|---|---|---|---|---|
| 0 | 18 | 8 | 307.0 | 130 |
| 1 | 15 | 8 | 350.0 | 165 |
| 2 | 18 | 8 | 318.0 | 150 |
| 3 | 16 | 8 | 304.0 | 150 |
| 4 | 17 | 8 | 302.0 | 140 |
| 5 | 15 | 8 | 429.0 | 198 |
| 6 | 14 | 8 | 454.0 | 220 |
| 7 | 14 | 8 | 440.0 | 215 |
| 8 | 14 | 8 | 455.0 | 225 |
| 9 | 15 | 8 | 390.0 | 190 |

# Python: numpy

In[1]:
```
import numpy as np
```

In[2]:
```
np.random.seed(42)
```

In[3]:
```
np.arange(0, 10, 1)
```

Out[1]:
```
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
```

In[4]:
```
np.isnan([0, 1, np.nan, 3])
```

Out[2]:
```
array([False, False,  True, False])
```

In[5]:
```
np.amin([1, 0, 3, 2])
```

Out[3]:
```
0
```

Out[6]:
```
np.argmin([1, 0, 3, 2])
```

Out[4]:
```
1
```

In[7]:
```
np.nanmin([1, 0, 3, np.nan])
```

Out[5]:
```
0
```

# Python: statsmodels

```r
path <- '/Users/esten/Downloads/Auto.csv'
data <- read.csv(path)

model <- lm(mpg ~ cylinders + displacement +
                  horsepower + weight +
                  acceleration + year,
            data=data)
summary(model)
```

In[1]:

```python
import statsmodels.formula.api as smf

path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

model = smf.ols(
    formula='mpg ~ cylinders + displacement +'
               'horsepower + weight + '
               'acceleration + year',
    data=df
)
fit = model.fit()
print(fit.summary())
```

```
Coefficients:
                Estimate Std. Error Pr(>|t|)
(Intercept)  -1.454e+01  4.764e+00  0.00244 **
cylinders    -3.299e-01  3.321e-01  0.32122
displacement  7.678e-03  7.358e-03  0.29733
horsepower   -3.914e-04  1.384e-02  0.97745
weight       -6.795e-03  6.700e-04  < 2e-16 ***
acceleration  8.527e-02  1.020e-01  0.40383
year          7.534e-01  5.262e-02  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’
      0.05
```

Out[1]:

```
coef   std err   P>|t|   [0.025  0.975]
-----------------------------------------------
Intercept      -14.5353  4.764  0.002  -23.90  -5.16
cylinders       -0.3299  0.332  0.321   -0.98   0.32
displacement     0.0077  0.007  0.297   -0.00   0.02
horsepower      -0.0004  0.014  0.977   -0.02   0.02
weight          -0.0068  0.001  0.000   -0.00  -0.00
acceleration     0.0853  0.102  0.404   -0.11   0.28
year             0.7534  0.053  0.000    0.65   0.85
===============================================
```

# Python: scikit-learn

In[1]:

```
from sklearn.linear_model import LinearRegression

path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

predictors = ['cylinders', 'displacement', 'horsepower',
              'weight', 'acceleration', 'year']
target = 'mpg'

model = LinearRegression()
model.fit(df[predictors], df[target])
model.summary()
```

# Python: scikit-learn

In[1]:

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from sklearn.linear_model import LinearRegression

path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

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              'weight', 'acceleration', 'year']
target = 'mpg'

model = LinearRegression()
model.fit(df[predictors], df[target])
model.summary()
```

Out[1]:

```
---------------------------------------------------------------------------
AttributeError                            Traceback (most recent call last)
Cell In[52], line 13
     11 model = LinearRegression()
     12 model.fit(df[predictors], df[target])
---> 13 model.summary()

AttributeError: 'LinearRegression' object has no attribute 'summary'
```

# Python: scikit-learn

In[1]:

```python
from sklearn.linear_model import LinearRegression

path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

predictors = ['cylinders', 'displacement', 'horsepower',
              'weight', 'acceleration', 'year']
target = 'mpg'

model = LinearRegression()
model.fit(df[predictors], df[target])

# Print model coefficients
print(f'Intercept: {model.intercept_}')
print(f'Coefficients: {model.coef_}')

# Print model residuals
predictions = model.predict(df[predictors])
residuals = df[target] - predictions
print(f'Residuals: {residuals.values[:5]}...')
```

Out[1]:

```
Intercept: -14.53525048050604
Coefficients: [-3.29859089e-01  7.67843024e-03 -3.91355574e-04 -6.79461791e-03
    8.52732469e-02  7.53367180e-01]
Residuals: [2.91708096 0.92742531 2.46368456 0.46552549 1.71359255]...
```

# Python: scikit-learn

## APIs of scikit-learn objects

To have a uniform API, we try to have a common basic API for all the objects. In addition, to avoid the proliferation of framework code, we try to adopt simple conventions and limit to a minimum the number of methods an object must implement.

Elements of the scikit-learn API are described more definitively in the Glossary of Common Terms and API Elements.

### Different objects

The main objects in scikit-learn are (one class can implement multiple interfaces):

**Estimator:** The base object, implements a `fit` method to learn from data, either:

```
estimator = estimator.fit(data, targets)
```

or:

```
estimator = estimator.fit(data)
```

**Predictor:** For supervised learning, or some unsupervised problems, implements:

```
prediction = predictor.predict(data)
```

Classification algorithms usually also offer a way to quantify certainty of a prediction, either using `decision_function` or `predict_proba`:

```
probability = predictor.predict_proba(data)
```

**Transformer:** For filtering or modifying the data, in a supervised or unsupervised way, implements:

```
new_data = transformer.transform(data)
```

When fitting and transforming can be performed much more efficiently together than separately, implements:

```
new_data = transformer.fit_transform(data)
```

**Model:** A model that can give a goodness of fit measure or a likelihood of unseen data, implements (higher is better):

```
score = model.score(data)
```

https://scikit-learn.org/stable/developers/develop.html

# Python: scikit-learn

In[1]:
```python
from sklearn.linear_model import LinearRegression

path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

predictors = ['cylinders', 'displacement', 'horsepower',
              'weight', 'acceleration', 'year']
target = 'mpg'

model = LinearRegression()
model.fit(df[predictors], df[target])

# Print model coefficients
print(f'Intercept: {model.intercept_}')
print(f'Coefficients: {model.coef_}')

# Print model residuals
predictions = model.predict(df[predictors])
residuals = df[target] - predictions
print(f'Residuals: {residuals.values[:5]}...')
```

Out[1]:
```
Intercept: -14.53525048050604
Coefficients: [-3.29859089e-01  7.67843024e-03 -3.91355574e-04 -6.79461791e-03
    8.52732469e-02  7.53367180e-01]
Residuals: [2.91708096 0.92742531 2.46368456 0.46552549 1.71359255]...
```

# Python: scikit-learn

In[1]:

```
from sklearn.svm import SVR

path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

predictors = ['cylinders', 'displacement', 'horsepower',
              'weight', 'acceleration', 'year']
target = 'mpg'

model = SVR(kernel='linear')
model.fit(df[predictors], df[target])

# Print model coefficients
print(f'Intercept: {model.intercept_}')
print(f'Coefficients: {model.coef_}')

# Print model residuals
predictions = model.predict(df[predictors])
residuals = df[target] - predictions
print(f'Residuals: {residuals.values[:5]}...')
```

Out[1]:

```
Intercept: [-35.38646279]
Coefficients: [[-1.0526357   0.05910105  -0.03667206  -0.00831565   0.56218046
      0.96851648]]
Residuals: [3.0266171  0.62154228  3.10666275  1.34695011  3.07475274]...
```

# Coding tips: Separation of concerns

In[1]:

```
# Read and clean data
path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

# Split data
train = df.iloc[:int(len(df) * 0.8)]
validation = df.iloc[int(len(df) * 0.8):]

# Define input and output variables
predictors = ['cylinders', 'displacement', 'horsepower',
              'weight', 'acceleration', 'year']
target = 'mpg'

# Define necessary data structures for state
chosen_predictors = []
mses = []

while len(predictors) > 0:
    best_predictor = {'mse': float('inf'), 'predictor': None}

    for predictor in set(predictors) - set(chosen_predictors):
        potential_predictors = chosen_predictors + [predictor]

        # Fit and evaluate model
        model = LinearRegression()
        model.fit(train[potential_predictors], train[target])
        predictions = model.predict(validation[potential_predictors])
        test_mse = np.mean((validation[target] - predictions) ** 2)

        # Compare model with previous best
        if test_mse < best_predictor['mse']:
            best_predictor = {'mse': test_mse, 'predictor': predictor}

    # Update state
    chosen_predictors.append(best_predictor['predictor'])
    mses.append(best_predictor['mse'])
    predictors = [p for p in predictors if p != best_predictor['predictor']]
```

# Coding tips: Separation of concerns

In[1]:

Setup

```
# Read and clean data
path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

# Split data
train = df.iloc[:int(len(df) * 0.8)]
validation = df.iloc[int(len(df) * 0.8):]

# Define input and output variables
predictors = ['cylinders', 'displacement', 'horsepower',
              'weight', 'acceleration', 'year']
target = 'mpg'

# Define necessary data structures for state
chosen_predictors = []
mses = []
```

Selection

```
while len(predictors) > 0:
    best_predictor = {'mse': float('inf'), 'predictor': None}

    for predictor in set(predictors) - set(chosen_predictors):
        potential_predictors = chosen_predictors + [predictor]
```

Modelling

```
        # Fit and evaluate model
        model = LinearRegression()
        model.fit(train[potential_predictors], train[target])
        predictions = model.predict(validation[potential_predictors])
        test_mse = np.mean((validation[target] - predictions) ** 2)
```

```
        # Compare model with previous best
        if test_mse < best_predictor['mse']:
            best_predictor = {'mse': test_mse, 'predictor': predictor}
```

Housekeeping

```
    # Update state
    chosen_predictors.append(best_predictor['predictor'])
    mses.append(best_predictor['mse'])
    predictors = [p for p in predictors if p != best_predictor['predictor']]
```

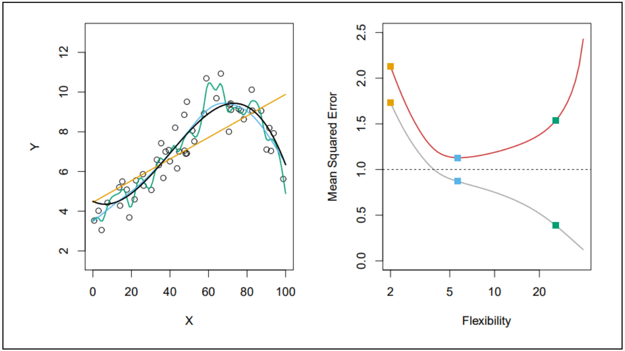
# Coding tips: Separation of concerns

In[1]:

```
# Read and clean data
path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

# Split data
train = df.iloc[:int(len(df) * 0.8)]
validation = df.iloc[int(len(df) * 0.8):]

# Define input and output variables
predictors = ['cylinders', 'displacement', 'horsepower',
                'weight', 'acceleration', 'year']
target = 'mpg'

# Define necessary data structures for state
chosen_predictors = []
mses = []

def fit_and_evaluate_model(model: LinearRegression, train: pd.DataFrame,
                           validation: pd.DataFrame, variables: List[str],
                           target: str):
    """ Fit a given model on a training dataset using a given set of variables
    and return MSE from a validation dataset. """
    model = LinearRegression()
    model.fit(train[potential_predictors], train[target])
    predictions = model.predict(validation[potential_predictors])

    return np.mean((validation[target] - predictions) ** 2)

while len(predictors) > 0:
    best_predictor = {'mse': float('inf'), 'predictor': None}

    for predictor in set(predictors) - set(chosen_predictors):
        potential_predictors = chosen_predictors + [predictor]
        test_mse = fit_and_evaluate_model(LinearRegression(), train, validation,
                                          variables=potential_predictors,
                                          target=target)

        # Compare model with previous best
        if test_mse < best_predictor['mse']:
            best_predictor = {'mse': test_mse, 'predictor': predictor}

    # Update state
    chosen_predictors.append(best_predictor['predictor'])
    mses.append(best_predictor['mse'])
    predictors = [p for p in predictors if p != best_predictor['predictor']]
```

Modelling

# Regularization: Motivation



# Regularization: Out-of-sample testing

# Regularization: Methods

1. Variable selection
    a. Best subset selection
    b. Forward stepwise selection
    c. Backward stepwise selection
2. Shrinkage
    a. LASSO
    b. Ridge Regression
    c. Elastic net
3. Dimensionality reduction
    a. Principal Component Regression
    b. Partial Least Squares

# Variable selection: Outline

Problem

We have a set of predictors $P = \{x_0, x_1, ...\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

# Variable selection: Outline

Problem

We have a set of predictors $P = \{x_0, x_1, ...\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

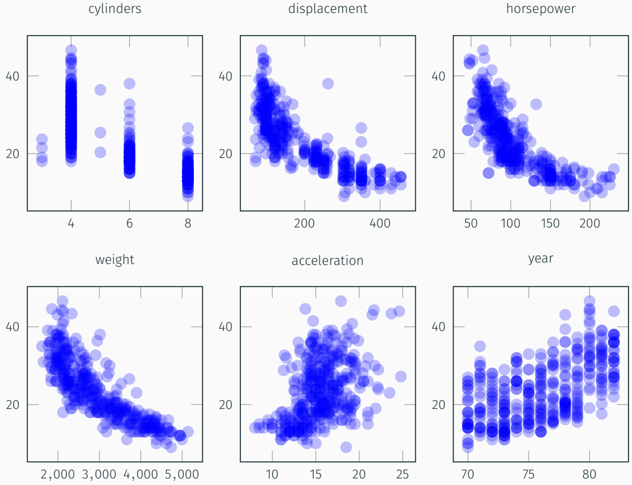
Motivation

1. Simplify interpretation
2. Reduce model complexity (overfitting)

# Variable selection: Outline

Problem

We have a set of predictors $P = \{x_0, x_1, \ldots\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.



# Variable selection: Best subset selection

Problem

We have a set of predictors $P = \{x_0, x_1, ...\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

Solution

Train models on all subsets $p$ and select the best one.

# Variable selection: Best subset selection

Problem

We have a set of predictors $P = \{x_0, x_1, \ldots\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

Solution

In[1]:

```
import numpy as np

from itertools import chain, combinations
from sklearn.linear_model import LinearRegression

subsets = list(chain.from_iterable(combinations(predictors, r) \
                                   for r in range(len(predictors)+1)))

best = {'mse': float('inf'), 'subset': None}

for subset in subsets:
    if len(subset) == 0:
        continue

    model = LinearRegression()
    model.fit(train[list(subset)], train[target])
    predictions = model.predict(validation[list(subset)])
    mse = np.mean((predictions - validation[target]) ** 2)

    if mse < best['mse']:
        best = {'mse': mse, 'subset': subset}

print(f'MSE: {best["mse"]:.2f}, predictors: {best["subset"]}')
```

Out[1]:

```
MSE: 29.68, predictors: ('cylinders', 'displacement', 'horsepower', 'weight', 'year')
```

# Variable selection: Best subset selection

Problem

We have a set of predictors $P = \{x_0, x_1, ...\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

Solution

Train models on all subsets $p$ and select the best one.

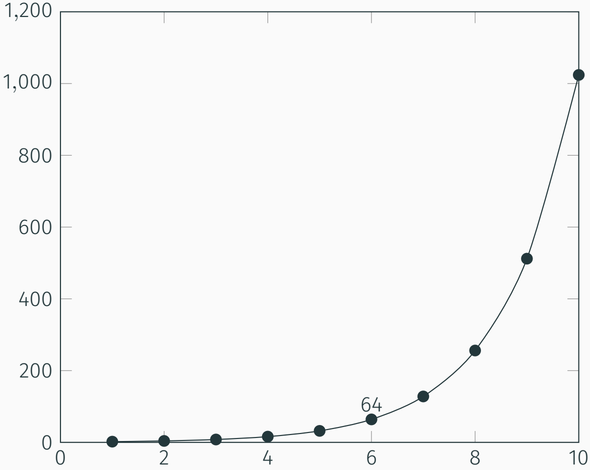
+ Positives

Guaranteed to find the optimal solution.
Simple implementation

- Drawbacks

Need to train many ($2^{|P|}$) models.

# Variable selection: Best subset selection



# Variable selection: Forward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, ...\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.

# Variable selection: Forward stepwise selection

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We have a set of predictors $P = \{x_0, x_1, ...\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.

$$\boxed{\begin{array}{c} y \sim \mathbb{1} \\ mse = 146.47 \end{array}}$$

# Variable selection: Forward stepwise selection

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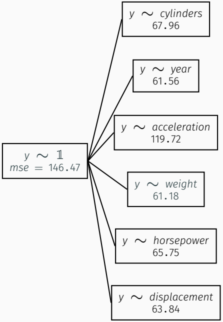

# Variable selection: Forward stepwise selection

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We have a set of predictors $P = \{x_0, x_1, \ldots\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

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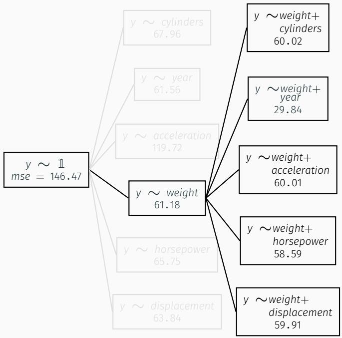

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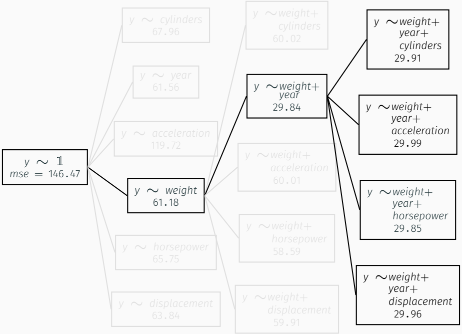

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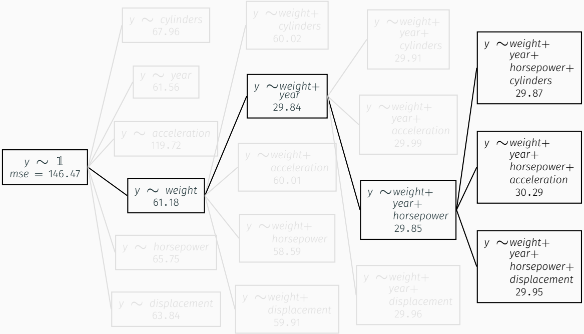

# Variable selection: Forward stepwise selection

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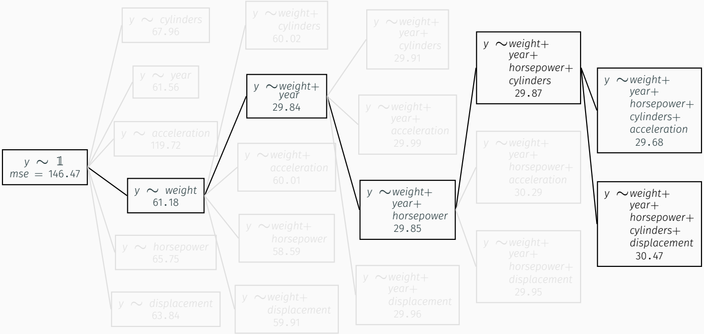
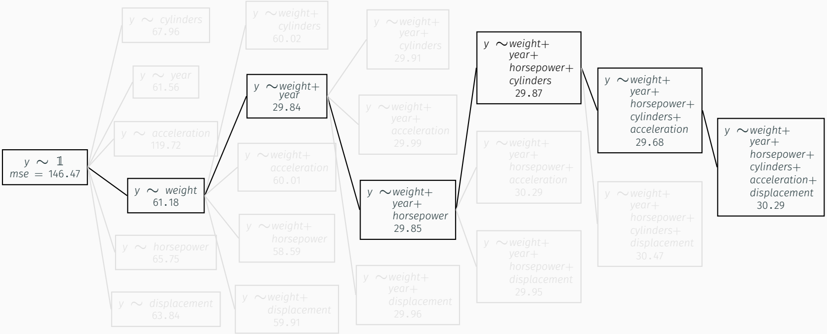


Problem

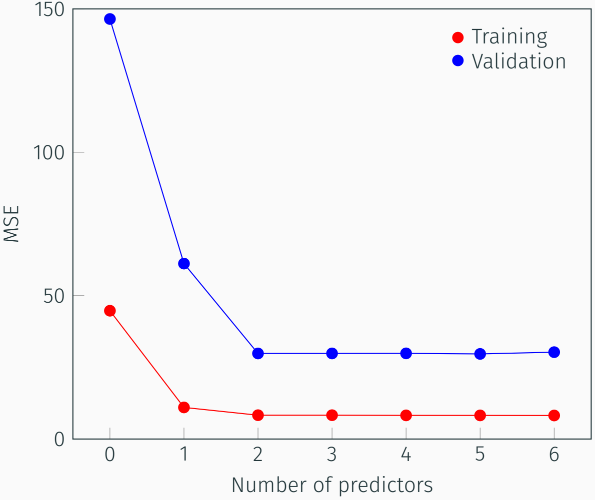
We have a set of predictors $P = \{x_0, x_1, ...\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

Solution

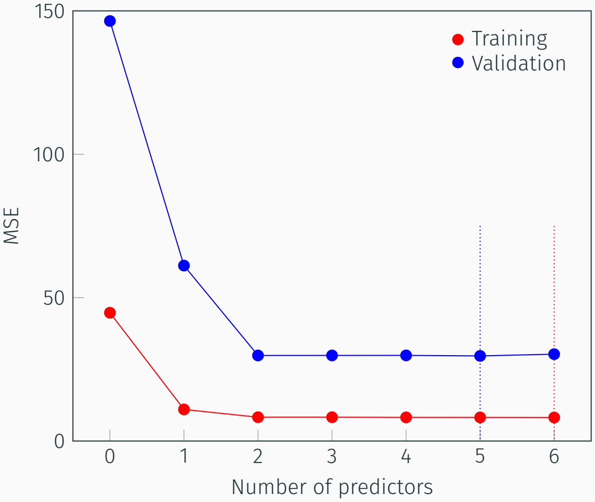
Start with no predictors. Iteratively add the predictor that yields the best model until all are included.



# Variable selection: Forward stepwise selection



# Variable selection: Forward stepwise selection



Problem

We have a set of predictors $P = \{x_0, x_1, ...\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

Solution

In[1]:
```
def fit_and_evaluate(train: pd.DataFrame, validation: pd.DataFrame,
                     predictors: List[str], target: str):
   model = LinearRegression()
   model.fit(train[predictors], train[target])

   train_predictions = model.predict(train[predictors])
   validation_predictions = model.predict(validation[predictors])

   return np.mean((train_predictions - train[target]) ** 2), \
          np.mean((validation_predictions - validation[target]) ** 2)

predictors = ['cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'year']
target = 'mpg'

train['intercept'] = 1
validation['intercept'] = 1
train_mse, validation_mse = fit_and_evaluate(train, validation,
                                             predictors=['intercept'],
                                             target=target)
print(f'[]: {validation_mse:.2f} ({train_mse:.2f})')

chosen_predictors = []

while len(chosen_predictors) < len(predictors):
   best_predictor = {'train_mse': None, 'validation_mse': float('inf'),
                     'predictor': None}

   for predictor in set(predictors) - set(chosen_predictors):
       train_mse, validation_mse = fit_and_evaluate(train, validation,
                                   predictors=chosen_predictors + [predictor],
                                   target=target)

       if validation_mse < best_predictor['validation_mse']:
           best_predictor = {'train_mse': train_mse, 'validation_mse': validation_mse, 'predictor': predictor}

   chosen_predictors.append(best_predictor['predictor'])

   print(f'{chosen_predictors}: {best_predictor["validation_mse"]:.2f} ({best_predictor["train_mse"]:.2f})')
```

# Variable selection: Forward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, ...\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.

\+ Positives

Need to train fewer models.

\- Drawbacks

Not guaranteed to find the optimal solution.

# Variable selection: Backward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, ...\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

Solution

Start with all predictors. Iteratively remove the predictor that yields the best model until all you have none left.

# Variable selection: Backward stepwise selection

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We have a set of predictors $P = \{x_0, x_1, ...\}$ and a target variable $y$, and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting $y$.

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Need to train fewer models.

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Not guaranteed to find the optimal solution.

# Shrinkage

# Shrinkage: Outline

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$

# Shrinkage: Outline

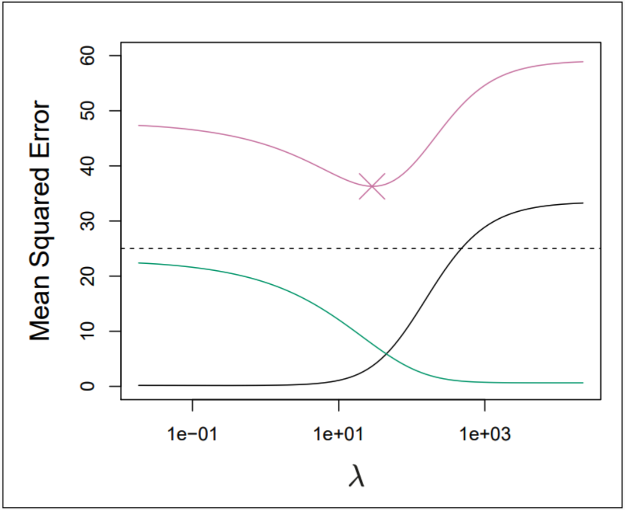
Out[1]:

| | coef | std err | P>\|t\| | [0.025 | 0.975] |
|---|---|---|---|---|---|
| Intercept | -14.5353 | 4.764 | 0.002 | -23.90 | -5.16 |
| cylinders | -0.3299 | 0.332 | 0.321 | -0.98 | 0.32 |
| displacement | 0.0077 | 0.007 | 0.297 | -0.00 | 0.02 |
| horsepower | -0.0004 | 0.014 | 0.977 | -0.02 | 0.02 |
| weight | -0.0068 | 0.001 | 0.000 | -0.00 | -0.00 |
| acceleration | 0.0853 | 0.102 | 0.404 | -0.11 | 0.28 |
| year | 0.7534 | 0.053 | 0.000 | 0.65 | 0.85 |

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$

$$mse = bias^2 + variance + irreducible\ error$$

# Shrinkage: Outline



# Shrinkage: Outline

$$salary \sim \beta_0 + \beta_1 * age$$

# Shrinkage: Outline

$$salary \sim \beta_0 + \beta_1 * age$$

$$salary \sim 3000000 + 10000 * age \qquad salary \sim 6000000 + 0 * age$$

# Shrinkage: Ridge regression

$$loss_{mse} = \sum_{i=0}^{n} \left( y_i - \sum_{j=0}^{p} \beta_j x_{ij} \right)^2$$

# Shrinkage: Ridge regression

$$loss_{ridge} = \sum_{i=0}^{n} \left( y_i - \sum_{j=0}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^{p} \beta_j^2$$

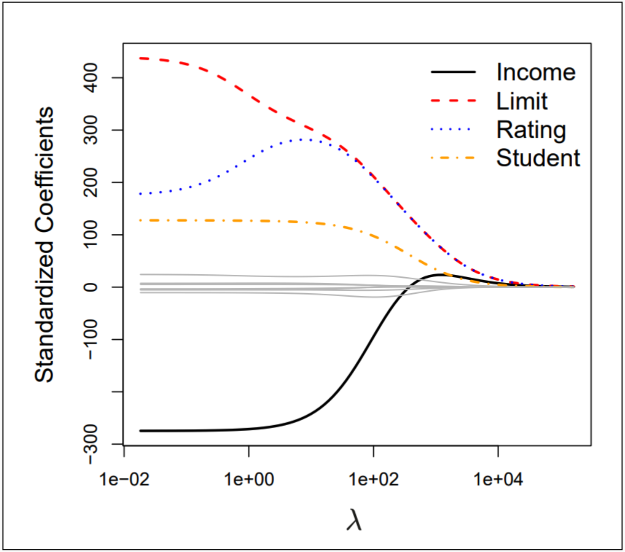
# Shrinkage: Ridge regression

$$loss_{ridge} = \sum_{i=0}^{n}\left(y_i - \sum_{j=0}^{p}\beta_j x_{ij}\right)^2 \boxed{+\lambda\sum_{j=0}^{p}\beta_j^2}$$

$$\Downarrow$$

$$\lambda \to \infty \Rightarrow \beta \to 0$$

# Shrinkage

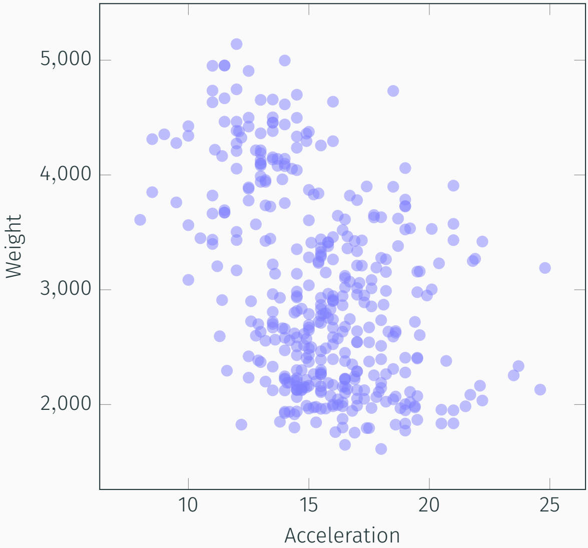


Income
Limit
Rating
Student
Standardized Coefficients
400
300
200
100
0
−100
−300
1e−02
1e+00
1e+02
1e+04
λ

# Shrinkage: Ridge regression

$$loss_{ridge} = \sum_{i=0}^{n} \left( y_i - \sum_{j=0}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^{p} \beta_j^2$$

# Shrinkage: Feature standardization



z-score standardization

# Shrinkage: Feature standardization

z-score standardization

$$\mathsf{x} = \frac{\mathsf{x} - \mu_X}{\sigma_X^2}$$

# Shrinkage: Feature standardization

## z-score standardization

$$x = \frac{x - \mu_x}{\sigma_x^2}$$

In[1]:
```
for col in predictors:
    print(f'{col}: {np.mean(df[col]):.2f} ({np.std(df[col]):.2f})')

# z-score standardization
for col in predictors:
    df[col] = (df[col] - np.mean(df[col])) / np.std(df[col])

for col in predictors:
    print(f'{col} after: {np.mean(df[col]):.2f} ({np.std(df[col]):.2f})')
```

## z-score standardization

$$x = \frac{x - \mu_x}{\sigma_x^2}$$

In[1]:
```python
for col in predictors:
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```

Out[1]:
```
cylinders: 5.47 (1.70)
displacement: 194.41 (104.51)
horsepower: 104.47 (38.44)
weight: 2977.58 (848.32)
acceleration: 15.54 (2.76)
year: 75.98 (3.68)
cylinders after: -0.00 (1.00)
displacement after: -0.00 (1.00)
horsepower after: -0.00 (1.00)
weight after: -0.00 (1.00)
acceleration after: 0.00 (1.00)
year after: -0.00 (1.00)
```

# Shrinkage: Ridge regression

http://localhost:8888/notebooks/notebooks/Live%20coding.ipynb

# Shrinkage: Ridge regression

$$loss_{ridge} = \sum_{i=0}^{n} \left( y_i - \sum_{j=0}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^{p} \beta_j^2$$

Regularization through shrinking the model covariates towards zero.

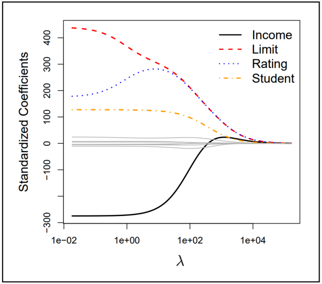
# Shrinkage: LASSO

$$loss_{ridge} = \sum_{i=0}^{n} \left( y_i - \sum_{j=0}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^{p} \beta_j^2$$

$$loss_{lasso} = \sum_{i=0}^{n} \left( y_i - \sum_{j=0}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^{p} |\beta_j|$$

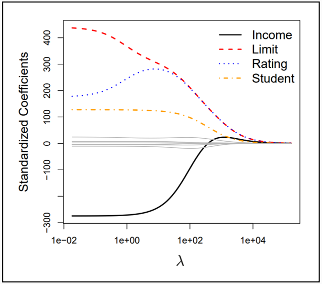
# Shrinkage: LASSO

Ridge

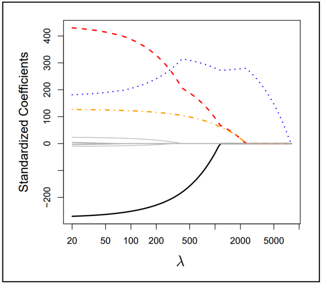


# Shrinkage: LASSO

Ridge

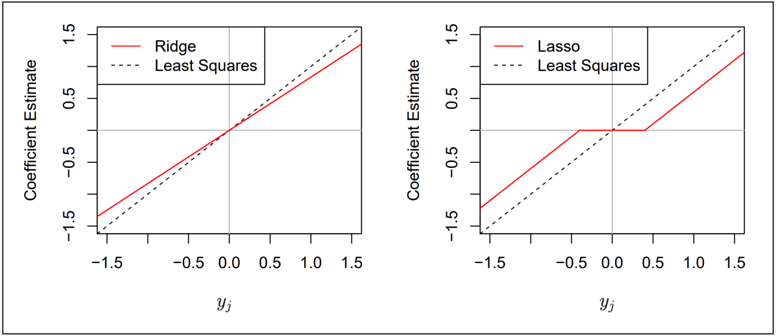



LASSO

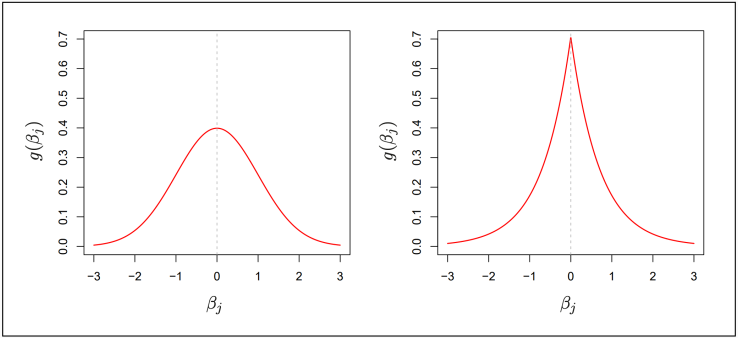


Python coefficients

# Shrinkage: LASSO



# Shrinkage: LASSO



# Shrinkage: Summary

$$loss_{mse} = \sum_{i=0}^{n} \left( y_i - \sum_{j=0}^{p} \beta_j x_{ij} \right)^2$$

Fits the **best** model to the data.

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Fits the **best** model to the data while **shrinking** coefficients towards zero.

# Shrinkage: Summary

$$loss_{mse} = \sum_{i=0}^{n}\left(y_i - \sum_{j=0}^{p}\beta_j x_{ij}\right)^2$$

Fits the **best** model to the data.

$$loss_{ridge} = \sum_{i=0}^{n}\left(y_i - \sum_{j=0}^{p}\beta_j x_{ij}\right)^2 + \lambda\sum_{j=0}^{p}\beta_j^2$$

Fits the **best** model to the data while **shrinking** coefficients towards zero.

$$loss_{lasso} = \sum_{i=0}^{n}\left(y_i - \sum_{j=0}^{p}\beta_j x_{ij}\right)^2 + \lambda\sum_{j=0}^{p}|\beta_j|$$

Fits the **best** model to the data while **shrinking** coefficients towards zero such that some variables are effectively **removed**.

# Dimensionality reduction: Outline

# Dimensionality reduction: Principal component analysis

# Dimensionality reduction: Principal component regression