EMPIRICAL MODE DECOMPOSITION

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separate signal into components to gain new insight into the inherent features

component, i.e., a portion of
the complete signal

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signal dictates decomposition,
adaptive basis system

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the complete signal

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separate signal into components to gain new
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EMD vs other methods

Objective: data decomposition to obtain components of the data which help you analyze/ understand data inherent features

Other commonly used methods:

- Fourier Transform (FT)/ Wavelet Transform (WT)

- Singular Value Decomposition (SVD)
  - Principal Component Analysis (PCA)
  - Proper Orthogonal Decomposition (POD)

- Dynamical Mode Decomposition (DMD)

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EMD vs other methods

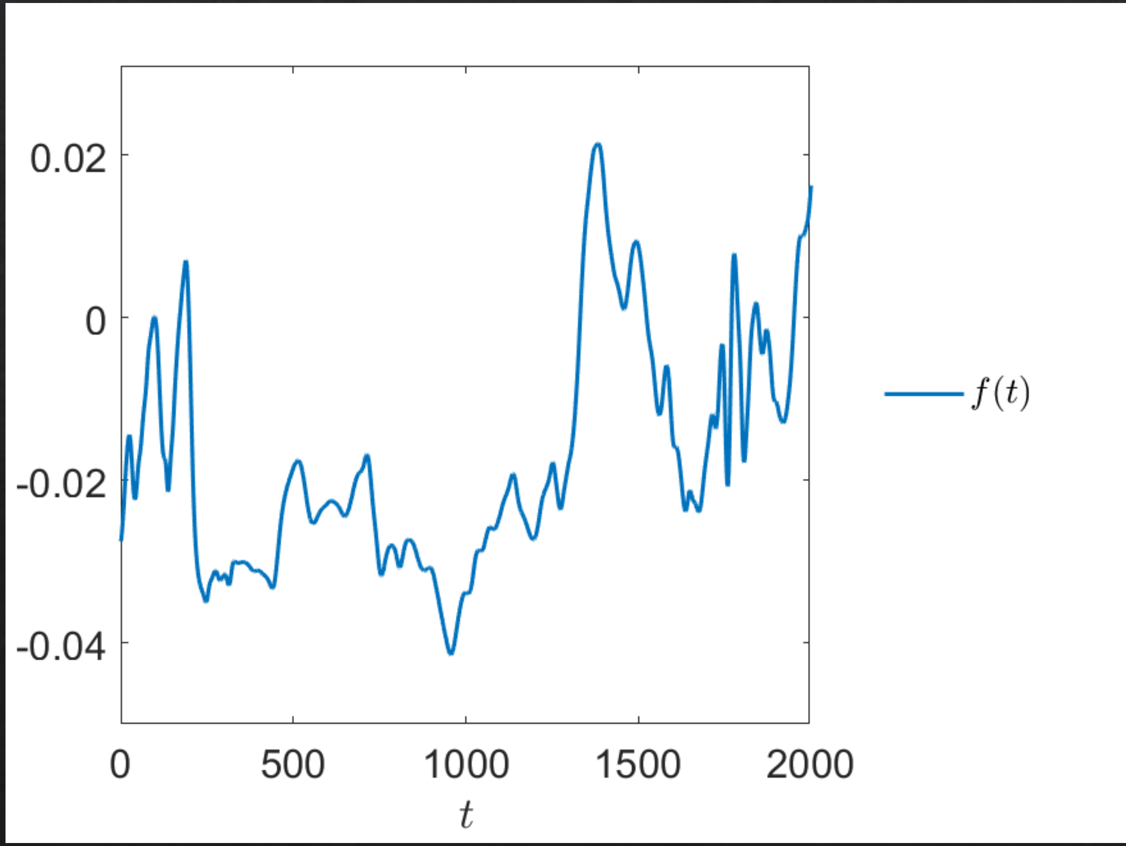
Objective: data decomposition to obtain components of the data which help you analyze/understand data inherent features

Properties of EMD:

- Ability to process non-linear and non-stationary data
- Basis system dictated by data (no prescribed system)
  - Modes adaptively biased towards locally dominant frequencies/scales
  - Physically meaningful modes

The starting point

of the one-dimensional, univariate EMD



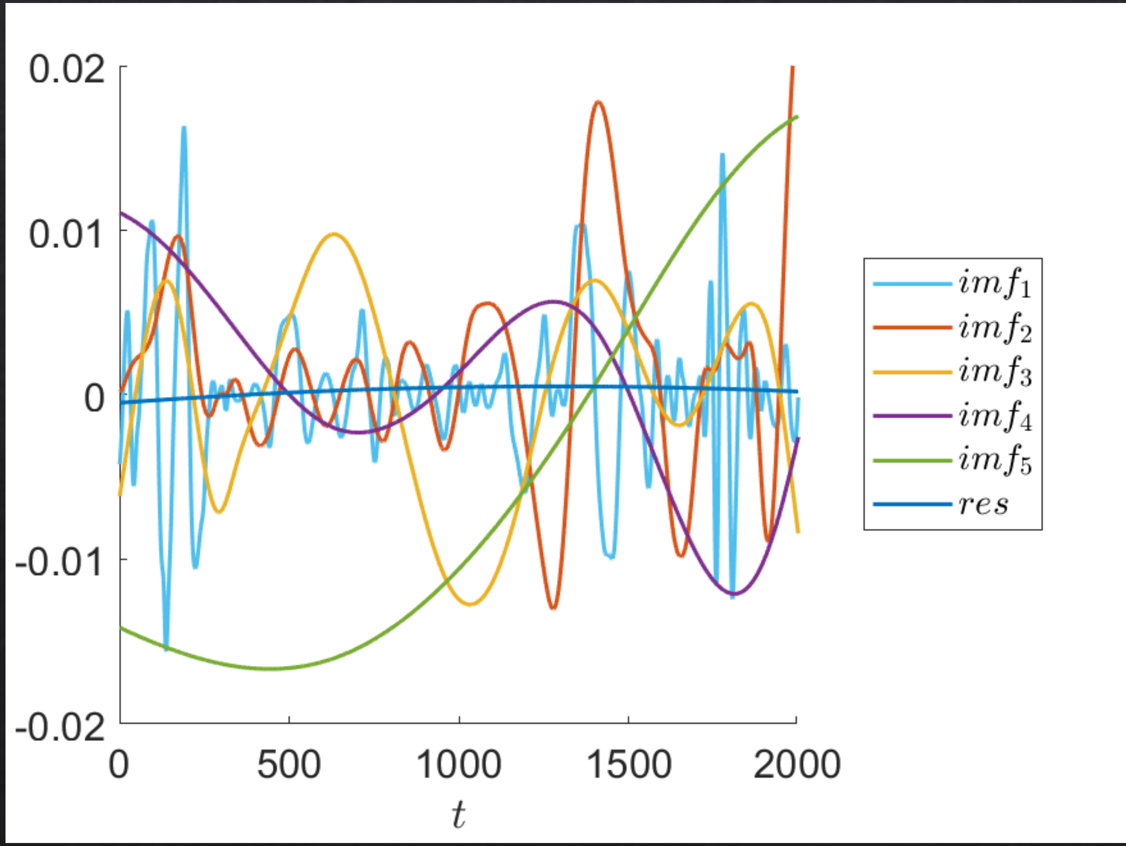
Huang et al. (1998). *The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis*. Proceedings of the Royal Society of London. Series A: mathematical, physical and engineering sciences, 454(1971), 903-995.

Starting point:
a one-dimensional, univariate signal $f(t)$*

*it could equally be a spatial signal

The outcome

of the one-dimensional, univariate EMD



The EMD provides Intrinsic Mode Functions (IMFs) and a residual.

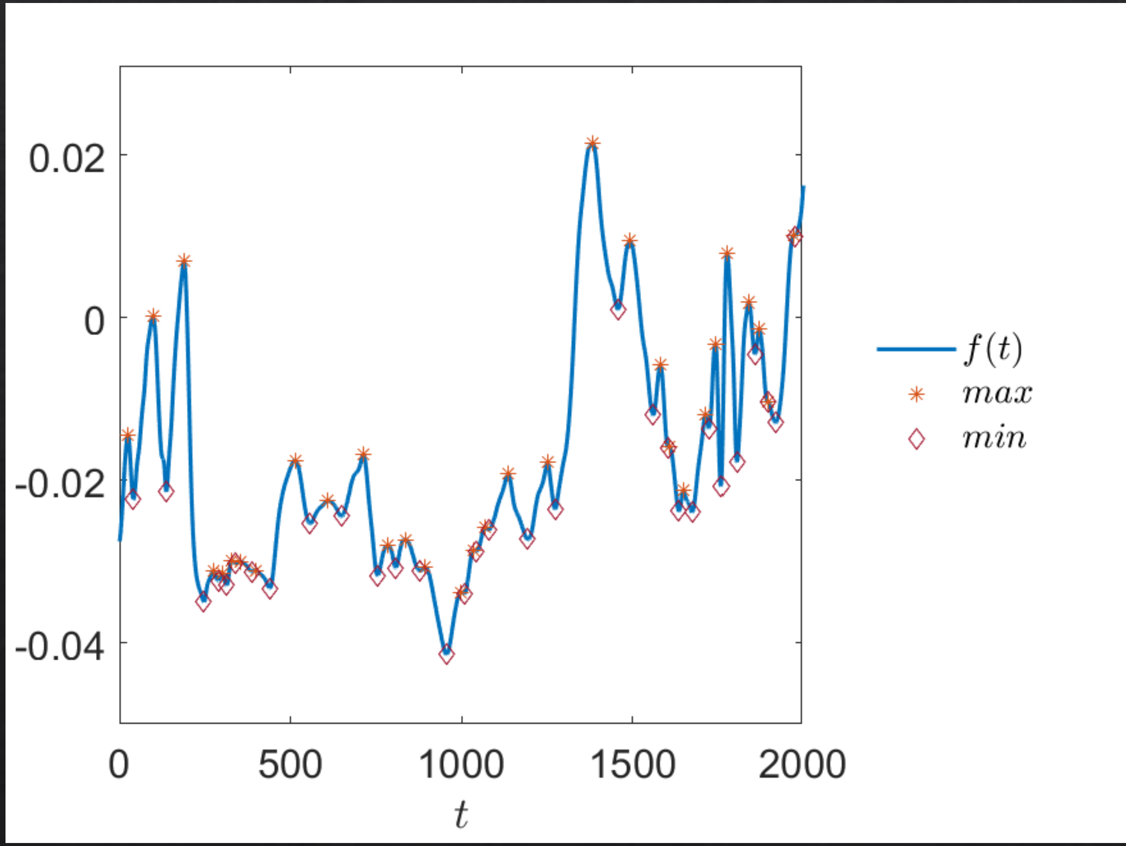
A higher mode number contains larger scales/ smaller frequencies.

The sum of all modes and the remaining residual resemble the original signal.

$$f(t) = \sum_i imf_i + res$$

High-level overview

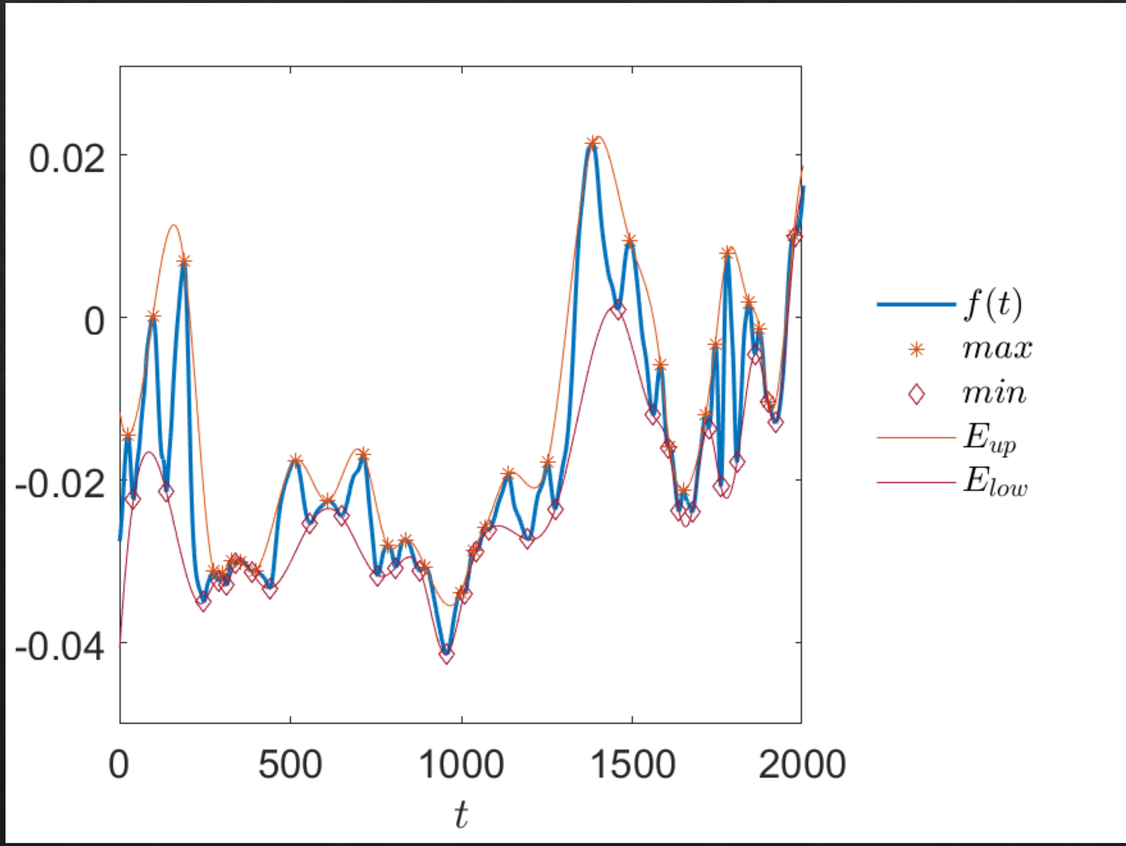
of the one-dimensional, univariate EMD



1. Find local extrema of the signal

High-level overview

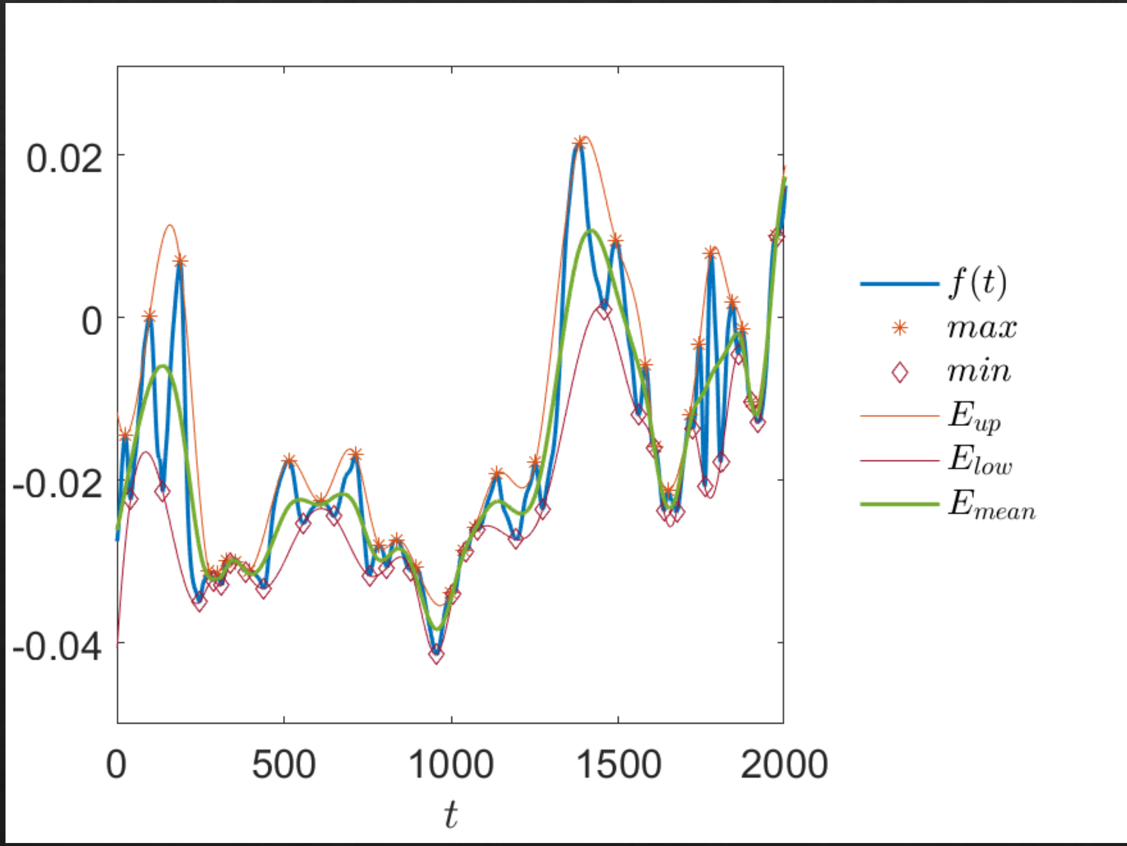
of the one-dimensional, univariate EMD



1. Find local extrema of the signal
2. Fit maxima and minima to an individual envelope
   - Maxima → $E_{up}(t)$
   - Minima → $E_{low}(t)$

High-level overview

of the one-dimensional, univariate EMD



1. Find local extrema of the signal
2. Fit maxima and minima to an individual envelope
   – Maxima → $E_{up}(t)$
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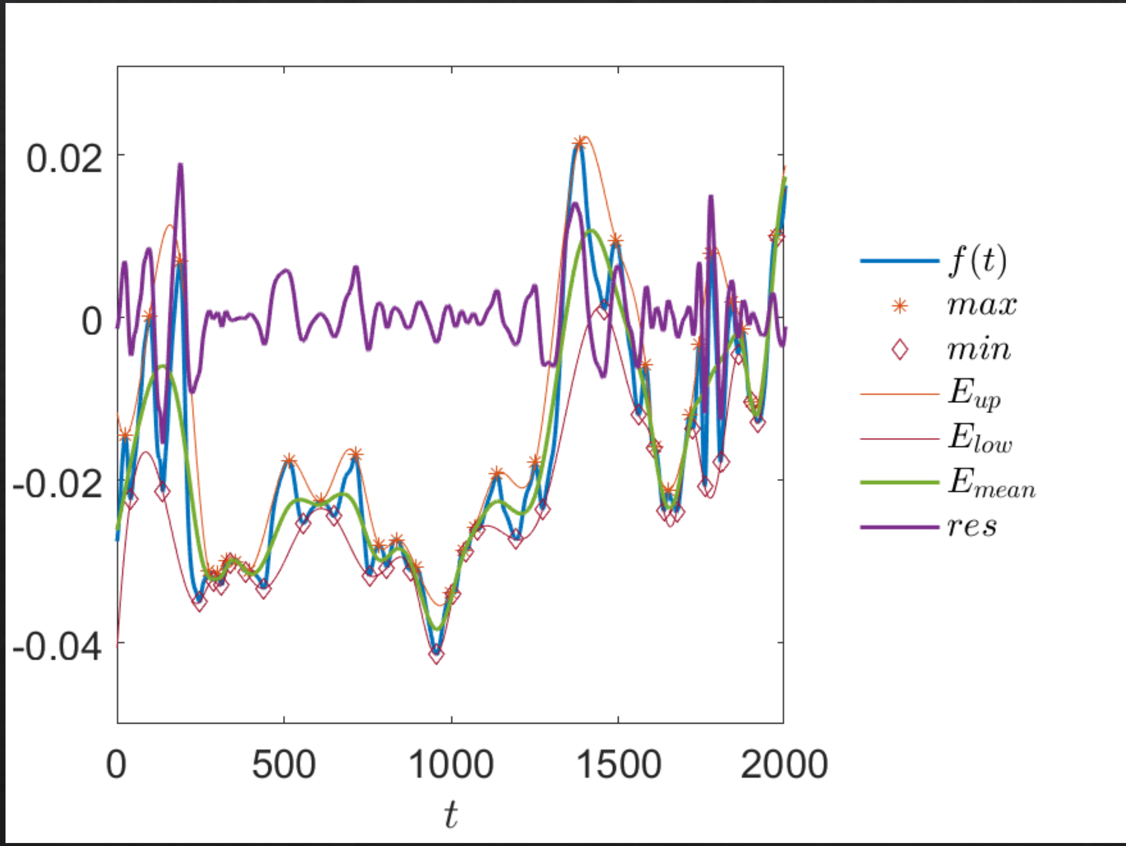
3. Determine mean of upper and lower envelope
   $E_{mean}(t) = (E_{up}(t) + E_{low}(t))/2$

High-level overview

of the one-dimensional, univariate EMD



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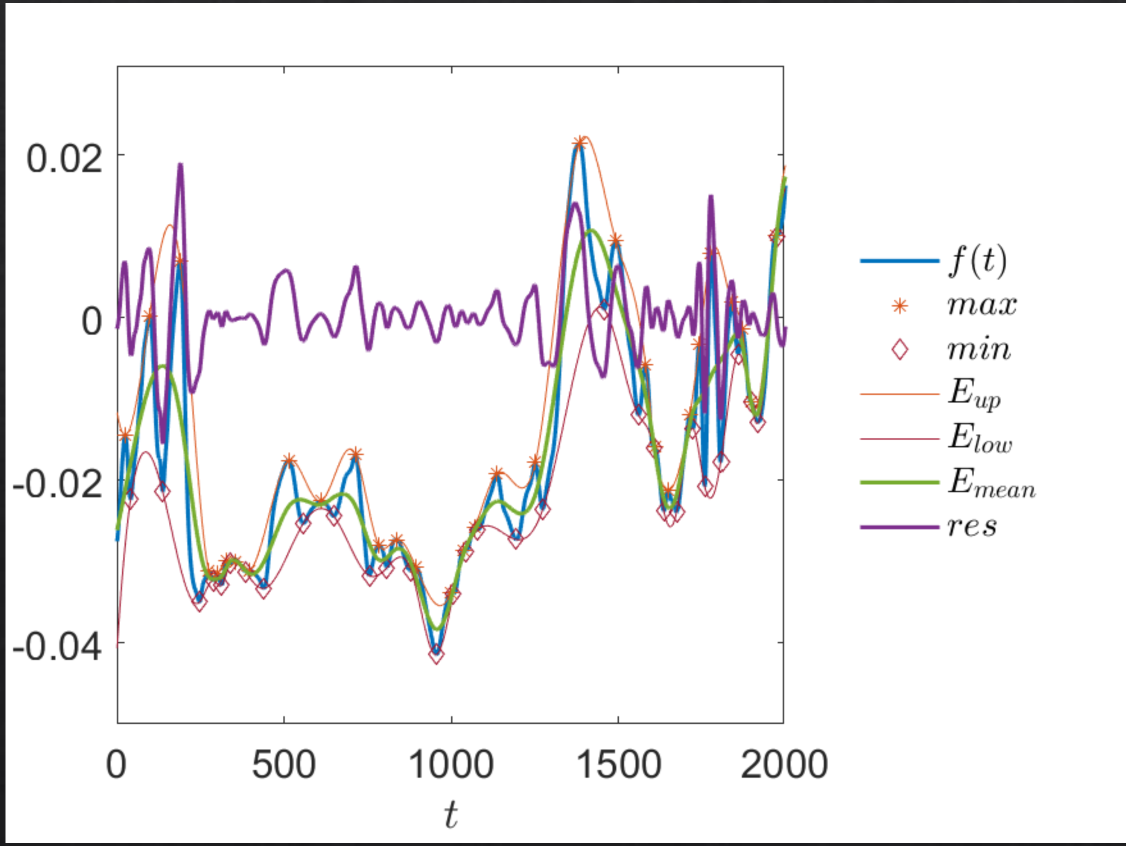
3. Determine mean of upper and lower envelope
   $E_{mean}(t) = (E_{up}(t) + E_{low}(t))/2$
4. Determine residual
   $res(t) = f(t) - E_{mean}(t)$

High-level overview

of the one-dimensional, univariate EMD



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5. Check stopping criterion
   $\sum_t \frac{(res(t)-f(t))^2}{f(t)^2} < \epsilon$

High-level overview

of the one-dimensional, univariate EMD

The residual does not satisfy the IMF properties yet → we need another iteration using $res(t)$

$res(t)$

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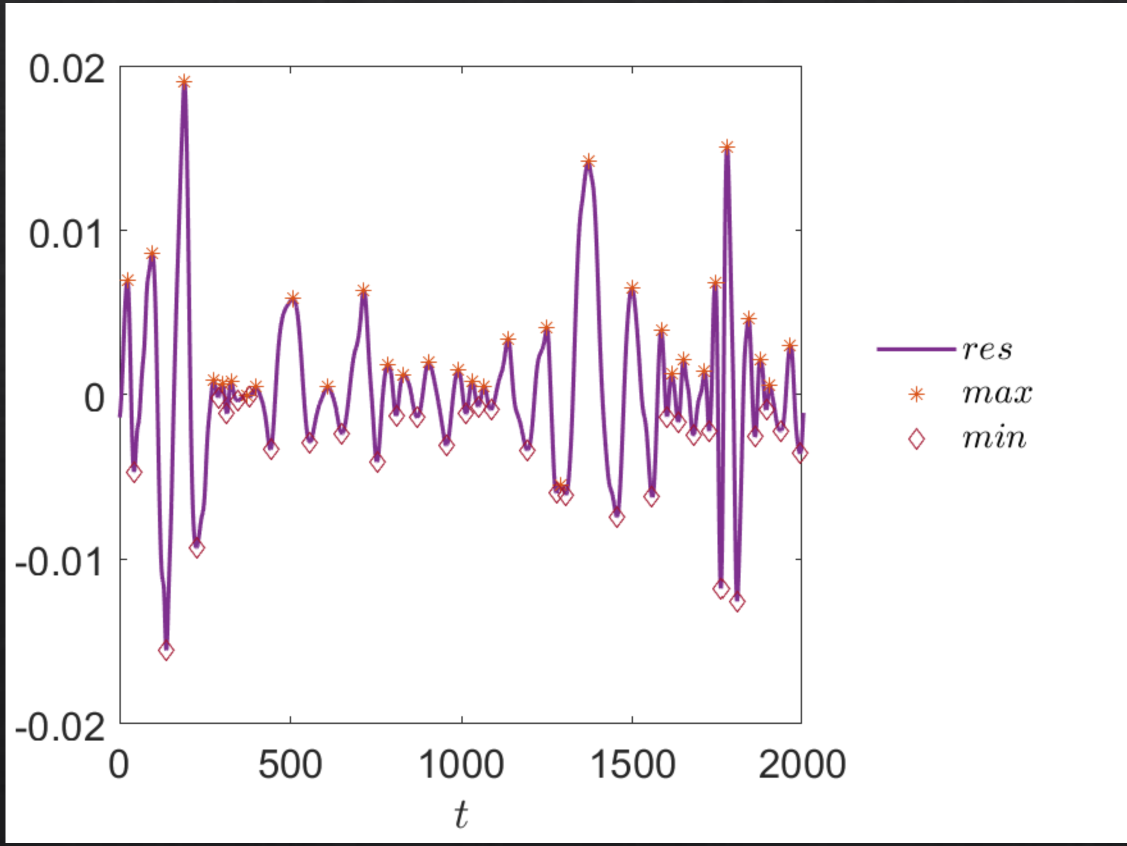
   $res(t) = f(t) - E_{mean}(t)$
5. Check stopping criterion
   $\sum_t \frac{(res(t)-f(t))^2}{f(t)^2} < \epsilon$

if false

High-level overview

of the one-dimensional, univariate EMD



$res(t)$

1. Find local **extrema** of the signal
2. Fit maxima and minima to an individual envelope
   - Maxima → $E_{up}(t)$
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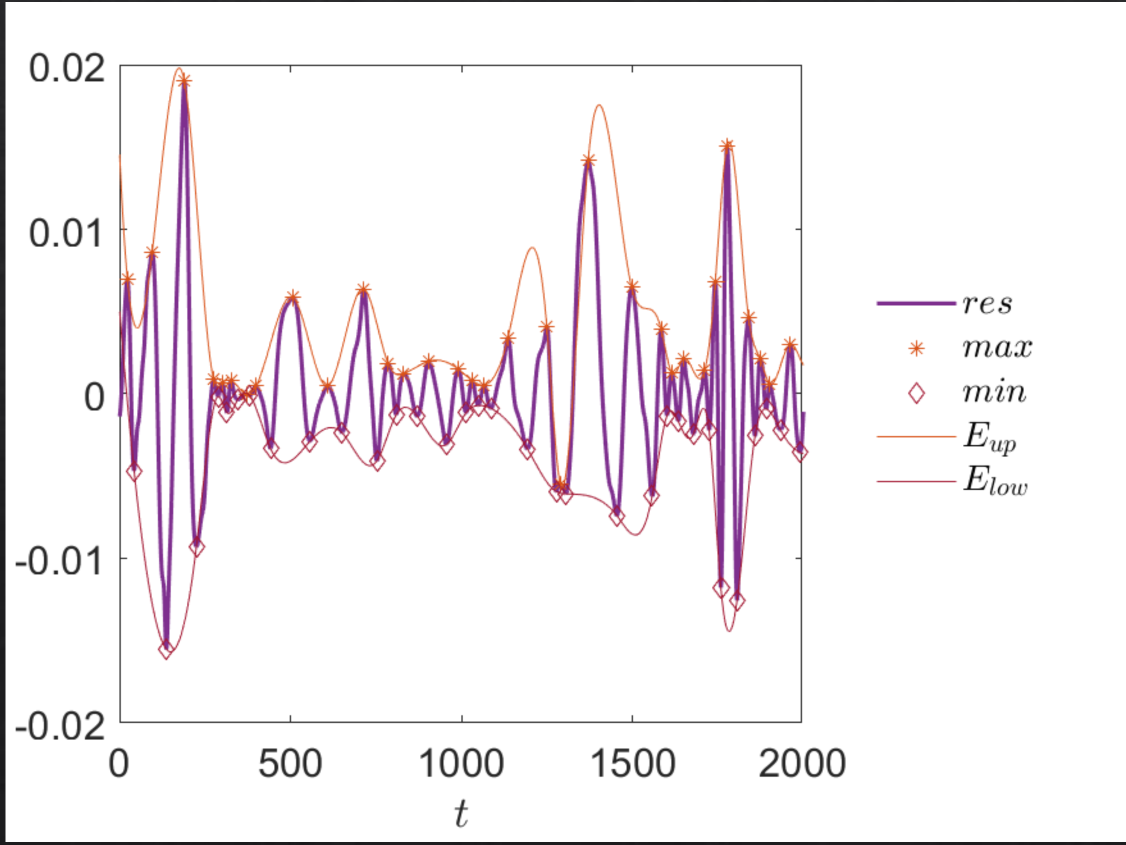
3. Determine mean of upper and lower envelope
   $E_{mean}(t) = (E_{up}(t) + E_{low}(t))/2$
4. Determine residual
   $res_2(t) = res - E_{mean}(t)$
5. Check stopping criterion
   $\sum_t \frac{(res_2(t) - res(t))^2}{res^2(t)} < \epsilon$

High-level overview

of the one-dimensional, univariate EMD



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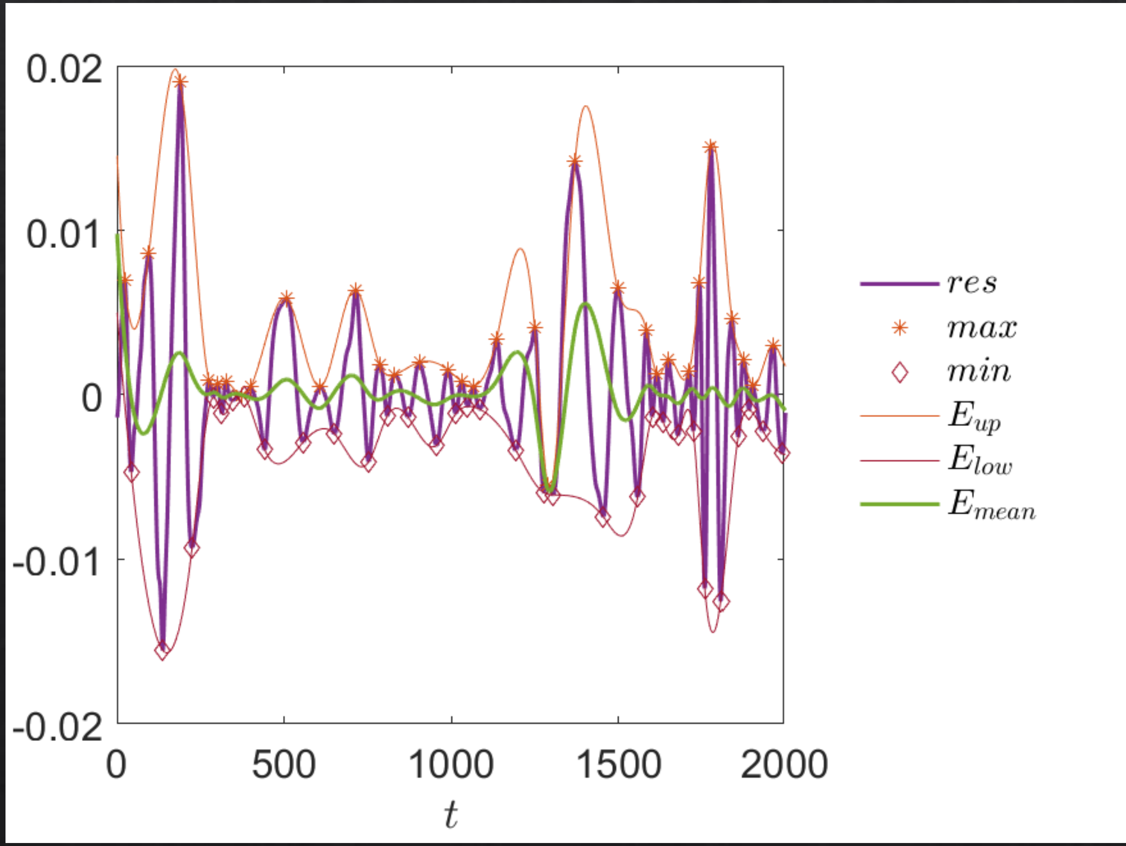
5. Check stopping criterion $\sum_t \frac{(res_2(t) - res(t))^2}{res^2(t)} < \epsilon$

High-level overview

of the one-dimensional, univariate EMD



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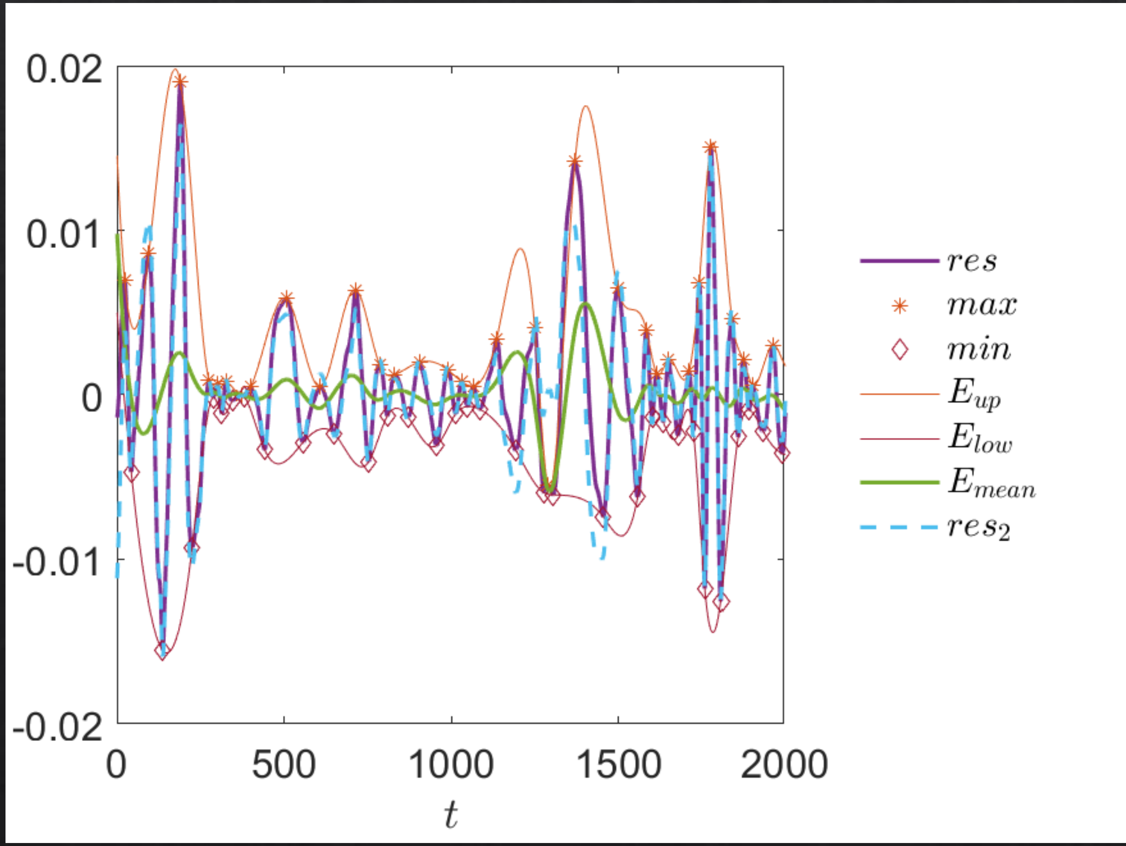

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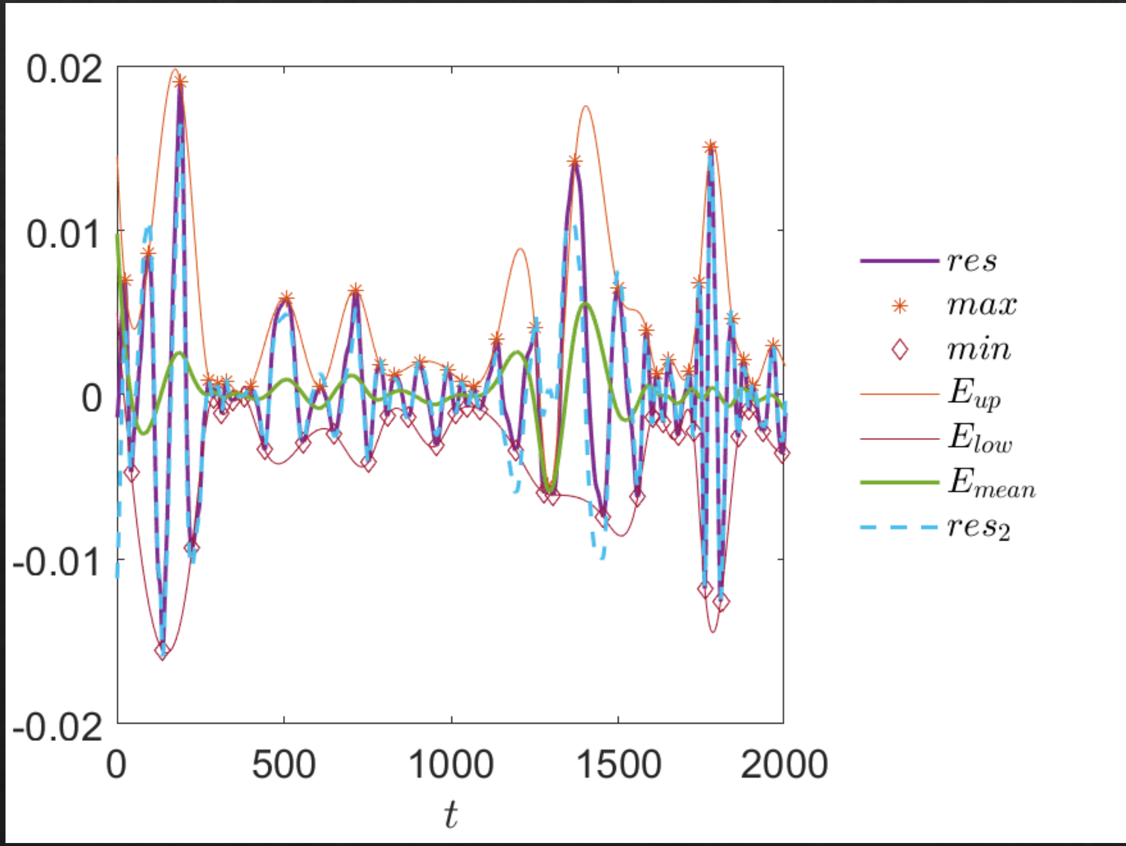

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High-level overview

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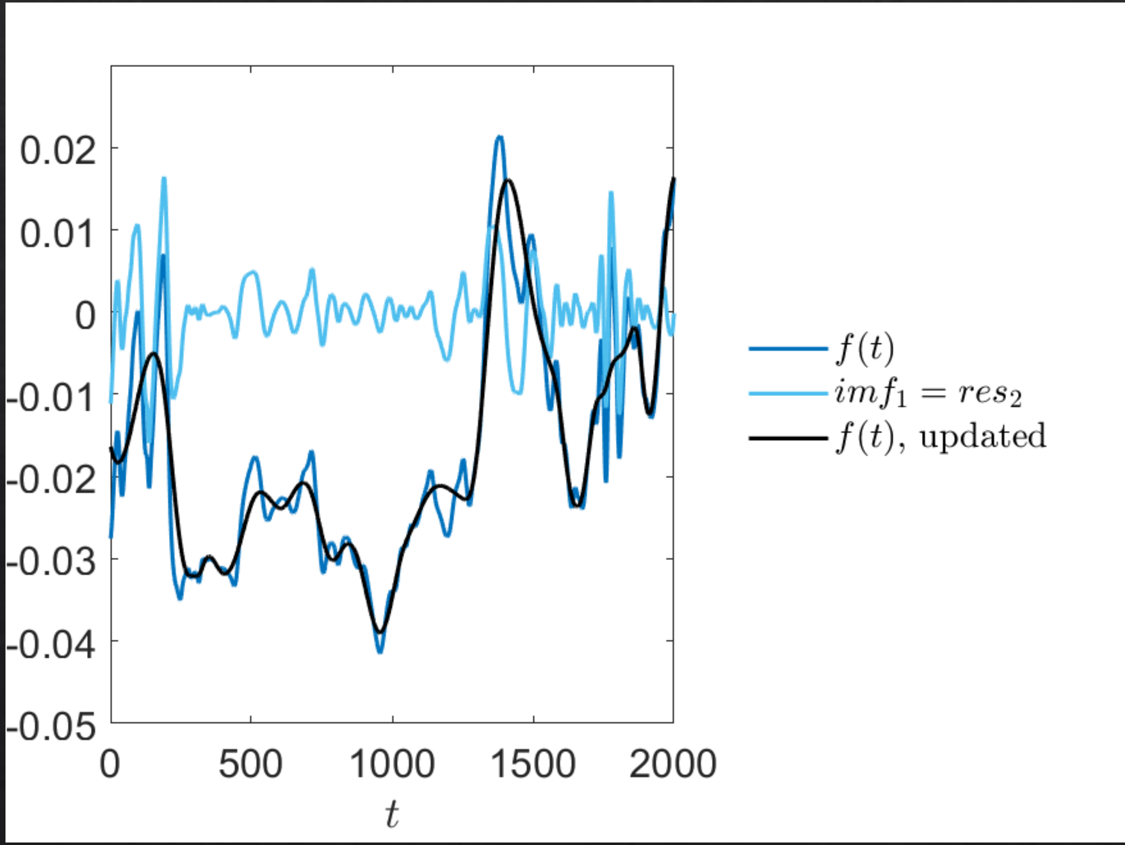

$f(t) \leftarrow f(t) - imf_1(t)$

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4. Determine residual
   $res(t) = f(t) - E_{mean}(t)$
5. Check stopping criterion
   $\sum_t \frac{(res_2(t) - res(t))^2}{res^2(t)} < \epsilon$

if true

$imf_1(t) = res(t)$

High-level overview

of the one-dimensional, univariate EMD

Inner loop: iterate until residual possesses IMF characteristics

$res_{i+1}(t)$

1. Find local extrema of the signal
2. Fit maxima and minima to an individual envelope
   - Maxima → $E_{up}(t)$
   - Minima → $E_{low}(t)$
3. Determine mean of upper and lower envelope
   $E_{mean}(t) = (E_{up}(t) + E_{low}(t))/2$
4. Determine residual
   $res_{i+1}(t) = res_i - E_{mean}(t)$
5. Check stopping criterion
   $\sum_t \frac{(res_{i+1}(t) - res_i(t))^2}{res_i^2(t)} < \epsilon$

if false

High-level overview

of the one-dimensional, univariate EMD

Inner loop: iterate until residual possesses IMF characteristics

Outer loop: iterate until all IMFs are extracted from the signal

$f(t) \leftarrow f(t) - imf_j(t)$

$res_{i+1}(t)$

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4. Determine residual
   $res_{i+1}(t) = res_i - E_{mean}(t)$
5. Check stopping criterion
   $\sum_t \frac{(res_{i+1}(t) - res_i(t))^2}{res_i^2(t)} < \epsilon$

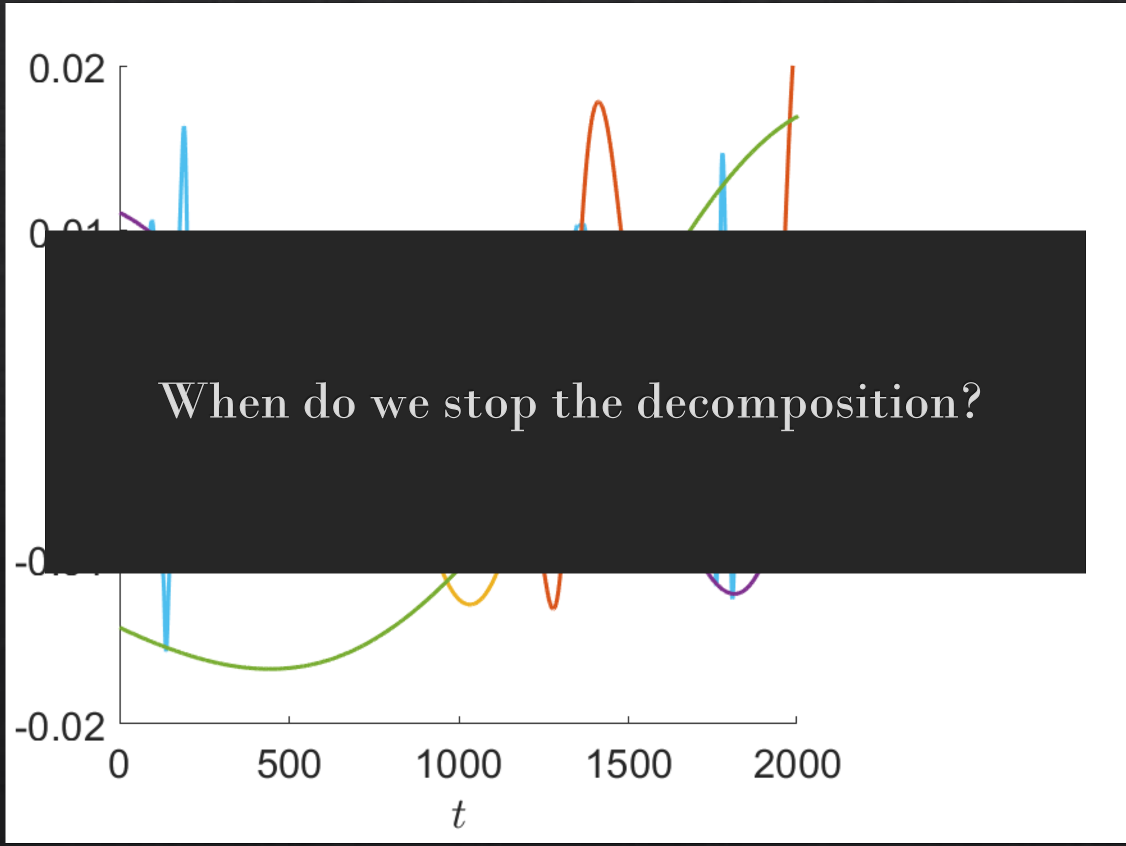
if true

$imf_j(t) = res_{i+1}(t)$

if false

High-level overview

of the one-dimensional, univariate EMD

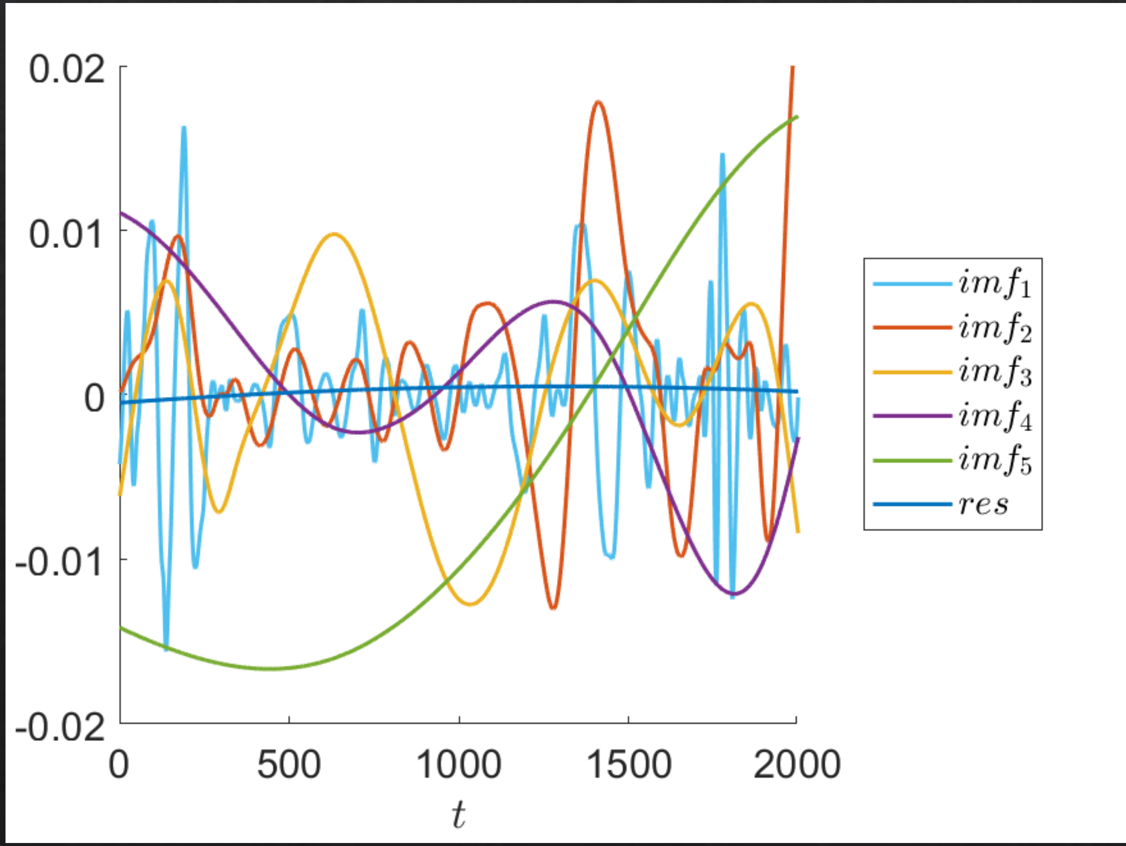


When do we stop the decomposition?

The decomposition stops when the residual approaches a monotonic function.

The outcome

of the one-dimensional, univariate EMD



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Examples: ocean wave data

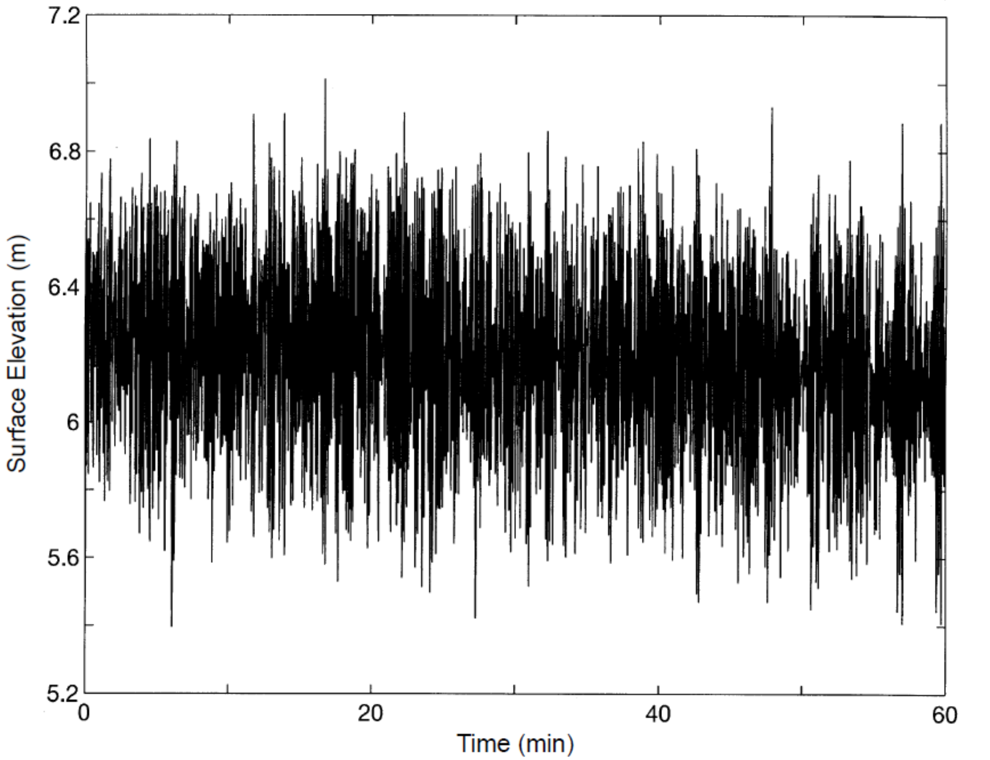

Figure 58. Field wave data measured at a tidal station covering a time span of 1 h. Because of the tides, the mean is not zero.

Huang et al. (1998). *The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis*. Proceedings of the Royal Society of London. Series A: mathematical, physical and engineering sciences, 454(1971), 903-995.

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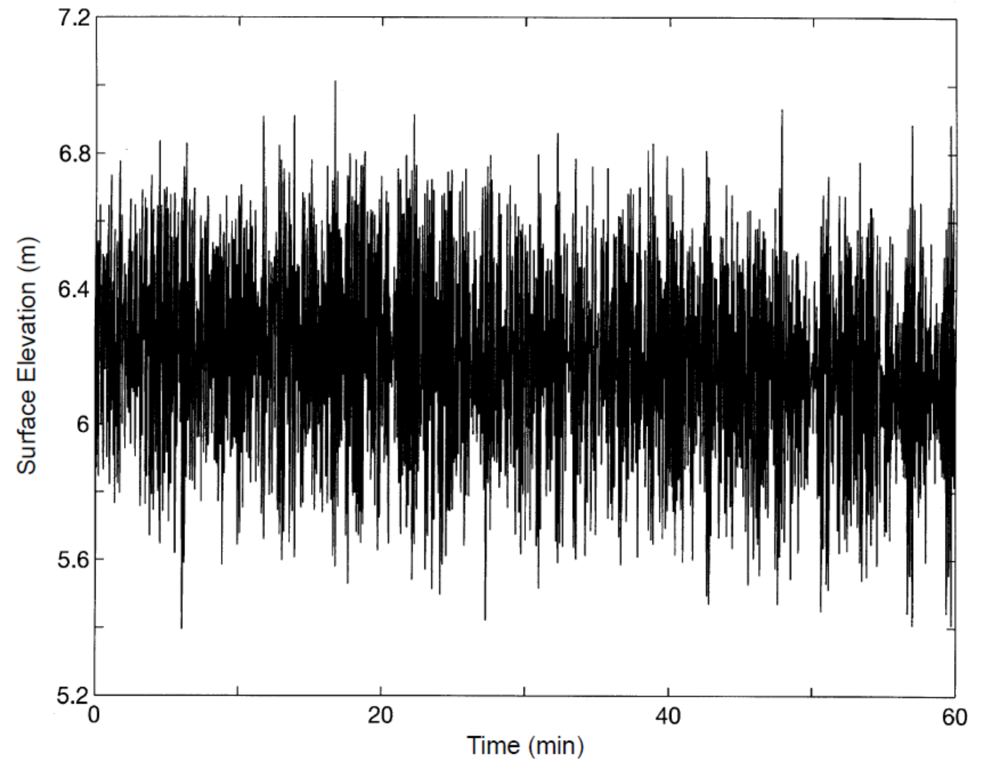

Figure 58. Field wave data measured at a tidal station covering a time span of 1 h. Because of the tides, the mean is not zero.

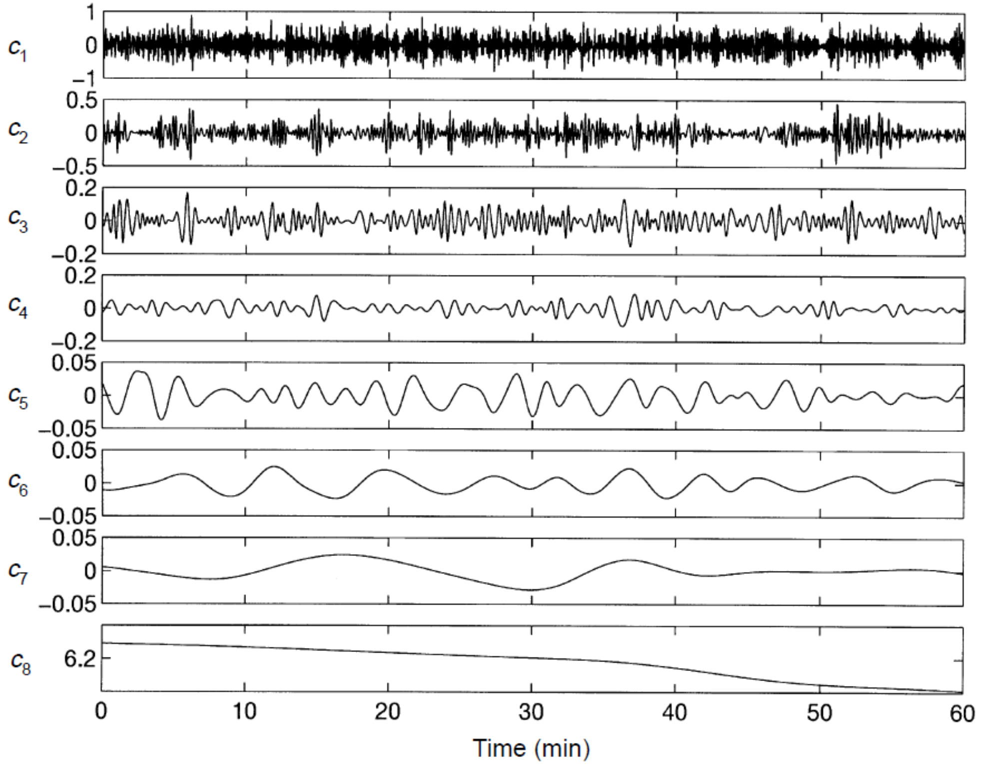

Figure 59. The IMF components derived from the data shown in figure 58: there are eight components with the last one showing the tidal range.

Huang et al. (1998). *The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis*. Proceedings of the Royal Society of London. Series A: mathematical, physical and engineering sciences, 454(1971), 903-995.

Examples: ocean wave data

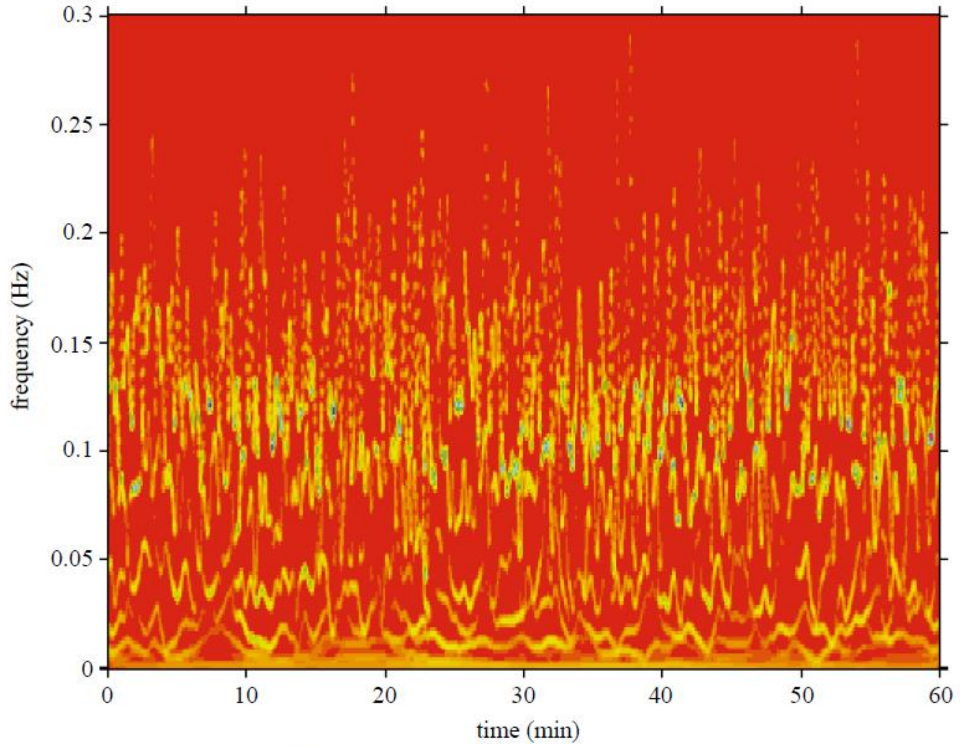

Figure 60. The 9 × 9 smoothed Hilbert spectrum for the data given in figure 58. The spectrum is extremely nodular, an indication that the wave is not stationary.

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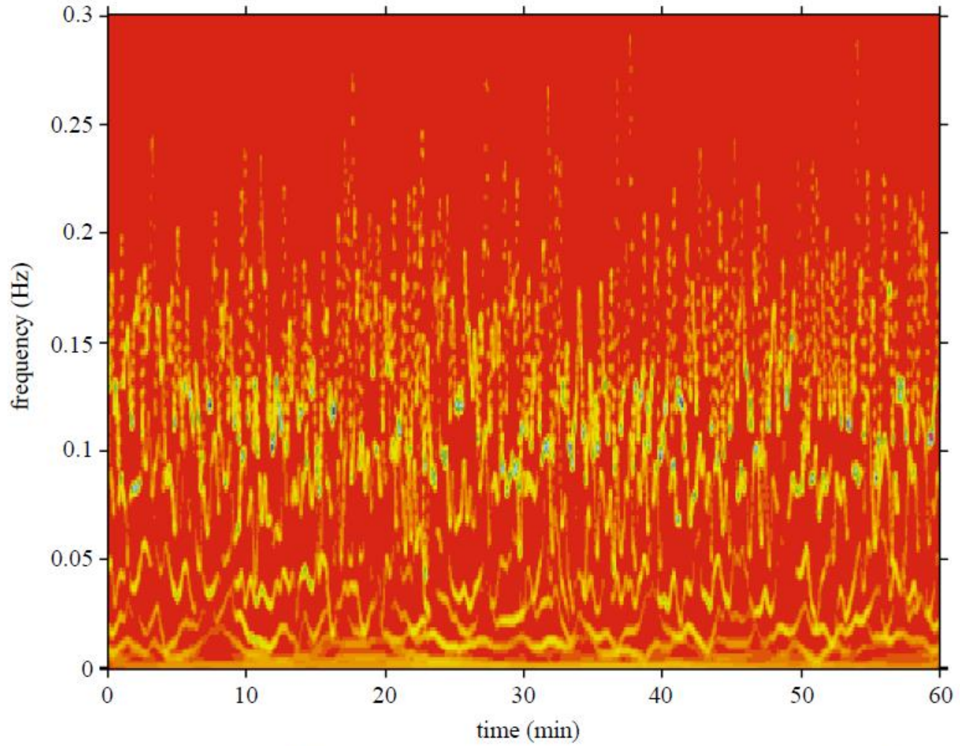

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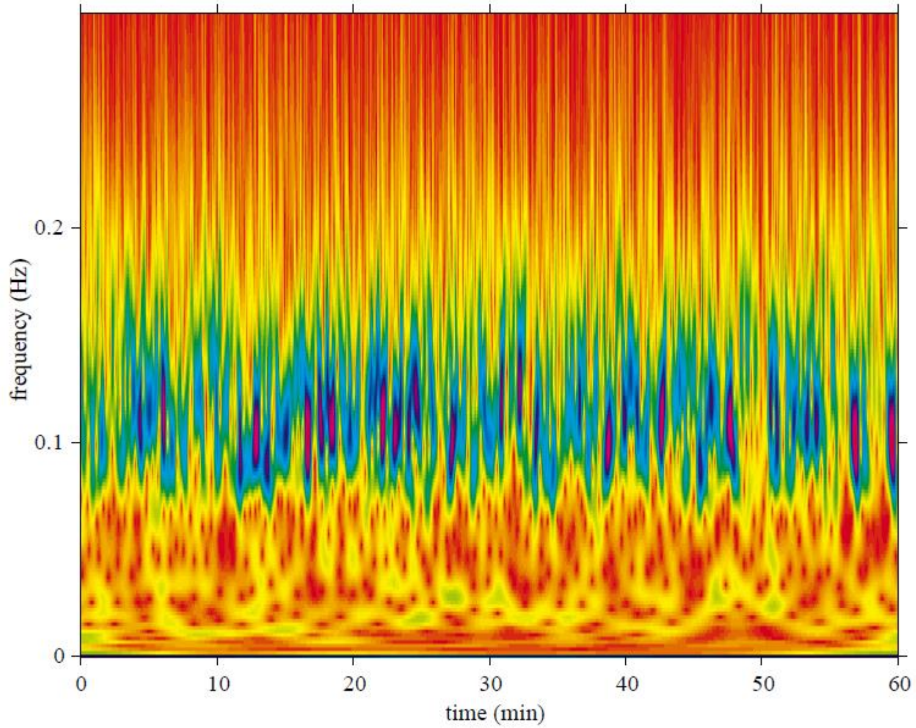
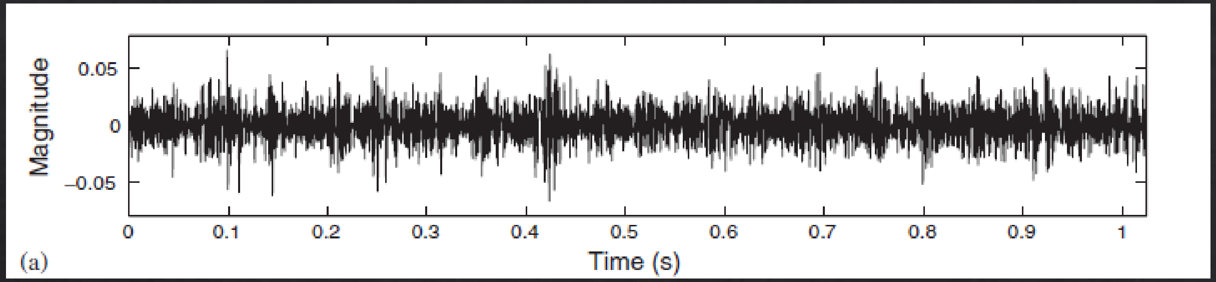


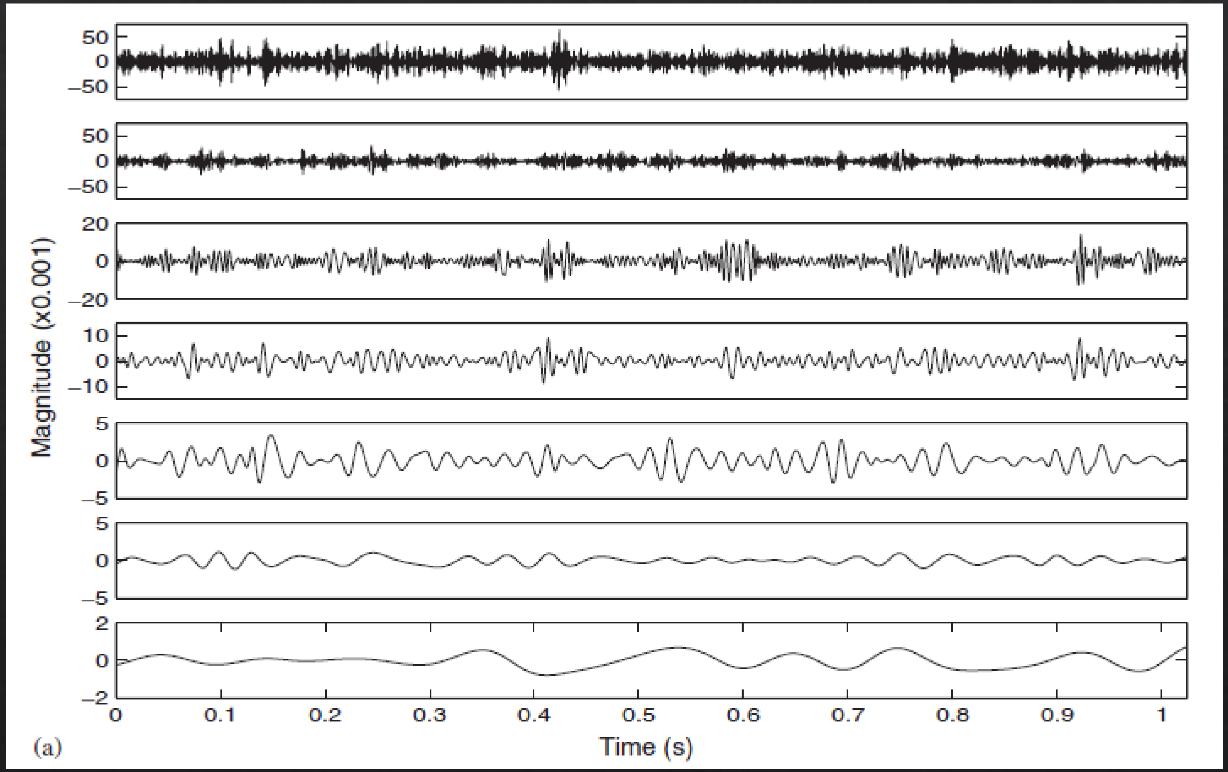
Figure 61. The Morlet wavelet spectrum of the data given in figure 58. Though the spectrum is more continuous, the energy distribution is still a function of time. Furthermore, there is a wider energy smearing in frequency space by the leakage and the harmonics.

Huang et al. (1998). *The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis.* Proceedings of the Royal Society of London. Series A: mathematical, physical and engineering sciences, 454(1971), 903-995.

Examples: gearbox fault diagnosis



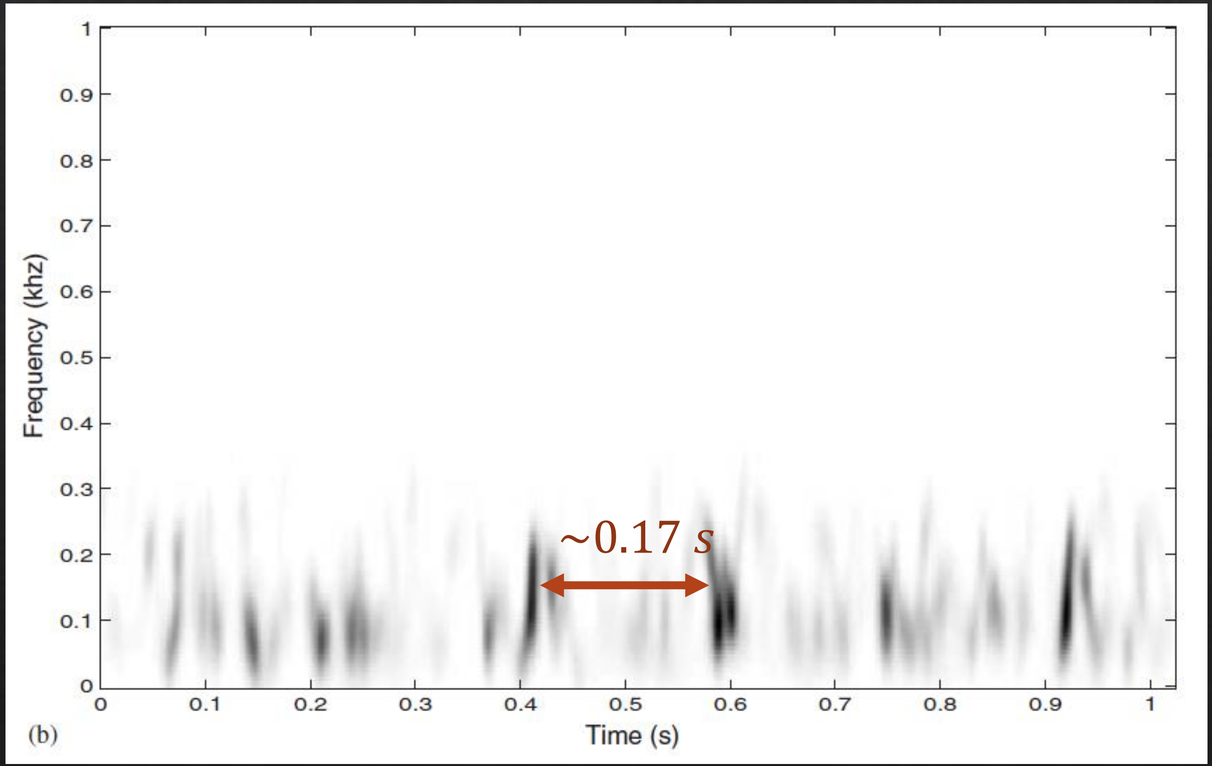
A vibration signal collected 1 min before the tooth was broken (fig. 5)



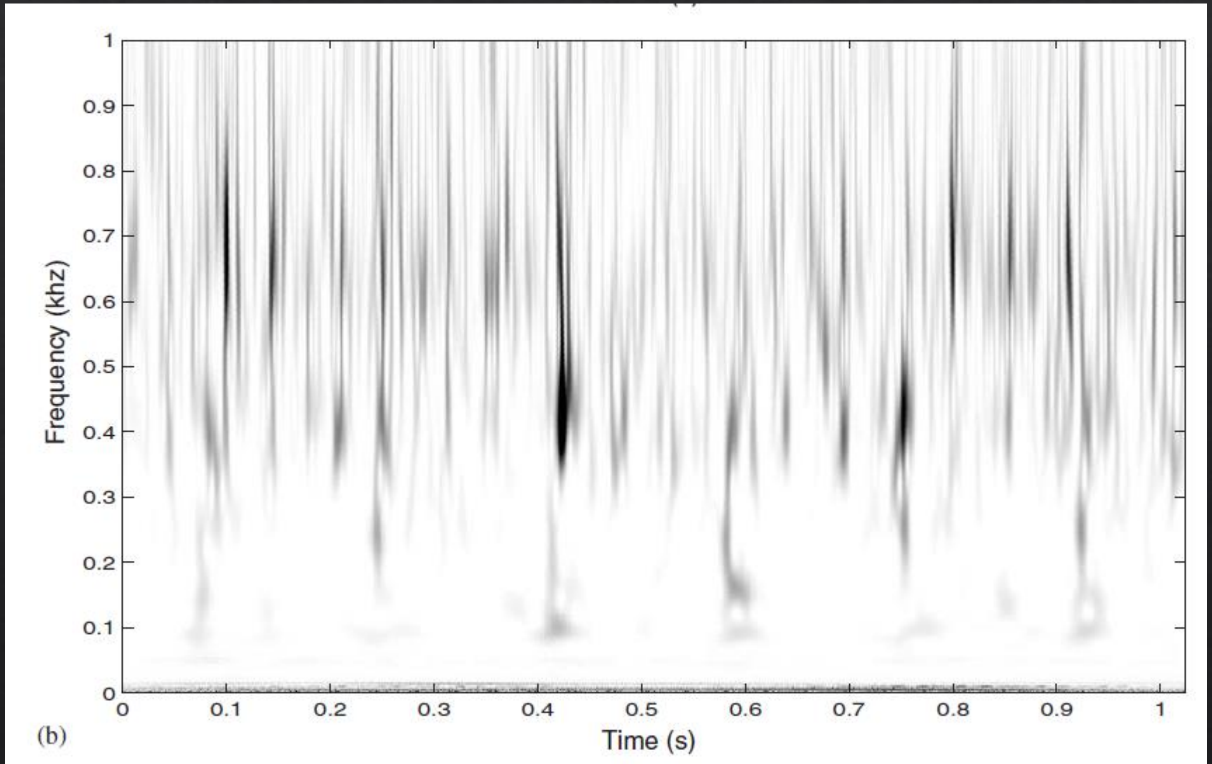
Corresponding IMFs (fig. 7)

Liu, B., Riemenschneider, S., & Xu, Y. (2006). *Gearbox fault diagnosis using empirical mode decomposition and Hilbert spectrum*. **Mechanical Systems and Signal Processing, 20(3), 718-734.**

Examples: gearbox fault diagnosis



Hilbert spectrum using the 3rd to the 6th IMF (fig. 7)



Wavelet spectrum (fig. 8)

Liu, B., Riemenschneider, S., & Xu, Y. (2006). *Gearbox fault diagnosis using empirical mode decomposition and Hilbert spectrum*. **Mechanical Systems and Signal Processing, 20(3), 718-734.**

Examples: ECG signals

Automatic Motion and Noise Artifact Detection in Holter ECG Data Using Empirical Mode Decomposition and Statistical Approaches

Publisher: IEEE Cite This PDF

Jinseok Lee ; David D. McManus ; Sneh Merchant ; Ki H. Chon All Authors

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Computers in Biology and Medicine
Volume 42, Issue 1, January 2012, Pages 83-92

Empirical mode decomposition based ECG enhancement and QRS detection

Saurabh Pal [a], Madhuchhanda Mitra [b]

Open Access Article

Arrhythmia ECG Noise Reduction by Ensemble Empirical Mode Decomposition

by Kang-Ming Chang

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Sensors **2010**, *10*(6), 6063-6080; https://doi.org/10.3390/s100606063

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Take-away

- Data decomposition separates a signal into components (“modes”) to gain new insight into inherent features

- The Empirical Mode Decomposition (EMD) can outperform other methods by it ability to
  - process non-linear and non-stationary data
  - produce physically meaningful modes
  - precisely identify features in combination with the Hilbert Transform

- The Intrinsic Mode Functions (IMFs) produced by the EMD are
  - obtained from an iterative process (sifting process)
  - based on the time lapse between consecutive extrema of the data
  - ordered from highest to lowest frequencies (smallest to largest scales)