

Spin-orbit driven emergent phases in quantum materials



Indra Dasgupta

*School of Physical Sciences
Indian Association for the Cultivation of Science
Kolkata, India
E-mail: sspid@iacs.res.in*

Funding SERB, TRC (DST)



*Workshop and International Conference on Electronic structure theory of
emergent spin orbit driven phenomenon (Nov 11-15, 2024 , IIT Bombay)*



Spin-Orbit Coupling & IIT Bombay



Sayantika Bhowal
Graduate Student (IACS)
Assistant Prof. IIT Bombay



Swarup Kr. Panda
Graduate Student (IACS)
MSc IIT Bombay
Associate Prof. Benett Univ



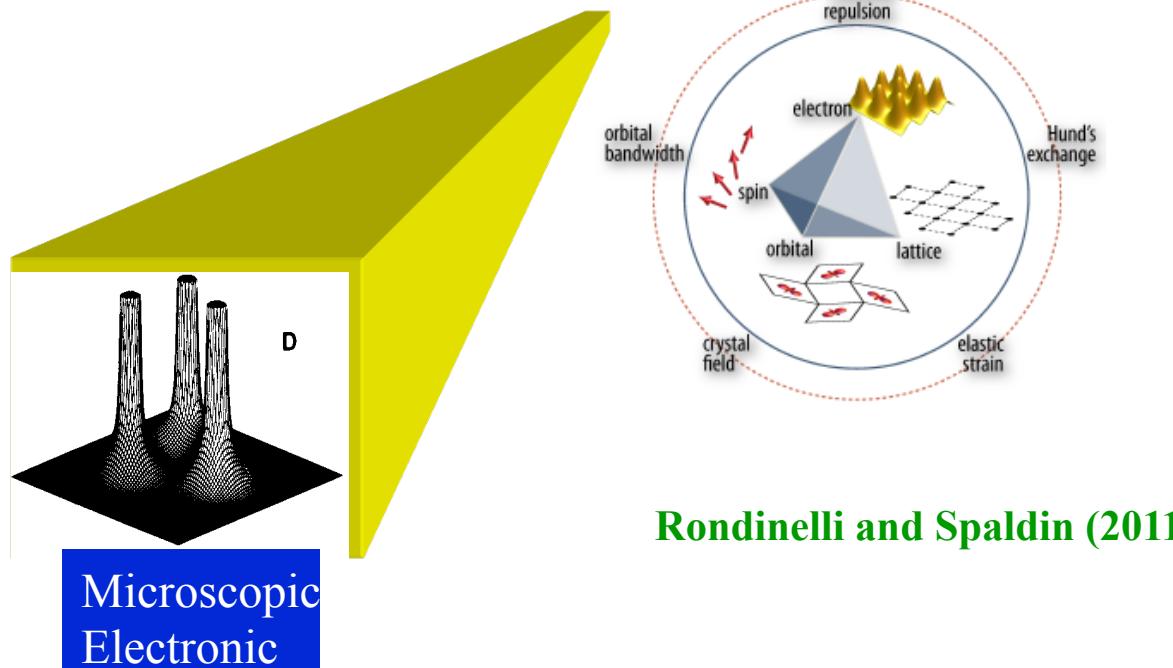
Ranjit Kr. Nanda
Graduate Student
PhD IIT Bombay
Prof. IIT Madras

(Half Heuslers)

Electronic Structure of Quantum Materials

Strong interplay of the spin, charge, orbital and lattice degrees freedom + Topology → Emergent Phases and functions

Quantum Description



Rondinelli and Spaldin (2011)

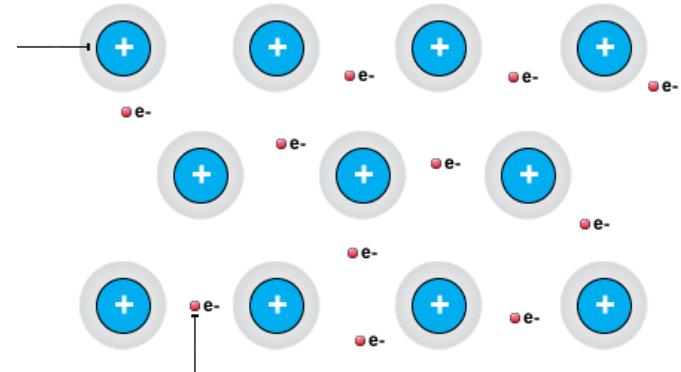
Starting Point: Independent Particle Approximation

Quantum Mechanics in Understanding Properties of Materials

$$H\Psi = E\Psi$$

$$H = \sum_i \left[-\frac{1}{2} \Delta_i^2 + V(r_i) \right] + \sum_{i < j} \frac{e^2}{|r_i - r_j|}$$

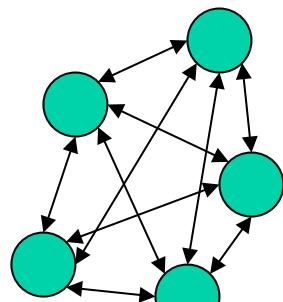
Non-interacting part Interacting part



- Density functional theory
- Reduction to an effective non-interacting system.
- Self-consistent solution to an one electron Schrödinger equation with a periodic potential.



Nobel Prize 1998



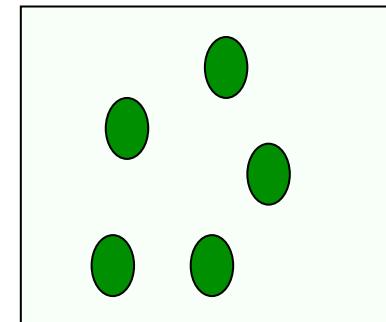
KOHN-SHAM APPROACH



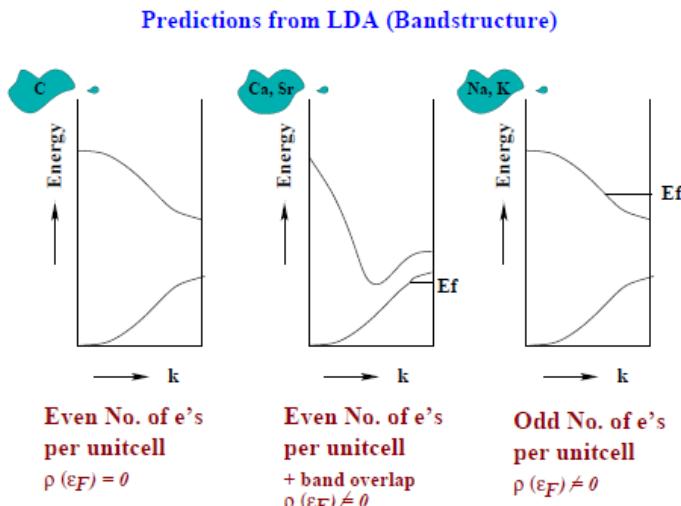
Non-Interacting, fictitious particles + Effective Potentials

KS-equation → $\left[-\frac{1}{2} \nabla^2 + V_H(n_{GS}; \vec{r}) + V_{ext}(n_{GS}; \vec{r}) + V_{xc}(n_{GS}; \vec{r}) \right] \Phi_i(\vec{r}) = \varepsilon_i \Phi_i(\vec{r})$

Talk by Prof. Biplab Sanyal



Success & Failure of DFT



Accordingly to LDA, odd no. of e's per unit cell always give rise to Metal.

Pauli Exclusion Principle

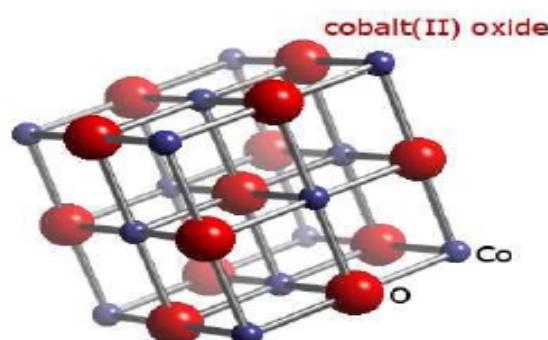
Success:

Electronic Structure of Graphene/Dirac Materials,
Topological Insulators

Identify Topological phases of Quantum Matter (Berry Phase):

Spin-orbit coupling is Important

Failure of Band Theory



Total No. of electrons = 9 + 6 = 15

Band theory predicts CoO to be metal, while it is the toughest insulator known

Failure of LDA → Failure of single particle picture
--> Importance of e-e interaction effects (Correlation)

Other Example

High Tc Cuprates
 $\text{La}_2\text{CuO}_4 \rightarrow$
Insulator predicted to be a metal

STRONG CORRELATION

Strongly Correlated Systems & Model Hamiltonian Approach

$$H = \sum_i \left[-\frac{1}{2} \Delta_i^2 + V(r_i) \right] + \sum_{i < j} \frac{e^2}{|r_i - r_j|}$$

Find some complete one-particle basis:

$$H = \sum_i \varepsilon_i n_i + \sum_{i \neq j} t_{ij} c_i^\dagger c_j + \frac{1}{2} \sum_{ijkl} v_{ijkl} c_i^\dagger c_j^\dagger c_l c_k$$

Non-interacting part (DFT)

Interacting Part (only partially takes into account
Correlation)



This model can be solved for small atoms and molecules, using, e.g. quantum chemical methods. But too complicated for systems we shall consider here:

Need to remove

→ Degrees of freedom (Retain only active orbitals)

→ Interaction terms

→ Model Hamiltonian with chemical realism

Key Ingredients of Quantum Matter Leading to Emergent Phases

- Crystal Field Splitting
- Kinetic Hopping (t)
- Onsite Coulomb Int. (U) & Hunds Exchange J_H
- Spin Orbit Interaction (λ)

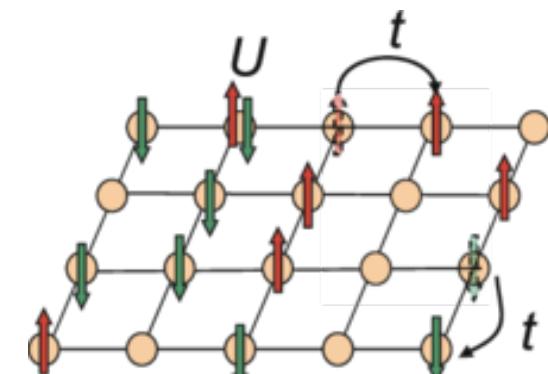
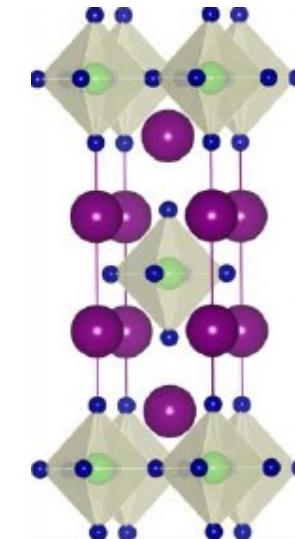
+

- Topology (Frustrated Lattice → Triangular, Kagome or Honeycomb)

Generic model Hamiltonian to start with.....

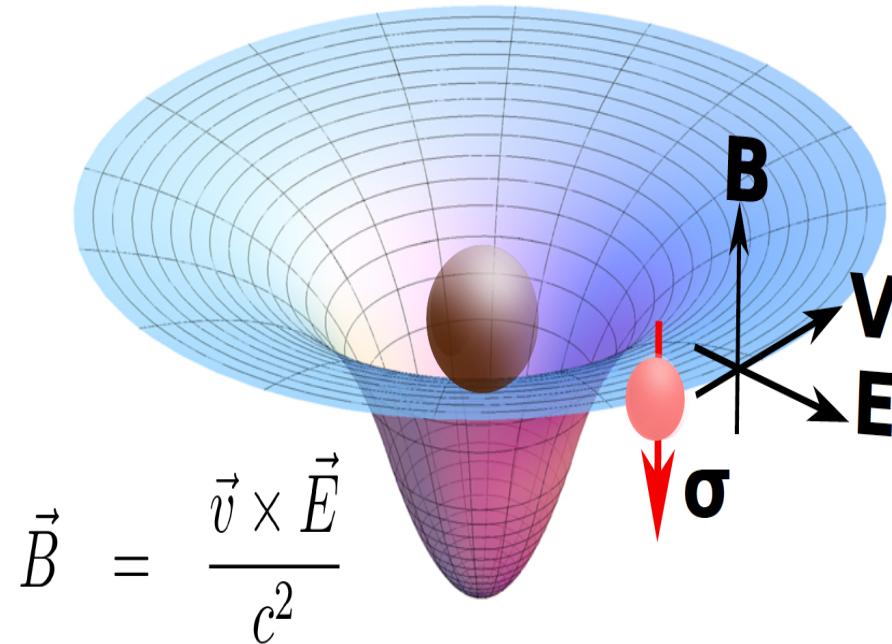
$$H = \sum t_{ij,\alpha\beta} C_{i\alpha}^\dagger C_{j\beta} + h.c + U \sum n_{i\alpha} (n_{i\alpha} - 1) + \lambda \sum L_i \cdot S_i$$

SrVO₃



Wikipedia

Spin-Orbit Coupling



The radial electric field produced by the nucleus appears as a magnetic field in the rest frame of the electron. This magnetic field interacts with the spin of the electron resulting in SOC

There can be other sources of electric field particularly in Non-centrosymmetric solids leading to Rashba-Dresselhaus effect

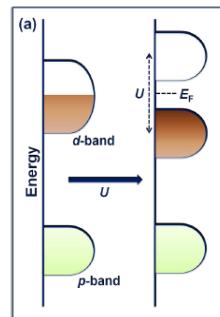
Emergent Phases:

3d TMOs
Multiferroics
Quantum Spin Systems

$$H = \sum t_{ij,\alpha\beta} C_{i\alpha}^\dagger C_{j\beta} + h.c + U \sum n_{i\alpha} (n_{i\alpha} - 1) + \lambda \sum L_i \cdot S_i$$

U/t

“Traditional” Mott insulator



$j_{eff}=1/2$ Mott Insulator

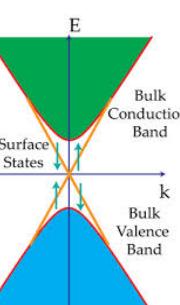
Iridates

Unconventional Magnetism
Spin-liquid

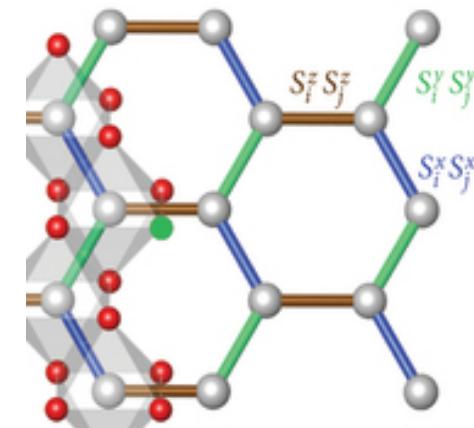
Simple Materials

λ/t

Topological Insulator, Rasba Systems



$$H_{spin} = \sum J_{ij} S_i \cdot S_j + K S_i^\gamma S_j^\gamma$$



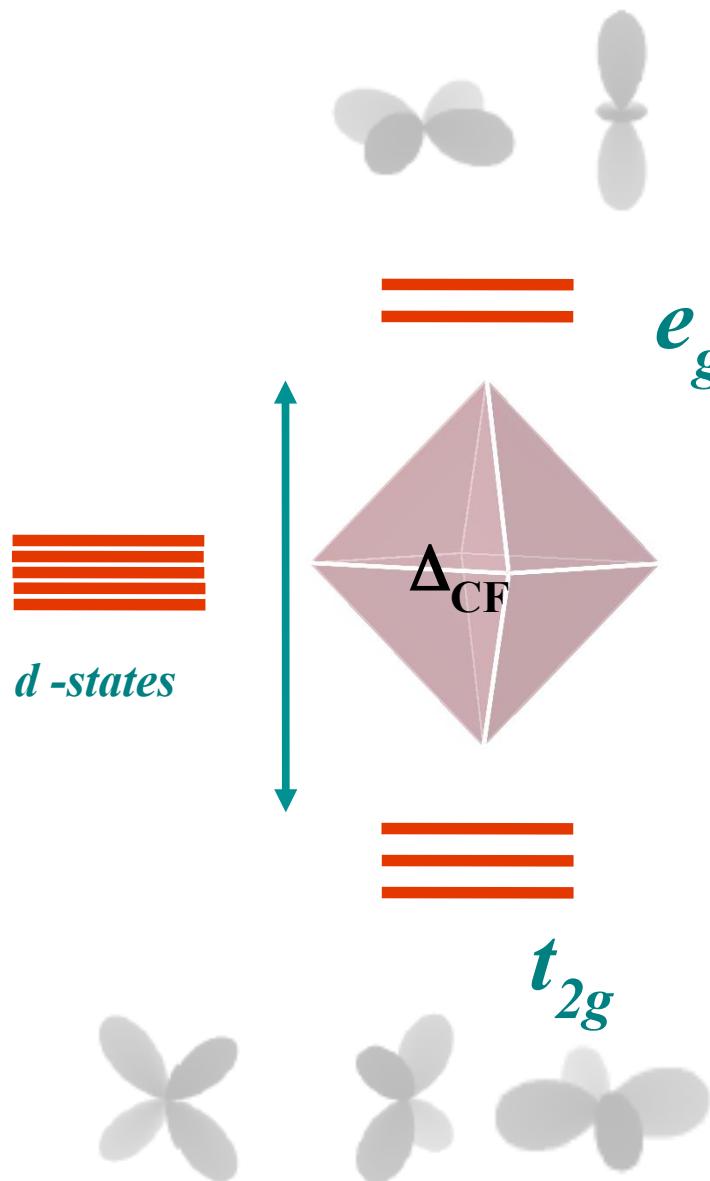
QSL state with gapped/gapless excitation

Third Axis : Disorder

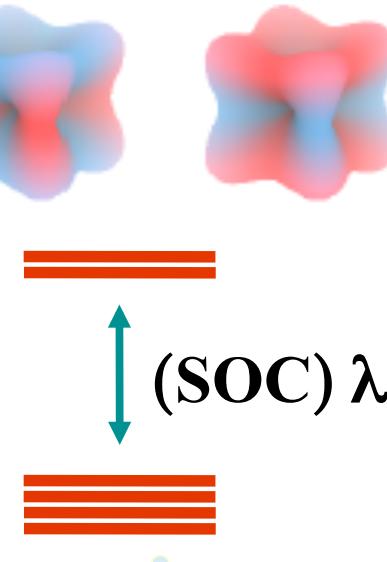
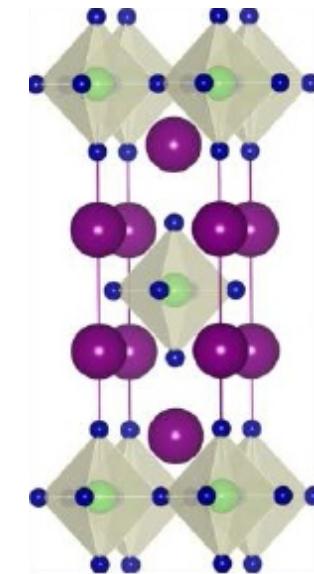
Ann. Rev. Cond. Mat. Physics, Vol. 5: 57-82 (2014)

Nat. Phys. 11, 444–445 (2015)
PRL 102, 017205 (2009)

Crystal Field and Spin-Orbit Coupling (Onsite Terms)



These parameters
can be estimated
from DFT calc.

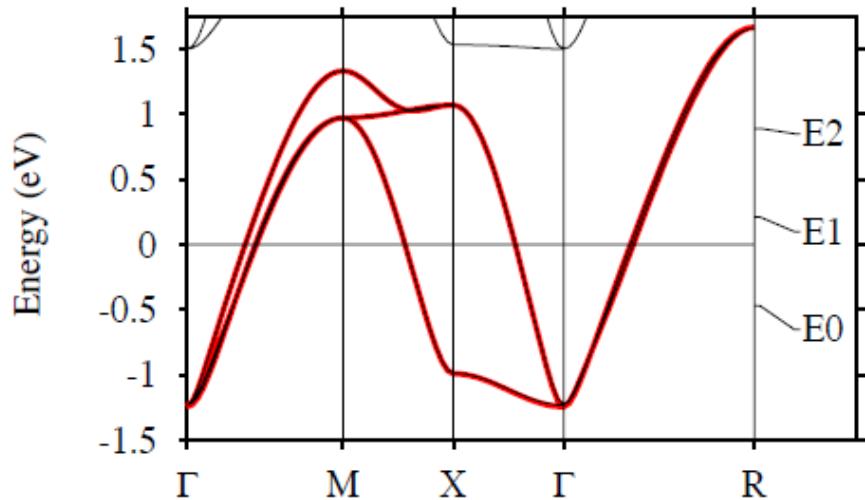
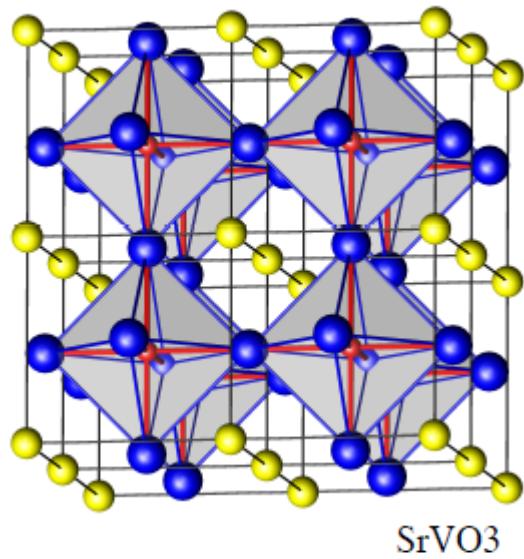


A diagram illustrating the formula for the magnetic field \vec{B} due to spin-orbit coupling. It shows a cone representing the momentum space path of a particle, with a vector \vec{v} pointing along the axis and a vector \vec{E} perpendicular to it. A magnetic field \vec{B} is shown as a vector pointing upwards, perpendicular to both \vec{v} and \vec{E} . A small red sphere with a black arrow σ indicates the direction of spin. The formula below the diagram is:

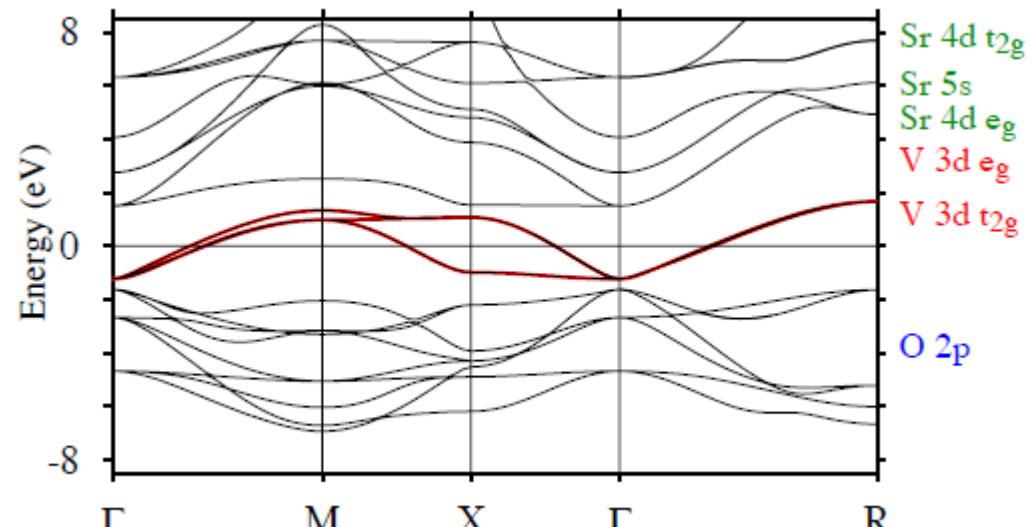
$$\vec{B} = \frac{\vec{v} \times \vec{E}}{c^2}$$

Few Band Tight-Binding Hamiltonian → Integrating out orbital dof

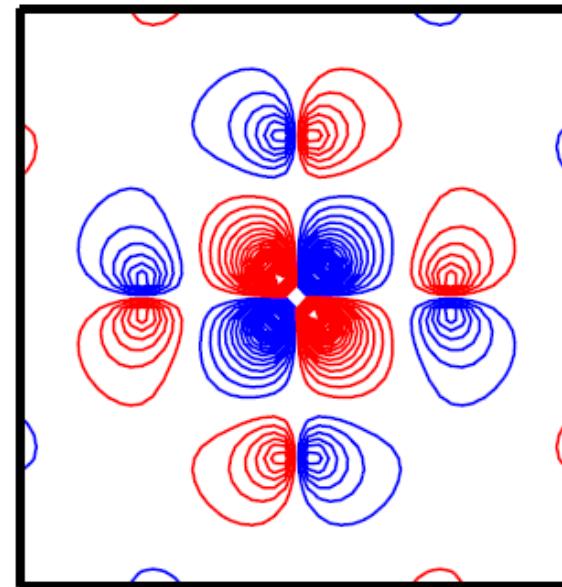
SrVO₃



Pavarini *et. al.* (2005)



t_{2g} Wannier Function



From LDA to minimal models

Löwdin downfolding: active (d) and passive (s, p).

$$H = \begin{pmatrix} H_{AA} & H_{AP} \\ H_{PA} & H_{PP} \end{pmatrix}$$

Integrate out weakly correlated/high energy states

$$H = H_{AA} + H_{AP}(\epsilon - H_{PP})^{-1}H_{AP}$$

Get rid of energy dependence.

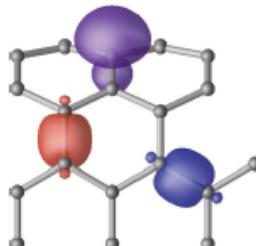
$$\tilde{H} = H_{AA} + H_{AP}(\epsilon_0 - H_{PP})^{-1}H_{PA}$$

Max-loc WFs \leftrightarrow “Exact” Tight-Binding

Compact mapping of Bloch states into local orbitals

$$\omega_n(\mathbf{r} - \mathbf{R}) = \frac{V}{8\pi^3} \int_{BZ} e^{-i\mathbf{k}\cdot\mathbf{R}} \psi_{n\mathbf{k}}(\mathbf{r}) d\mathbf{k}$$

$$\psi_{n\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{N_R}} \sum_{\mathbf{R}} e^{i\mathbf{k}\cdot\mathbf{R}} \omega_n(\mathbf{r} - \mathbf{R})$$



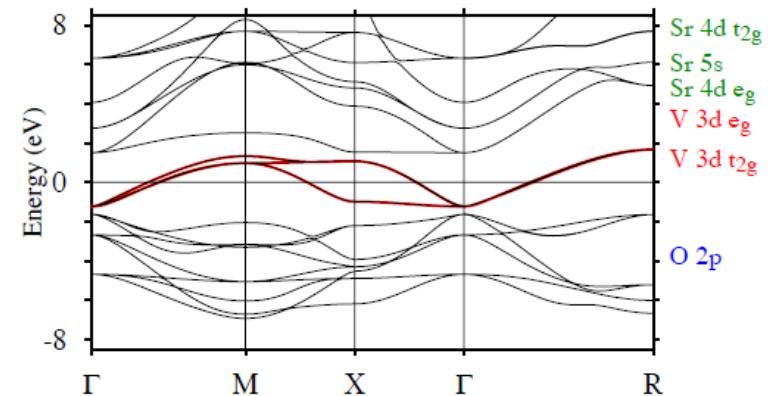
Multiband case:

$$w_n(\mathbf{r} - \mathbf{R}) = \frac{V}{8\pi^3} \int_{BZ} e^{-i\mathbf{k}\cdot\mathbf{R}} \sum_m U_{mn}^{(k)} \psi_{m\mathbf{k}}(\mathbf{r}) dk$$

SrVO₃ → Active Orbitals t_{2g}, Passive → Rest

NMTO Downfolding Method

O.K Andersen & T. Saha-Dasgupta



**Max. Localized WF
D. Vanderbilt**

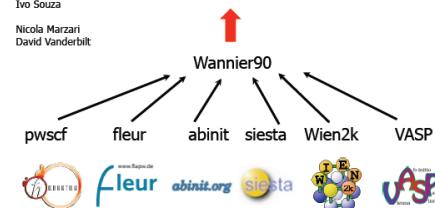
PRB 56, 12847 (1997)
Rev. Mod. Phys 84, 1419 (2012)

Wannier90 code

Authors
Arash Mostofi
Jonathan Yates
Giovanni Pizzi
Ivo Souza

Nicola Marzari
David Vanderbilt

Wannier interpolation,
transport, etc.



Strong Correlation..

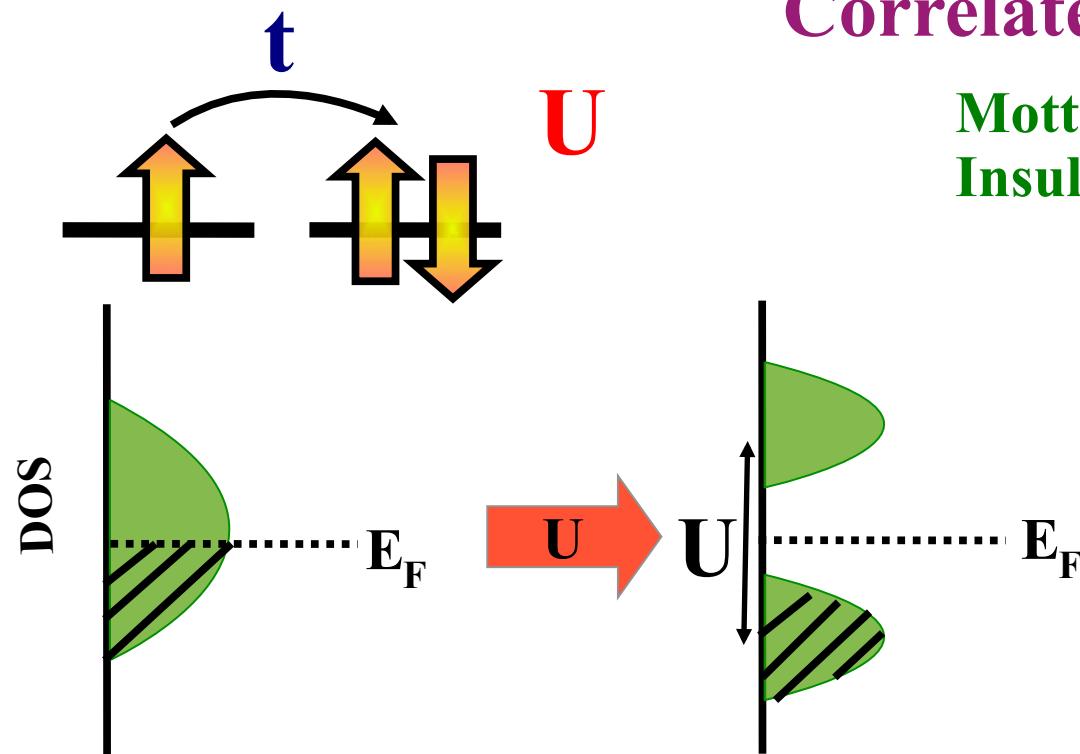
Unpaired electron → metallic character

Hubbard
Model

$$H = \sum t_{ij,\alpha\beta} C_{i\alpha}^\dagger C_{j\beta} + h.c + U \sum n_{i\alpha} (n_{i\alpha} - 1)$$

Correlated System

Mott
Insulator



3d TMO-High T_c Cuprates:
➤ Half filled systems, Cu: d^9
➤ Insulator!

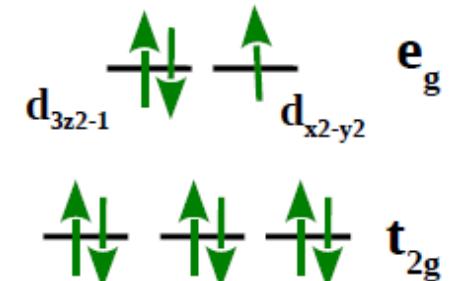
Hubbard Model

$$H = \sum t_{ij} [c_{i\sigma}^+ c_{j\sigma} + c_{j\sigma}^+ c_{i\sigma}] + U \sum n_{i\uparrow} n_{i\downarrow}$$

Mottronics

Hubbard Model to Heisenberg Model

1 Orbital and 1 electron / site \rightarrow High Tc Cuprates (d^9)



$U \gg t$, Consider t as a perturbation (Strong Coupling Expansion)

$$H_{Heis} = \sum J_{ij} S_i S_j$$

$$J=4t^2/U$$

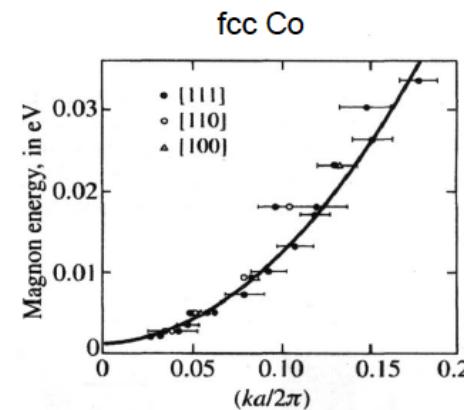
Spin Wave Analysis (Magnons)

$$S^z = S - a^\dagger a$$

$$S^+ = \sqrt{2S} \sqrt{1 - \frac{a^\dagger a}{2S}} a = \sqrt{2S} \left(1 - \frac{a^\dagger a}{4S} \right) a + \dots$$

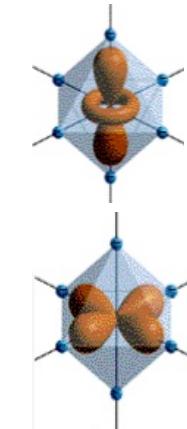
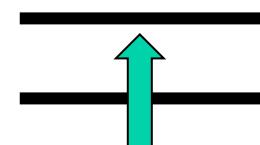
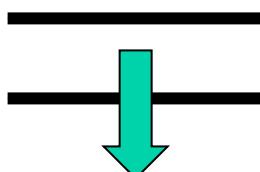
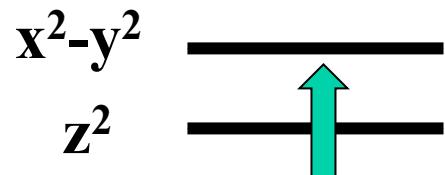
$$S^- = \sqrt{2S} a^\dagger \sqrt{1 - \frac{a^\dagger a}{2S}} = \sqrt{2S} a^\dagger \left(1 - \frac{a^\dagger a}{4S} \right) + \dots$$

$$\hat{\mathcal{H}} \sim J \sum_{\langle i,j \rangle} a_i^\dagger a_j + h.c.$$



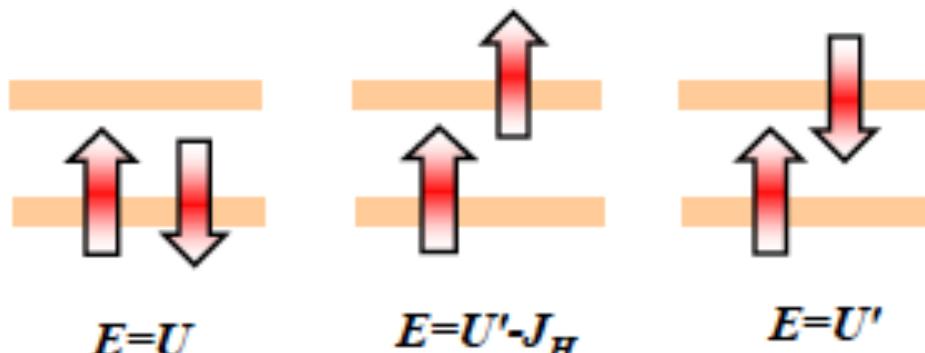
Interactions in a Multiband Hubbard Model

Single site : 2 Orbitals per site (e_g Orbitals)



Cooperative JT Interaction / Orbital Ordering

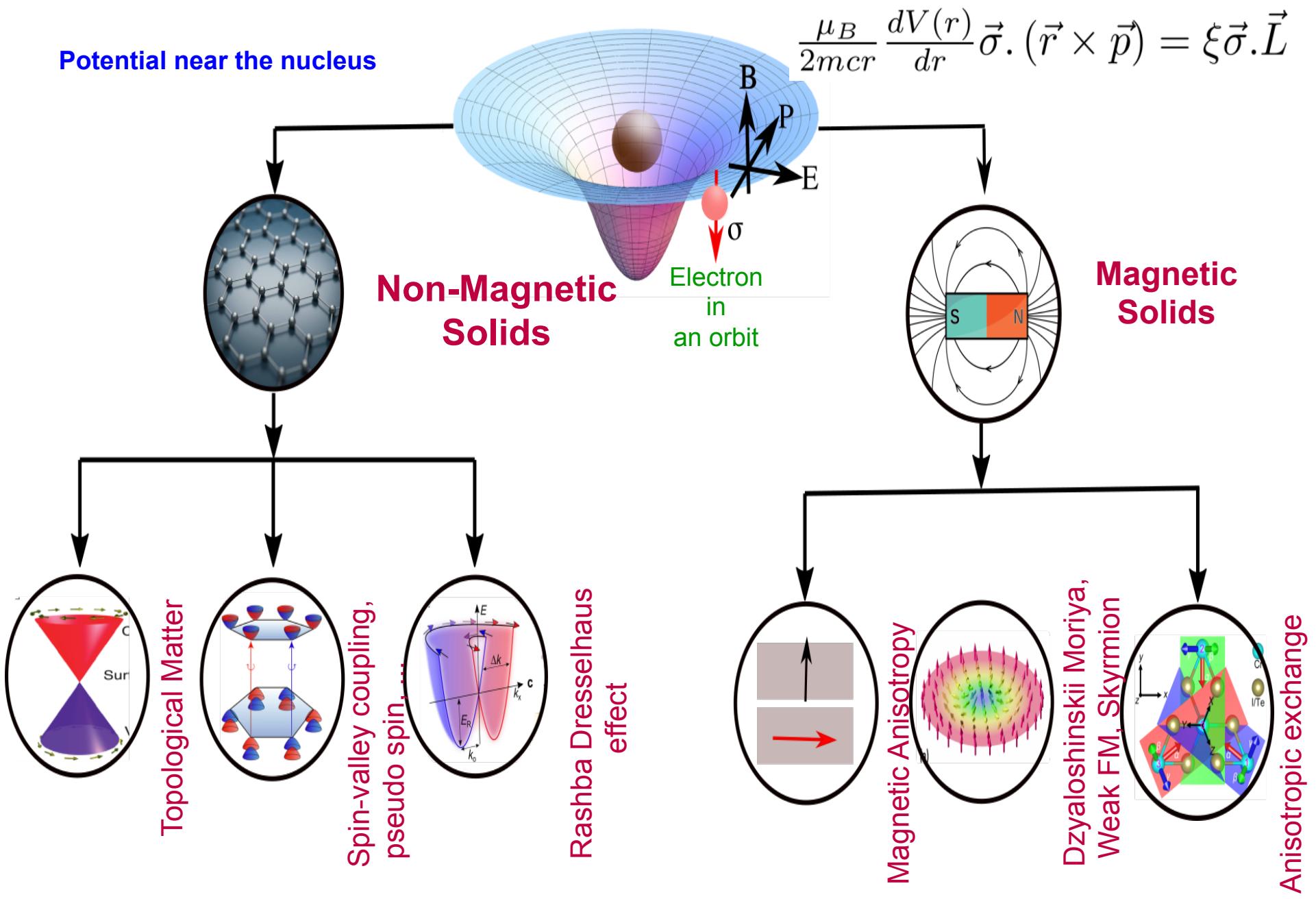
After hopping three possible configurations



U' = Inter-orbital
Coulomb
Interaction

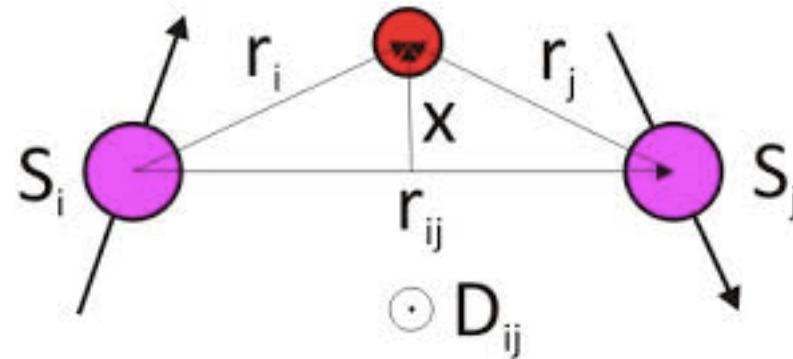
J_H = Hund's
Exchange

Spin-orbit coupling



Exchange Interaction and Spin-Orbit Coupling

Dzyaloshinskii–Moriya interaction



Anti-symmetric exchange interaction → Canting of Spins

$$H_{DM} = \sum D_{ij} \cdot S_i \times S_j$$

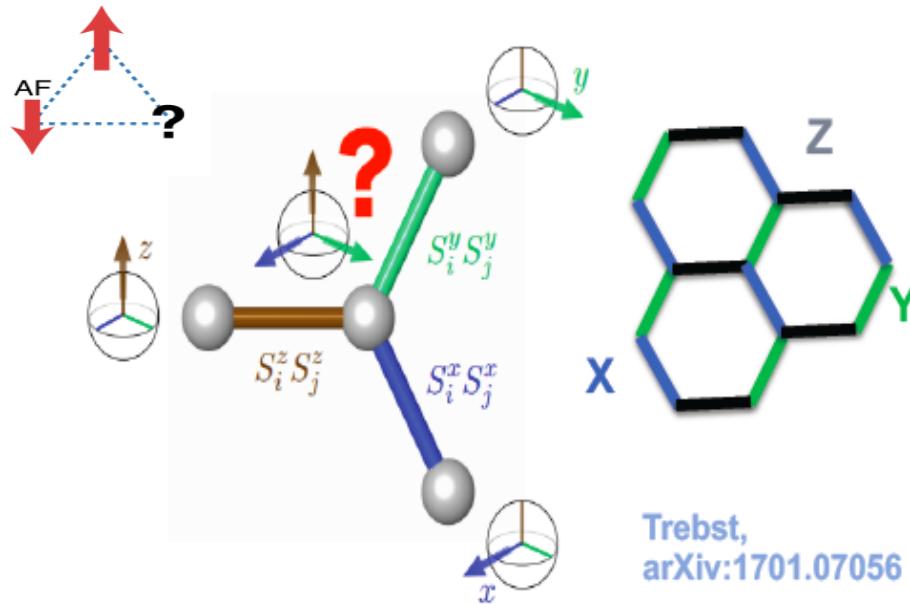
Requirements:

- (a) Spin Orbit Coupling
- (b) Lack of Inversion Symmetry

SOC can lead to single ion anisotropy (easy axis)
Heisenberg Model → Ising Model

Kitaev model

- Bond-directional dependent Ising coupling → exchange frustration
- Ground state for $S=1/2$ moments on a 2D honeycomb lattice is an exactly solvable quantum spin liquid (QSL)
- Fractionalized spin excitations in the QSL are topologically protected gauge flux and Majorana fermion pairs



Trebst,
arXiv:1701.07056



Available online at www.sciencedirect.com

SCIENCE  DIRECT[®]

Annals of Physics 321 (2006) 2–111

ANNALS
of
PHYSICS
www.elsevier.com/locate/aop

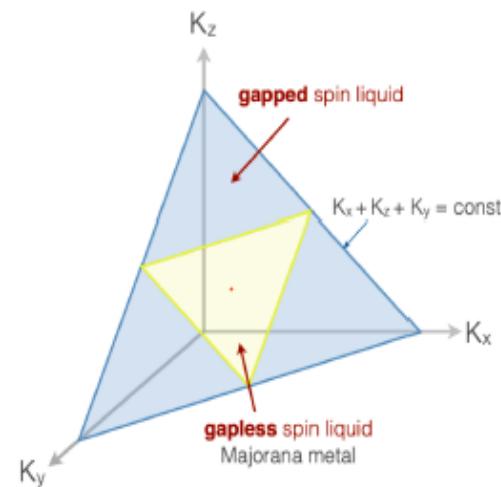
Anyons in an exactly solved model and beyond

Alexei Kitaev *

California Institute of Technology, Pasadena, CA 91125, USA

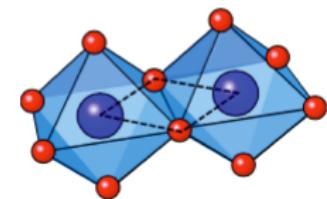
Received 21 October 2005; accepted 25 October 2005

Alexei Kitaev



Material Realization of Kitaev Spin Liquid

Proposed physical realization: Edge-sharing octahedra in honeycomb $J_{\text{eff}} = \frac{1}{2}$ Mott insulators leads to bond directional, Kitaev-like exchange



Honeycomb systems with edge sharing octahedra

Multiband Hubbard Model + SOC

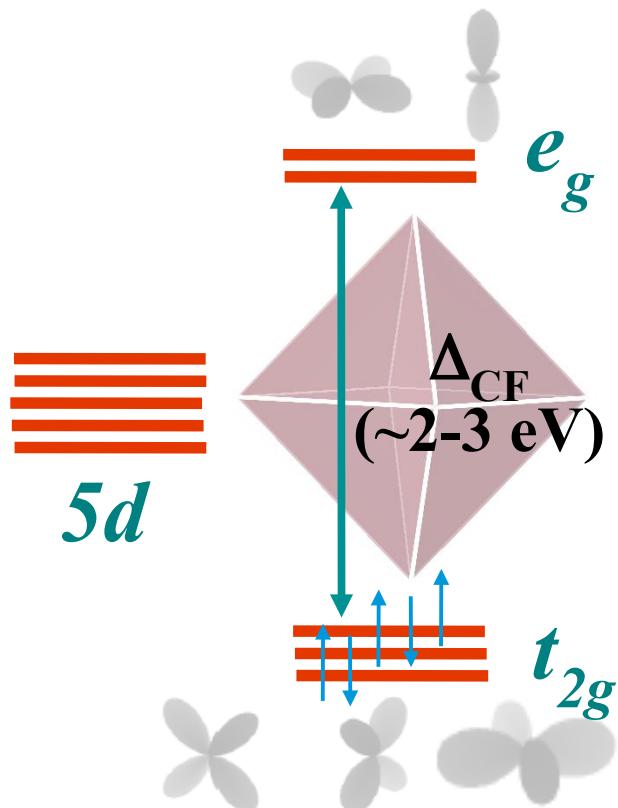
Search for Novel Quantum Materials

SOC Induced Novel Insulating State in $5d$ transition metal Oxides:

Traditional Belief:

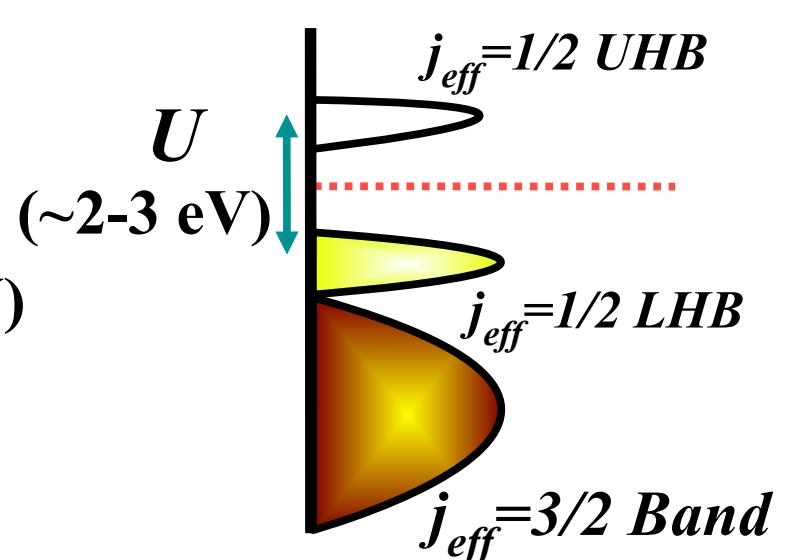
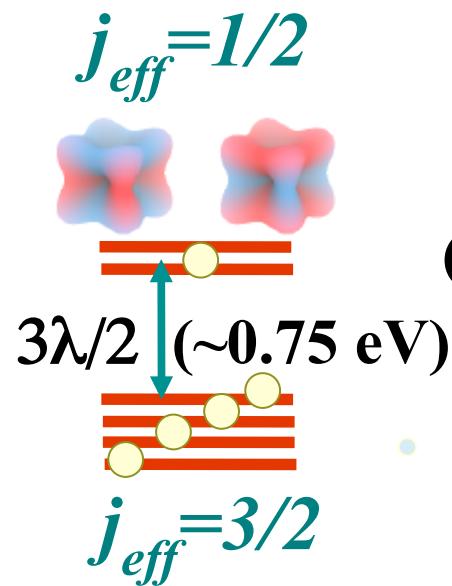
$5d$ systems have large bandwidth
→ U/t small
→ uncorrelated metal (Boring!!!)

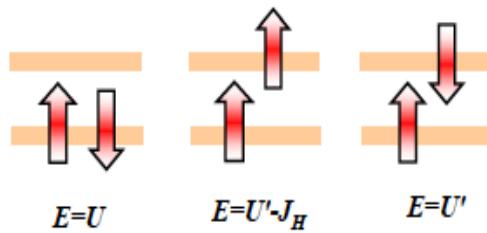
CaIrO_3 , Sr_2IrO_4 ,
 Na_2IrO_3 having d^5
configuration are
insulator!



$j_{\text{eff}}=1/2$ Mott insulator

Phys. Rev. Lett 101, 076402 (2008)





A Multiband Hubbard Hamiltonian:

$$H = H_\Delta + H_{\text{int}} + H_{\text{SOC}} + H_t$$

$$H_\Delta = \sum_i \sum_{l,m} e_{l,m} d_{i,l\sigma}^\dagger d_{i,m\sigma}$$

Crystal field
Interaction term

Spin-orbit
coupling

$$H_t = \sum_{l,m,\sigma} \sum_{i \neq j} t_{l,m}^{ij} d_{i,l\sigma}^\dagger d_{j,m\sigma}$$

$$H_{\text{SOC}} = \frac{i\lambda}{2} \sum_i \sum_{lmn} \epsilon_{lmn} \sum_{\sigma\sigma'} \sigma_{\sigma\sigma'}^n d_{i,l\sigma}^\dagger d_{i,m\sigma'}$$

$$H_{\text{int}} = U_d \sum_i \sum_l n_{i,l\uparrow} n_{i,l\downarrow} + \frac{(U'_d - J_H)}{2} \sum_i \sum_{\substack{l,m \\ l \neq m}} n_{i,l\sigma} n_{i,m\sigma} + \frac{U'_d}{2} \sum_i \sum_{\substack{l,m \\ l \neq m \\ \sigma \neq \sigma'}} n_{i,l\sigma} n_{i,m\sigma'} - \frac{J_H}{2} \sum_i \sum_{\substack{l,m \\ l \neq m}} (d_{i,m\uparrow}^\dagger d_{i,m\downarrow} d_{i,l\downarrow}^\dagger d_{i,l\uparrow} - d_{i,m\downarrow}^\dagger d_{i,m\uparrow} d_{i,l\uparrow}^\dagger d_{i,l\downarrow} + \text{h.c.})$$

Kanamori Hamiltonian

**Mott Insulators in the Strong Spin-Orbit Coupling Limit:
From Heisenberg to a Quantum Compass and Kitaev Models**

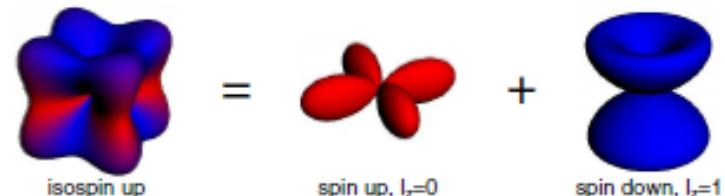
G. Jackeli^{1,*} and G. Khaliullin¹

¹*Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-70569 Stuttgart, Germany*
(Received 21 August 2008; published 6 January 2009)

Spin –Orbital Entangled $J_{\text{eff}} = \frac{1}{2}$ States

$$|\uparrow\rangle = \phi_{3\alpha} = \frac{1}{\sqrt{3}}(d_{xy\uparrow} + d_{yz\downarrow} + id_{zx\downarrow}),$$

$$|\downarrow\rangle = \phi_{3\beta} = \frac{1}{\sqrt{3}}(d_{xy\downarrow} - d_{yz\uparrow} + id_{zx\uparrow}).$$

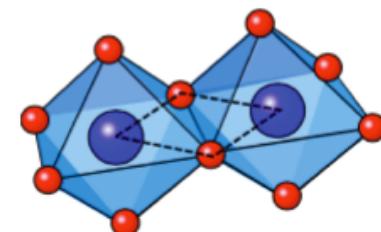


Isospin up state

Strong Coupling expansion in the $J_{\text{eff}} = \frac{1}{2}$ manifold U, $\lambda \gg t$

Spin model: Kitaev Interaction

$$H = \sum_{\alpha\beta(\gamma)} \epsilon_{ij} [K S_i^\gamma S_j^\gamma + \Gamma (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha)]$$



Edge sharing octahedra in a Honeycomb lattice

Realization of novel Magnetic States → Spin Liquids ²³

Unusual spin dynamics in the low-temperature magnetically ordered state of $\text{Ag}_3\text{LiIr}_2\text{O}_6$

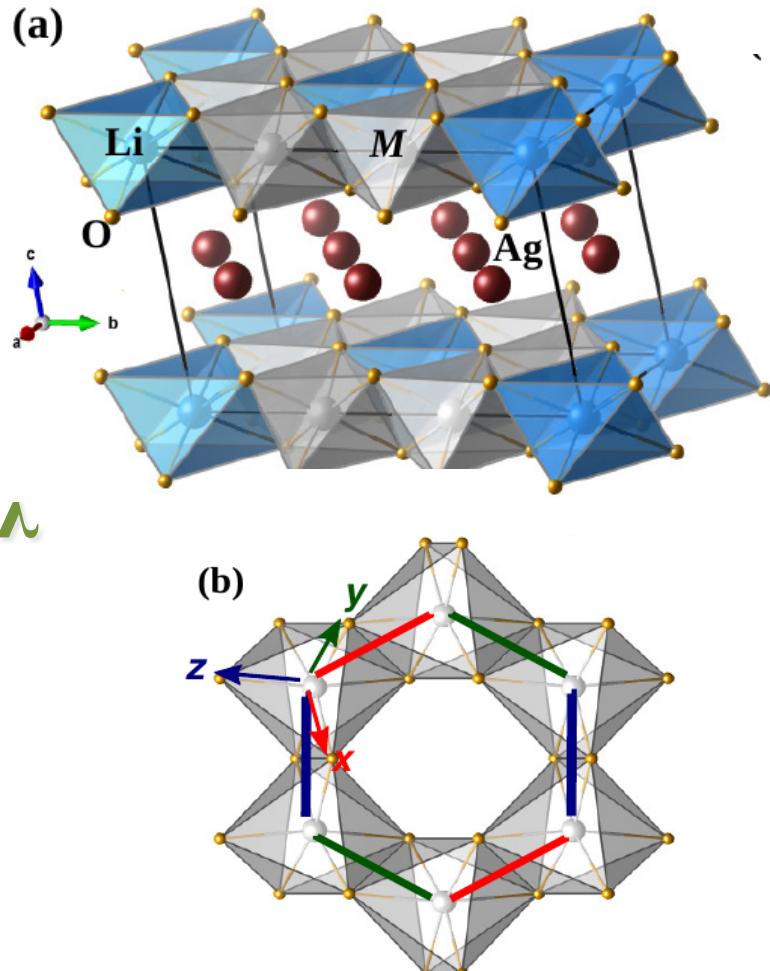
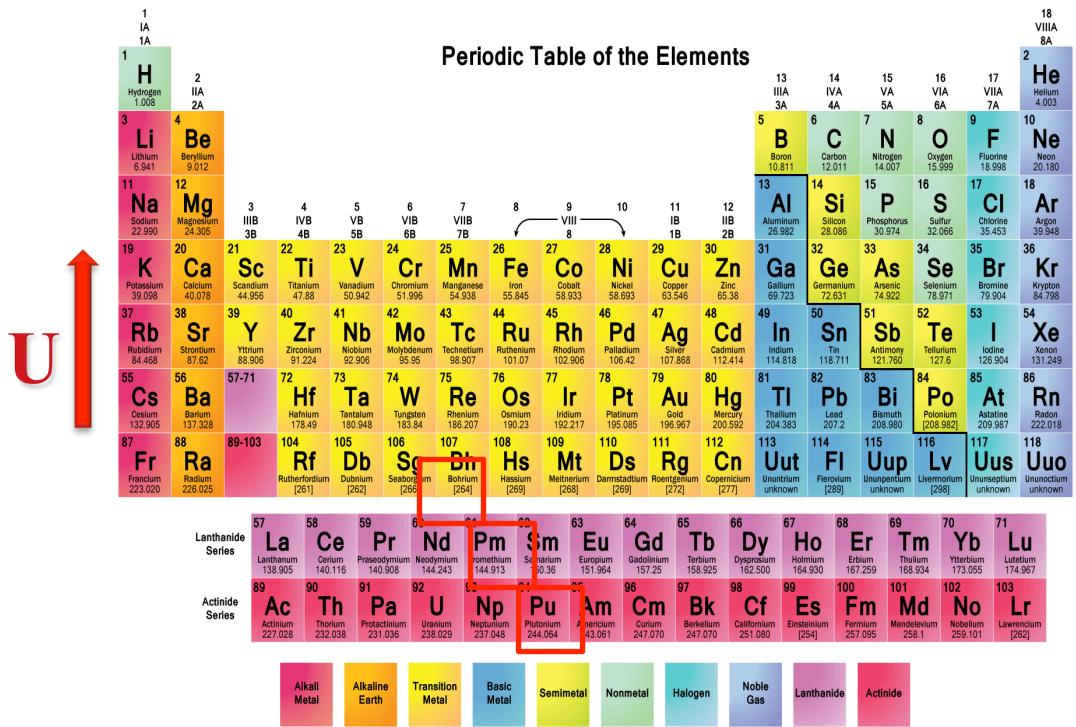
Atasi Chakraborty,^{1,*} Vinod Kumar^{1,2,*} Sanjay Bachhar^{1,2}, N. Büttgen,³ K. Yokoyama^{1,4}, P. K. Biswas,⁴ V. Siruguri,⁵ Sumiran Pujari^{1,2}, I. Dasgupta,¹ and A. V. Mahajan^{1,2,†}



Theory (DFT) : Atasi Chakraborty , Mainz (Germany)

Expt : (NMR, muSR) Vinod Kumar, Sanjay Bachhar & A.V. Mahajan (IIT-B)

$\text{Ag}_3\text{LiIr}_2\text{O}_6$

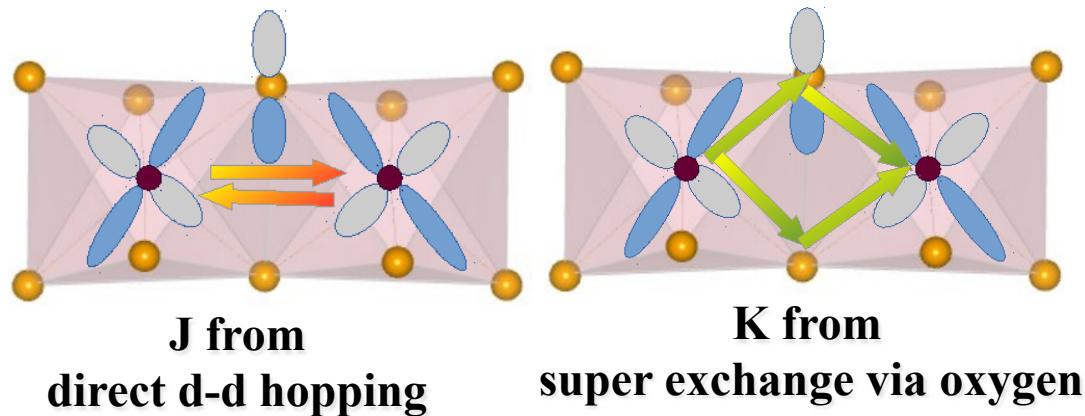


Monoclinic [space group C2/m]

$\text{Ag}_3\text{LiIr}_2\text{O}_6$

Honeycomb network: dominant SOC ($5d^5$) + Spin 1/2 $\xrightarrow{\hspace{1cm}}$ Kitaev exchange

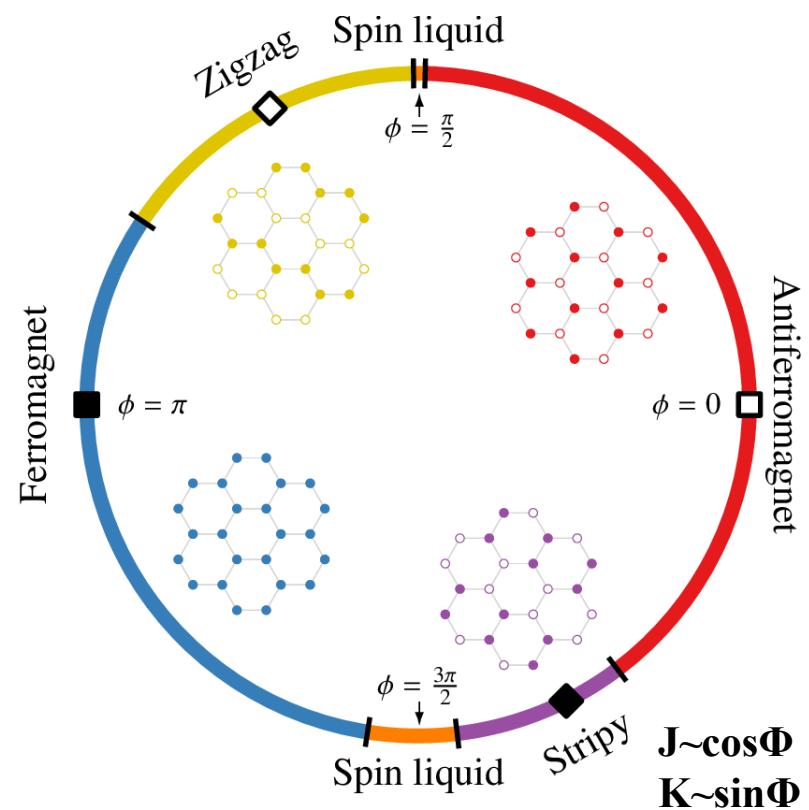
$$H = \sum_{\langle i,j \rangle \in \gamma} (J S_i \cdot S_j + K S_i^\vee S_j^\vee)$$



Competing energy scale

Long range ordering

QSL



A spin-orbital-entangled quantum liquid on a honeycomb lattice

K. Kitagawa^{1*}, T. Takayama^{2*}, Y. Matsumoto², A. Kato¹, R. Takano¹, Y. Kishimoto³, S. Bette², R. Dinnebier², G. Jackeli^{2,4} & H. Takagi^{1,2,4}

PHYSICAL REVIEW LETTERS 123, 237203 (2019)

Thermodynamic Evidence of Proximity to a Kitaev Spin Liquid in Ag₃LiIr₂O₆

Faranak Bahrami,¹ William Lafargue-Dit-Hauret,^{2,3} Oleg I. Lebedev,⁴ Roman Movshovich,⁵ Hung-Yu Yang,¹ David Broido,¹ Xavier Rocquefelte,² and Fazel Tafti^{1,*}

¹Department of Physics, Boston College, Chestnut Hill, Massachusetts 02467, USA

²Univ Rennes, CNRS, ISCR (Institut des Sciences Chimiques de Rennes) UMR 6226, F-35000 Rennes, France

³Physique Théorique des Matériaux, CESAM, Université de Liège, B-4000 Sart Tilman, Belgium

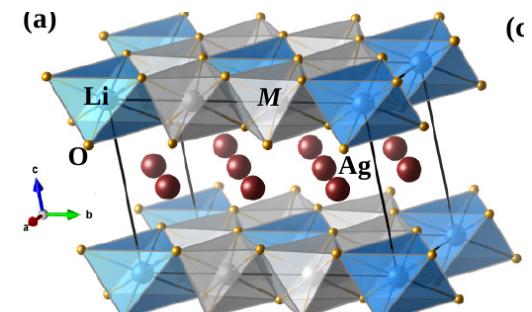
⁴Laboratoire CRISMAT, ENSICAEN-CNRS UMR6508, 14050 Caen, France

⁵MPA-CMMS, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA



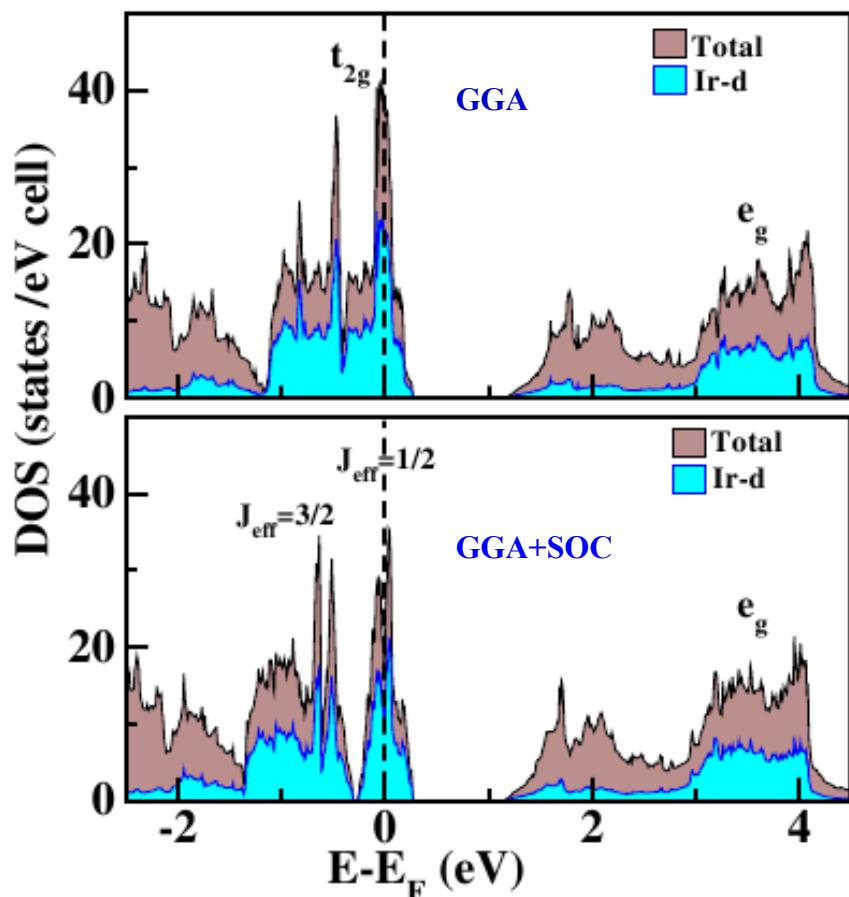
(Received 27 May 2019; revised manuscript received 4 September 2019; published 3 December 2019)

Is Ag₃LiIr₂O₆ a Kitaev Spin Liquid ?



Hamiltonian within bond basis

$$H = \sum_{\langle ij \rangle \in \alpha\beta(\gamma)} [J^\alpha S_i^\alpha S_j^\alpha + J^\beta S_i^\beta S_j^\beta + J^\gamma S_i^\gamma S_j^\gamma + \Gamma(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) + D(S_i^\alpha S_j^\beta - S_i^\beta S_j^\alpha)]$$



$$\mathbf{S}_i = \begin{pmatrix} S_i^x \\ S_i^y \\ S_i^z \end{pmatrix} \quad \hat{H} = \mathbf{s}_i \cdot \tilde{\mathbf{J}}_{ij} \cdot \mathbf{s}_j$$

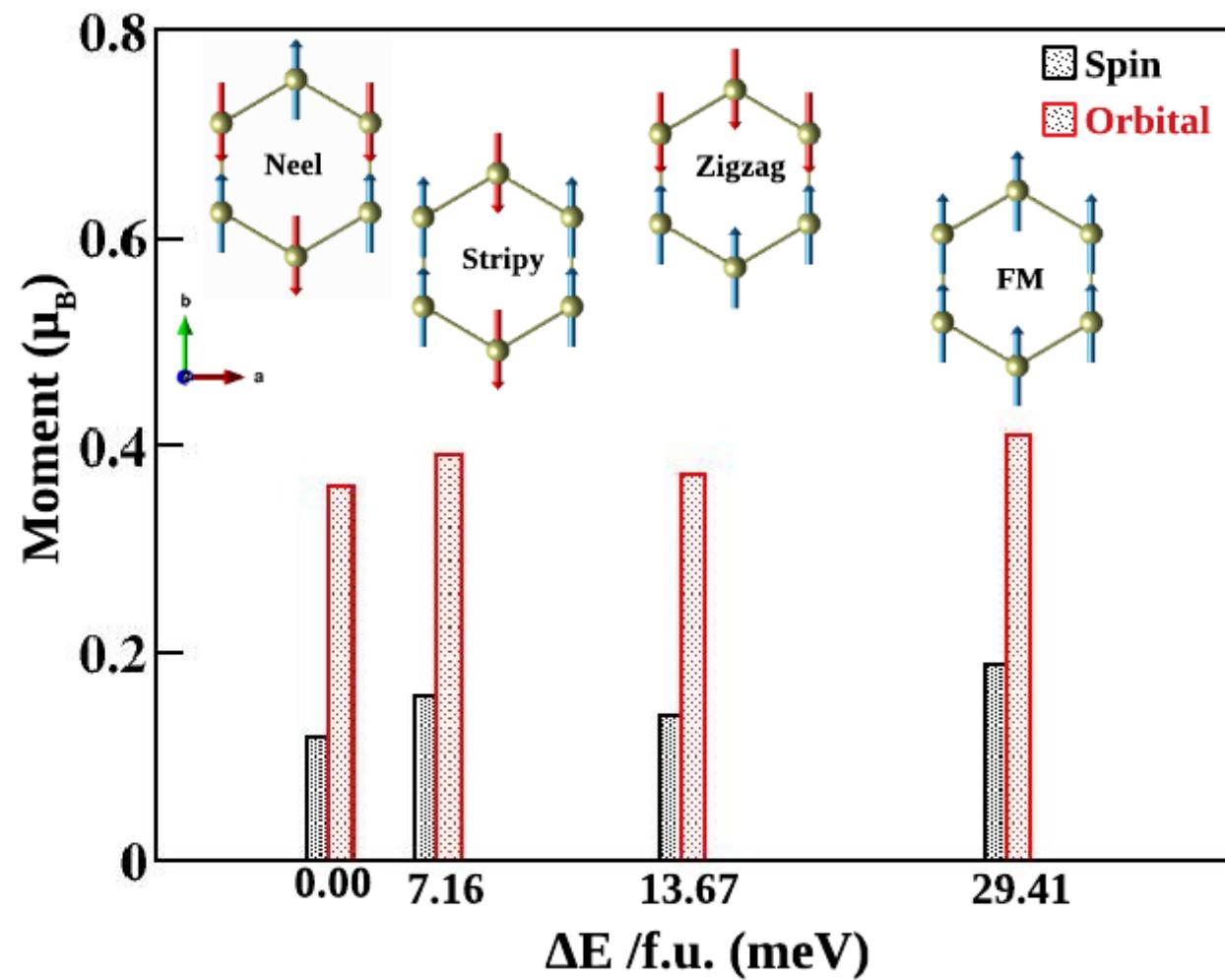
$$\tilde{\mathbf{J}} = \begin{pmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix}^{(2)}$$

For the Z bond

$J_z = 5.5$ meV (AFM)
 $K_z = -5.1$ meV (FM),
 $\Gamma_z = -1.3$ meV (FM),
 $\Gamma_z' = -3.0$ meV (FM)

J is comparable to K
-> Ordering

Ground state magnetism



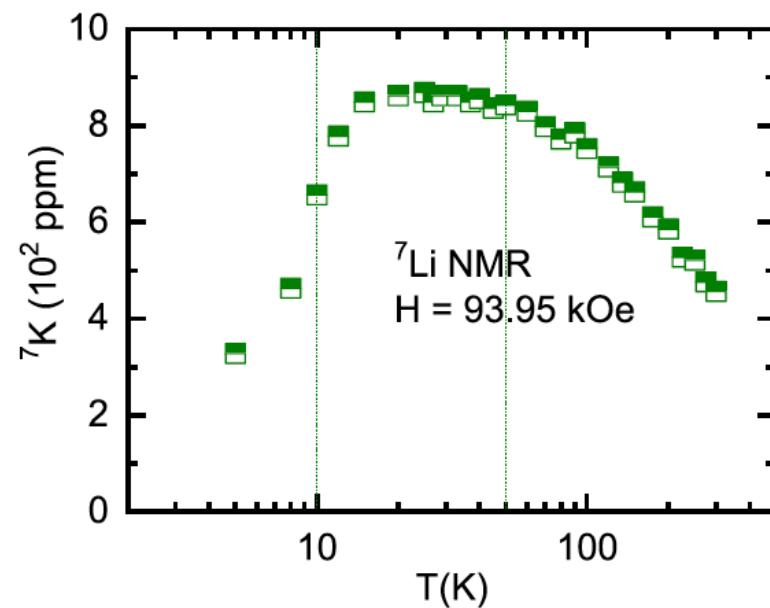
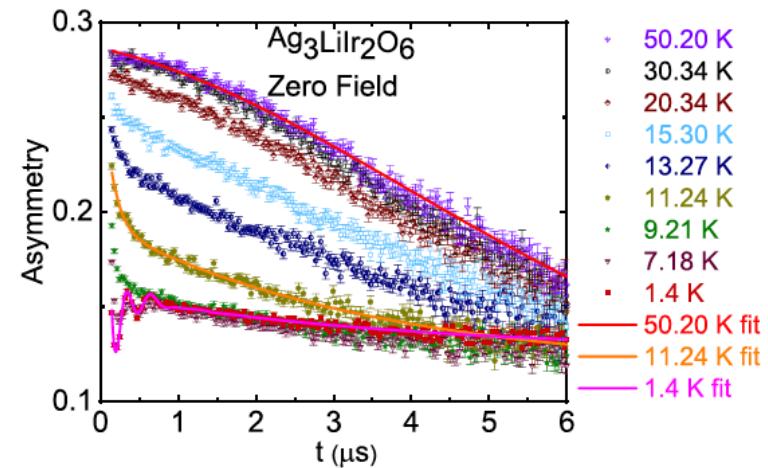
Stable AFM long range ordered ground state

A. V. Mahajan (IIT Bombay) Ordering confirmed by muSR measurements

Experimental Scenario

Oscillations in μ SR relaxation data below $\sim 9\text{K}$ \rightarrow Magnetic Ordering.
Analysis of muon data in the ordered state complemented with DFT of Muon stopping sites points toward incommensurate Neel and stripe ordered magnetic domains

${}^7\text{Li}$ NMR shift shows 2 anomalies
10K \sim onset of long range ordering.
Broad maximum at 50K \sim onset of SRO



Spin Liquids

PRL 116, 097205 (2016)

PHYSICAL REVIEW LETTERS

week ending
4 MARCH 2016

Origin of the Spin-Orbital Liquid State in a Nearly $J = 0$ Iridate $\text{Ba}_3\text{ZnIr}_2\text{O}_9$

Abhishek Nag,¹ S. Middey,^{2†} Sayantika Bhowal,³ S. K. Panda,^{2§} Roland Mathieu,⁴ J. C. Orain,⁵ F. Bert,⁵ P. Mendels,⁵ P. G. Freeman,^{6,7} M. Mansson,^{6,8} H. M. Ronnow,⁶ M. Telling,⁹ P. K. Biswas,¹⁰ D. Sheptyakov,¹¹ S. D. Kaushik,¹² Vasudeva Siruguri,¹² Carlo Meneghini,¹³ D. D. Sarma,¹⁴ Indra Dasgupta,^{2,3,*} and Sugata Ray^{1,2,†}

PHYSICAL REVIEW LETTERS 125, 267202 (2020)

Geometric Frustration

Gapless Quantum Spin Liquid in the Triangular System $\text{Sr}_3\text{CuSb}_2\text{O}_9$

S. Kundu^{1,*} Aga Shahee¹, Atasi Chakraborty^{1,2}, K. M. Ranjith^{1,3}, B. Koo^{1,3}, Jörg Sichelschmidt^{1,3}, Mark T. F. Telling,⁴ P. K. Biswas,⁴ M. Baenitz,³ I. Dasgupta^{1,2}, Sumiran Pujari¹, and A. V. Mahajan^{1,†}

PHYSICAL REVIEW B 104, 115106 (2021)

Exchange Frustration (Kitaev)

Unusual spin dynamics in the low-temperature magnetically ordered state of $\text{Ag}_3\text{LiIr}_2\text{O}_6$

Atasi Chakraborty,^{1,*} Vinod Kumar^{1,2,*}, Sanjay Bachhar^{1,2}, N. Büttgen,³ K. Yokoyama^{1,4}, P. K. Biswas,⁴ V. Siruguri,⁵ Sumiran Pujari^{1,2}, I. Dasgupta,¹ and A. V. Mahajan^{1,2,†}

**Is there any other source of
Spin-Orbit Coupling ?**

Collaboration



Kunal Dutta (IACS)



Subhadeep Bandyopadhyay (IACS)
→ Univ of Leige', Belgium



Atanu Paul (IACS)
→ Bar –Ilan University, Israel

Phys. Rev. B 101, 014109 (2020)
Phys Rev B 103, 014105 (2021)

Phys Rev B 108, 245146 (2023)

Symmetry and Band Splitting

Crystal inversion symmetry : $\varepsilon(\vec{k} \uparrow) = \varepsilon(-\vec{k} \uparrow)$

Time reversal symmetry : $\varepsilon(\vec{k} \uparrow) = \varepsilon(-\vec{k} \downarrow)$

Kramer's degeneracy : $\varepsilon(\vec{k} \uparrow) = \varepsilon(\vec{k} \downarrow)$

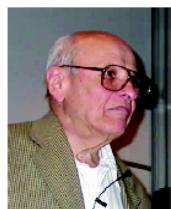
**Breaking Time reversal symmetry
applying external magnetic field**

$$\varepsilon(\vec{k} \uparrow) \neq \varepsilon(-\vec{k} \downarrow)$$

Breaking inversion symmetry

(Can be broken by applying
external Electric field)

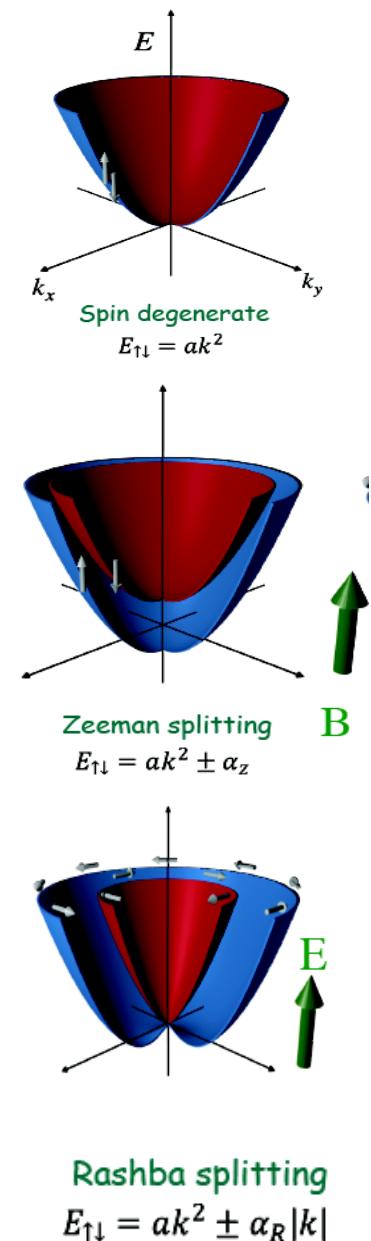
$$\varepsilon(\vec{k} \uparrow) \neq \varepsilon(-\vec{k} \uparrow)$$



Emmanuel I. Rashba (1927-)

Ref: Rashba, Sov. Phys. Solid State (1960)

The Rashba-Dresselhaus Effect



Rashba-Dresselhaus Spin-Orbit Coupling

No Inversion Symmetry →

Potential gradient from positively charged nucleus generates an electric field $E = E z$

Electron sees a magnetic field in rest frame :

$$B = (v \times E) / c^2$$

B field couples to electron's spin moment (M_s)

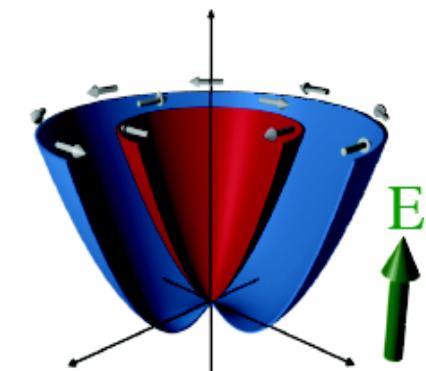
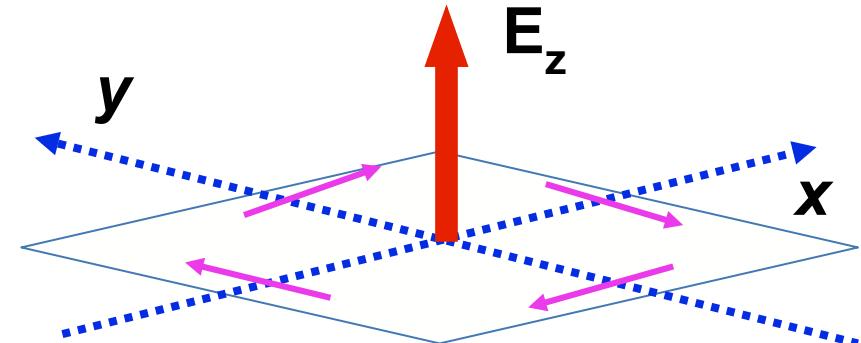
$$H = M_s \cdot B = (g\mu_B \sigma/2) \cdot (v \times E / c^2) \\ = \alpha \sigma \cdot (k \times z) = B(k) \cdot \sigma$$

σ : Pauli spin matrices

→ $B(k)$: Momentum dependent magnetic field
 k dependence of B is determined by symmetry

$$H_R = \alpha_R \vec{\sigma} \cdot (\vec{k} \times \hat{z}) = \alpha_R (\sigma_x k_y - \sigma_y k_x)$$

$$B(k) = (k_y, -k_x)$$



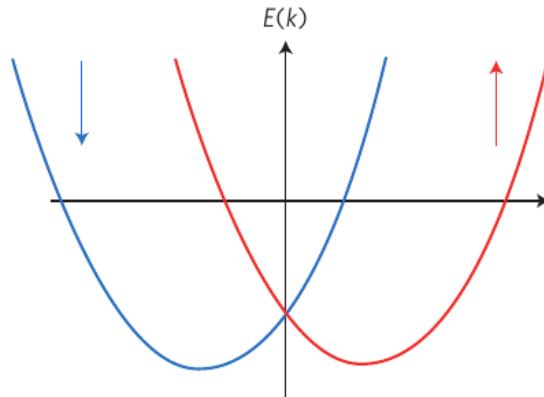
$$E_{\uparrow\downarrow} = ak^2 \pm \alpha_R |k|$$

*Momentum dependent splitting of bands:
Rashba Splitting*

Provides an electric field control over the spin degrees of freedom (*Spintronics*)

Rashba effect

Hamiltonian: $H_R = \alpha_R (\sigma_x k_y - \sigma_y k_x)$

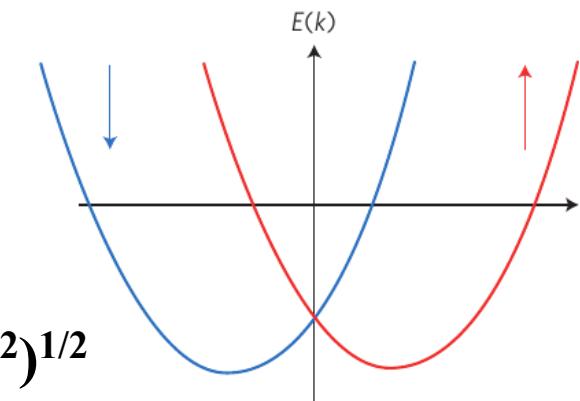


Eigen values

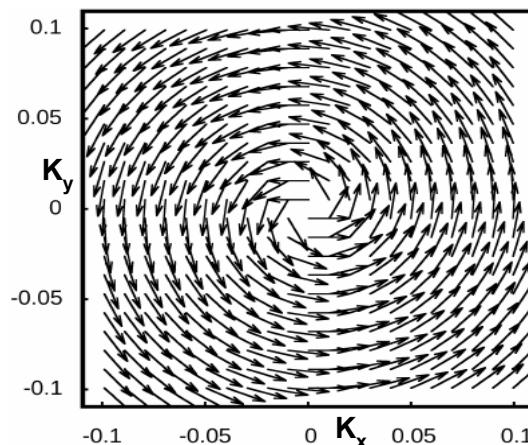
$$E = E_0 + \hbar^2 / 2m \cdot \alpha (k_x^2 + k_y^2)^{1/2}$$

Dresselhaus effect

Hamiltonian: $H_D = \alpha_D (\sigma_x k_y + \sigma_y k_x)$



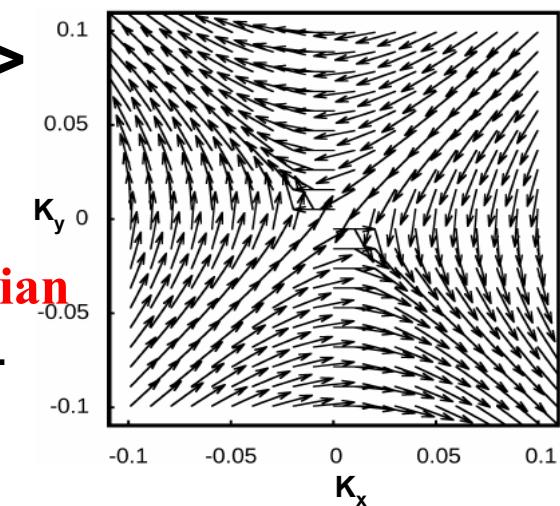
Spin-texture: Orientation of the expectation value of the spin components on the momentum plane



Rashba Spintexture

$$\langle \sigma_i \rangle = \langle \Psi | \sigma_i | \Psi \rangle$$

Ψ is the eigen function of
Rashba/Dresselhaus Hamiltonian
 σ 's are the Pauli spin matrices.



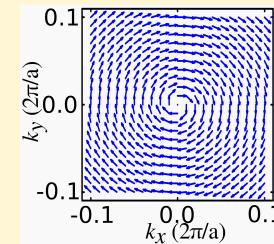
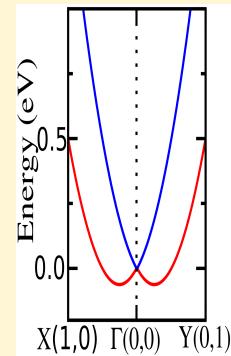
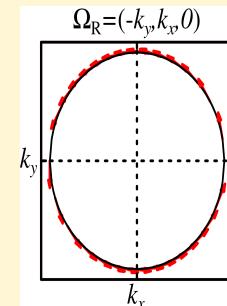
Dresselhaus Spintexture

Field[B(k)]

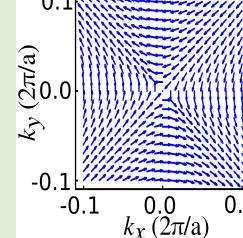
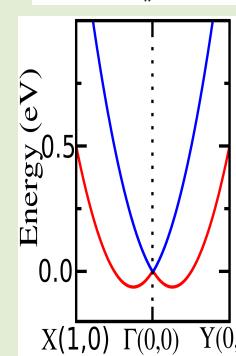
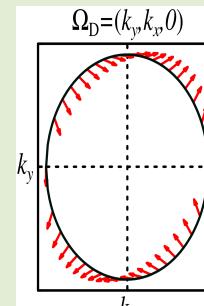
Band Dispersion

Spin Texture

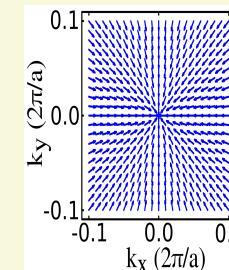
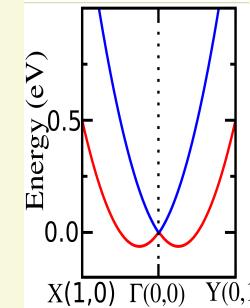
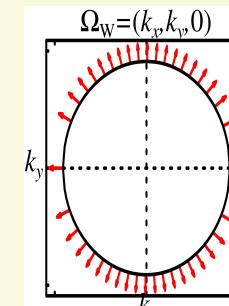
Rashba effect



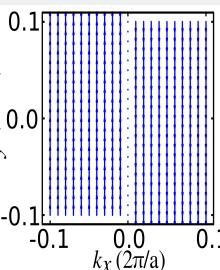
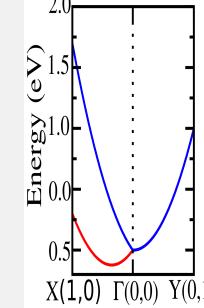
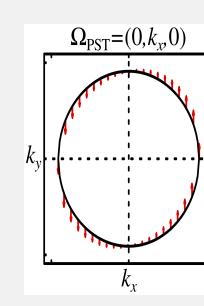
Dresselhaus effect



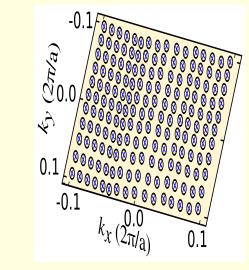
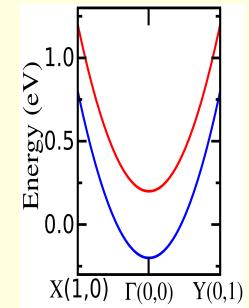
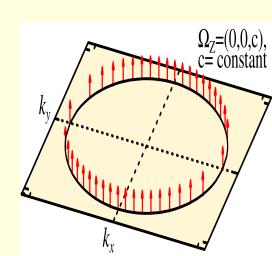
Weyl effect



Persistent



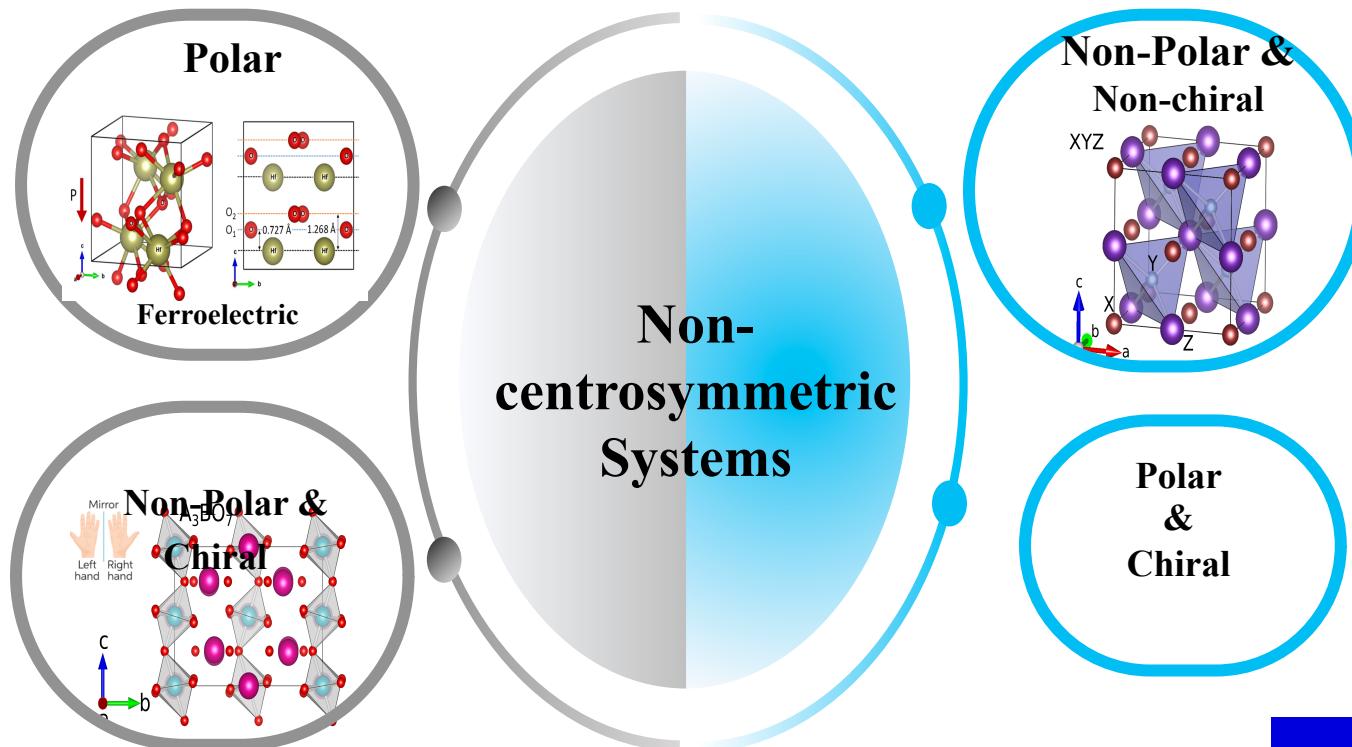
Zeeman effect



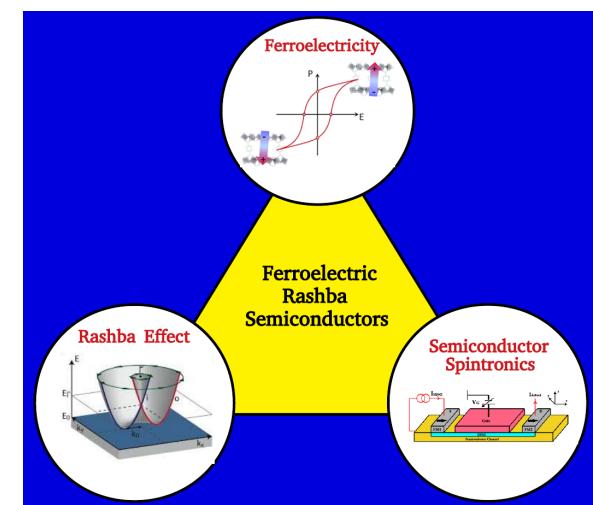
- ◆ Field (B) and ST are determined by the local symmetry of the BZ (little group)
- ◆ PSH robust against spin dependent disorder leads to infinite spin lifetime

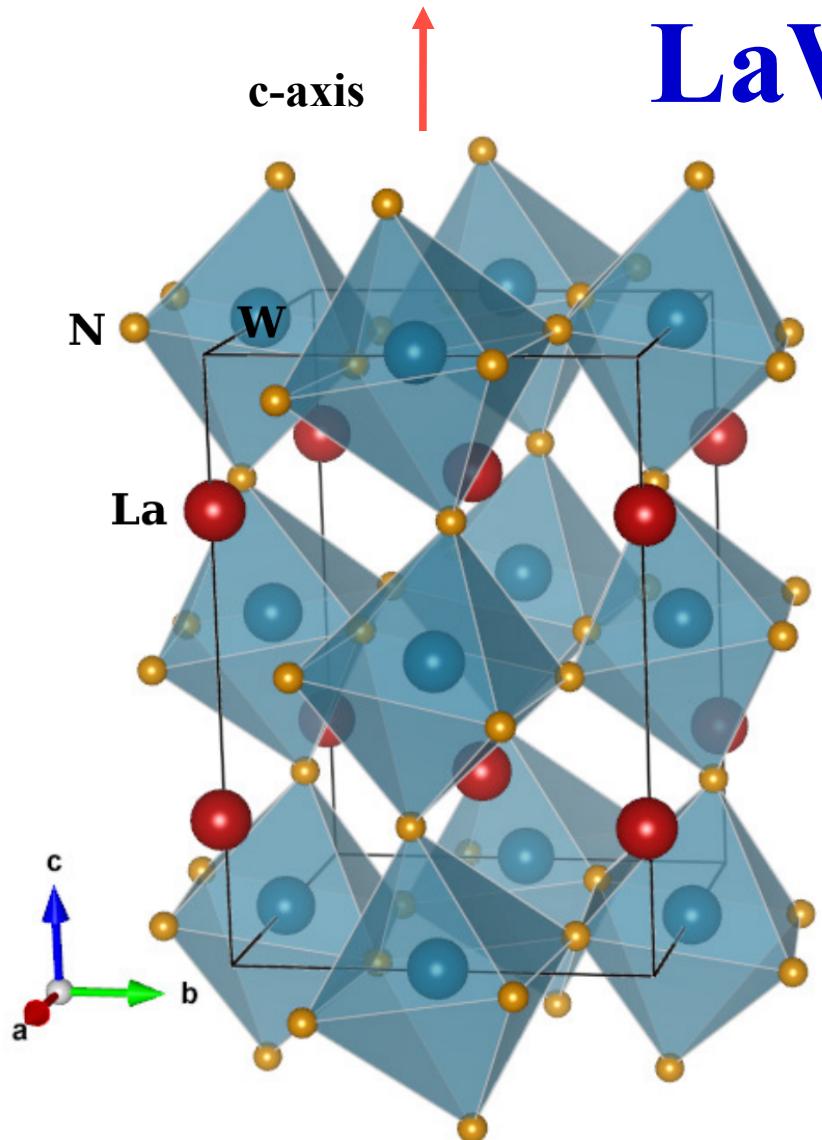
How to design PSH ? Try $\alpha_R = \alpha_D$ RMP (2017) Strained LaAlO₃/SrTiO₃ (001)
 Exploit non-symmorphic Symmetry in some bulk systems ? Nat. Comm (2018)
 Zeeman like splitting of bands induced by electric field ?

Search for Rashba Systems



Ferroelectric Materials:
Reversal of Spin Texture
upon application of electric field





LaWN_3

Pna₂₁ phase

Primitive orthorhombic unit cell
with $a = 5.5703 \text{ \AA}^0$, $c = 5.5992 \text{ \AA}^0$

Space Group = Pna₂₁, No:- 33
(nonsymmorphic)

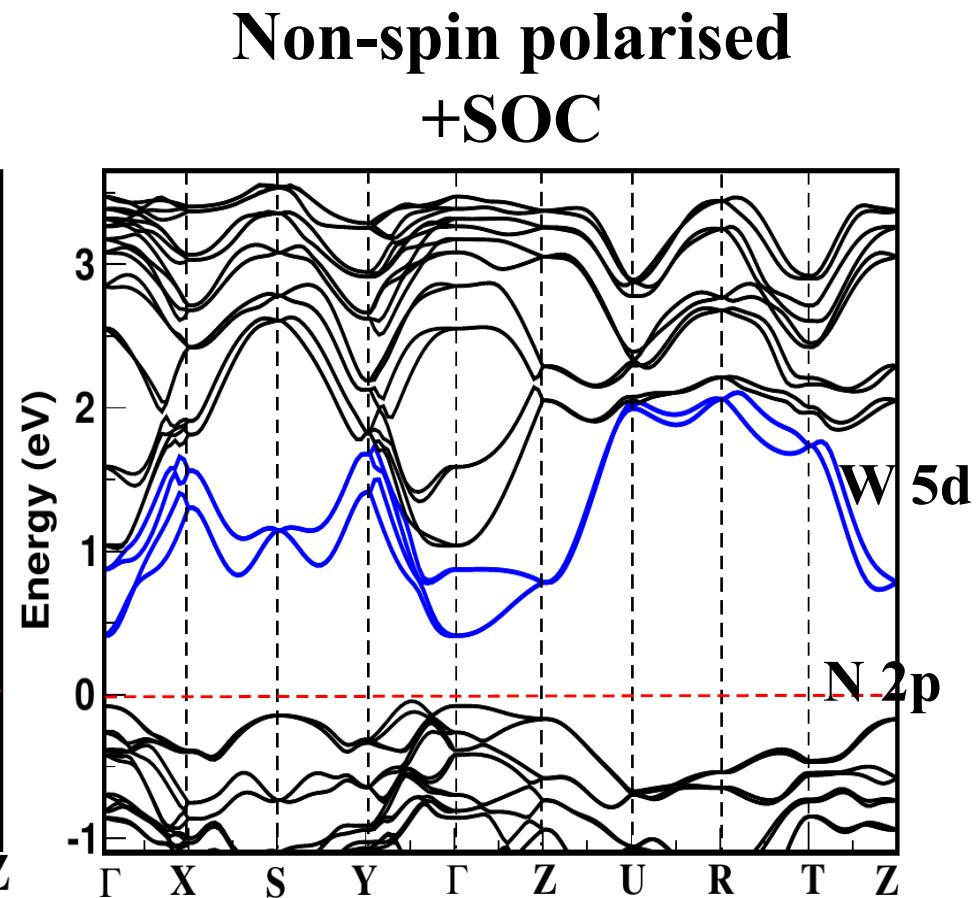
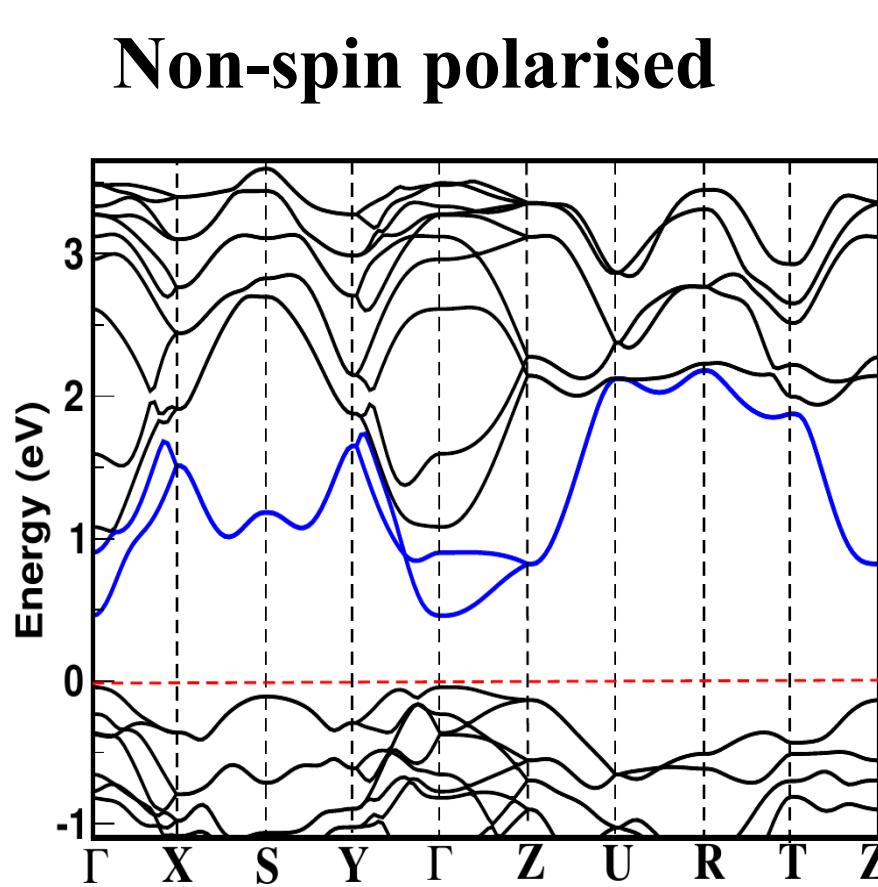
Unit cell contains 4 f.u. W and
N forms octahedral network.

This phase is **non-centrosymmetric**
with **C_{2v}** Point group symmetry.

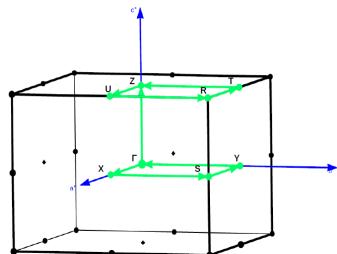
System has ferroelectric polarisation of around **20 $\mu\text{C}/\text{cm}^2$**
along **(001)** direction.

Ref: PRB 95,014111(2017)

Effect of Spin orbit coupling



No splitting along Γ -Z direction of SOC included
bands (parallel to polarisation axis)
Splitting along Γ -X direction
(perp. to polarisation axis)



This is consistent
with Rashba-
Dresselhaus effect

Role of Symmetry

Point group Symmetry (C_{2v}) operations of the system are :

Operation	(k_x, k_y, k_z)	$(\sigma_x, \sigma_y, \sigma_z)$
E	(k_x, k_y, k_z)	$(\sigma_x, \sigma_y, \sigma_z)$
C2	$(-k_x, -k_y, k_z)$	$(-\sigma_x, -\sigma_y, \sigma_z)$
$i\Sigma_x$	$(-k_x, k_y, k_z)$	$(\sigma_x, -\sigma_y, -\sigma_z)$
$i\Sigma_y$	$(k_x, -k_y, k_z)$	$(-\sigma_x, \sigma_y, -\sigma_z)$

Rashba term, $\alpha_R(\sigma_y k_x - \sigma_x k_y)$

Dresselhaus term, $\alpha_D(\sigma_y k_x + \sigma_x k_y)$

Invariant under all symmetry operations

We can expect both Rashba and Dresselhaus coupling in the system

No term containing out of plane spin component is invariant under all symmetry operations

Rashba or Dresselhaus?



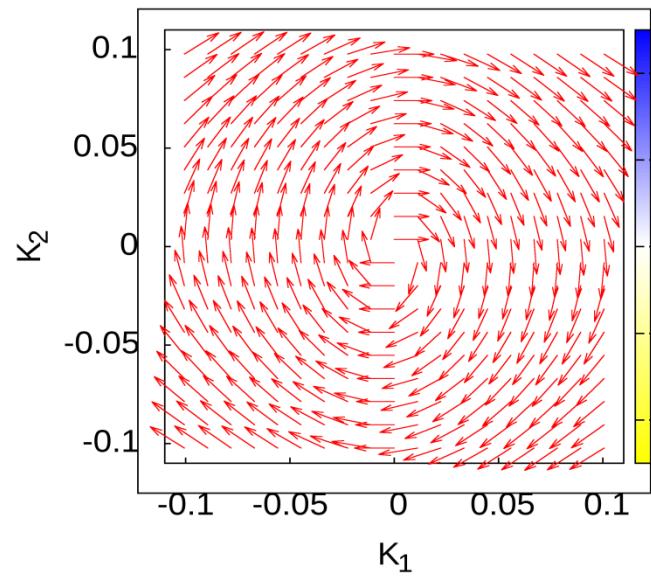
c

Orientation of the spin texture is opposite to each other.
(Helicity is opposite)

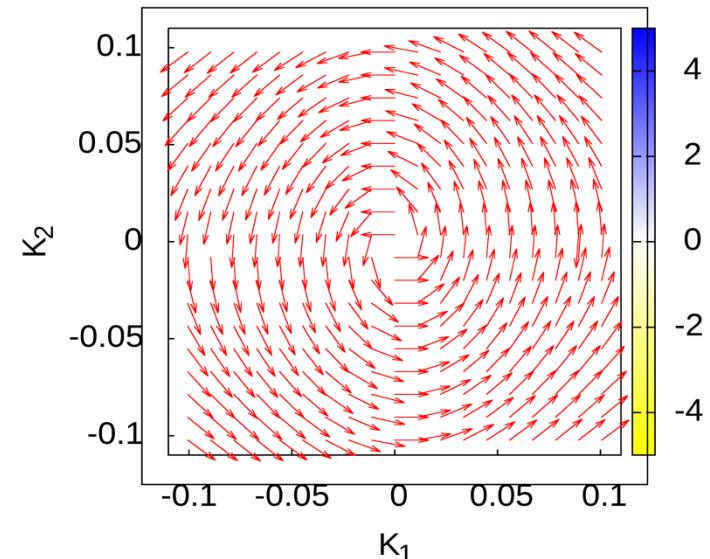


No out of plane component of spin, and spin is always \perp to momentum suggests it is Rashba effect

Inner Branch



Outer Branch



Model Hamiltonian

$$H_\Gamma(k) = E_0(k) + \alpha k_x \sigma_y + \beta k_y \sigma_x$$

Eigenvalue equation of the Hamiltonian,

$$E_\Gamma(k) = E_0(k) \pm \sqrt{\alpha^2 k_x^2 + \beta^2 k_y^2}$$

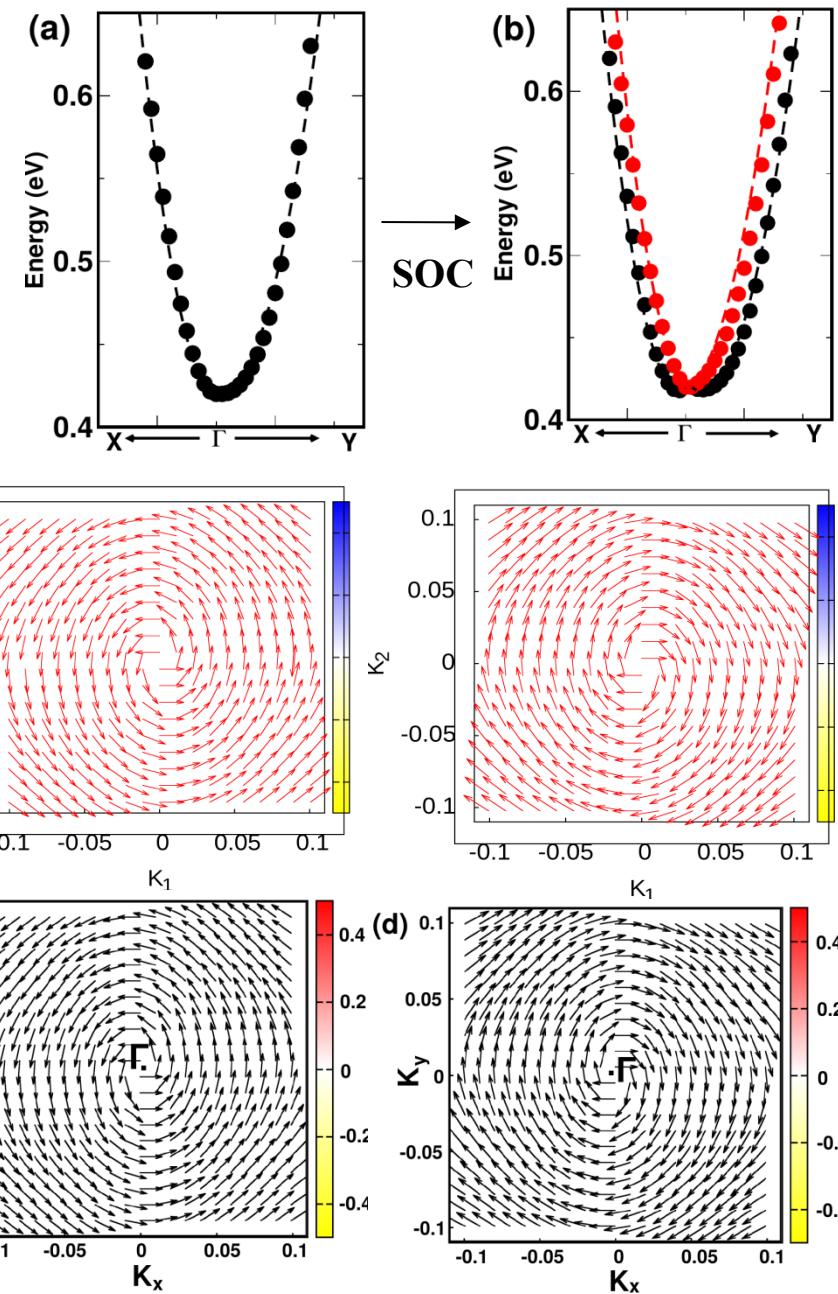
DFT

Where Rashba parameter (α_R) and Dresselhaus parameter $s(\alpha_D)$ fitting DFT bands are:

k.p model

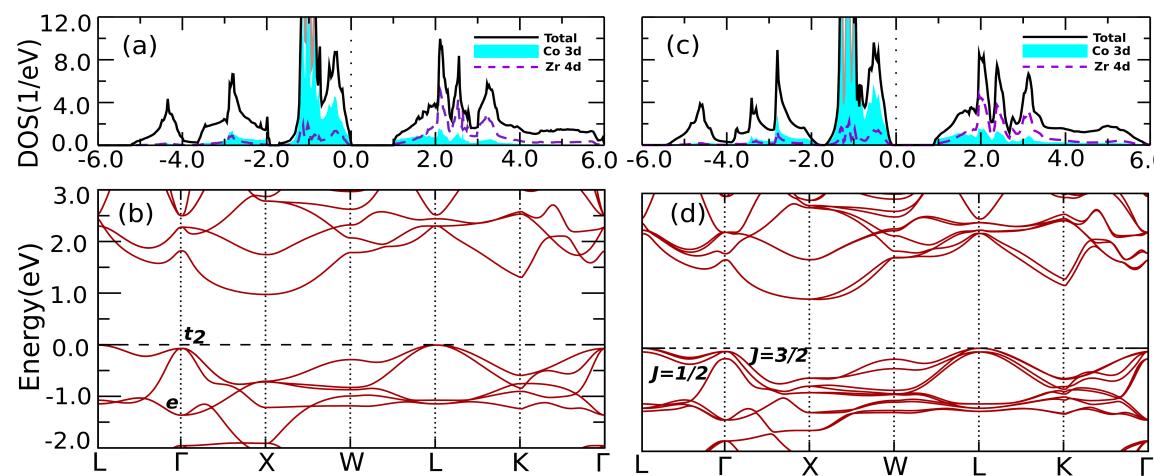
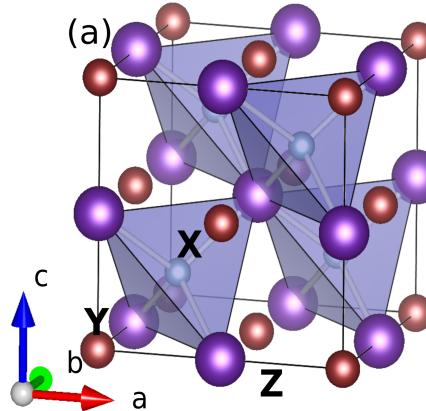
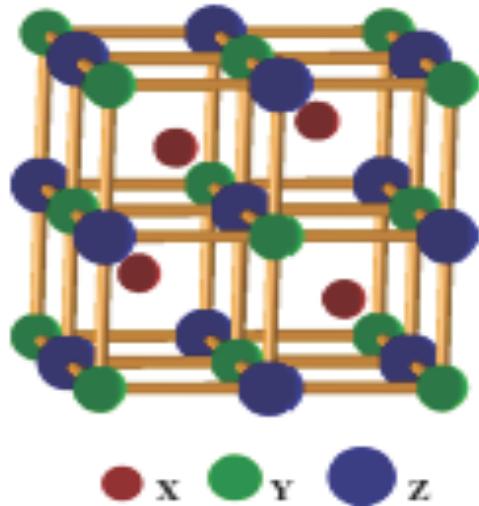
$$\alpha_R = 0.32 \text{ eV.A}^0, \quad \alpha_D = 0.014 \text{ eV.A}^0$$

Clearly Rashba effect dominates over Dresselhaus effect



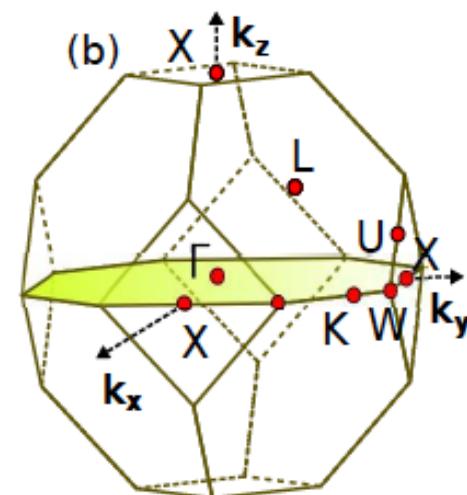
Non CentroSymmetric Half-Heusler Compounds XYZ (CoZrBi)

The unit cell is fcc lattice with three atom as basis.



18-electron half-Heusler CoZrBi
 Co electron configuration [Ar]3d⁷ 4s²
 Zr electron configuration [Kr]4d² 5s²
 Bi electron configuration [Xe]4f¹⁴ 5d¹⁰ 6s² 6p³

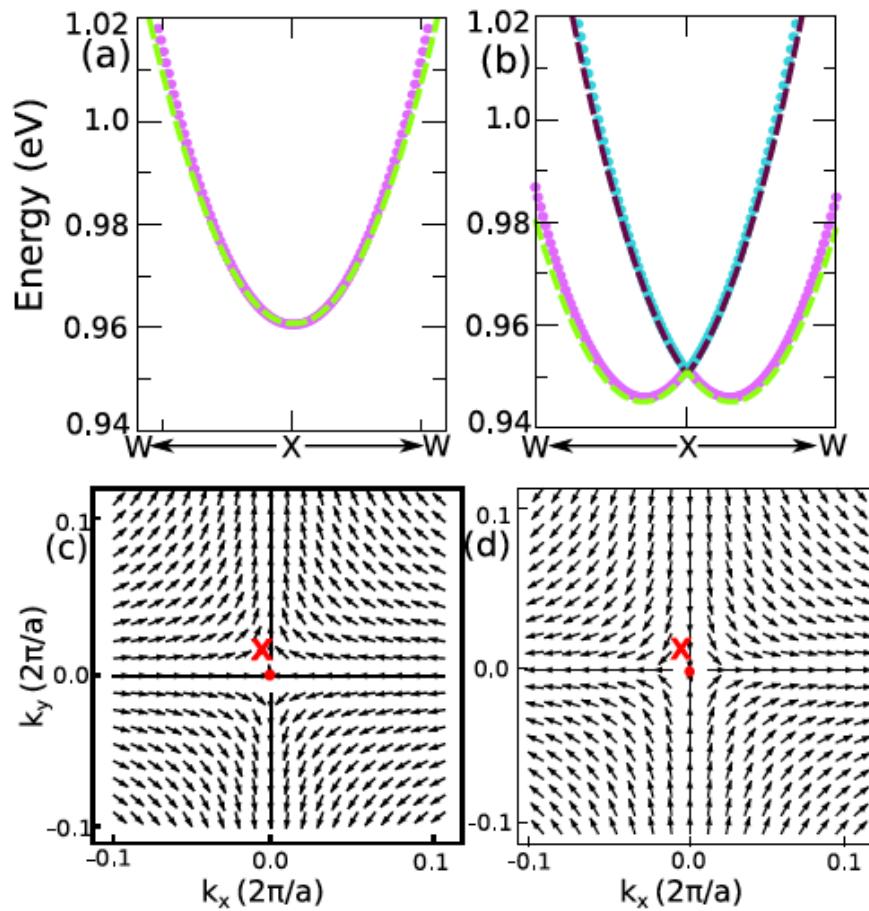
Total number of valence electron = 9+4+5 = 18



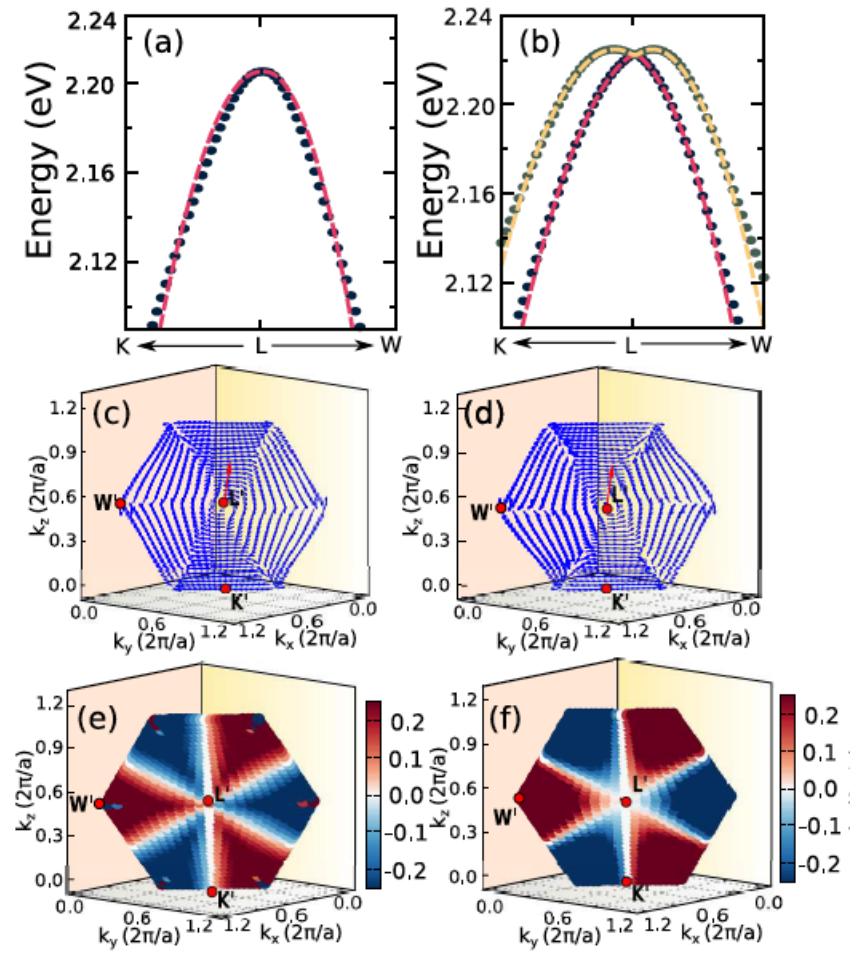
Local wavevector point group symmetry (little group) plays an important role

Dresselhaus and Rashba Effect in non-polar non-centrosymmetric CoZrBi

Non polar X point



Polar L point

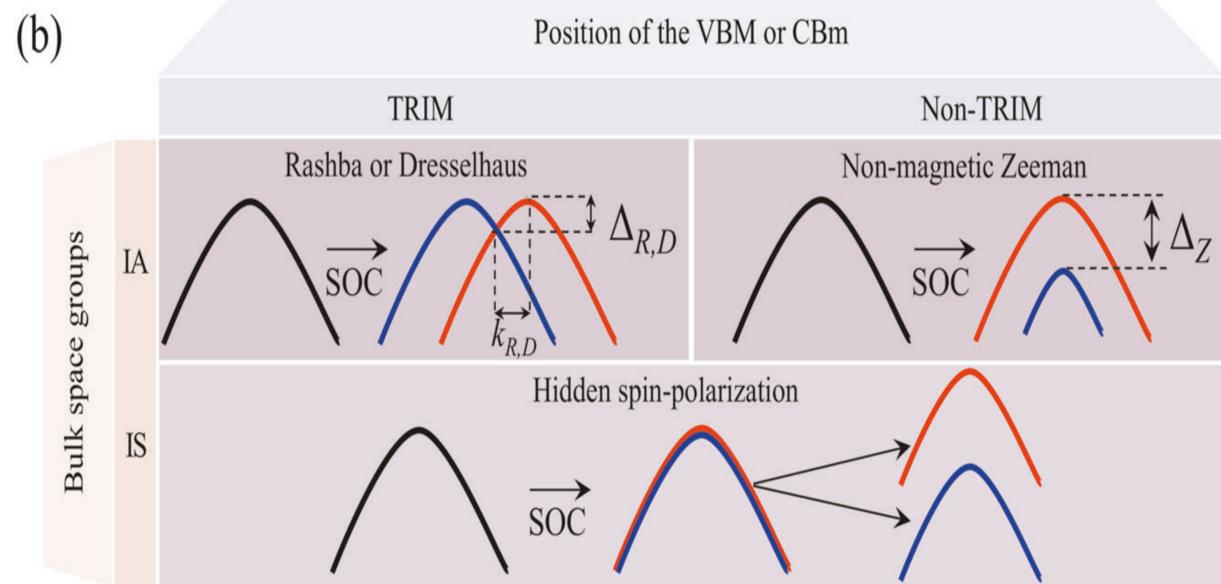
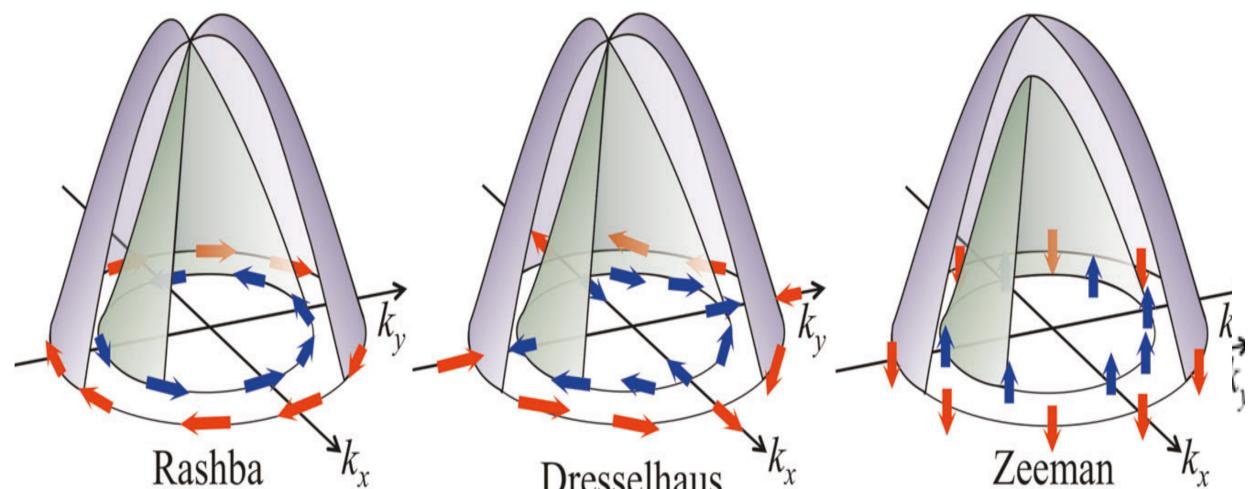
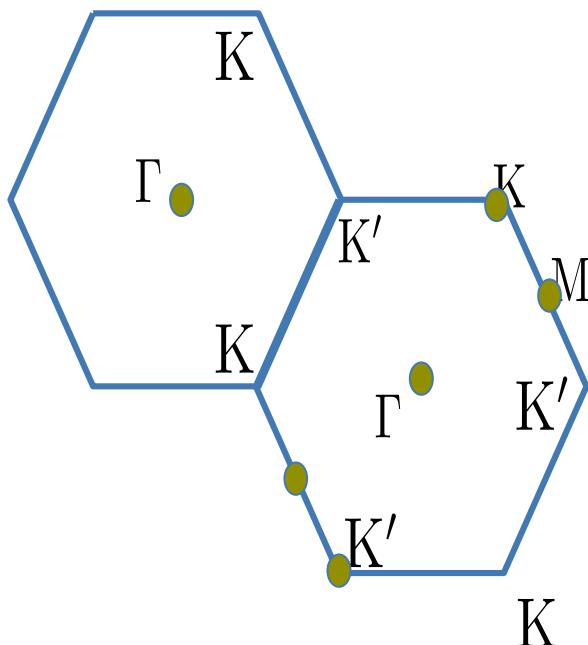


Dresselhaus Effect (CBM)

Crystallographic Space Group Symmetry vs Wave vector point group symmetry
(Little Group)

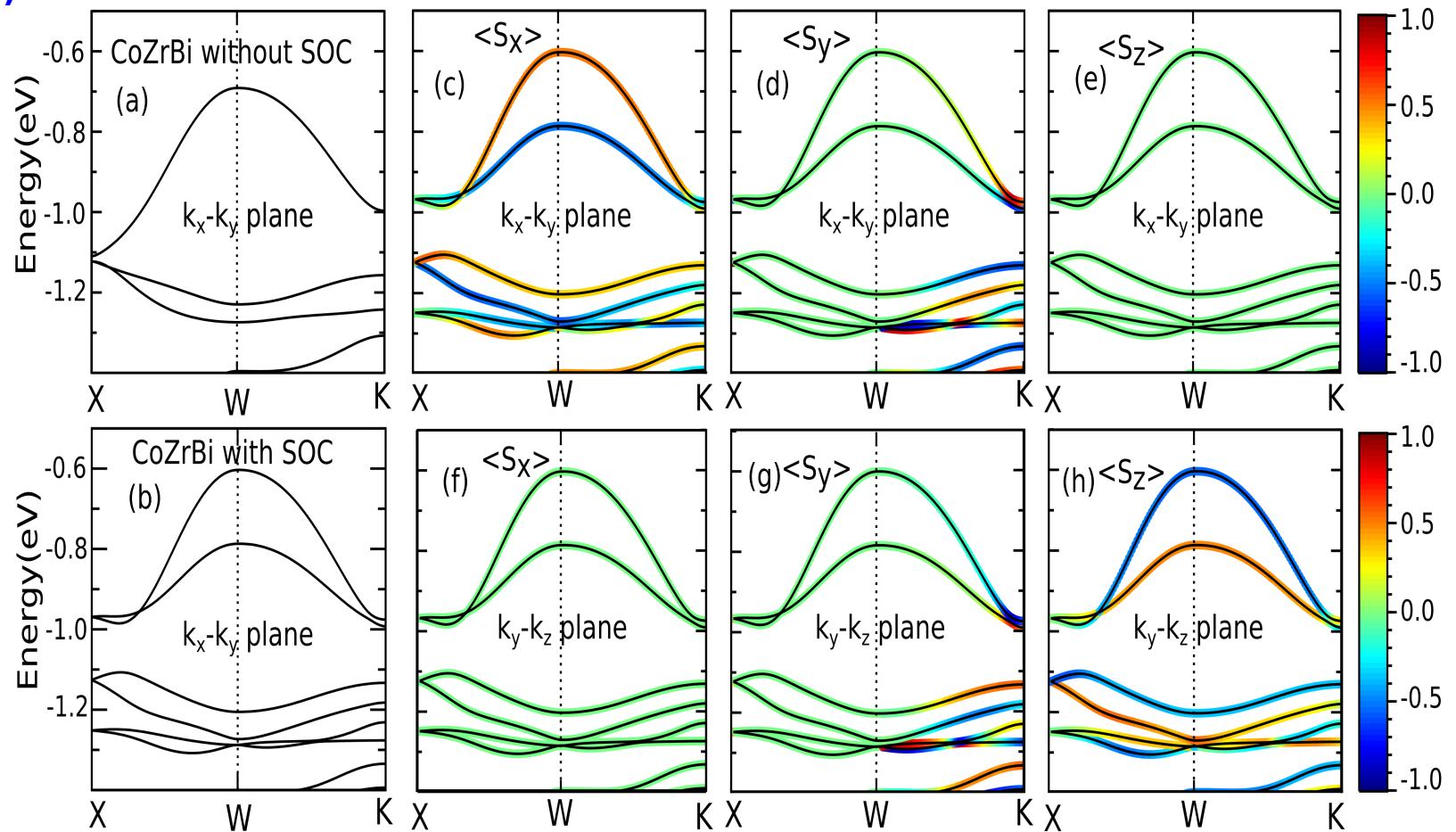
Rashba Effect (CB)

Time Reversal Invariant Momenta(TRIM)



18electron
: (ZrCoBi)

Zeeman Effect



In the k_x - k_y plane along the X-W-K direction, the S_x component of the spin expectation value shows a prominent contribution.

In the k_x - k_y plane along the X-W-K direction, the S_z component of the spin expectation value shows a prominent contribution.

Model Hamiltonian for Zeeman Spin splitting

$$H = H_0 + \vec{\Omega}_D(\mathbf{k}) \cdot \vec{\sigma}$$

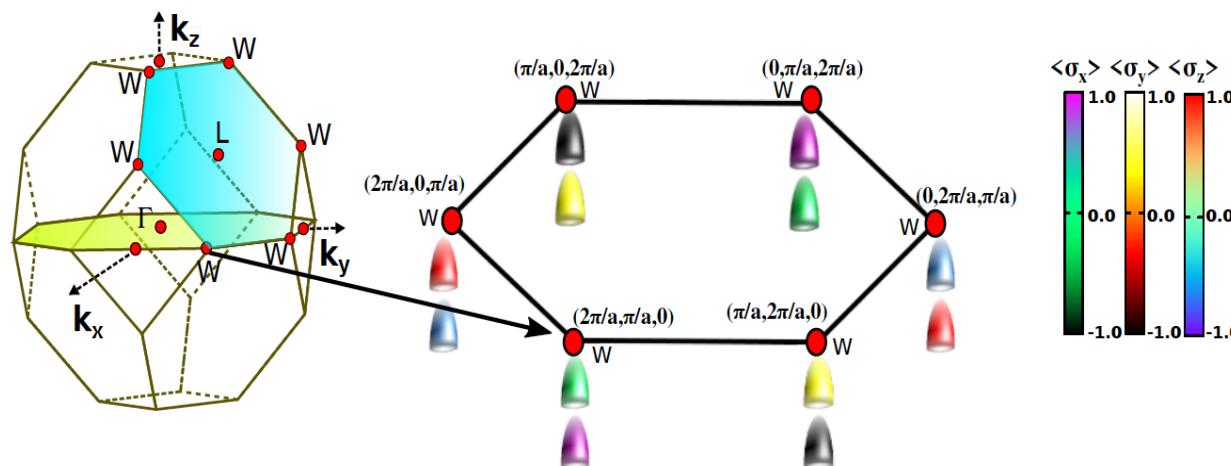
$$H_0 = \frac{\hbar^2(k_x^2 + k_y^2)}{2m^*} \mathbf{1}$$

$$\Omega_D(\mathbf{k}) = \lambda_D(k_x(k_y^2 - k_z^2), k_y(k_z^2 - k_x^2), k_z(k_x^2 - k_y^2))$$

In k_x - k_y plane $k_z=0$. Effective magnetic field along W-X direction is,

$$\Omega_D(\mathbf{k}) \approx (k_x k_y^2, 0, 0) \quad k_x \ll (k_y = \frac{2\pi}{a})$$

S_x expectation value of spin survives along the path X-W-K in k_x - k_y plane



SOC ties spin splitting to the valley index.

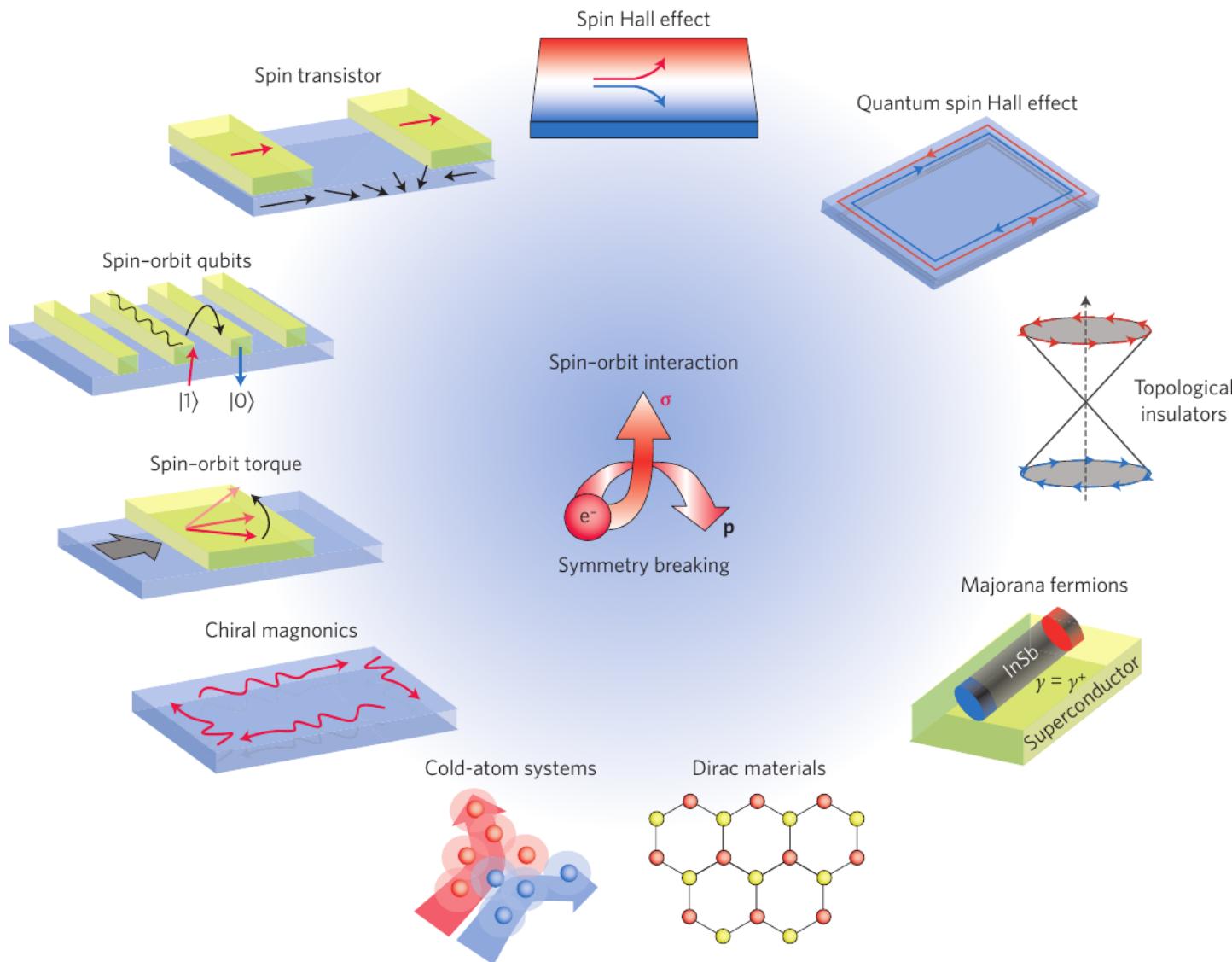
Charge carriers between valleys should have simultaneous spin flip and momentum transfer.

This favors long valley life-time for valleytronics application.

Spin splitting controlled by Electric field

Ideal for spin valleytronics

Emergent Phenomena in Spin Orbit Coupled Systems



Ref: A. Manchon et al Nat. Mat. 14, 871 (2015)

Thank You