Assignment 6: Lorenz attractor

1 Overview

In this assignment you will write a short document about the Lorenz attractor, a classic example of a chaotic dynamical system. You will write code in Python to implement the equations for the Lorenz system in order to generate graphics to include in your document. The system of equations that we will model are as follows:

$$\begin{aligned} \frac{dx}{dt} &= a(y-x) \\ \frac{dy}{dt} &= x(b-z)-y \\ \frac{dz}{dt} &= xy-cz \end{aligned}$$

These equations describe the change of position for a point (x, y, z) in time given some parameters a, b, and c. For the purposes of this assignmnet, you should let $a = 10$, $b = 28$, and $c = \frac{8}{3}$.

2 Tasks

2.1 Write a function `lorenz`

Write a python function `lorenz` that takes as arguments a point in $\mathbb{R}^3$, the parameters a, b, and c, as well as a time step dt. The input represents the point at time t and the output should be the point at $t + dt$. Specifically, it should return $(x + dt * dx, y + dt * dy, z + dt * dz)$ for an input (x, y, z).

2.2 Evolve `lorenz` for 1000 steps

Given some starting point (x_0, y_0, z_0) and your function `lorenz` that evolves the system by one time step dt, iterate the function to produce a sequence of points:

$$seq = [(x_0, y_0, z_0), (x_1, y_1, z_1), \cdots, (x_n, y_n, z_n)]$$

The sequence should be either a single n-by-3 or 3-by-n NumPy array. You should start from $(x_0, y_0, z_0) = (1, 1, 1)$ with a time step $dt = 0.01$.

2.3 Plot a projection of the iterated sequence

Using the PyPlot library from Matplotlib, plot two of the dimensions of the sequence against each other. For example, the y coordinates plotted versus the z coordinates for a sequence starting at $(x, y, z) = (1, 1, 1)$ looks like the plot in Figure 1.

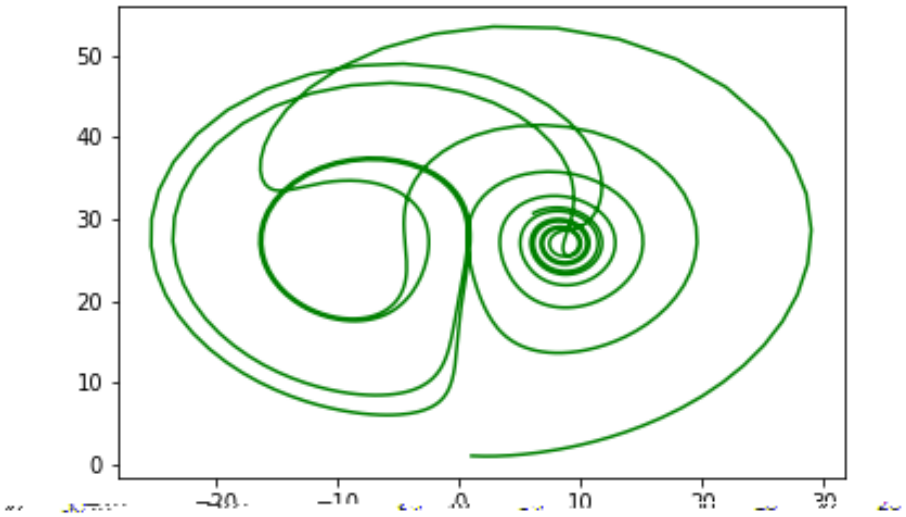

Figure 1: Lorenz attractor.

2.4 Write a short LaTeX report about your work

Write a brief 2-4 page document (10pt font, 1 inch margins) that includes:

- A brief description of the Lorenz attractor. Look it up online and briefly summarize what you find (e.g., Who invented it? Where did it come from?). Use footnotes or BibTex entries to reference your sources.
- Include two figures (no more than 3/4 of the text width in size each) showing a plot of the sequence you produced (e.g., Figure 1) for two different starting positions (x_0, y_0, z_0). Refer to the figures from within your text and indicate what starting points were used for each.
- A brief explanation of how your code works.

3 Turn in

Create a directory `assn6` in your class github repository and place the following files within:

- A jupyter notebook containing your code, plots, and some example outputs.
  - Note: this assignment should not result in a huge amount of code. I would expect that when you are done, you will have around 25-35 lines of code overall. This assignment will require you to think a bit about how to structure your solution though.
- The .tex file for your report as well as the figures that are included by your document.
  - Remember: to include graphics in LaTeX you should use the graphicx package and the includegraphics command.