Intersect Algorithm

1. Generating rays
   1.1) $p_{film}(p_x, p_y, p_z) = (2\frac{x}{x_{max}} - 1, 1 - 2\frac{y}{y_{max}}, -1)$
   1.2) $(M_{scale}M_{rot}M_{trans})^{-1} = M_{trans}^{-1}M_{rot}^{-1}M_{scale}^{-1}$
   To intersect with objects in pre-normalization world space, it's easier to have direction vector from eye to point on film plane to also be in pre-normalization world space. We need the transformation to transform points in canonical view volume back to untransformed world space.
   1.3) $ray = p_{eye} + t\frac{p_{world} - p_{eye}}{||p_{world} - p_{eye}||}$

2. Cone-Ray Intersection
   2.1) cone body
   Let $Q = (x, y, z)$. Equation of unit cone is: $f(Q) = x^2 + z^2 - g(y)^2 = 0$
   Since $-0.5 \leq y \leq 0.5$, $g(y) = (1/4 - y/2)^2$.
   Now, letting $(x, y, z) = (p_x + d_x t, p_y + d_y t, p_z + p_z t)$:
   $f(Q) = (p_x + d_x t)^2 + (p_z + d_z t)^2 - ((1/4 - p_y/2) - d_y t/2)^2 = 0$
   $= p_x^2 + 2d_x p_x t + d_x^2 t^2 + p_z^2 + 2d_z p_z t + d_z^2 t^2 - (1/4 - p_y/2)^2 + (1/4 - p_y/2)d_y t - d_y^2 t^2/4 = 0$
   $(d_x^2 + d_z^2 - d_y^2/4)t^2 + (2d_x p_x + 2d_z p_z + 1/4 d_y - d_y p_y/2)t + (p_x^2 + p_z^2 - 1/16 + 1/4 p_y - p_y^2/4) = 0$

   ———ANSWER———
   Using quadratic equation:
   $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where
   $a = d_x^2 + d_z^2 - d_y^2/4$
   $b = 2d_x p_x + 2d_z p_z + 1/4 d_y - d_y p_y/2$
   $c = p_x^2 + p_z^2 - 1/16 + 1/4 p_y - p_y^2/4$
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   2.2) cone cap
   Letting $y = -1/2$ and $x^2 + z^2 - 1/4 \leq 0$:
   $p_y + t d_y = -1/2$

   ———ANSWER———
   $t = \frac{-1/2 - p_y}{d_y}$
   and t should satisfy following inequality: $(p_x + t d_x)^2 + (p_z + t d_z)^2 - 1/4 \leq 0$
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   Here, we must note that we could obtain multiple values for t. However, the smallest non-negative t represents the intersection nearest to eye with an object.

3. Illuminating Samples
   3.1) $\vec{n}_{world} = (M^{-1})^t \vec{n}_{object}$
   3.2) $\vec{n} \cdot \vec{L}$ is equivalent to cosine of incident angle of light (angle between light direction and normal). It accounts for reduction in light intensity due to incident angle.

4. Finally...
   4.1) Lighting is the process of computing the colors of points as seen by viewer, while shading is a process that involves interpolation of color of points using the colors of points with known illumination.