PSY9511: Seminar 4

Model selection, validation and testing

Esten H. Leonardsen September 15, 2025



Outline

- 1. Loss functions and performance metrics
- 2. Strategies for model evaluation
 - · Training and validation split
 - · (Stratification)
 - · (Leave-one-out cross-validation)
 - Cross-validation
 - Bootstrap
- 3. The theoretical basis for model evaluation
- 4. Strategies for model selection and evaluation
 - Train/validation/test split
 - · Nested cross-validation
- 5. Assignment 4



Motivation

- Reporting inflated modelling performance due to suboptimal testing practices
- 2. Using inappropriate measures for model performance (often accuracy)



Motivation

- Reporting inflated modelling performance due to suboptimal testing practices
- 2. Using inappropriate measures for model performance (often accuracy)



We believe that our models are better than they actually are, which yields false conclusions



Loss functions versus performance metrics

Commonalities

- · Allows us to evaluate the performance of a model
- Typically on the form $f(y, \hat{y})$

Loss functions

 Tailored specifically for mathematical optimization of models

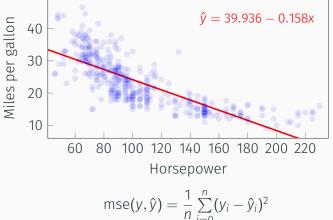
Performance metrics

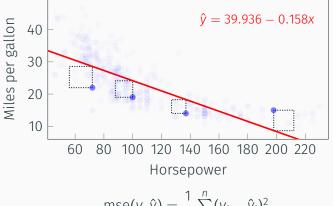
Tailored specifically for interpretation of model performance by humans

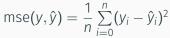


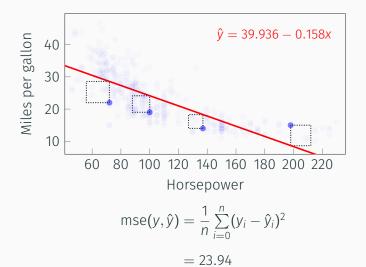
$$mse(y, \hat{y}) = \frac{1}{n} \sum_{i=0}^{n} (y_i - \hat{y}_i)^2$$



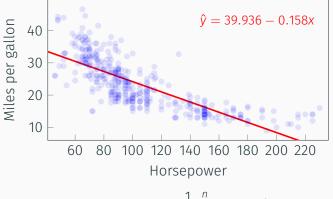








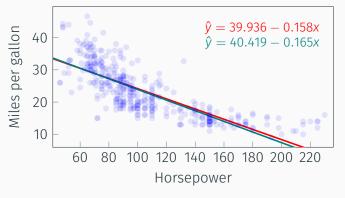




$$mse(y, \hat{y}) = \frac{1}{n} \sum_{i=0}^{n} (y_i - \hat{y}_i)^2$$

$$mae(y, \hat{y}) = \frac{1}{n} \sum_{i=0}^{n} |y_i - \hat{y}_i|$$





$$mse(y, \hat{y}) = \frac{1}{n} \sum_{i=0}^{n} (y_i - \hat{y}_i)^2$$

$$mae(y, \hat{y}) = \frac{1}{n} \sum_{i=0}^{n} |y_i - \hat{y}_i|$$



Loss functions

- Different loss functions measures different properties of the model fit
- Optimizing for different loss functions will yield different parameter estimates

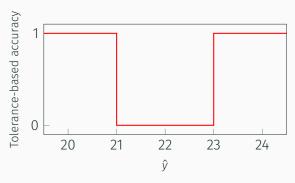
Tolerance-based accuracy:

A prediction is considered correct if it is within a predefined margin of error from the true value

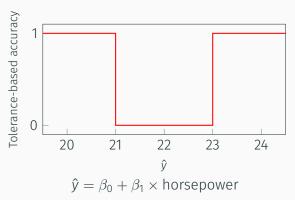
accuracy*
$$(y, \hat{y}) = \begin{cases} 0 & \text{if } |y - \hat{y}| < \text{tol} \\ 1 & \text{else} \end{cases}$$

mpg	horsepower
22	72

mpg	horsepower
22	72



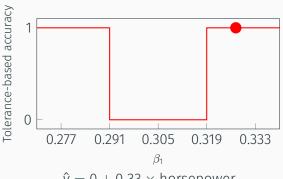
mpg	horsepower
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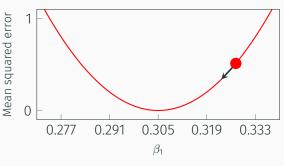


mpg	horsepower
22	72





mpg	horsepower
22	72



$$\hat{y} = 0 + 0.33 \times \text{horsepower}$$



Loss functions

- Different loss functions measures different properties of the model fit
- Optimizing for different loss functions will yield different parameter estimates
- Should be differentiable to allow for mathematical optimization

$$mse(y, \hat{y}) = \frac{1}{n} \sum_{i=0}^{n} (y_i - \hat{y}_i)^2$$

$$OR$$

$$mae(y, \hat{y}) = \frac{1}{n} \sum_{i=0}^{n} |y_i - \hat{y}_i|$$



$$\frac{1}{n} \sum_{i=0}^{n} (y_i - \hat{y}_i)^2$$

Mean squared error (MSE)

- + Can be used as a loss function
- + Widely used
- ? Penalizes large errors
- ? Interpretation
- Depends on scale

$$\sqrt{\frac{1}{n}\sum_{i=0}^{n}(y_i-\hat{y}_i)^2}$$

Root mean squared error (RMSE)

- + Can be used as a loss function
- + More interpretable than MSE, total loss ≈ individual errors
- ? Penalizes large errors
- Depends on scale

$$\frac{1}{n}\sum_{i=0}^{n}|y_i-\hat{y}_i|$$

Mean absolute error (MAE)

- + Can be used as a loss function (but not as suitable as MSE/RMSE)
- + More interpretable than MSE/RMSE, total loss = average error
- ? Many small errors are as bad as one large error
- Depends on scale



$$\frac{\sum_{i=1}^{n} (y_i - \bar{y})(\hat{y}_i - \bar{\hat{y}})}{\sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2 \sum_{i=1}^{n} (\hat{y}_i - \bar{\hat{y}})^2}}$$

Pearson correlation coefficient (r)

- + Scale independent
- ? Only suitable for linear relationships
- Should not be used as a loss function
- Does not care about whether the predictions are close to the true values

$$1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2}$$

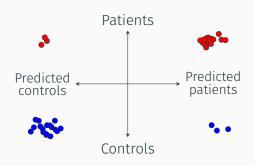
Proportion of variance explained (r^2)

- + Scale independent
- Interpretable
- ? Only suitable for linear relationships
- Should not be used as a loss function

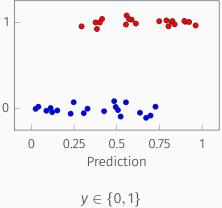




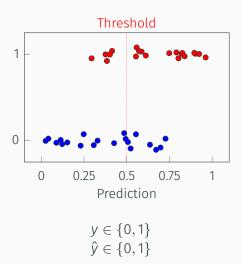
 $y \in \{Patients, Controls\}$

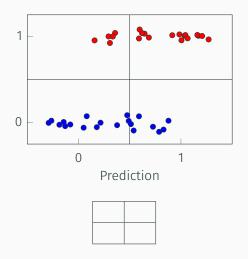


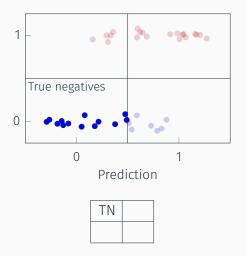
 $y \in \{Patients, Controls\}$ $\hat{y} \in \{Patients, Controls\}$

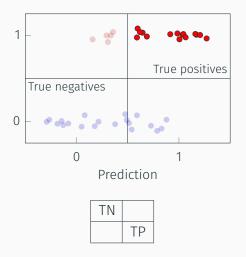


$$y \in \{0, 1\}$$
$$\hat{y} \in [0, 1]$$

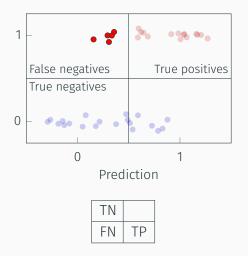




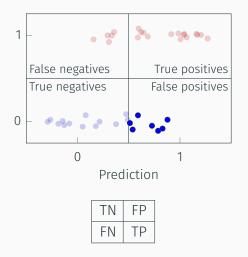














Confusion matrix:

Binary classification metrics:

- Many metrics rely on thresholding the predictions to obtain binary predictions.
- Although not a metric per se, the confusion matrix is a very useful tool to understand model behaviour, and should always be looked at (and preferably reported).

$$-(y \log(\hat{y}) + (1-y) \log(1-\hat{y}))$$

Logloss

- + Does not rely on thresholding
- + Can be used as a loss function (and very often is)
- Not very interpretable



$$\frac{\mathit{TP} + \mathit{TN}}{\mathit{TP} + \mathit{TN} + \mathit{FP} + \mathit{FN}}$$

Accuracy

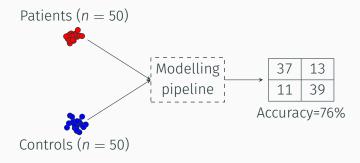
- + Interpretable
- Does not account for different costs of misclassification

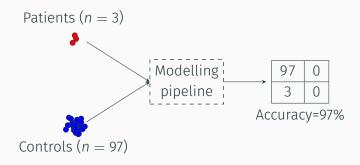
$$\frac{TP + TN}{TP + TN + FP + FN}$$

Accuracy

- + Interpretable
- Does not account for different costs of misclassification

What is the major pitfall of accuracy as a metric?





$$\frac{\mathit{TP} + \mathit{TN}}{\mathit{TP} + \mathit{TN} + \mathit{FP} + \mathit{FN}}$$

Accuracy

- + Interpretable
- Does not account for different costs of misclassification
- Does not handle imbalanced classes

$$\frac{TP}{TP+FN}$$

True positive rate (sensitivity)

- + Interpretable, calculates the proportion of cases that are detected
- + Useful when the cost of false negatives is high (Population-wide screening for severe disease)
- Very one-sided (should be used alongside other metrics)



$$\frac{TN}{TN+FP}$$

True negative rate (specificity)

- + Interpretable, calculates the proportion of controls that are detected
- Useful when the cost of false positives is high (Intrusive treatment of rare and mild conditions)
- Very one-sided (should be used alongside other metrics)

$$\frac{TP}{TP+FP}$$

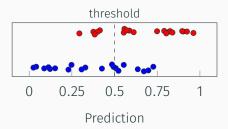
Positive predictive value (PPV, precision)

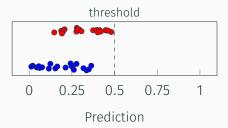
- + Interpretable, calculates the proportion of predicted cases that are actually cases
- + Useful when the cost of false positives is high (Selection of participants for expensive clinical trials)
- Very one-sided (should be used alongside other metrics)

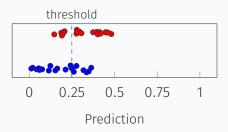
$$\frac{\frac{TP}{TP+FN} + \frac{TN}{TN+FP}}{2}$$

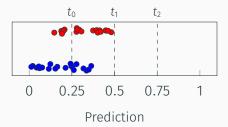
Balanced accuracy

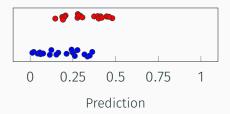
- + Interpretable, behaves similarly to regular accuracy.
- + Takes into account imbalanced classes



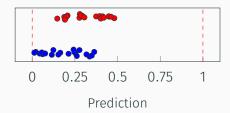




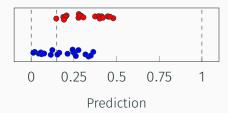




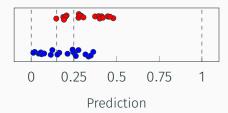
threshold	TPR	FPR



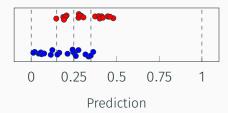
threshold	TPR	FPR
0	1	1
1	0	0



threshold	TPR	FPR
0	1	1
0.15	0.95	0.5
1	0	0



threshold	TPR	FPR
0	1	1
0.15	0.95	0.5
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0.35	0.2	0.0
1	0	0



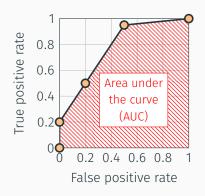
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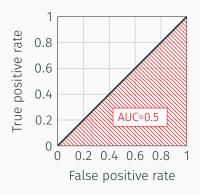
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Area under the receiver operating characteristic curve (AUC/AUROC)

- A performance metric that does not rely on a correct classification threshold
- Measures whether the predictions are ranked correctly (e.g. patients have a higher prediction than controls)
- Handles class imbalance (relatively well) and is commonly reported in the literature



Loss functions and performance metrics: Summary

Performance metrics and loss functions measure the performance of a predictive model

- There is a range of alternatives that can be used, each capturing a different aspect of a model's performance
- It is good practice to report more than one metric
- For regression, MSE is a common loss function with nice mathematical properties and MAE is an intuitive performance metric
- For classification, log-loss is the most common loss function for probabilistic classifiers and AUC is a widely used metric that is easy to interpret, handles class imbalance (to some degree), and is not reliant on the choice of classification threshold

Loss functions and performance metrics: Summary

http://localhost:8888/notebooks/notebooks% 2FClassification%20metrics.ipynb



Strategies for model evaluation



Model evaluation: Rationale

Statistical inference:

Goal: In-sample quantification

Predictive modelling:

Goal: Out-of-sample generalization

Model evaluation: Rationale

How can we test how good our model is on unseen data and be certain that this predictive performance holds if we present even more new data

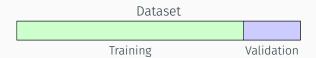


Model evaluation: Validation set

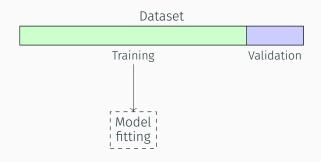


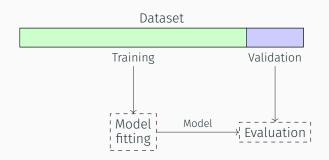


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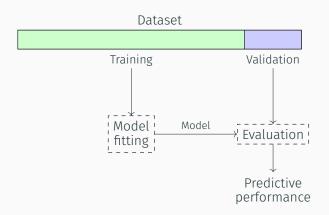


Model evaluation: Validation set





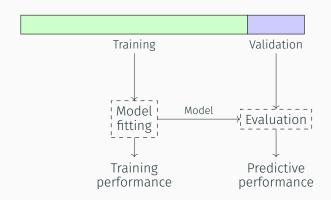




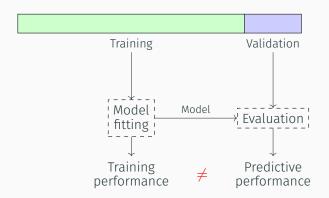


In the validation set approach we split the dataset into two subsets (commonly $\sim 80\%/20\%$), use the first for training the model and the second to test its performance.

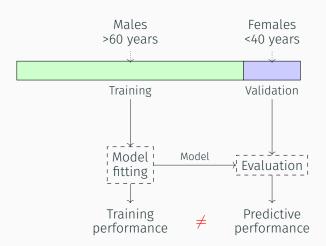
- + Accurate estimate of out-of-sample error
- + Simple
- Variable results depending on the exact split
- Only uses a subset of data for training models
- Gives a point estimate of the error, without confidence intervals











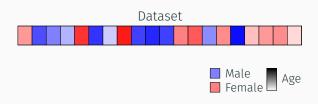


Stratification:

Ensuring all folds of the dataset are similar with respect to some given characteristics.

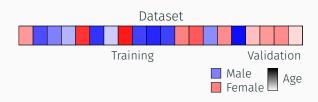
Dataset

```
In[1]: df = ...
```



```
In[1]: df = ...
```



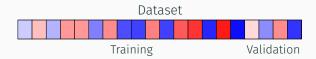


```
In[1]: df = ...
    train = df.iloc[:int(len(df) * 0.8)]
    validation = df.iloc[int(len(df) * 0.8):]
```



```
In[1]: df = ...
    df = df.sort_values(['sex', 'age'])
```

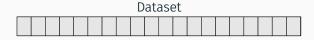


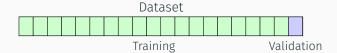


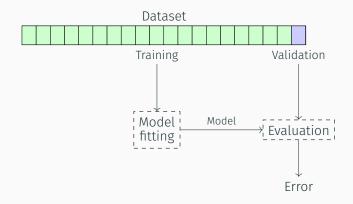
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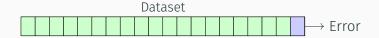
- Helps alleviate the risk of training performance >> validation performance
- Always stratify on target variable first
- Also good idea to stratify on other core characteristics, e.g. sex and age

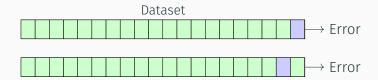


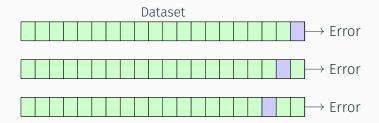






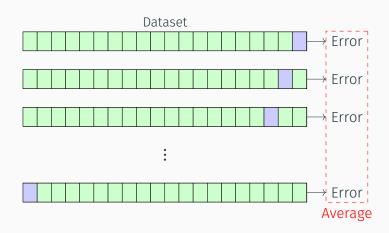












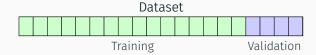


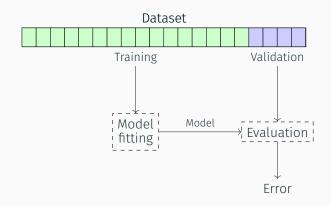
Fits *n* models for *n* datapoints, each time leaving a single datapoint out for testing.

- + Uses all data to train models
- + Not dependent on arbitrary data splits
- + Unbiased (with regards to the full dataset)
- Computationally expensive
- Effectively gives a point estimate of the error
- All models are going to be trained on > 99% overlapping data \rightarrow highly correlated



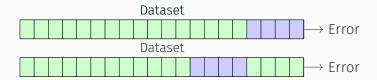


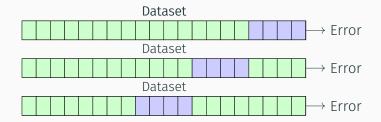


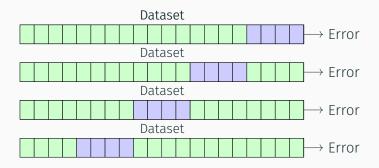




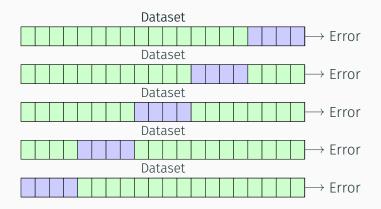




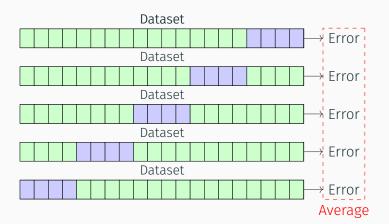










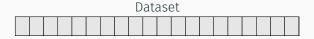




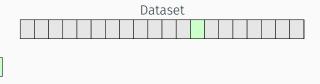
Fits k (usually $k \in \{5, 10\}$) models for n > k datapoints, each leaving n/k datapoints for out-of-sample testing.

- + Uses all data to train models
- + Yields multiple estimates of out-of-sample error
- Different choices of k (and exact splits) yields different results
- No longer a single model from which information (e.g. parameter estimates and p-values) can be derived

Model evaluation: Bootstrapping

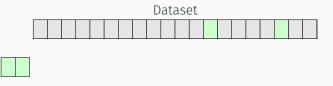


Model evaluation: Bootstrapping

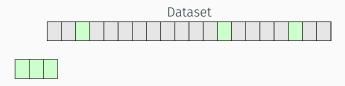




Model evaluation: Bootstrapping



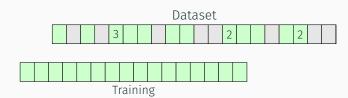


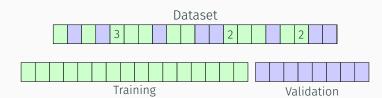


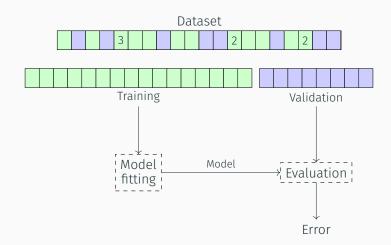




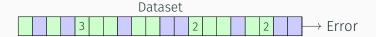


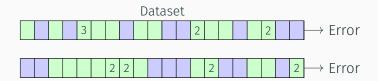




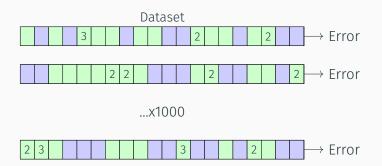




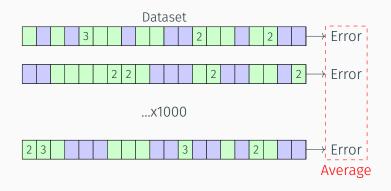










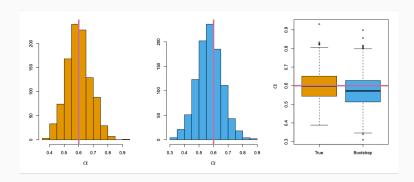




Fits b models with m datapoints (typically m < n), sampled from the original dataset with replacement.

- + Uses all data to train models
- + Provides a dense distribution of model performances
- + Versatile: Can be used for other things, e.g. getting a confidence interval for model parameters
- Different choices of b (and exact splits) yields different results





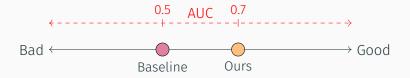


Why do we want to evaluate our model?

- We want to show that our model is better than random guessing
- 2. We want to show that our model is better than another model









http://localhost:8888/notebooks/notebooks%2FModel%
20variability.ipynb





There is going to be variability in our model's performance (and possibly the baseline).

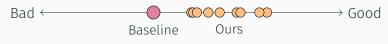
Is our model significantly better?





Approach 1:

Is the mean of the distribution of performances from our model (with regards to variability that is **unrelated** to efficacy) significantly higher than the point-estimate baseline?



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Approach 2:

Is the point-estimate performance of our model significantly higher than the mean of the baseline distribution?



Approach 2:

Is the point-estimate performance of our model significantly higher than the mean of the baseline distribution?

Age	Sex	Feature	Outcome	
25	Male	0.53	1	
38	Female	-0.76	1	
45	Male	0.89	1	
33	Female	-0.21	1	Modelling _
29	Male	0.12	1	\longrightarrow Frror
41	Female	-0.68	0	pipeline ;
56	Male	0.45	0	
52	Female	-0.32	0	
31	Male	0.91	0	
48	Female	-0.15	0	



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Approach 2:

Is the point-estimate performance of our model significantly higher than the mean of the baseline distribution?



Approach 2:

Is the point-estimate performance of our model significantly higher than the mean of the baseline distribution?



Approach 3:

Is the mean of the distribution of performances from our model significantly higher than the mean of the distribution of baseline performances?

Model evaluation: Summary

- · Model evaluation should always happen out-of-sample
- If n is big (\geq 10000), a train/validation split can be sufficient
- For smaller samples, k-fold cross-validation with 5 \leq k \leq 10 is a good trade-off between bias and variance
- The bootstrap is an effective way of getting confidence intervals for model performance and parameters
- Cross-validation (or bootstrapping) will produce a distribution of model performances (although caution the correlation)
- Permutation testing can produce a distribution of baseline performances



Model selection and evaluation



Model selection and evaluation: Rationale

- Model evaluation via cross-validation is sufficient if we want to estimate the out-of-sample error of a known model.
- Very often we want to know whether a set of predictors are informative for an outcome given the best possible model.
- In that case, we have to both **choose the best model**, and **estimate its performance**.
- If we choose the model based on regular cross-validation, the performance estimate will (likely) be inflated



Model selection and evaluation: Rationale

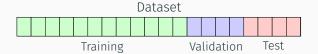
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- If we choose the model based on regular cross-validation, the performance estimate will (likely) be inflated
- ightarrow We need a more advanced strategy



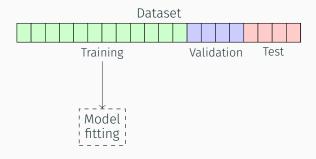
Model selection and evaluation: Train/validation/test



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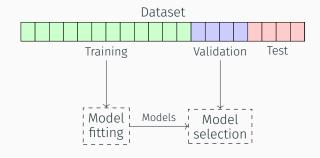


Model selection and evaluation: Train/validation/test



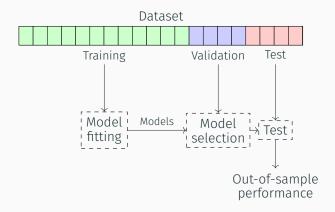


Model selection and evaluation: Train/validation/test

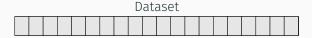




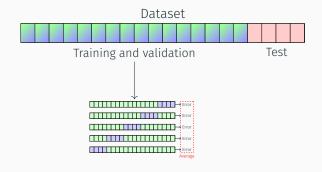
Model selection and evaluation: Train/validation/test



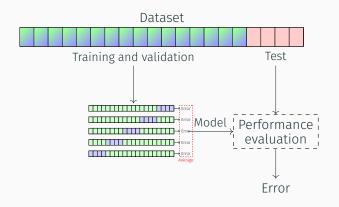




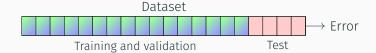


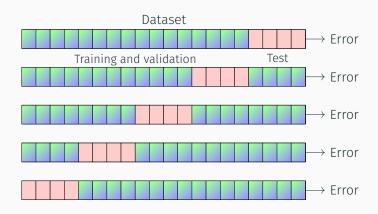


















Performs k outer cross-validations, each performing k-1 inner cross validations, and uses the best models from the inner loop to predict in the outer loop.

- + Uses all data to train models
- + Unbiased estimate of out-of-sample error
- Very computationally expensive
- Hard to interpret beyond performance: We now have either k, or k² models that might behave in different ways

Model selection and evaluation: Summary

- Whenever a modelling choice is made on the basis of performance in a dataset, we have to assume the performance achieved by the chosen model is inflated
- If n is big (\geq 10000), a single train/validation/test split is often sufficient
- When possible, use nested cross-validation to select the best model and estimate the out-of-sample error

Model selection and evaluation: Summary

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- Whenever a modelling choice is made on the basis of performance in a dataset, we have to assume the performance achieved by the chosen model is inflated
- If n is big (\geq 10000), a single train/validation/test split is often sufficient
- When possible, use nested cross-validation to select the best model and estimate the out-of-sample error

Assignment 4

- 1. Download the Heart-dataset from the book
- 2. Fit and validate a model using a train/validation split
- 3. Fit and validate multiple models using cross-validation
- 4. (Optional): Fit and validate multiple models using the bootstrap

