# TD6 – Rate Models

## Practical Information

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**TD Material**
https://github.com/esther-poniatowski/2223_UlmM2_ThNeuro

**Goals of the TD**

This TD aims to study the **Ring Model**, originally introduced for "orientation coding" in the visual cortex. More generally, this model could apply for any network encoding a circular one-dimensional variable.

**Ring Model**

The model consists of a network of neurons responsive to a one-dimensional stimulus $\theta$ which spans the range $[0, 2\pi]$. Each neuron is characterized by a preferred stimulus value. Therefore, neurons can be conceptually aligned along a ring, their position being assimilated to their preferred stimulus value.

In the limit of a large number of neurons and with a uniform distribution of preferred stimuli in the range $[0, 2\pi]$, the network is assumed to be continuous. Specifically, for each value of $\theta$, there exists one neuron preferring this stimulus value. In this view, the neural activity can be written as a continuous function $m(\theta, t)$.

Each neuron receives two types of inputs :

- External inputs which depend on the neuron's position : $h(\theta)$.
- Recurrent inputs which depend on the activity of the whole network. The connection strength between two neurons is set by the distance between their preferred stimuli $\theta_1$ and $\theta_2$ :

$$J_0 + J_1 \cos(\theta_1 - \theta_2)$$

The activity of the network evolves according to the following dynamics :

$$\frac{\mathrm{d}m(\theta, t)}{\mathrm{d}t} = -m(\theta, t) + f[I(\theta, t)] \tag{1}$$

$$\begin{cases} f(x > 0) = x \\ f(x < 0) = 0 \end{cases} \tag{2}$$

## 1 Input current & Uniform state

① In the general case, express the total input current received by a neuron preferring a stimulus value $\theta$.

The network is submitted to a constant, uniform and positive external current $h(\theta) = h_0$, which is sufficient for inducing a positive, uniform network activity $m(\theta, t) = m_0$.

② In this particular case, express the current received by each neuron. Deduce the equilibrium network activity $m_0$. How does it depend on the connectivity parameters ?

## 2 Description through order parameters

To determine whether this uniform state is stable, the network's activity is assumed to be perturbed around its equilibrium :

$$m(\theta, t) = m_0 + \delta m(\theta, t)$$

In this context, the network's evolution can be studied through the evolution of two order parameters :

$$M(t) = \frac{1}{2\pi} \int_0^{2\pi} \delta m(\theta', t) \, \mathrm{d}\theta' \tag{3}$$

$$C(t) = \frac{1}{2\pi} \int_0^{2\pi} \delta m(\theta', t) e^{i\theta'} \, \mathrm{d}\theta' \tag{4}$$

③ Interpret the order parameters $M$ and $C$.

The perturbation is assumed to be uniform :

$$\delta m(\theta, t) = \epsilon$$

④ Compute the values of $M$ and $C$.

## 3 Bumpy perturbation

Now, the perturbation is assumed to be a small bump centered around an angle $\phi$ :

$$\delta m(\theta, t) = \epsilon \cos(\theta - \phi) = \epsilon \, \frac{e^{i(\theta-\phi)} + e^{-i(\theta-\phi)}}{2}$$

⑤ Compute the values of $M$ and $C$.

⑥ Linearize the dynamics of the activity around $m_0$ and express it as a function of $M(t)$ and $C(t)$.

⑦ Derive the differential equations governing the evolution of the order parameters.

⑧ Determine the conditions under which the uniform activity stable. What happens when either of these conditions is not met ?

Consider the case $J_0 < 1$, $J_1 < 2$.
The network is submitted to an external input with weak modulation :

$$h(\theta) = h_0 + \epsilon \cos(\theta) \qquad \epsilon \ll 1$$

⑨ Determine is the evolution of activity profile of the network through the order parameters.

⑩ Justify that the activity profile at equilibrium could a priori be written under the form :

$$m(\theta, t) = m_0 + m_1 \cos(\theta)$$

⑪ Determine conditions on the connectivity parameters so that the network amplifies the input, i.e.

$$\frac{m_1}{m_0} > \frac{\epsilon}{h_0}$$