

PSY9511: Seminar 3

Regularization and variable selection

Esten H. Leonardsen

11.09.24

1. Assignment 2
2. Regularization
 - Variable selection
 - Shrinkage (+ live coding 🎉)

Assignment 2



UNIVERSITETET
I OSLO

Non-numeric horsepower: Expectations Interpretation of year-variable Type of prediction

Assignment 2: Data splitting

```
In[1]: import pandas as pd

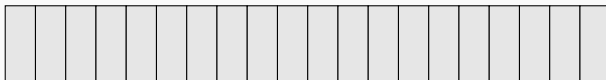
df = pd.DataFrame(...)
train = df.sample(frac=0.8)
test = df.sample(frac=0.2)
```



Assignment 2: Data splitting

```
In[1]: import pandas as pd

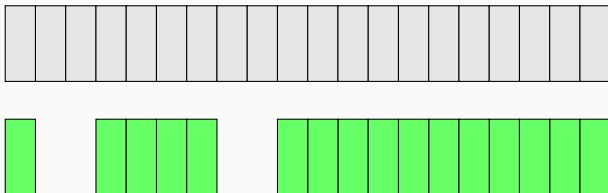
df = pd.DataFrame(...)
train = df.sample(frac=0.8)
test = df.sample(frac=0.2)
```



Assignment 2: Data splitting

```
In[1]: import pandas as pd

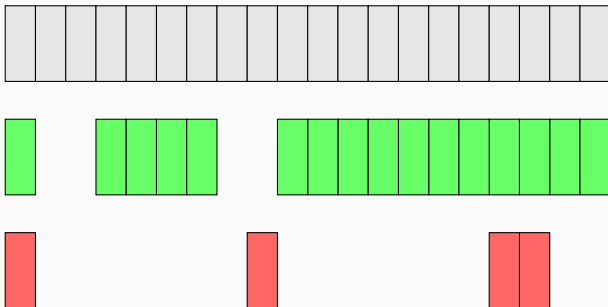
df = pd.DataFrame(...)
train = df.sample(frac=0.8)
test = df.sample(frac=0.2)
```



Assignment 2: Data splitting

```
In[1]: import pandas as pd

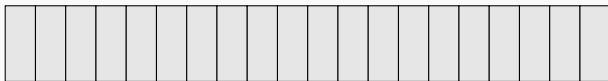
df = pd.DataFrame(...)
train = df.sample(frac=0.8)
test = df.sample(frac=0.2)
```



Assignment 2: Data splitting

```
In[1]: import pandas as pd

df = pd.DataFrame(...)
train = df.sample(frac=0.8)
test = df.drop(train.index)
```



Assignment 2: Random seeds

```
In[1]: import pandas as pd

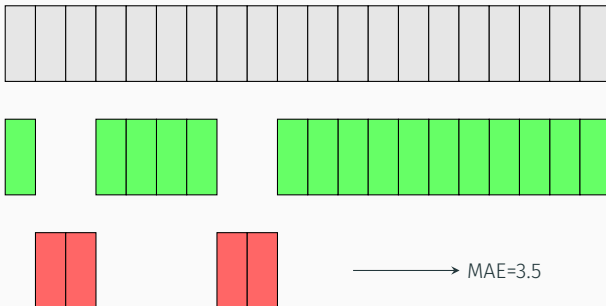
df = pd.DataFrame(...)
train = df.sample(frac=0.8)
test = df.drop(train.index)
```



Assignment 2: Random seeds

```
In[1]: import pandas as pd

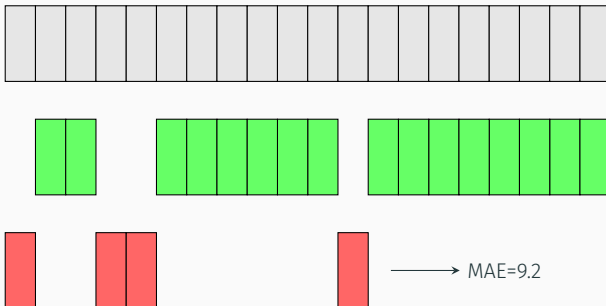
df = pd.DataFrame(...)
train = df.sample(frac=0.8)
test = df.drop(train.index)
```



Assignment 2: Random seeds

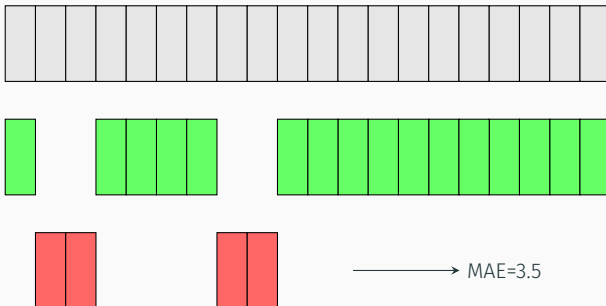
```
In[1]: import pandas as pd

df = pd.DataFrame(...)
train = df.sample(frac=0.8)
test = df.drop(train.index)
```



Assignment 2: Random seeds

```
In[1]: import pandas as pd
import numpy as np
np.random.seed(42)
df = pd.DataFrame(...)
train = df.sample(frac=0.8)
test = df.drop(train.index)
```



Assignment 2: Log-odds vs probability vs class

```
model <- glm(muscle ~ year + weight, data=df, family='binomial')  
predict(model, df)
```



Assignment 2: Log-odds vs probability vs class

```
model <- glm(muscle ~ year + weight, data=df, family='binomial')  
predict(model, df)
```

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1.2460 | 1.9245 | 1.0019 | 0.9911 | 1.0485 | 4.2506 | 4.1465 | 4.5522 | 2.4889 | 1.4578 | 1.6223 |



Assignment 2: Log-odds vs probability vs class

```
model <- glm(muscle ~ year + weight, data=df, family='binomial')  
predict(model, df)
```

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1.2460 | 1.9245 | 1.0019 | 0.9911 | 1.0485 | 4.2506 | 4.1465 | 4.5522 | 2.4889 | 1.4578 | 1.6223 |

```
predict(model, df, type="response")
```

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.7766 | 0.8726 | 0.7314 | 0.7293 | 0.7405 | 0.9853 | 0.9844 | 0.9895 | 0.9233 | 0.8112 | 0.8352 |



Assignment 2: Log-odds vs probability vs class

```
In[1]: model = LogisticRegression()  
model.fit(df[['year', 'weight']], df['muscle'])  
model.predict(df[['year', 'weight']])
```

```
Out[1]: array([0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0])
```

```
In[1]: model.predict_proba(df[['year', 'weight']])
```

```
Out[1]: array([[0.14, 0.86], [0.08, 0.92], [0.17, 0.83], [0.18, 0.82]])
```



Assignment 2: Eye test

```
model <- glm(muscle ~ year + weight, data=df, family='binomial')  
predict(model, df)
```

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1.2460 | 1.9245 | 1.0019 | 0.9911 | 1.0485 | 4.2506 | 4.1465 | 4.5522 | 2.4889 | 1.4578 | 1.6223 |



Assignment 2: Eye test

```
model <- glm(muscle ~ year + weight, data=df, family='binomial')
predict(model, df)
```

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1.2460 | 1.9245 | 1.0019 | 0.9911 | 1.0485 | 4.2506 | 4.1465 | 4.5522 | 2.4889 | 1.4578 | 1.6223 |

```
model <- lm(mpg ~ horsepower, df)
summary(model)
```

```
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 19.59412  0.96187 20.371 < 2e-16 ***
horsepower102 0.40588  4.08087  0.099  0.920840
horsepower103 0.70588  4.08087  0.173  0.862789
horsepower105 0.90588  1.49529  0.606  0.545091
```

Regularization

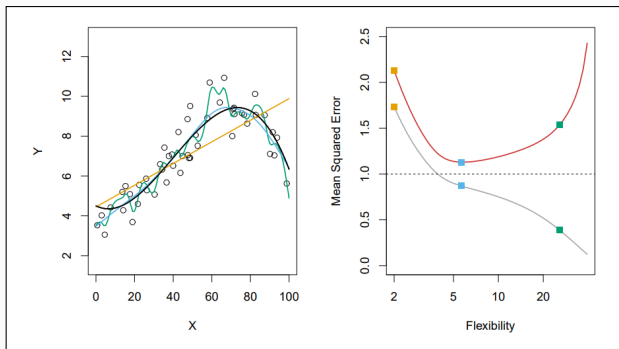


UNIVERSITETET
I OSLO

$$y \sim \beta_0 + \beta_1 * x_1 + \beta_2 * x_2$$

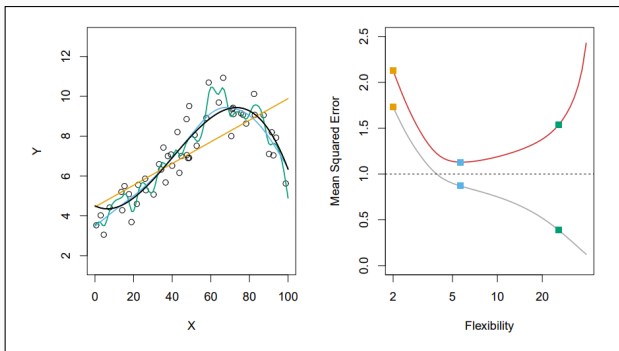
Regularization: Preparations

$$y \sim \beta_0 + \beta_1 * x_1 + \beta_2 * x_2$$



Regularization: Preparations

$$y \sim \beta_0 + \beta_1 * x_1 + \beta_2 * x_2 + \beta_3 * x_3 + \beta_4 * x_4 + \beta_5 * x_5 + \beta_6 * x_6$$



```
In[1]: import pandas as pd

df = pd.read_csv('/Users/esten/Downloads/Auto.csv')
train = df.iloc[:int(len(df) * 0.8)]
validation = df.iloc[int(len(df) * 0.8):]

print(f'Using len(train) samples for training')
print(f'Using len(validation) samples for validation')
```

```
Out[1]: Using 317 samples for training
Using 80 samples for validation
```


1. Variable selection
 - a. Best subset selection
 - b. Forward stepwise selection
 - c. Backward stepwise selection
2. Shrinkage
 - a. LASSO
 - b. Ridge Regression
3. Dimensionality reduction: Lecture 6 and self-study

1. Variable selection
 - a. Best subset selection
 - b. Forward stepwise selection
 - c. Backward stepwise selection
2. Shrinkage
 - a. **LASSO**
 - b. Ridge Regression
3. Dimensionality reduction: Lecture 6 and self-study

Variable selection



UNIVERSITETET
I OSLO

The number of predictors we are using in our model directly impacts model complexity.

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

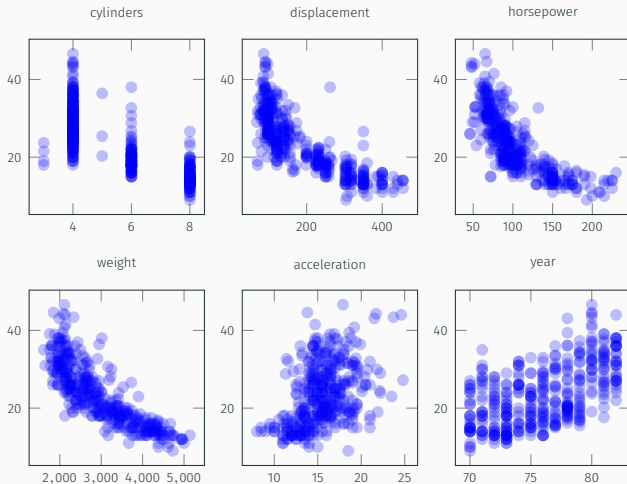
Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Motivation

To reduce model complexity (and therefore risk of overfitting), and to simplify subsequent interpretations.

Variable selection



Variable selection: Best subset selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Train models on all subsets p and select the best one.



Variable selection: Best subset selection

```
In[1]: import numpy as np

from itertools import chain, combinations
from sklearn.linear_model import LinearRegression

subsets = list(chain.from_iterable(combinations(predictors, r)
                                   for r in range(len(predictors)+1)))

best = {'mse': float('inf'), 'subset': None}

for subset in subsets:
    if len(subset) == 0:
        continue

    model = LinearRegression()
    model.fit(train[list(subset)], train[target])
    predictions = model.predict(validation[list(subset)])
    mse = np.mean((predictions - validation[target]) ** 2)

    if mse < best['mse']:
        best = {'mse': mse, 'subset': subset}

print(f'MSE: {best["mse"]:.2f}, predictors: {best["subset"]}')
```

```
Out[1]: MSE: 29.68, predictors: ('cylinders', 'displacement', 'horsepower', 'weight', 'year')
```



Variable selection: Best subset selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Train models on all subsets p and select the best one.

+ Positives

Guaranteed to find the optimal solution.

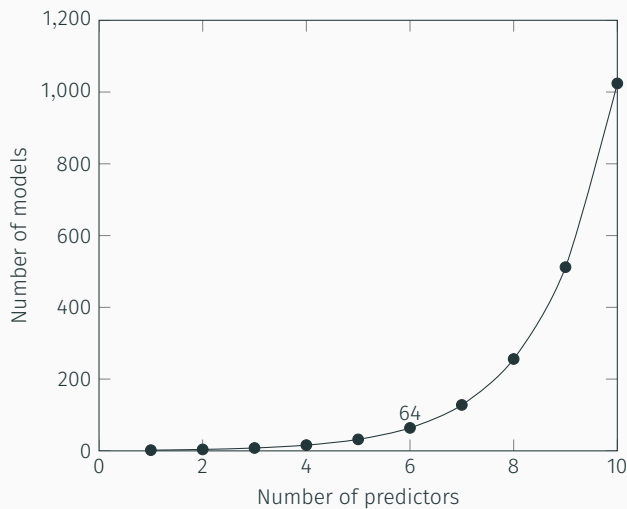
Simple implementation

- Drawbacks

Need to train many ($2^{|P|}$) models.



Variable selection: Best subset selection



Variable selection: Forward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.



Variable selection: Forward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.

| |
|------------------------------|
| $y \sim 1$ $mse = 146.47$ |
|------------------------------|



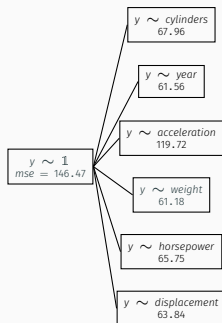
Variable selection: Forward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.



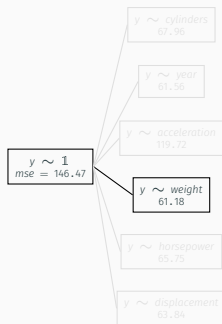
Variable selection: Forward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.



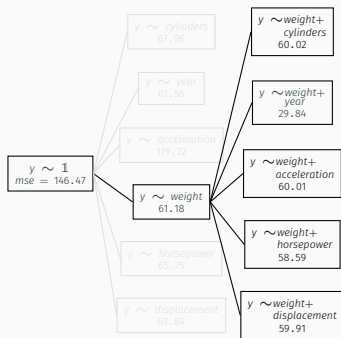
Variable selection: Forward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.



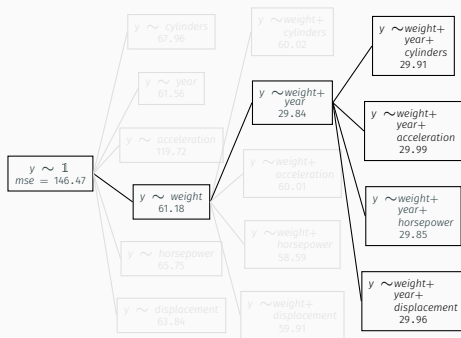
Variable selection: Forward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.



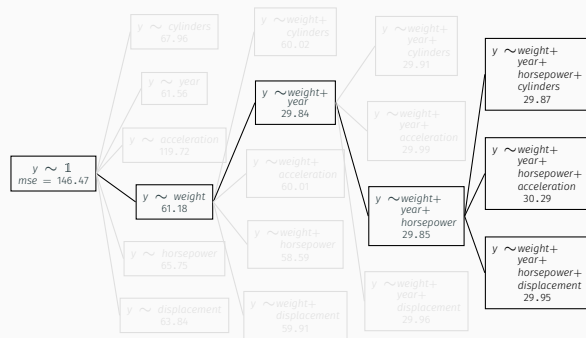
Variable selection: Forward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.



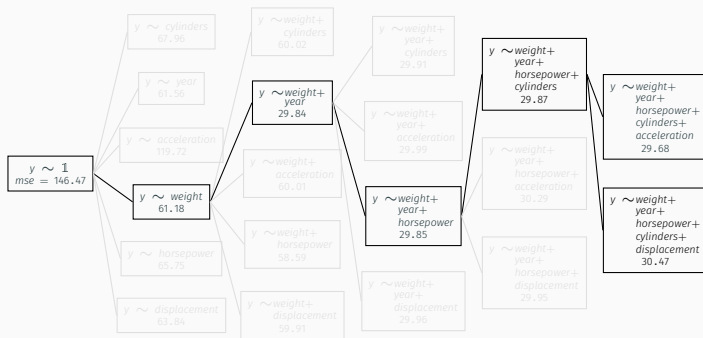
Variable selection: Forward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.



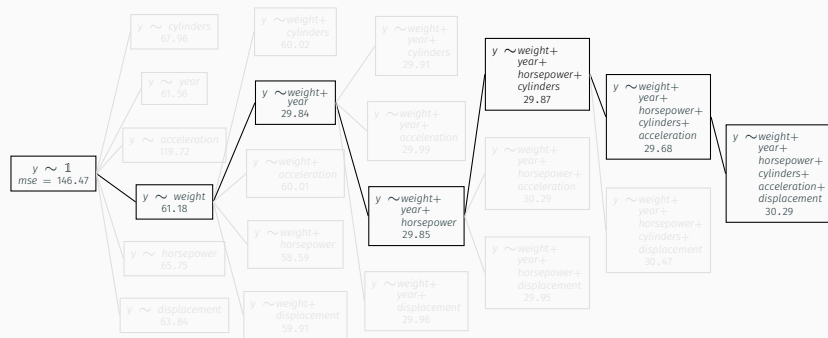
Variable selection: Forward stepwise selection

Problem

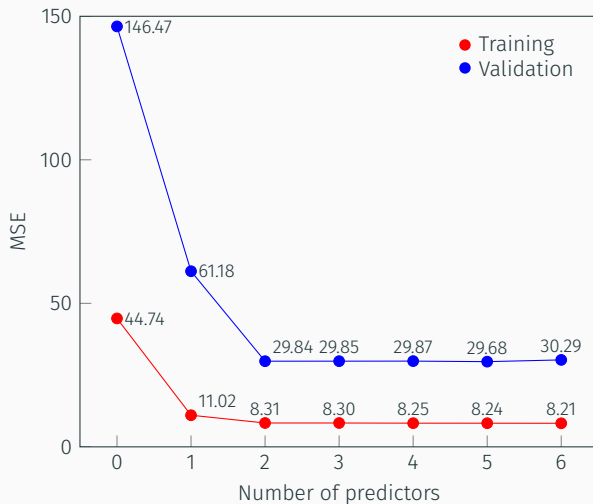
We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.



Variable selection: Forward stepwise selection



Variable selection: Forward stepwise selection

```
In[1]: def fit_and_evaluate(train: pd.DataFrame, validation: pd.DataFrame,
    predictors: List[str], target: str):
    model = LinearRegression()
    model.fit(train[predictors], train[target])

    train_predictions = model.predict(train[predictors])
    validation_predictions = model.predict(validation[predictors])

    return np.mean((train_predictions - train[target]) ** 2),
           np.mean((validation_predictions - validation[target]) ** 2)

predictors = ['cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'year']
target = 'mpg'

train['intercept'] = 1
validation['intercept'] = 1
train_mse, validation_mse = fit_and_evaluate(train, validation, predictors=['intercept'], target=target)
print(f'[]: {validation_mse:.2f} ({train_mse:.2f})')

chosen_predictors = []

while len(chosen_predictors) < len(predictors):
    best_predictor = {'train_mse': None, 'validation_mse': float('inf'),
                    'predictor': None}

    for predictor in set(predictors) - set(chosen_predictors):
        train_mse, validation_mse = fit_and_evaluate(train, validation, predictors=chosen_predictors + [predictor], target=target)

        if validation_mse < best_predictor['validation_mse']:
            best_predictor = {'train_mse': train_mse, 'validation_mse': validation_mse, 'predictor': predictor}

    chosen_predictors.append(best_predictor['predictor'])

print(f'{chosen_predictors}: {best_predictor["validation_mse"]:.2f} ({best_predictor["train_mse"]:.2f})')
```



Variable selection: Forward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with no predictors. Iteratively add the predictor that yields the best model until all are included.

+ Positives

Need to train fewer models.

- Drawbacks

Not guaranteed to find the optimal solution.



Variable selection: Backward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with all predictors. Iteratively remove the predictor that yields the best model until all you have none left.



Variable selection: Backward stepwise selection

Problem

We have a set of predictors $P = \{x_0, x_1, \dots\}$ and a target variable y , and we want to find the subset $p \subseteq P$ that yields the best (linear) model for predicting y .

Solution

Start with all predictors. Iteratively remove the predictor that yields the best model until all you have none left.

+ Positives

Need to train fewer models.

- Drawbacks

Not guaranteed to find the optimal solution.



Shrinkage



UNIVERSITETET
I OSLO

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$
$$\beta_n \rightarrow 0$$

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$
$$\beta_n \rightarrow 0$$

1. $\beta_1 = 0 \implies$ One less degree of freedom in our function

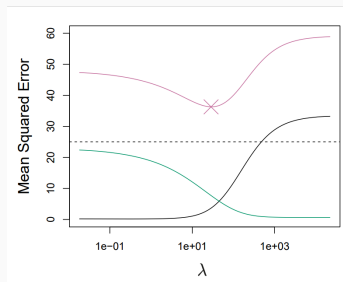
$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$
$$\beta_n \rightarrow 0$$

1. $\beta_1 = 0 \implies$ One less degree of freedom in our function

$$mse = bias^2 + variance + irreducible\ error$$

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$
$$\beta_n \rightarrow 0$$

1. $\beta_1 = 0 \implies$ One less degree of freedom in our function



$$mse = bias^2 + variance + irreducible\ error$$

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$
$$\beta_n \rightarrow 0$$

1. $\beta_1 = 0 \implies$ One less degree of freedom in our function
2. A little more bias \implies A lot less variance

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$
$$\beta_n \rightarrow 0$$

1. $\beta_1 = 0 \implies$ One less degree of freedom in our function
2. A little more bias \implies A lot less variance

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$

↑
↓

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$
$$\beta_n \rightarrow 0$$

1. $\beta_1 = 0 \implies$ One less degree of freedom in our function
2. A little more bias \implies A lot less variance
3. Parameters depend on each other \implies
Fewer degrees of freedom

$$loss_{mse} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2$$

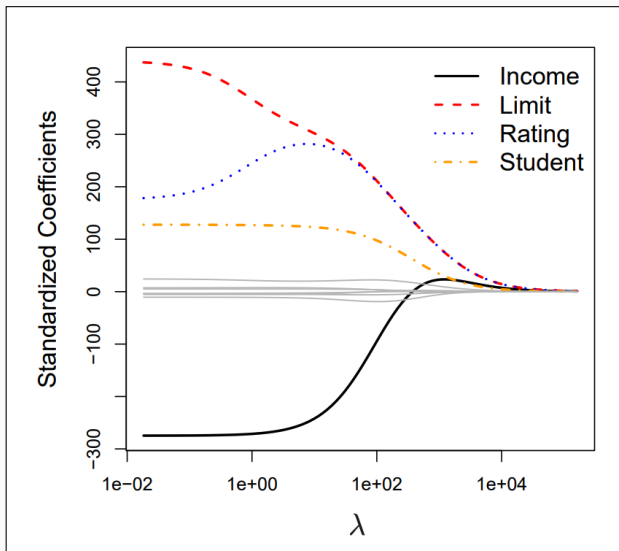
$$loss_{ridge} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p \beta_j^2$$

$$\text{loss}_{\text{ridge}} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p \beta_j^2$$

\Downarrow

$$\lambda \rightarrow \infty \Rightarrow \beta \rightarrow 0$$

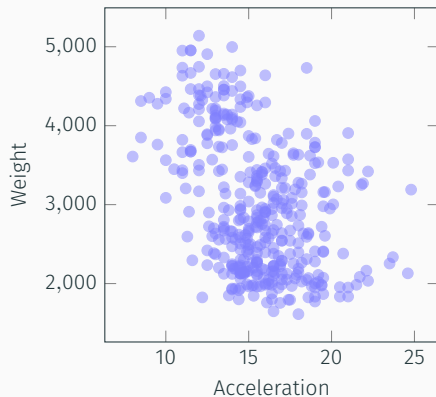
Shrinkage: Ridge regression



$$loss_{ridge} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p \beta_j^2$$

$$y \sim \beta_0 + \beta_1 x_1 + \beta_2 x_2, x_1 \in [0, 1], x_2 \in [0, 1000]$$

Shrinkage: Feature standardization



z-score standardization

z-score standardization

$$x = \frac{x - \mu_x}{\sigma_x}$$

z-score standardization

$$x = \frac{x - \mu_x}{\sigma_x}$$

```
In[1]: for col in predictors:
        print(f'{col}: {np.mean(df[col]):.2f} ({np.std(df[col]):.2f})')

        # z-score standardization
        for col in predictors:
            df[col] = (df[col] - np.mean(df[col])) / np.std(df[col])

        for col in predictors:
            print(f'{col} after: {np.mean(df[col]):.2f} ({np.std(df[col]):.2f})')
```

```
Out[1]: cylinders: 5.47 (1.70)
displacement: 194.41 (104.51)
horsepower: 104.47 (38.44)
cylinders after: -0.00 (1.00)
displacement after: -0.00 (1.00)
horsepower after: -0.00 (1.00)
```



$$loss_{ridge} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p \beta_j^2$$

$$\text{loss}_{\text{ridge}} = \sum_{i=0}^n \left(y_{ij} - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p \beta_j^2$$

Blackboard!

$$loss_{ridge} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p \beta_j^2$$

<http://localhost:8888/notebooks/notebooks/Live%20coding.ipynb>



$$loss_{ridge} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p \beta_j^2$$

Regularization by shrinking the model covariates towards zero.

$$loss_{ridge} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p \beta_j^2$$

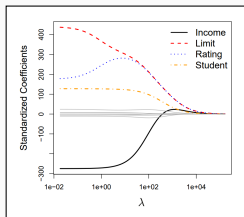
$$loss_{lasso} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p |\beta_j|$$

$$loss_{ridge} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p \beta_j^2$$

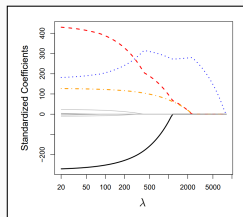
$$loss_{lasso} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p |\beta_j|$$

Shrinkage: LASSO

Ridge

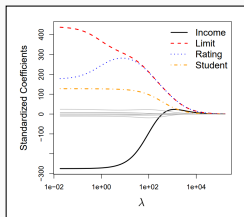


LASSO

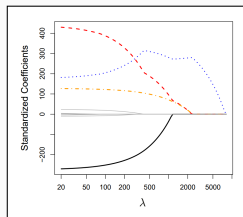


Shrinkage: LASSO

Ridge



LASSO

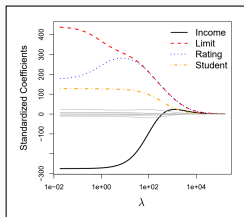


| Predictor | Ridge | LASSO |
|--------------|-------|-------|
| Intercept | 23.44 | 23.44 |
| Weight | -5.59 | -4.78 |
| Year | 2.75 | 2.00 |
| Acceleration | 0.19 | 0 |
| Displacement | 0.66 | 0 |

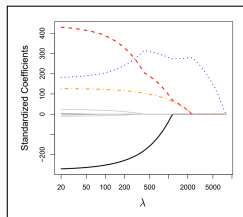


Shrinkage: LASSO

Ridge



LASSO

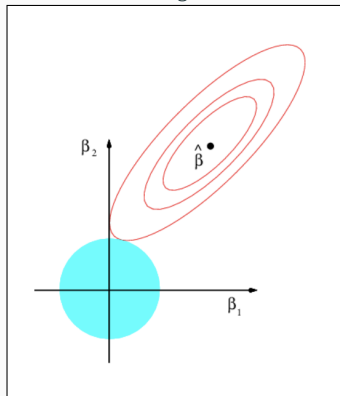


A coefficient of 0 does not mean the predictor has
no association with the outcome!

Lasso

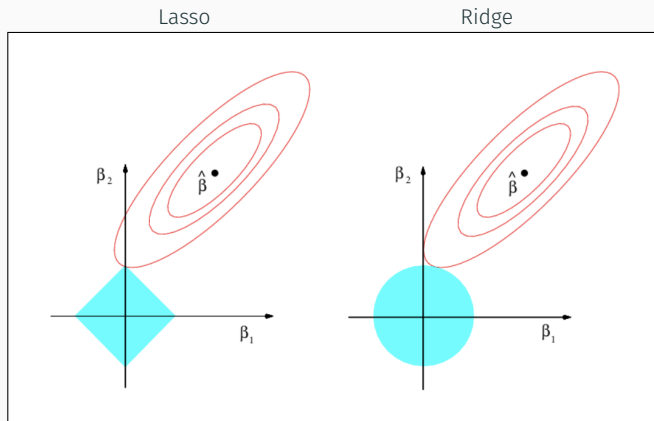
?

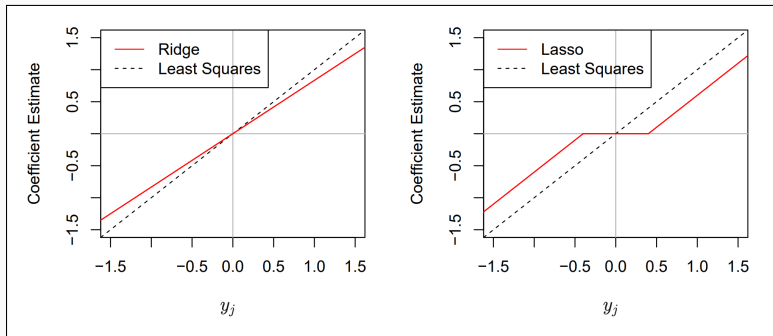
Ridge



Whiteboard! 🎉

Shrinkage: LASSO





$$loss_{mse} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2$$

Fits the **best** model
to the data.

$$loss_{mse} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2$$

Fits the **best** model to the data.

$$loss_{ridge} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p \beta_j^2$$

Fits the **best** model to the data while **shrinking** coefficients towards zero.

$$loss_{mse} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2$$

Fits the **best** model to the data.

$$loss_{ridge} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p \beta_j^2$$

Fits the **best** model to the data while **shrinking** coefficients towards zero.

$$loss_{lasso} = \sum_{i=0}^n \left(y_i - \sum_{j=0}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=0}^p |\beta_j|$$

Fits the **best** model to the data while **shrinking** coefficients towards zero such that some variables are effectively **removed**.

Assignment 3



UNIVERSITETET
I OSLO

https://uio.instructure.com/courses/53357/assignments/118667?module_item_id=962921

Coding tips: Separation of concerns

```
In[1]: # Read and clean data
path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

# Split data
train = df.iloc[:int(len(df) * 0.8)]
validation = df.iloc[int(len(df) * 0.8):]

# Define input and output variables
predictors = ['cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'year']
target = 'mpg'

# Define necessary data structures for state
chosen_predictors = []
mses = []

while len(predictors) > 0:
    best_predictor = {'mse': float('inf'), 'predictor': None}

    for predictor in set(predictors) - set(chosen_predictors):
        potential_predictors = chosen_predictors + [predictor]

        # Fit and evaluate model
        model = LinearRegression()
        model.fit(train[potential_predictors], train[target])
        predictions = model.predict(validation[potential_predictors])
        test_mse = np.mean((validation[target] - predictions) ** 2)

        # Compare model with previous best
        if test_mse < best_predictor['mse']:
            best_predictor = {'mse': test_mse, 'predictor': predictor}

    # Update state
    chosen_predictors.append(best_predictor['predictor'])
    mses.append(best_predictor['mse'])
    predictors = [p for p in predictors if p != best_predictor['predictor']]
```



Coding tips: Separation of concerns

In[1]:

```
# Read and clean data
path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

# Split data
train = df.iloc[:int(len(df) * 0.8)]
validation = df.iloc[int(len(df) * 0.8):]

# Define input and output variables
predictors = ['cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'year']
target = 'mpg'

# Define necessary data structures for state
chosen_predictors = []
mses = []

while len(predictors) > 0:
    best_predictor = {'mse': float('inf'), 'predictor': None}

    for predictor in set(predictors) - set(chosen_predictors):
        potential_predictors = chosen_predictors + [predictor]

        # Fit and evaluate model
        model = LinearRegression()
        model.fit(train[potential_predictors], train[target])
        predictions = model.predict(validation[potential_predictors])
        test_mse = np.mean((validation[target] - predictions) ** 2)

        # Compare model with previous best
        if test_mse < best_predictor['mse']:
            best_predictor = {'mse': test_mse, 'predictor': predictor}

    # Update state
    chosen_predictors.append(best_predictor['predictor'])
    mses.append(best_predictor['mse'])
    predictors = [p for p in predictors if p != best_predictor['predictor']]
```

Setup

Selection

Modelling

Housekeeping



Coding tips: Separation of concerns

```
In[1]: # Read and clean data
path = '/Users/esten/Downloads/Auto.csv'
df = pd.read_csv(path)

# Split data
train = df.iloc[:int(len(df) * 0.8)]
validation = df.iloc[int(len(df) * 0.8):]

# Define input and output variables
predictors = ['cylinders', 'displacement', 'horsepower', 'weight', 'acceleration', 'year']
target = 'mpg'

# Define necessary data structures for state
chosen_predictors = []
mses = []

def fit_and_evaluate(model: LinearRegression, train: pd.DataFrame, validation: pd.DataFrame, variables: List[str], target: str):
    """ Fit a given model on a training dataset using a given set of variables and return MSE from a validation dataset. """
    model.fit(train[potential_predictors], train[target])
    predictions = model.predict(validation[potential_predictors])

    return np.mean((validation[target] - predictions) ** 2)

while len(predictors) > 0:
    best_predictor = {'mse': float('inf'), 'predictor': None}

    for predictor in set(predictors) - set(chosen_predictors):
        potential_predictors = chosen_predictors + [predictor]
        test_mse = fit_and_evaluate(LinearRegression(), train, validation, variables=potential_predictors, target=target)

        # Compare model with previous best
        if test_mse < best_predictor['mse']:
            best_predictor = {'mse': test_mse, 'predictor': predictor}

    # Update state
    chosen_predictors.append(best_predictor['predictor'])
    mses.append(best_predictor['mse'])
    predictors = [p for p in predictors if p != best_predictor['predictor']]
```

Modelling

