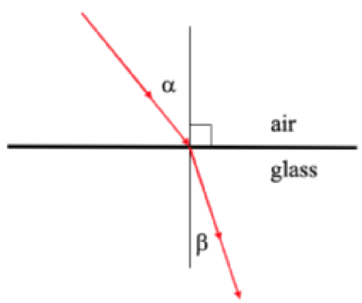
Report: Index of Refraction using Snell's law in Python

INTRODUCTION:

This report will provide a summary of the formulas used in, and the process of, analyzing the properties of our new glass product and its index of refraction. Data analysis is done using Python using basic error propagation and χ^2 fitting.

SUMMARY OF SNELL'S LAW:

When a light hits a glass interface at an angle α, it refracts and results in the ray with an angle β. These angles are related by Snell's Law, where n are the indices of refraction; n_{air} is assumed to be 1 and n_{glass} is the desired index of refraction of which I have found an acceptable value for:



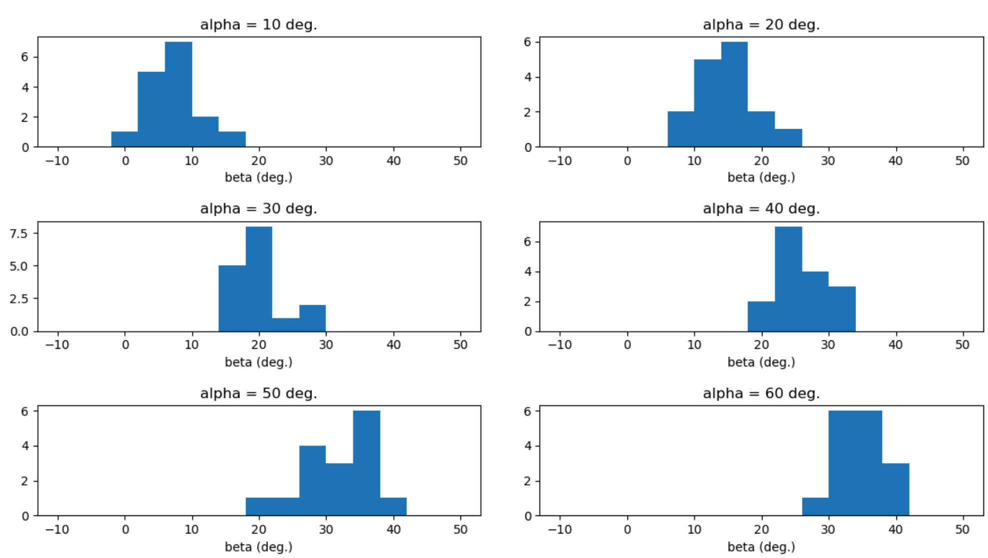
$$n_{air}\sin(\alpha) = n_{glass}\sin(\beta)$$

$$\sin(\beta) = \sin(\alpha)/n_{glass}$$

DATA TAKEN AND HISTOGRAMS

The data to be analyzed includes 6 sets of measurements, where each set is represented by a different α value and returns sixteen β measurements, giving a total of 96 measurements.

Making histograms of the data for each α set yields the following:



From this we can see the gradual shift in the mean of β as α changes.

COMPUTING VALUES FOR A SNELL'S LAW FIT

To prepare the data for a Snell's Law fit in in order to find n_{glass}, I had to find the values for $sin(\alpha)$ and $sin(\beta)$. First, all data values were converted to radians. I took the mean value of β for each 6 datasets, the standard deviation of β for each individual measurement of each dataset, then the standard deviation of the mean, or $\sigma(\beta)$ for each dataset, which was done by dividing the individual standard deviation by the root of 16 (as there are 16 data points in each set).

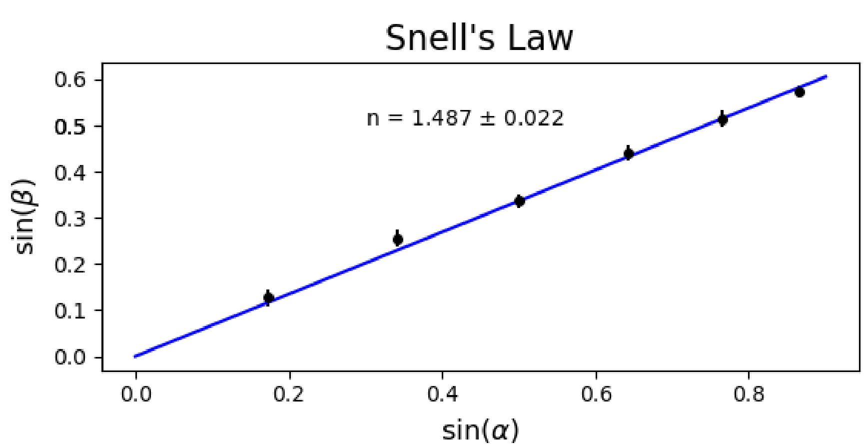
Now possessing the mean β and it's uncertainty, I propagated these twelve values through the sine function to get $sin(\beta)$ and $\sigma(sin(\beta))$, and made a table of all the relevant values:

Table of measurements (in Radians)

Alpha	Sin(Alpha)	Beta (mean)	Sin(Beta)	Unc. of Sin(Beta)
0.1745	0.1736	0.1275	0.1272	0.0187
0.3491	0.3420	0.2578	0.2549	0.0178
0.5236	0.5000	0.3433	0.3366	0.0154
0.6981	0.6428	0.4561	0.4404	0.0166
0.8727	0.7660	0.5396	0.5138	0.0180
1.0472	0.8660	0.6108	0.5735	0.0124

SNELL'S LAW FITTING

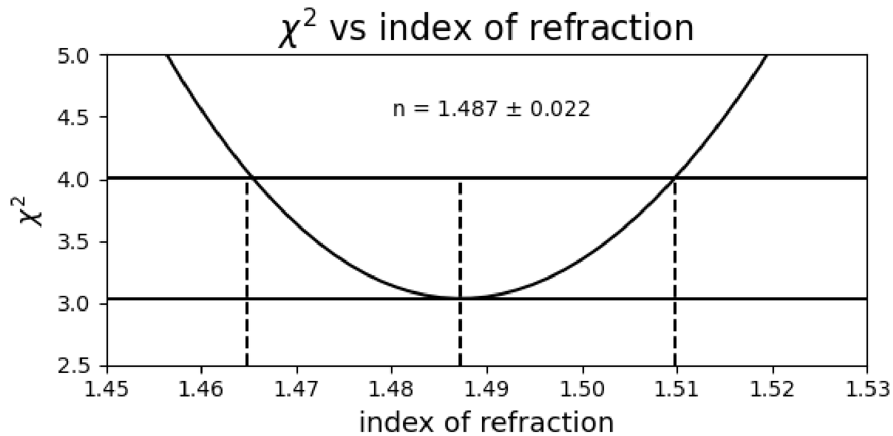
Next, I plotted the 6 $sin(\beta)$ points and the corresponding $\sigma(sin(\beta))$ errors, against $sin(\alpha)$, and created an algorithm to find the best fit linear line that would take into account both the points and their unique errors. The best estimate for the index of refraction is one over the slope of this line according to the Snell's Law formula. Since the algorithm uses n_{glass} as a parameter, this value is found when the fitting line is created. The uncertainty of the index of refraction is also provided:



DATA-FITTING AGREEMENT

To illustrate how well the fitted index of refraction agrees with the data given, I did a χ^2 analysis of the $sin(\beta)$ values generated by the new n_{glass} vs. the $sin(\beta)$ values from the data.
The χ^2 is equal to 3.0349, and with 5 degrees of freedom (due to 6 points minus 1 fitting parameter), the $P(\chi^2, v)$ value is 0.6946. To put this value into perspective, a value anywhere between 0.05 and 0.95 indicates a good agreement, so my value of the index of refraction is a good representation of the data.

A graph showing the χ^2 vs. the index of refraction is shown below. This illustrates how, as the 'guess' for the index moves farther from the accepted value, the χ^2 gets 'worse'. The middle dotted line shows the location of the best index of refraction, while the two lines parallel to it show the error of this value. As seen, the χ^2 curve increases to an unacceptable value as the index becomes less accurate:



CONCLUSION

The final acceptable value for the index of refraction, or n_{glass} is:

$$n_{glass} = 1.4873 \pm 0.0225$$