MULTIVARIATE EMPIRICAL MODE DECOMPOSITION

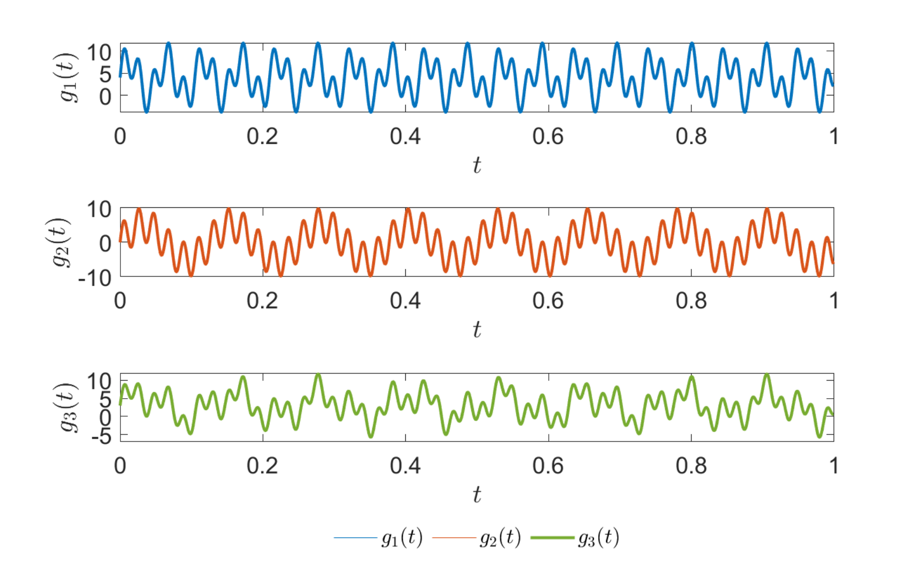
multiple one-dimensional signals

MULTIVARIATE EMPIRICAL MODE DECOMPOSITION

The main feature

of the one-dimensional MEMD

Objective: find and align common scales within multivariate data



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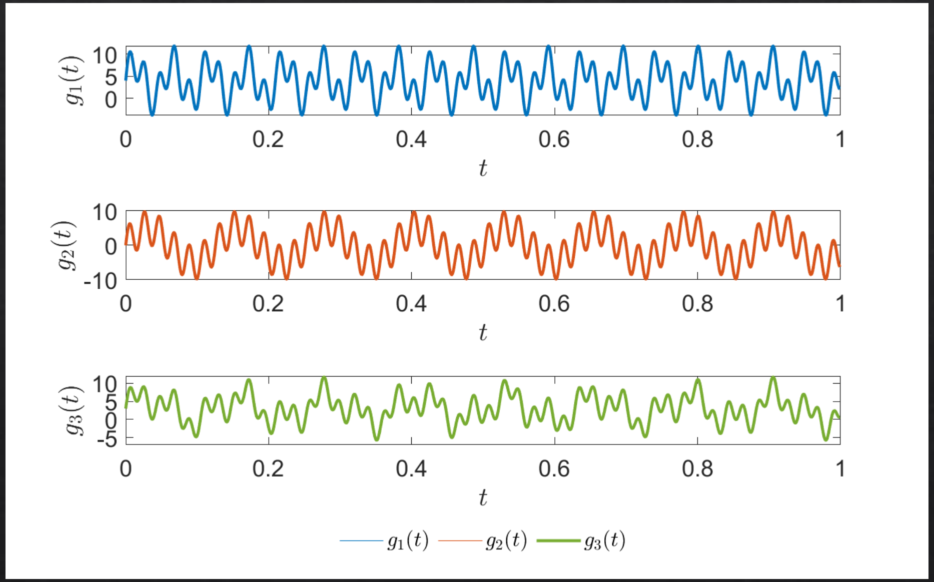
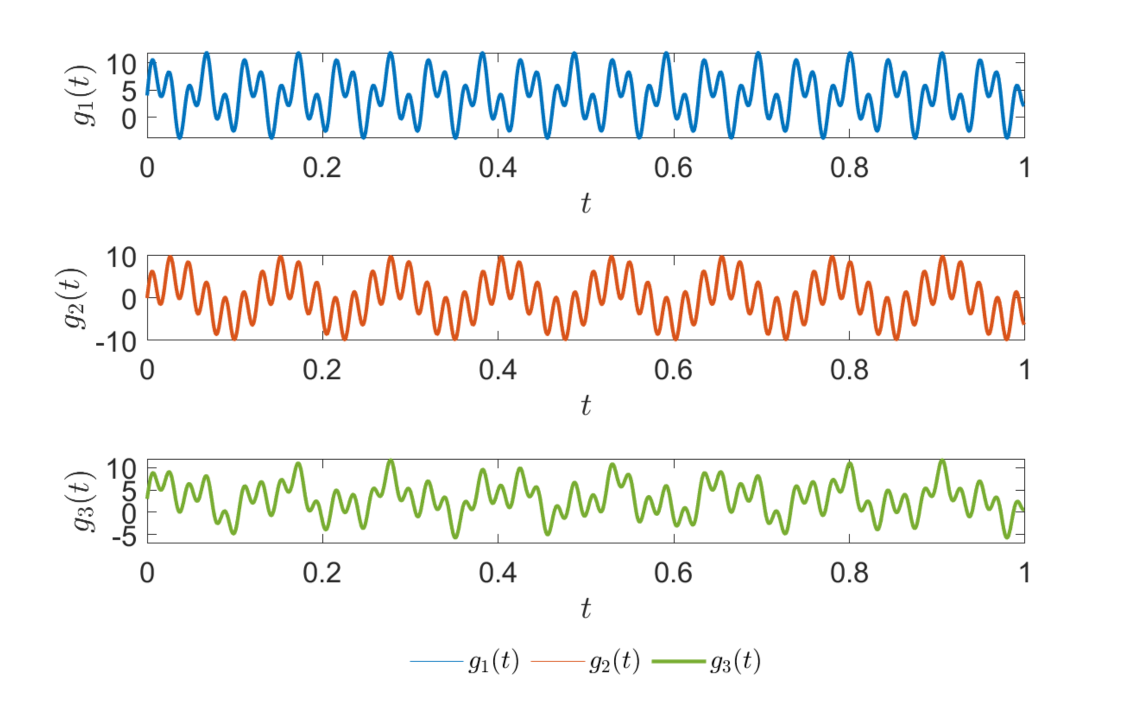


How is this achieved?

- The MEMD simultaneously decomposes all signals
- Scales common to two or more signals are found in equal-indexed IMFs
- This "mode alignment property" enables a profound analysis across multiple signals

The starting point

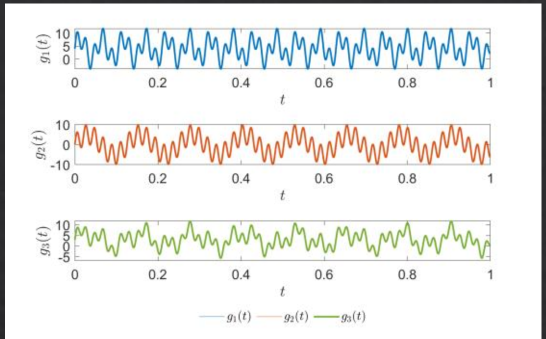
of the one-dimensional MEMD



Rehman & Mandić (2010). *Multivariate empirical mode decomposition*. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 466(2117), 1291-1302

Starting point:
several concatenated 1D signals
$\mathbf{g}(t) = \{g_1(t), g_2(t), \dots g_N(t)\}$*

*they could equally be spatial signals



The outcome

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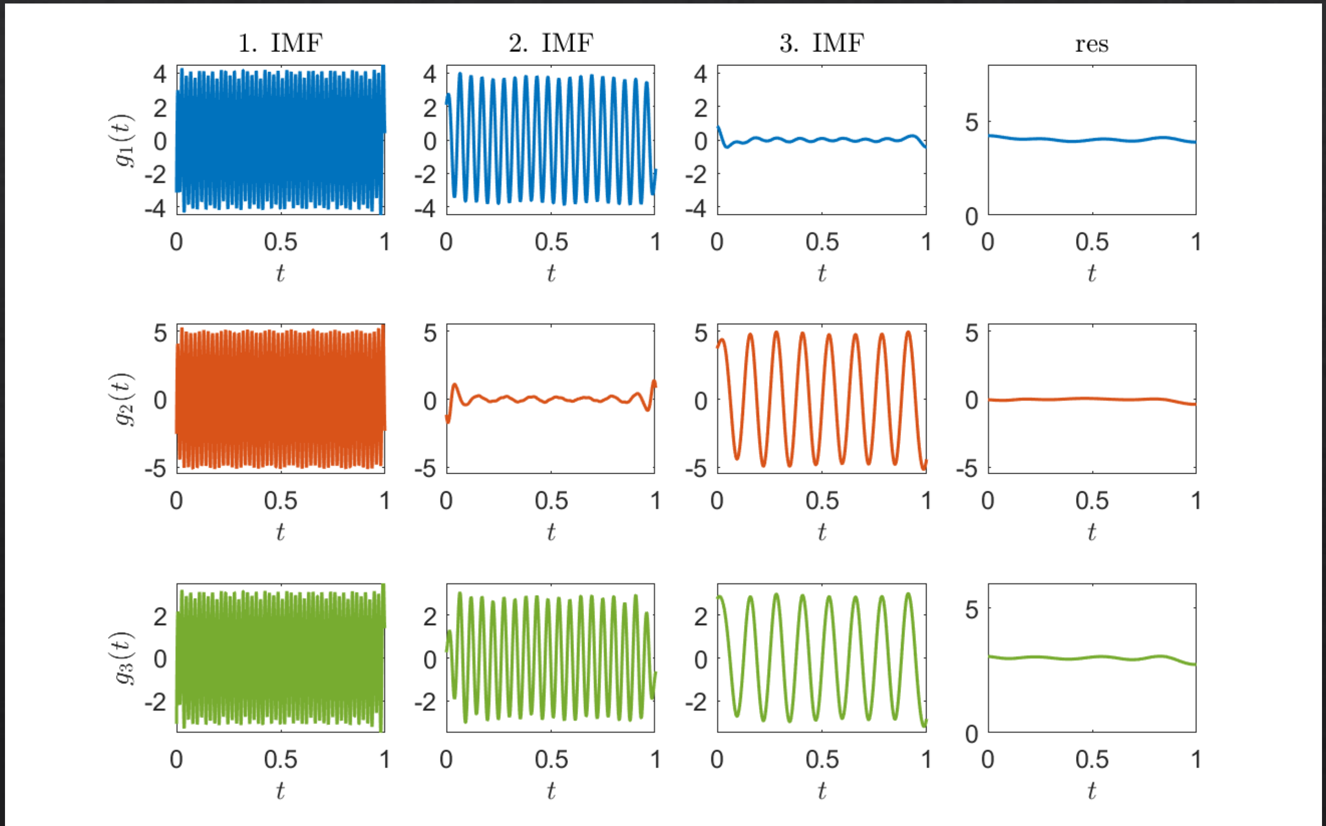
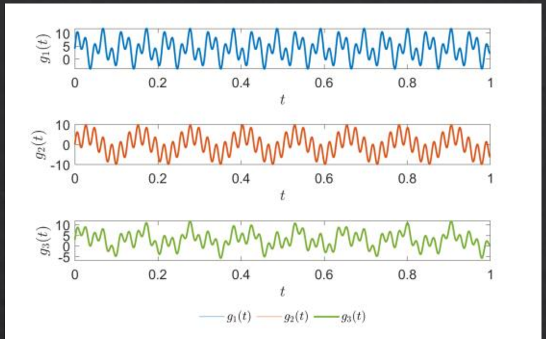


$f_1 = 300Hz \quad f_2 = 120Hz \quad f_3 = 50Hz \qquad const.$



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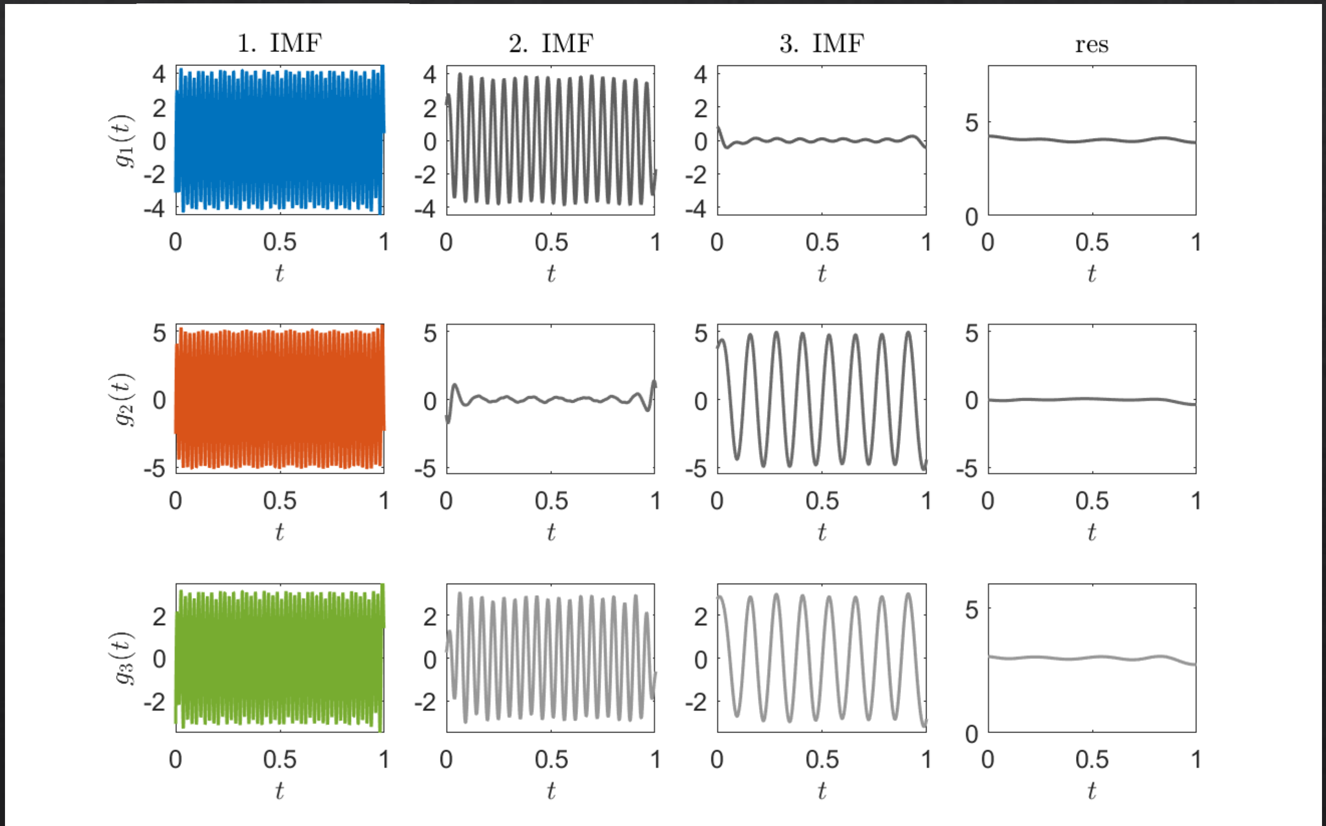

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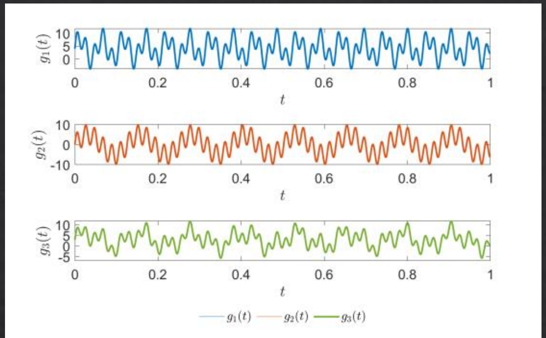

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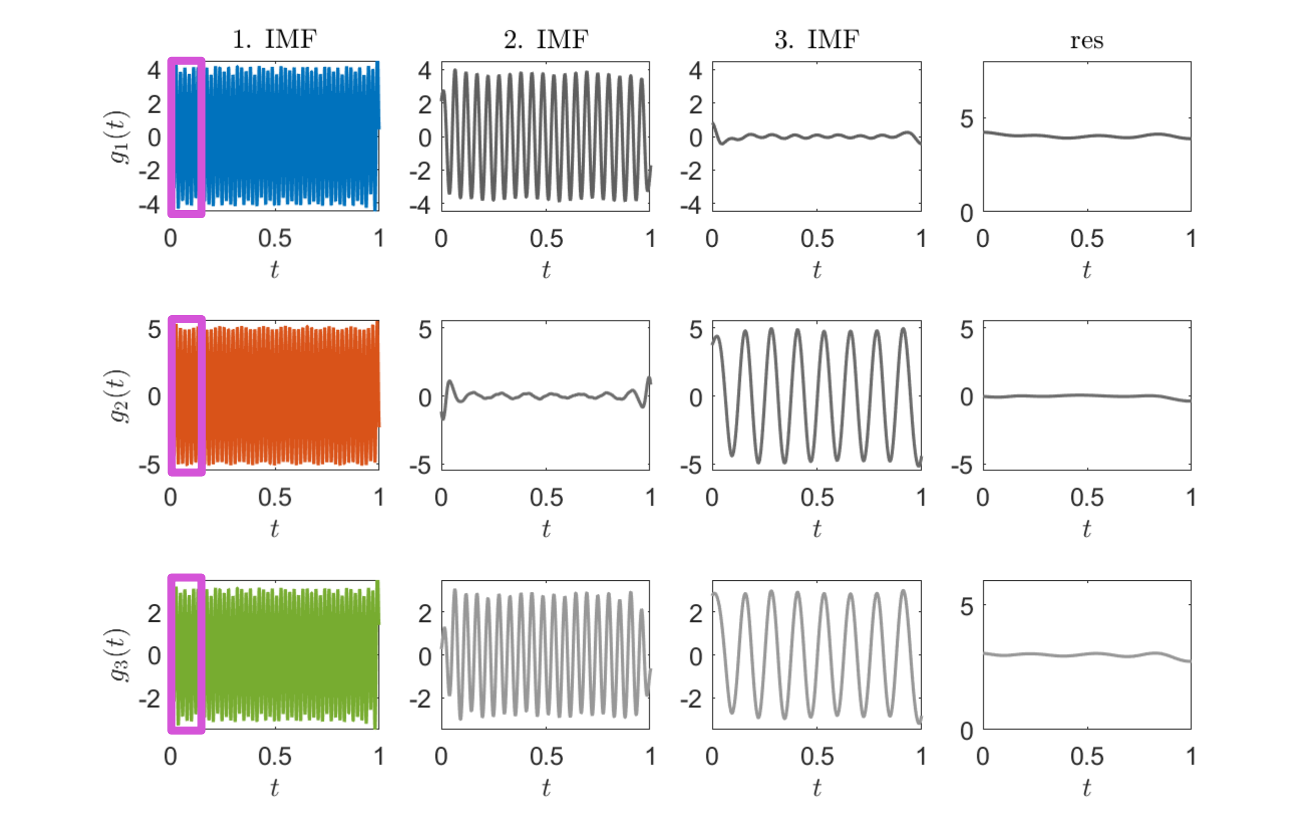

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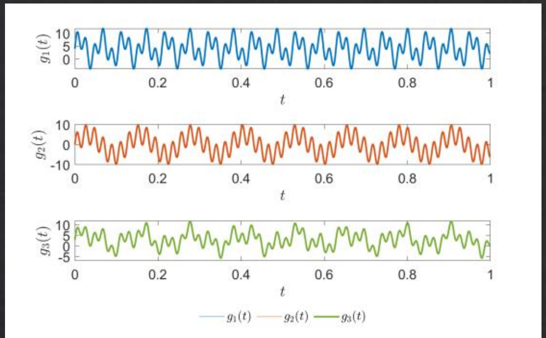

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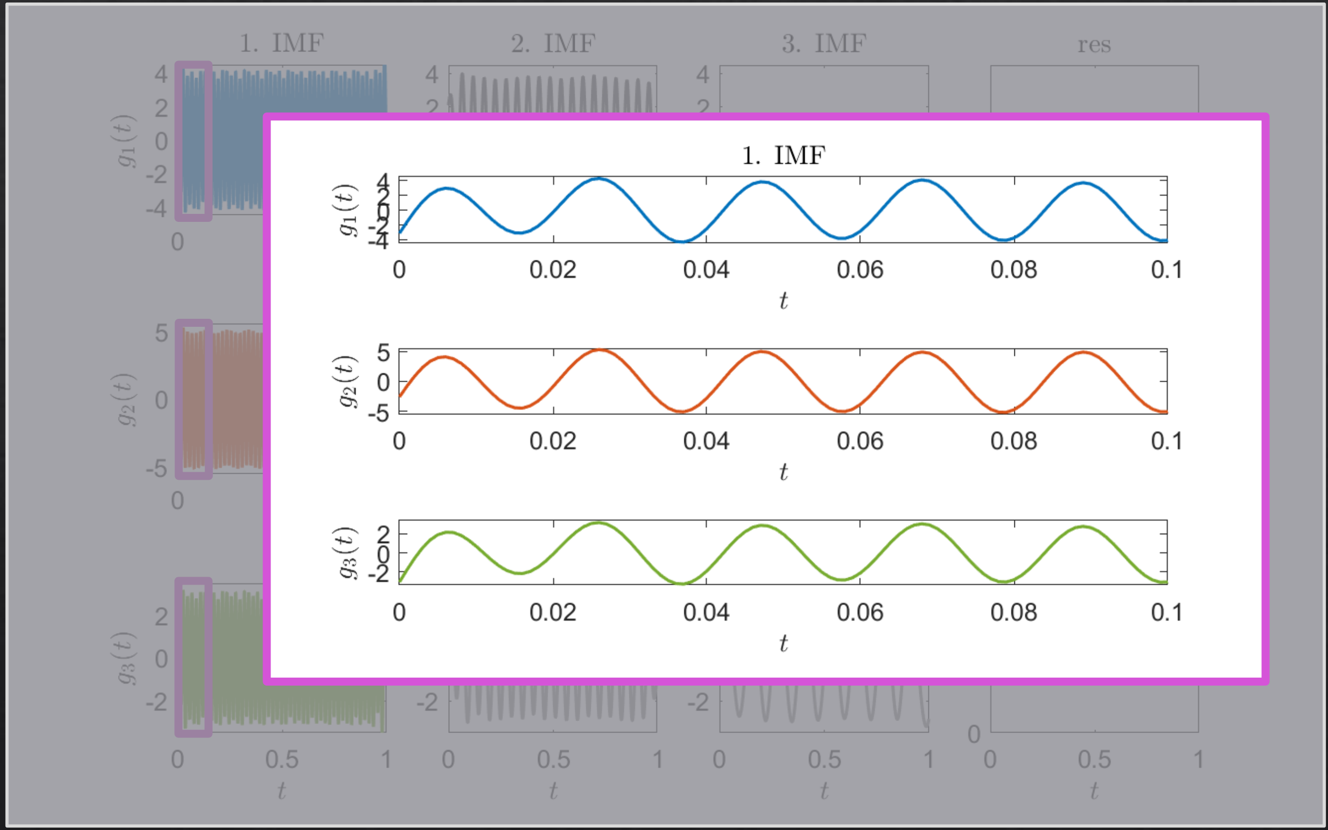

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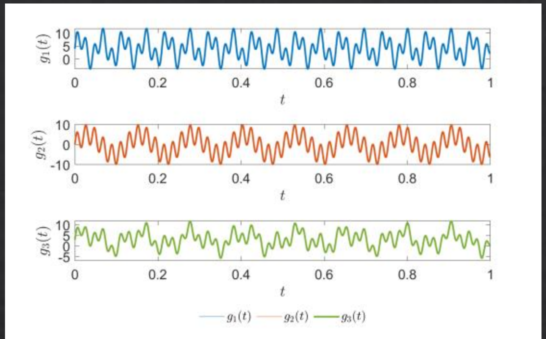

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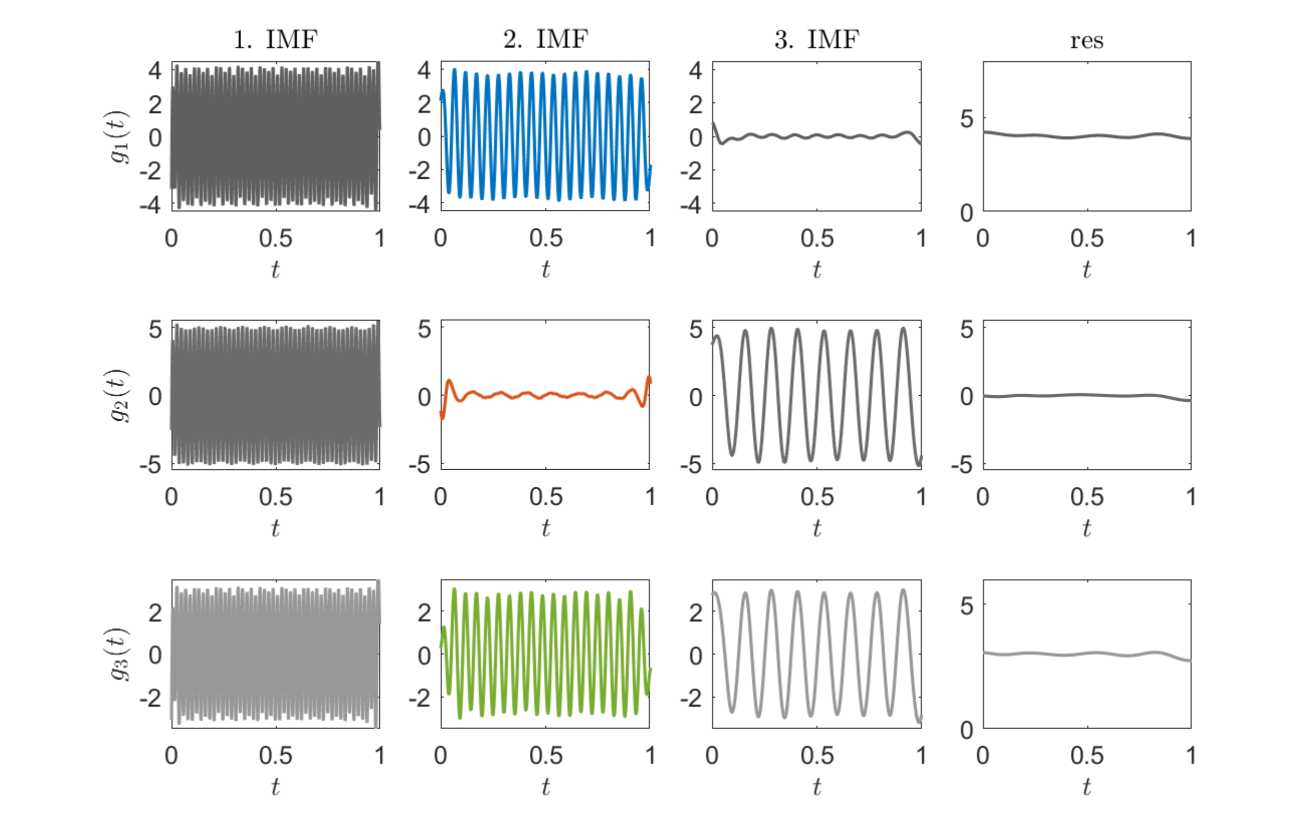

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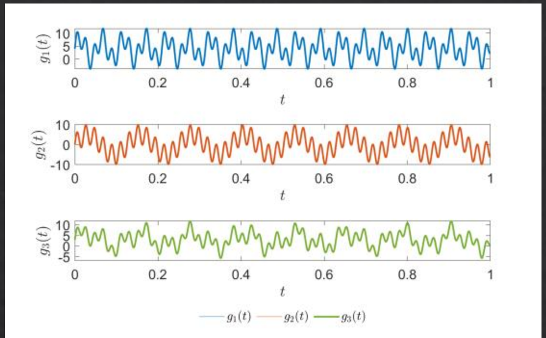

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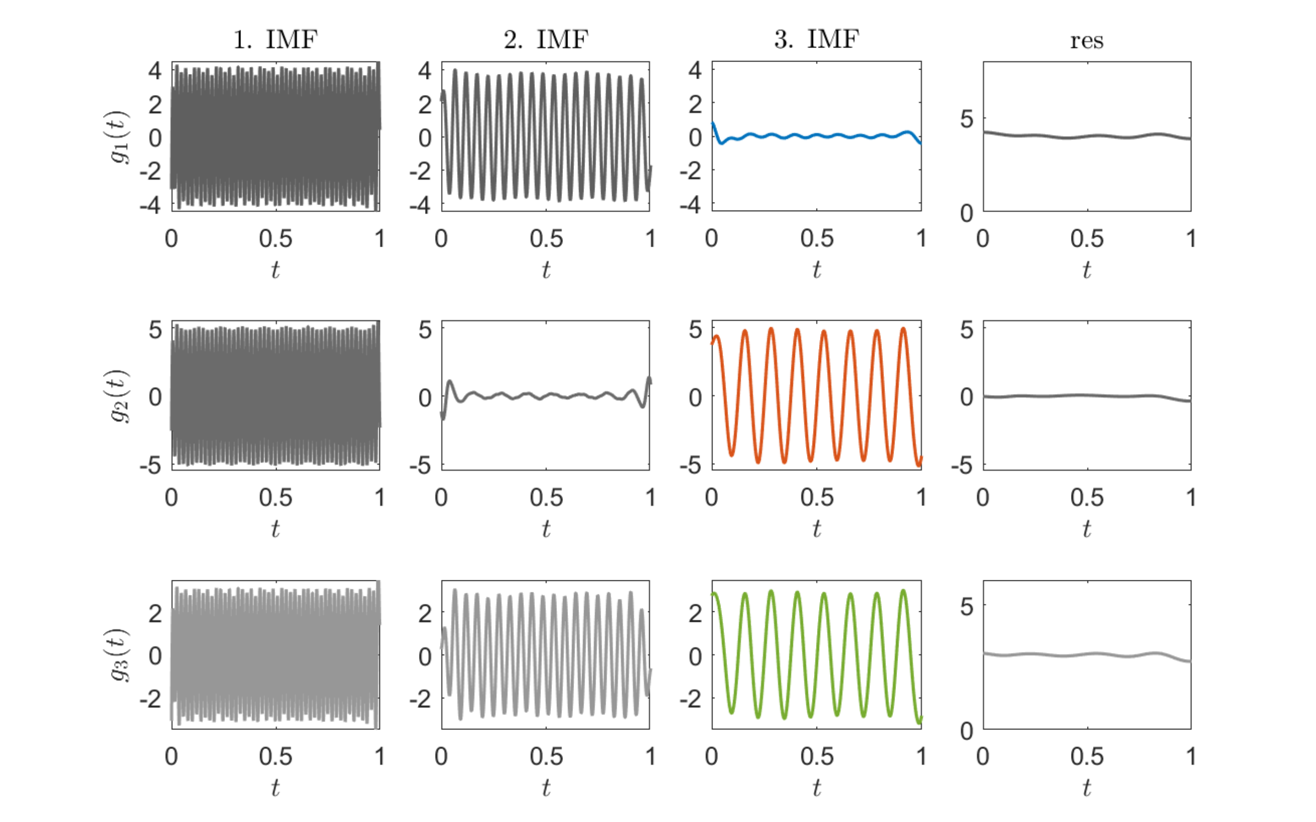

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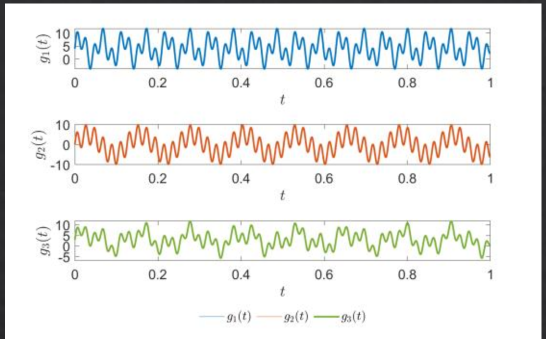

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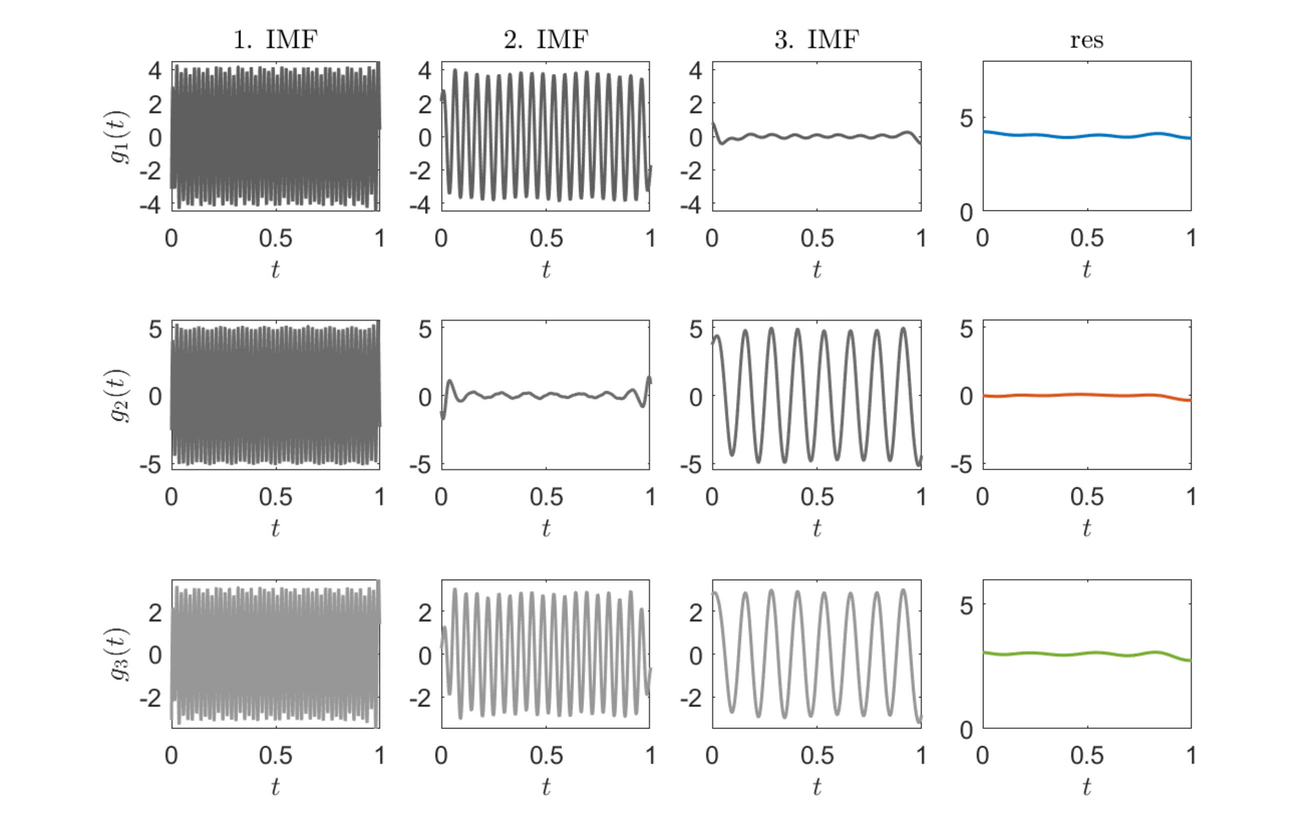

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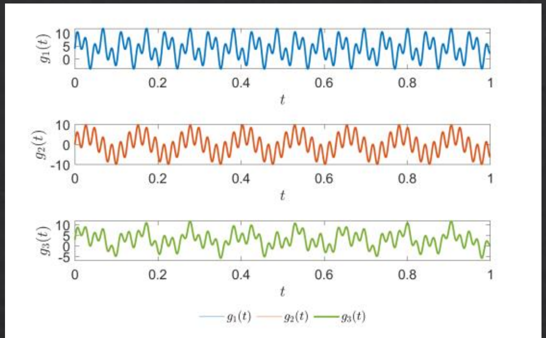

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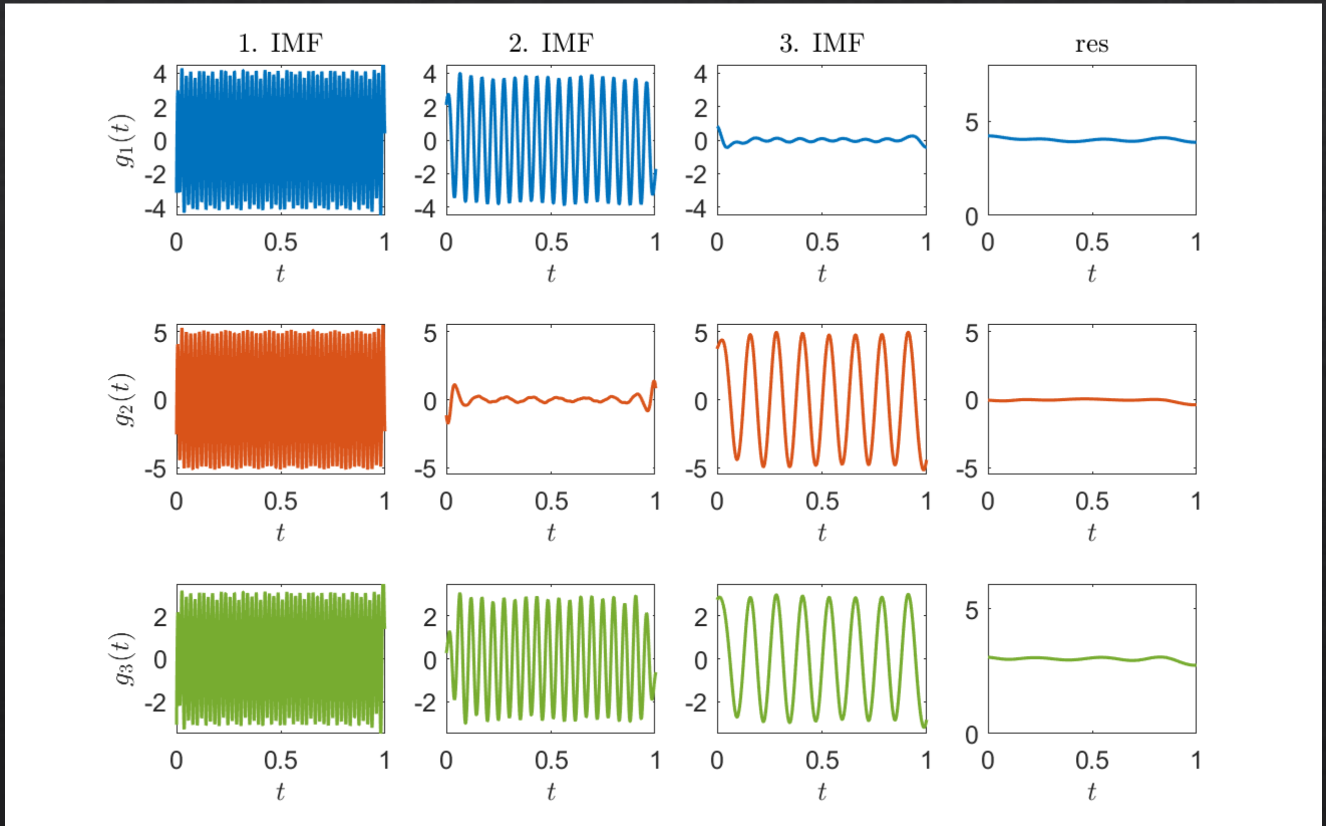

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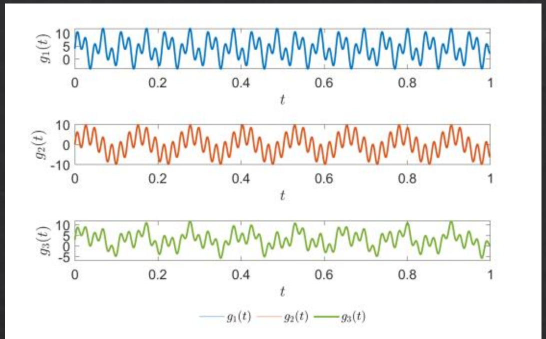

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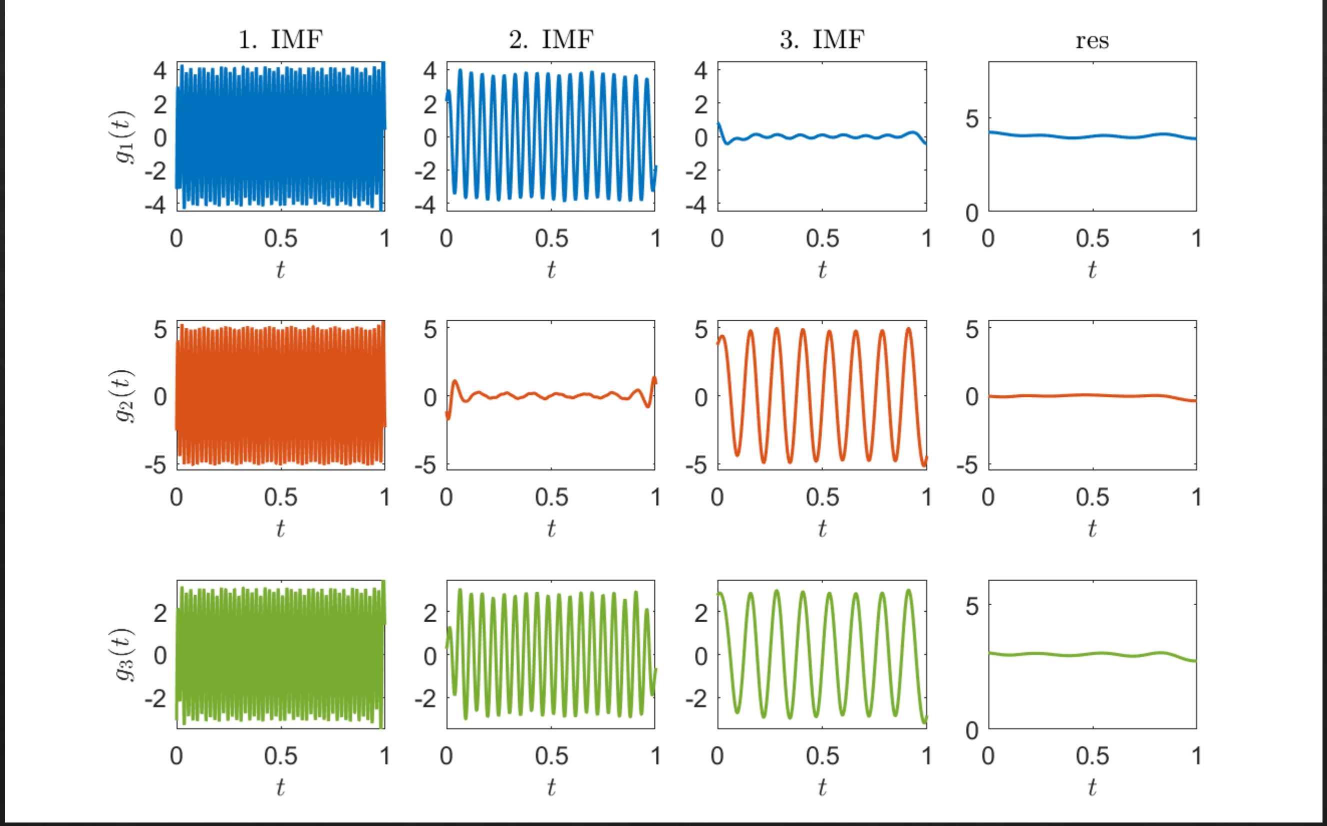

$$g_1(t) = 4 + 4sin(f_1 t) + 4sin(f_2 t) + \quad 0$$

$$g_2(t) = 0 + 5sin(f_1 t) + \quad 0 \quad + 5sin(f_3 t)$$

$$g_3(t) = 3 + 3sin(f_1 t) + 3sin(f_2 t) + 3sin(f_3 t)$$

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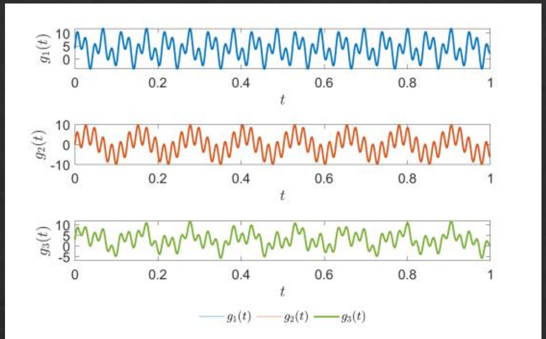

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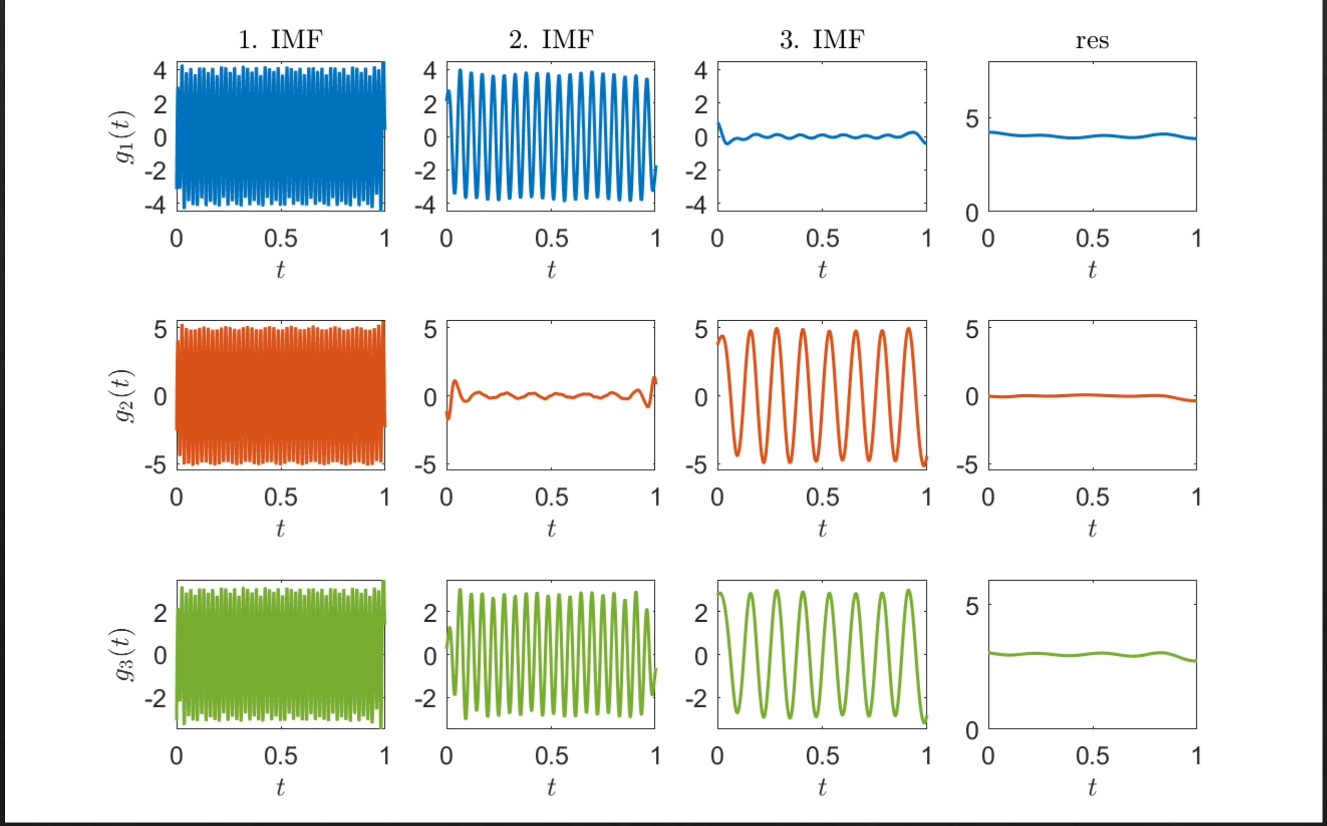

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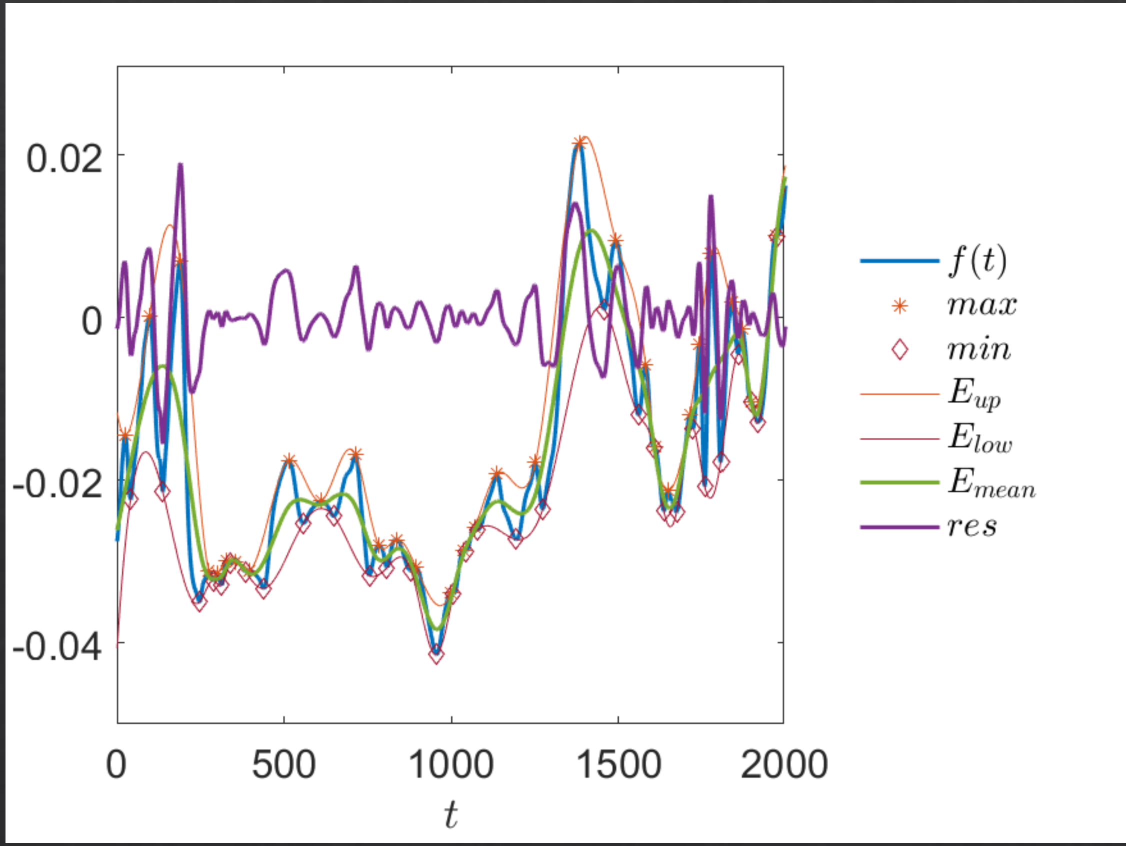
The MEMD provides Intrinsic Mode Functions (IMFs) and a residual for each variate.

A higher mode number contains larger scales/ smaller frequencies.

The sum of all modes and the remaining residual resemble the original signal.

$$g(t) = \sum_j imf_j + res$$

Recap: the univariate EMD



1. Find local extrema of the signal
2. Fit maxima and minima to an individual envelope
   - Maxima → $E_{up}(t)$
   - Minima → $E_{low}(t)$
3. Determine mean of upper and lower envelope
   $E_{mean}(t) = (E_{up}(t) + E_{low}(t))/2$
4. Determine residual
   $res_{i+1}(t) = res_i - E_{mean}(t)$
5. Check stopping criterion
   $\sum_t \frac{(res_{i+1}(t) - res_i(t))^2}{res_i^2(t)} < \epsilon$

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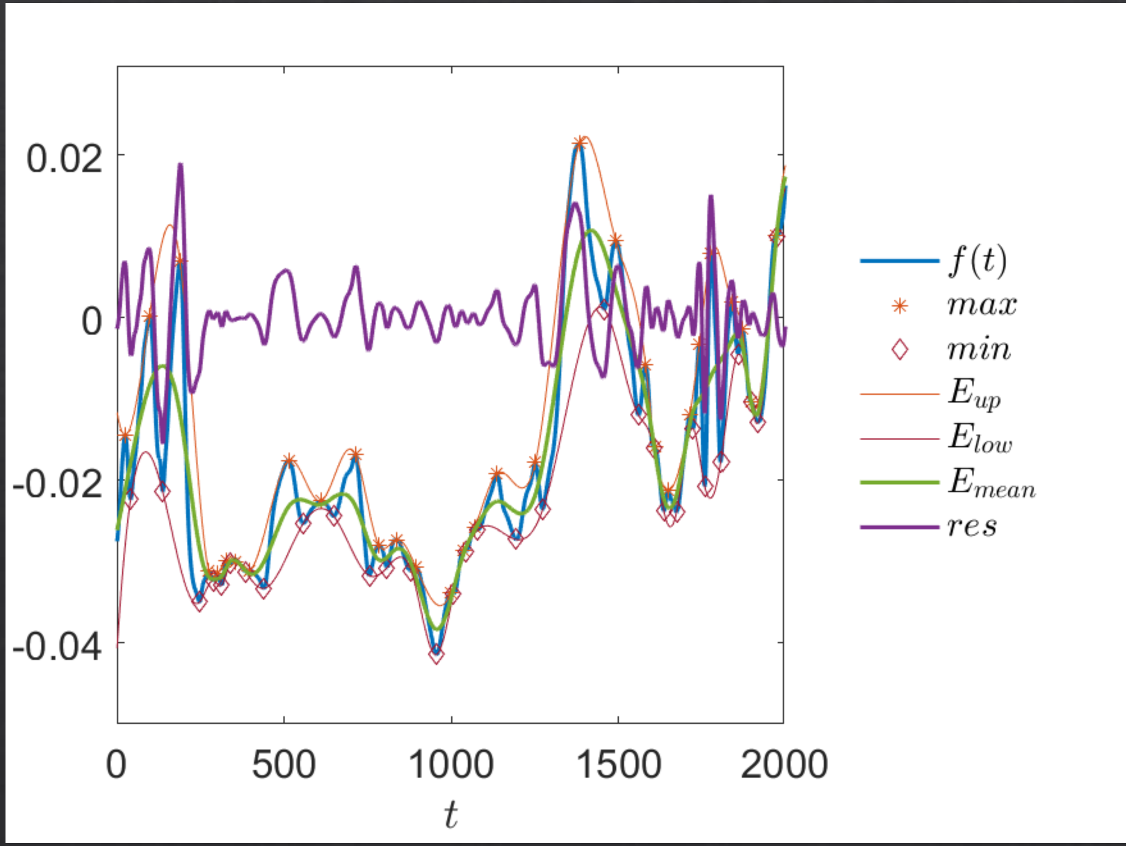

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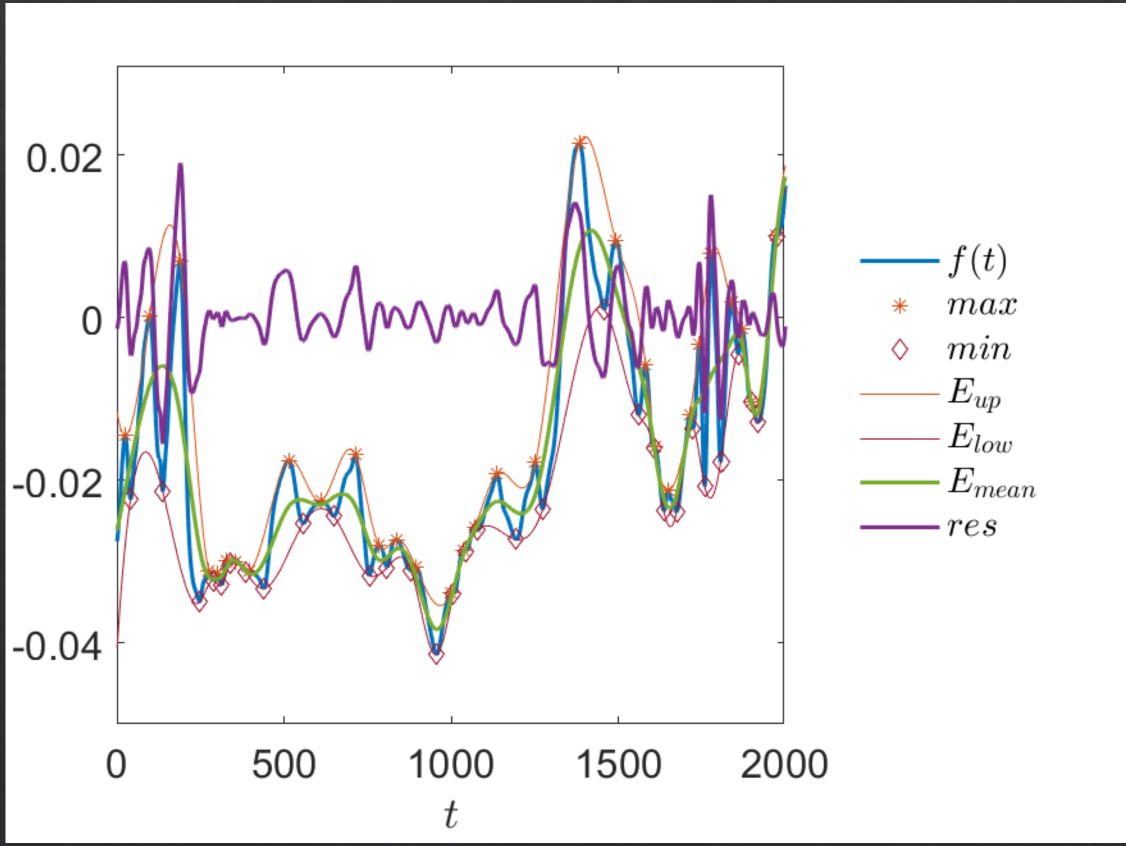

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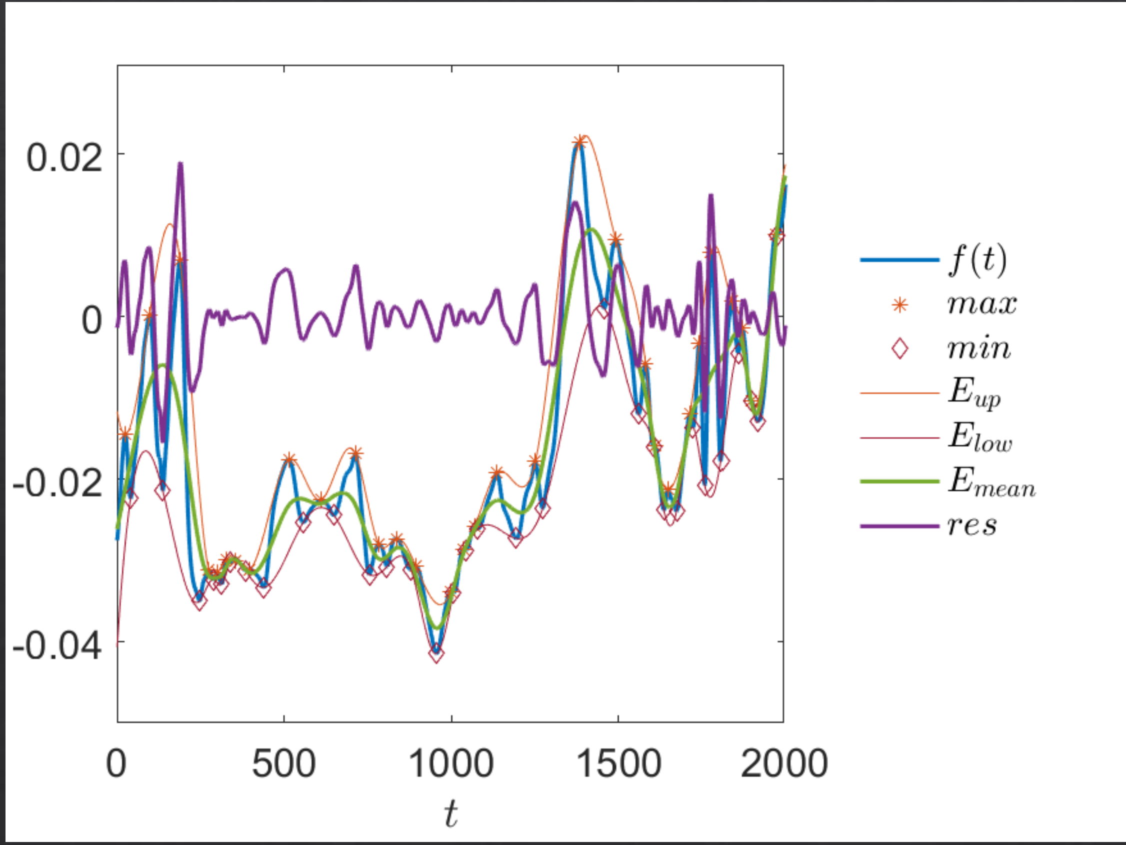

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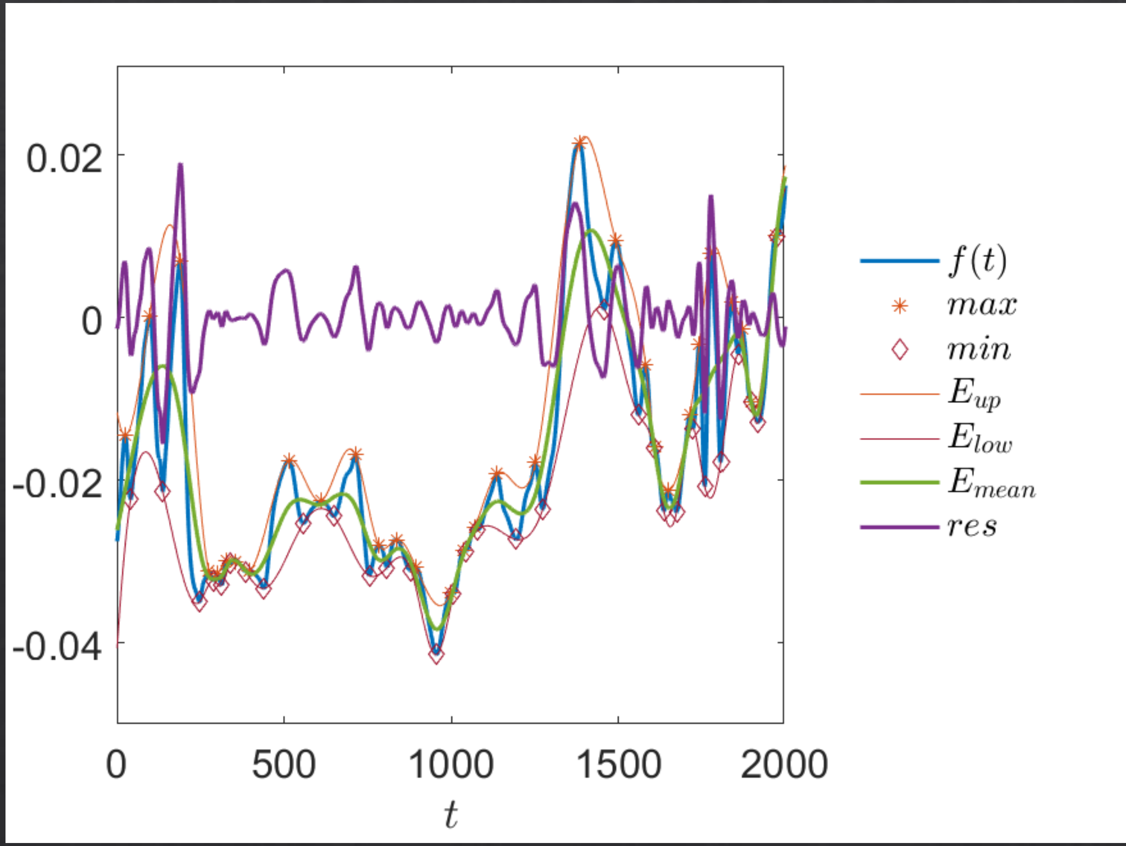

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Recap: the univariate EMD

Inner loop: iterate until residual possesses IMF characteristics

$res_{i+1}(t)$

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if false

Recap: the univariate EMD

Inner loop: iterate until residual possesses IMF characteristics

Outer loop: iterate until all IMFs are extracted from the signal

$f(t) \leftarrow f(t) - imf_j(t)$

$res_{i+1}(t)$

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if false

if true

$imf_j(t) = res_{i+1}(t)$

Univariate EMD vs MEMD

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Local extrema determination is not straightforward for multivariate signals

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Using real-valued signal projections to define multi-dimensional envelopes

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Univariate EMD vs MEMD

Local extrema determination is not straightforward for multivariate signals

Using real-valued signal projections to define multi-dimensional envelopes

0. Select uniform projection directions
1. Find local extrema of projections
2. Fit original data at extrema locations to multivariate envelopes $\{E^{\theta_k}(t)\}_{k=1}^{K}$
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Univariate EMD vs MEMD

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3. Average over all projection directions to get mean envelopes

$$E_{mean}(t) = \frac{1}{K}\sum_{k=1}^{K} E^{\theta_k}(t)$$

4. Determine residual

$$\boldsymbol{res}\,(t) = \boldsymbol{f}(t) - E_{mean}(t)$$

5. Check stopping criterion

Univariate EMD vs MEMD

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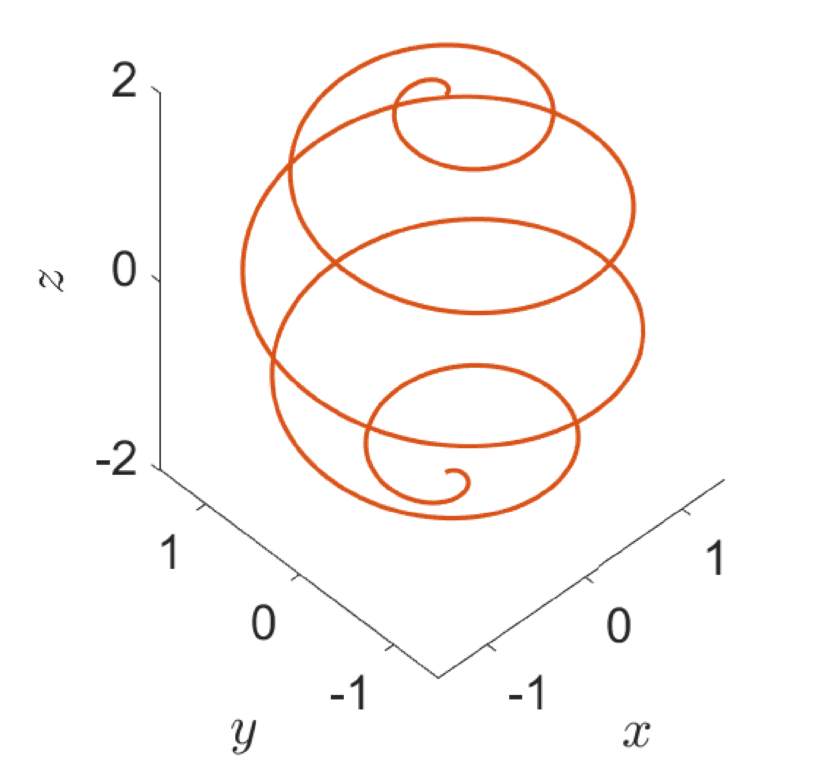
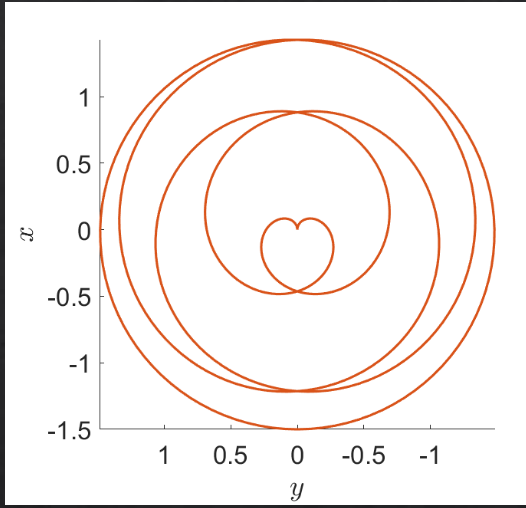
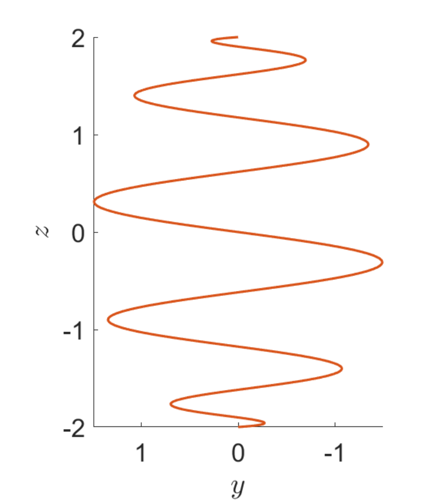
Trivariate signal ($N = 3$)

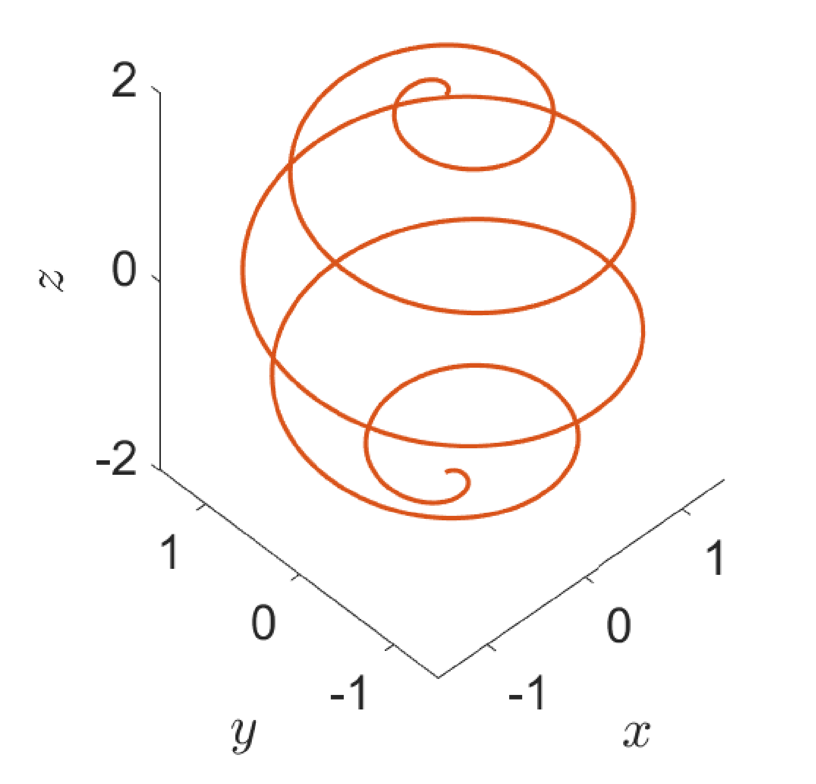






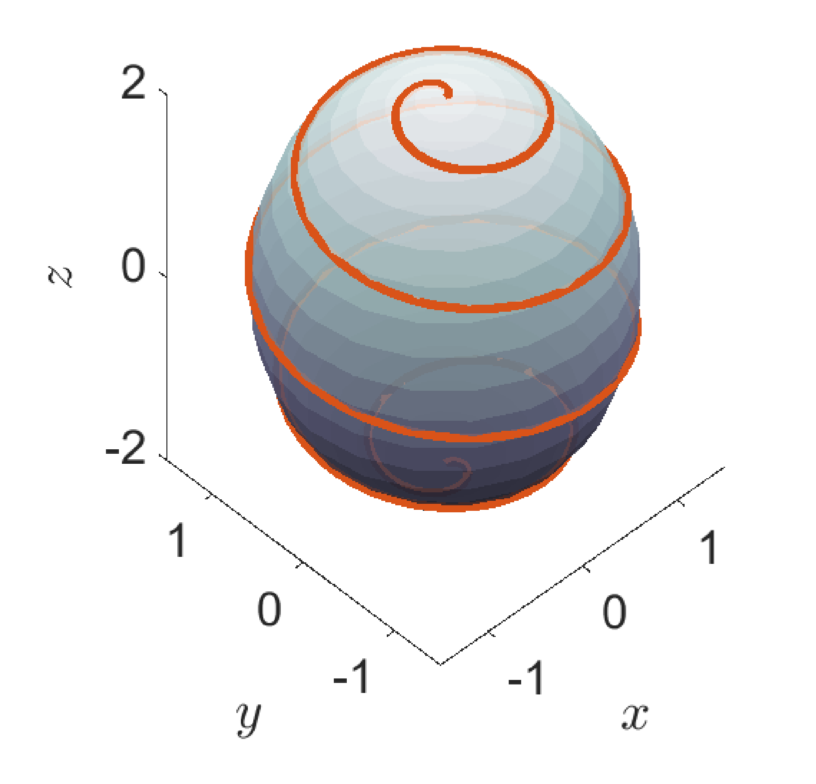
$$\mathbf{g}(t) = \begin{pmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \end{pmatrix} = \begin{pmatrix} 1.5sin(f_1 t) \cdot cos(10 f_1 t) \\ 1.5sin(f_1 t) \cdot sin(10 f_1 t) \\ 2cos(f_1 t) \end{pmatrix}$$

Multi-dimensional envelopes



Objective: find multi-dimensional envelope enclosing the signal to calculate local mean $E_{mean}(t)$

Multi-dimensional envelopes



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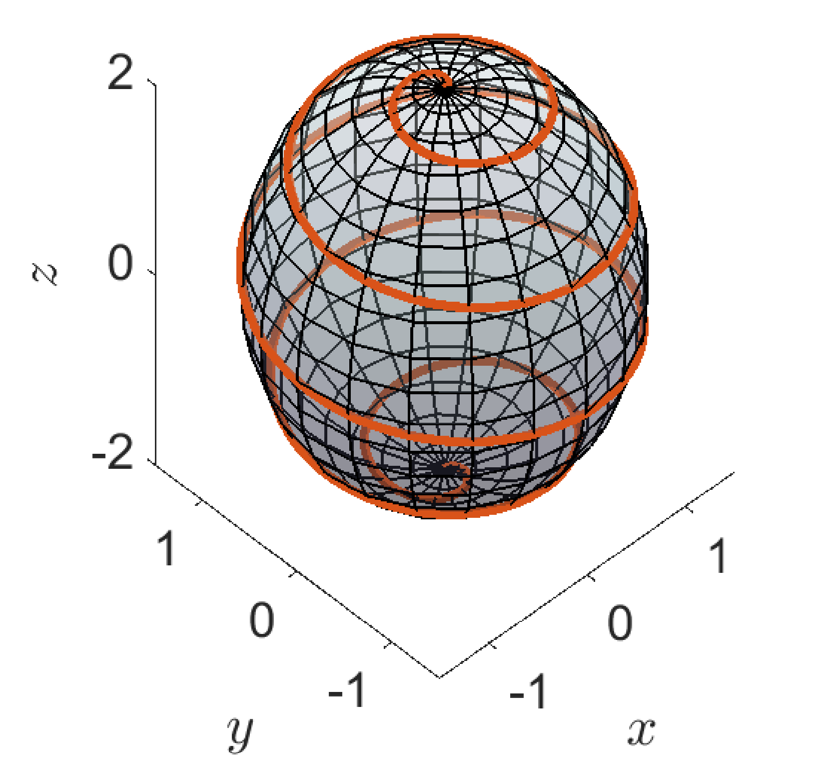

Objective: find multi-dimensional envelope enclosing the signal to calculate local mean $E_{mean}(t)$

Solution:

- approximate single N-D envelope by multiple envelopes across multiple directions K in N-D space
- as $K \to \infty$, the approximate local mean approaches the true local mean

Signal projections

<u>Objective:</u> find uniformly distributed direction vectors

Signal projections

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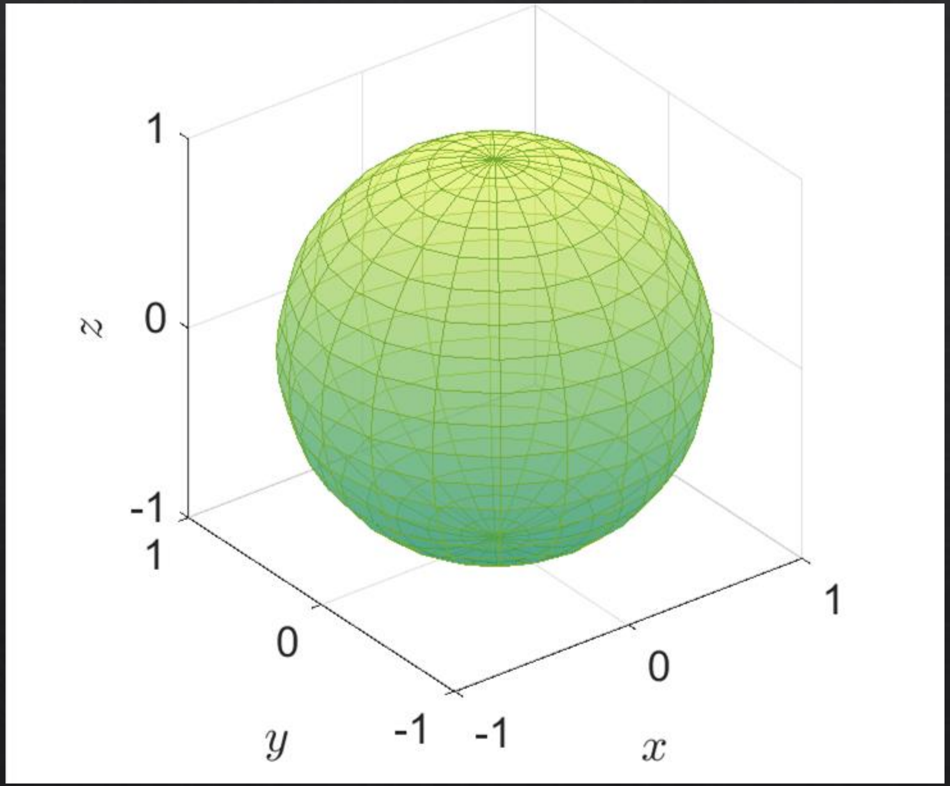
Solution:

- Each direction vector can be represented as a point on the unit $(N-1)$ sphere
- Objective can be transferred to finding a uniform sampling scheme on a hypersphere

Signal projections



Unit two-sphere

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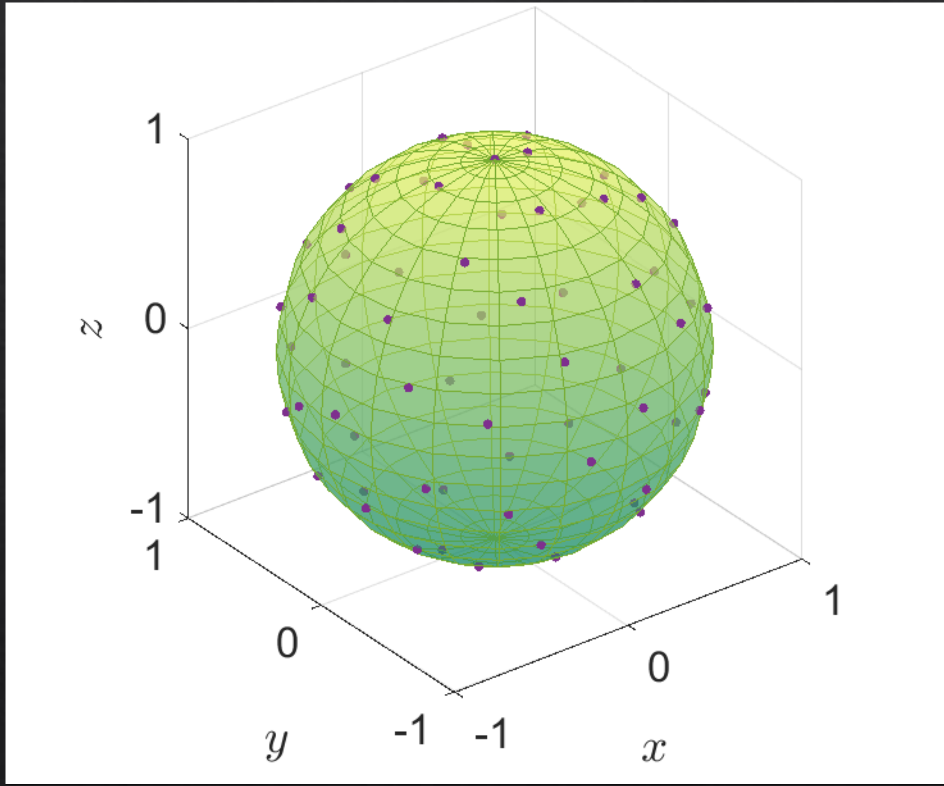
Signal projections



Unit two-sphere
with K = 64 direction points

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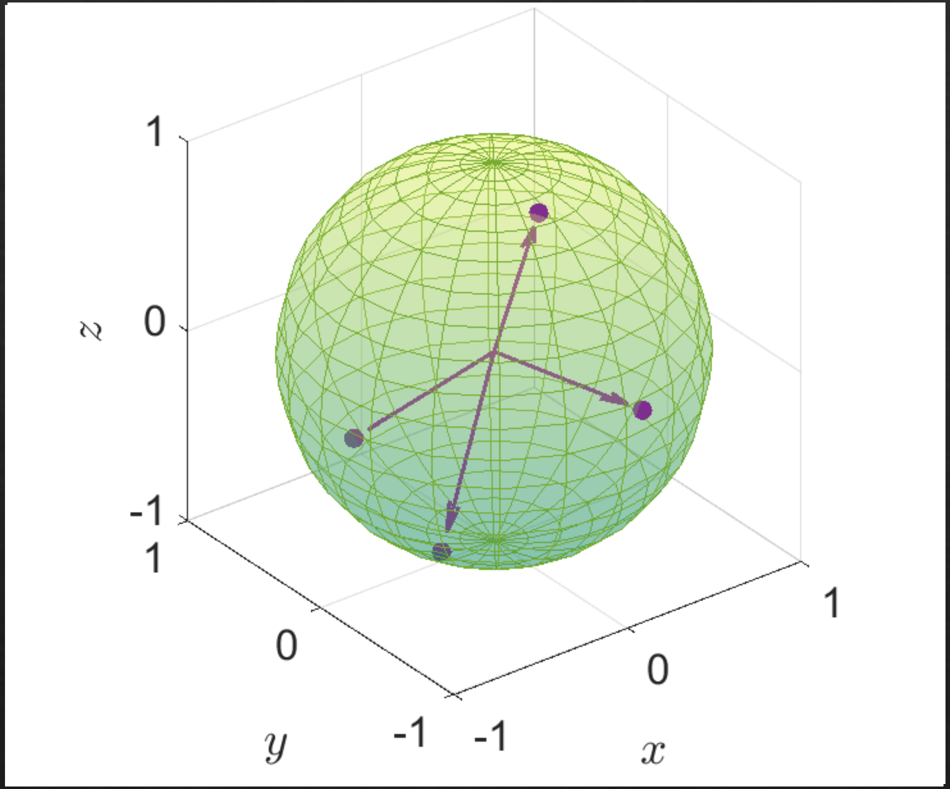
→ uniform sampling using low-discrepancy Hammersley sequence

Signal projections



Unit two-sphere
with exemplary direction vectors

Objective: find uniformly distributed direction vectors

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- Each direction vector can be represented as a point on the unit $(N-1)$ sphere
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→ uniform sampling using
low-discrepancy Hammersley sequence

→ direction vectors $\boldsymbol{x}^{\theta_k} = \{\boldsymbol{x}_1^k, \boldsymbol{x}_2^k, \dots, \boldsymbol{x}_N^k\}$ based on the angles θ_k corresponding to the points

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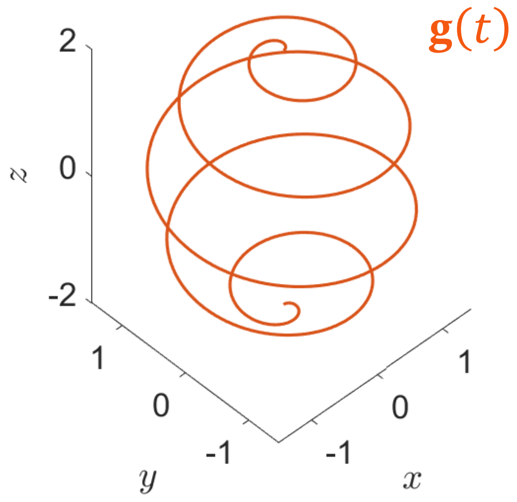
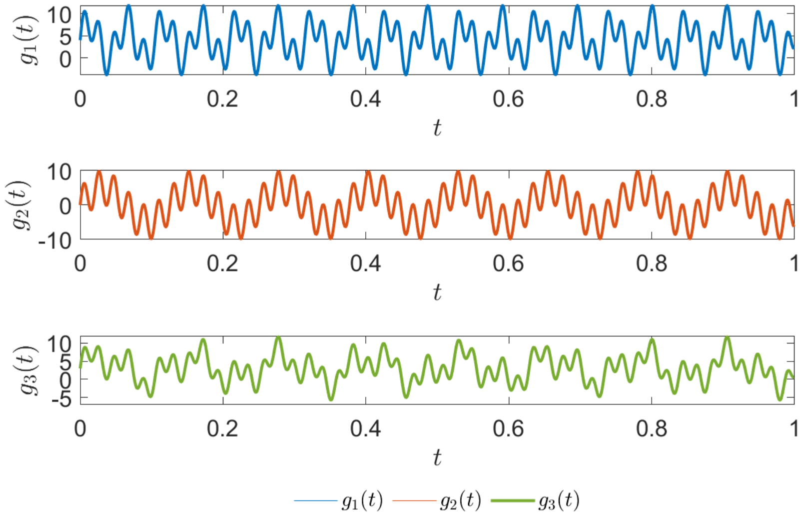


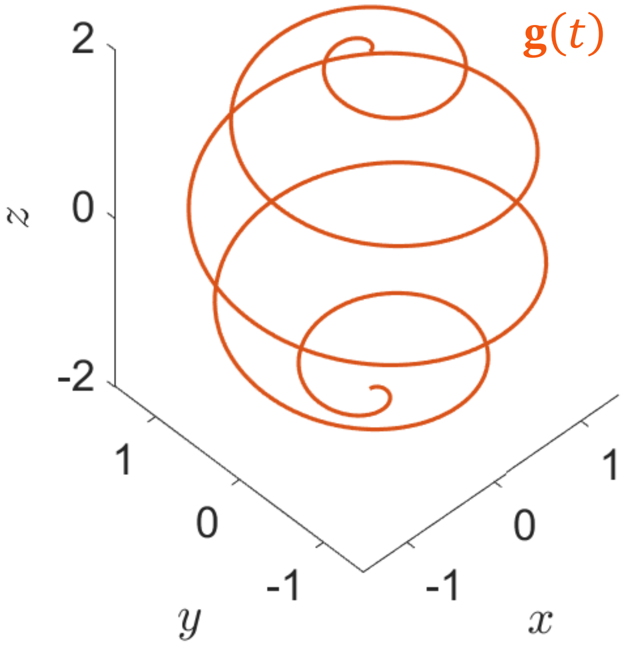


The main feature

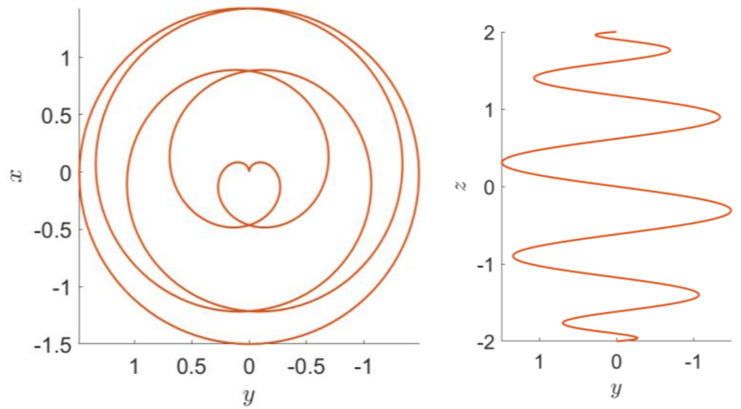
Objective: find and align common scales within multivariate data

How is this achieved?

- The MEMD simultaneously decomposes all signals
- Scales common to two or more signals are found in equal-indexed IMFs
- This "mode alignment property" enables a profound analysis across multiple signals



$$\mathbf{g}(t) = \begin{pmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \end{pmatrix} = \begin{pmatrix} 1.5sin(f_1 t) \cdot cos(10 f_1 t) \\ 1.5sin(f_1 t) \cdot sin(10 f_1 t) \\ 2cos(f_1 t) \end{pmatrix}$$



The main feature

Objective: find and align common scales within multivariate data

How is this achieved?

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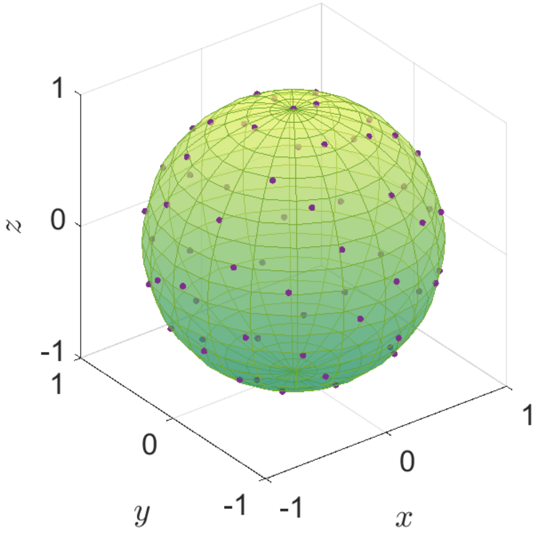
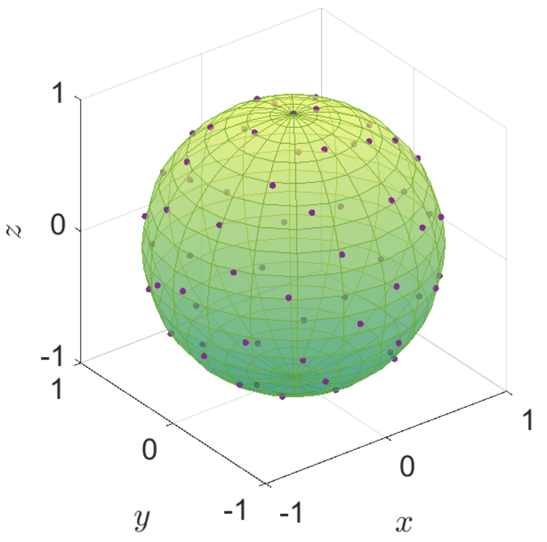
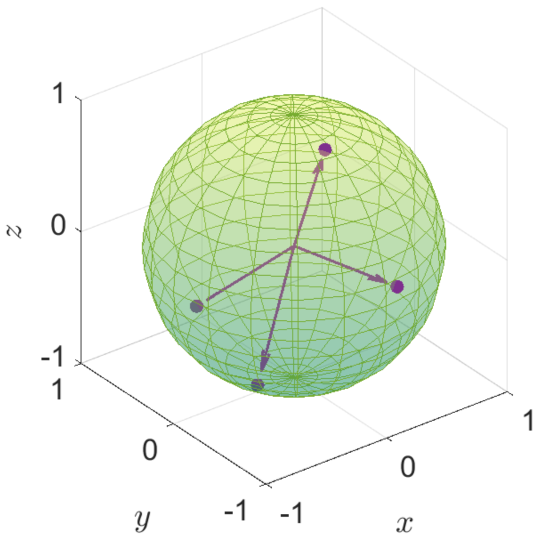


The algorithm

1. Choose a suitable pointset and determine direction vectors $\boldsymbol{x}^{\theta_k} = \{x_1^k, x_2^k, \dots, x_N^k\}$





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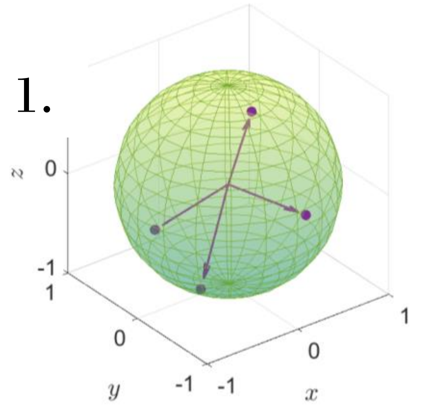

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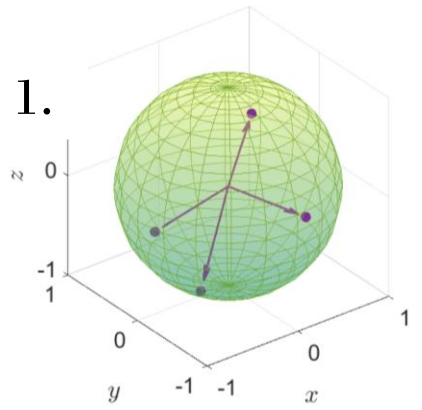
1.



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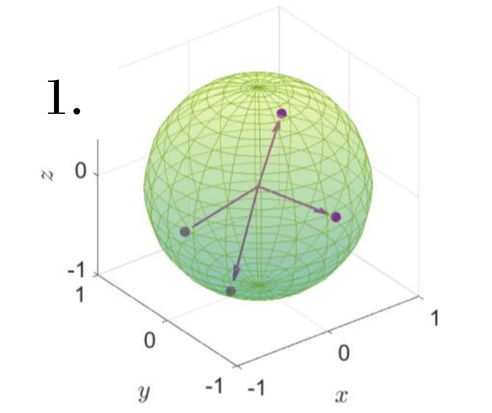

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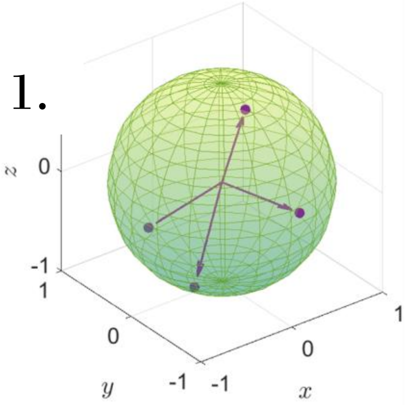
$$= \begin{bmatrix} g_1(t_1) & g_2(t_1) & g_3(t_1) \\ \dots & \dots & \dots \\ g_1(t_T) & g_2(t_T) & g_3(t_T) \end{bmatrix} \cdot \begin{pmatrix} x_1^{\theta_k} \\ x_2^{\theta_k} \\ x_3^{\theta_k} \end{pmatrix}$$

1.



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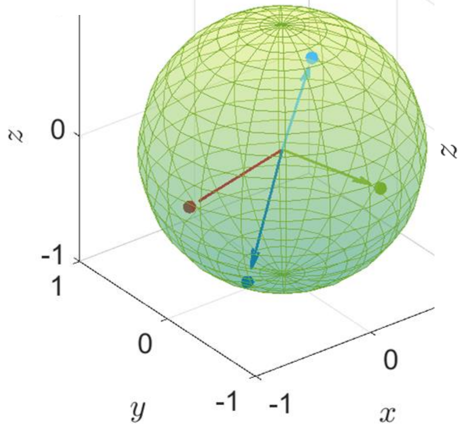
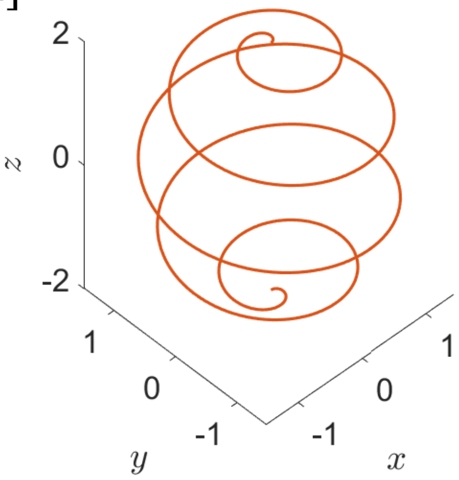
$$
\begin{aligned}
P^{\theta_k}(t) &= \boldsymbol{g}(\boldsymbol{t}) \cdot \boldsymbol{x}^{\boldsymbol{\theta}_k} \\
&= \begin{bmatrix} g_1(t_1) & g_2(t_1) & g_3(t_1) \\ \dots & \dots & \dots \\ g_1(t_T) & g_2(t_T) & g_3(t_T) \end{bmatrix} \cdot \begin{pmatrix} x_1^{\theta_k} \\ x_2^{\theta_k} \\ x_3^{\theta_k} \end{pmatrix} \\
&= \begin{pmatrix} g_1(t_1) \cdot x_1^{\theta_k} + g_2(t_1) \cdot x_2^{\theta_k} + g_3(t_1) \cdot x_3^{\theta_k} \\ \dots \\ g_1(t_T) \cdot x_1^{\theta_k} + g_2(t_T) \cdot x_2^{\theta_k} + g_3(t_T) \cdot x_3^{\theta_k} \end{pmatrix}
\end{aligned}
$$

→ from multivariate input data to K univariate projected signals

1. $k = [1, 10, 35, 61]$





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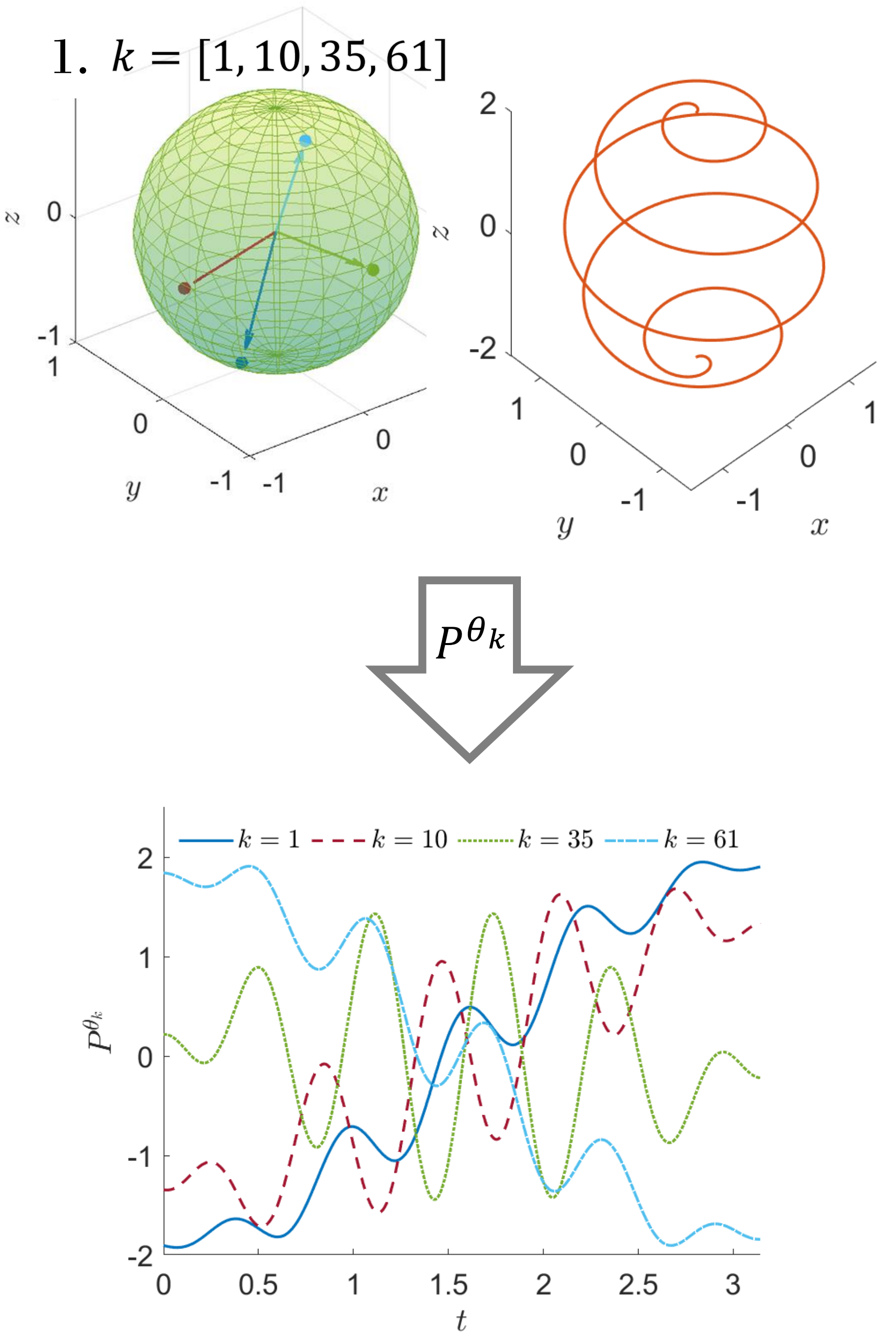

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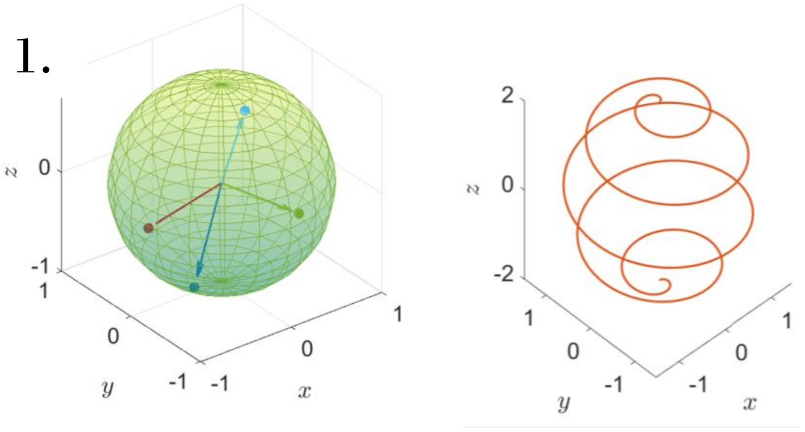

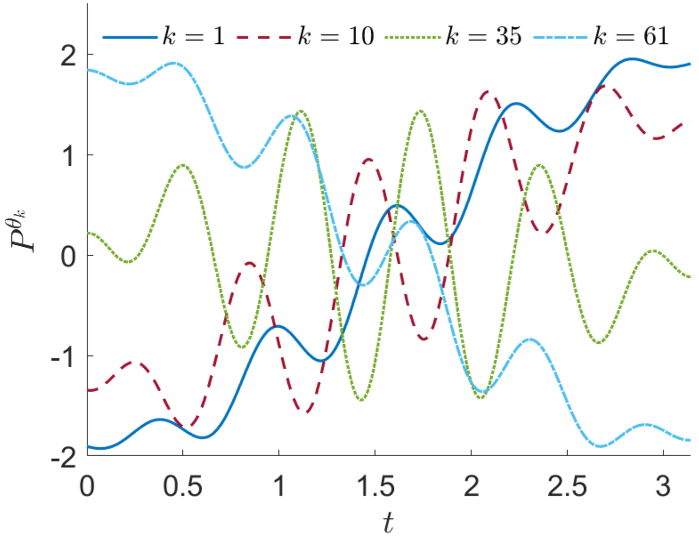

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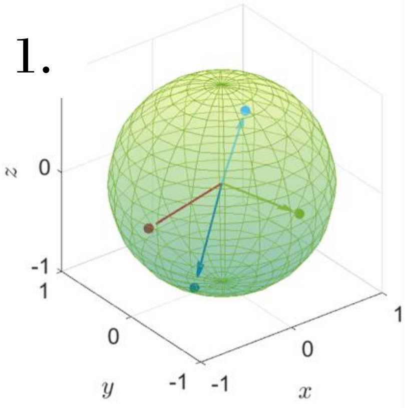

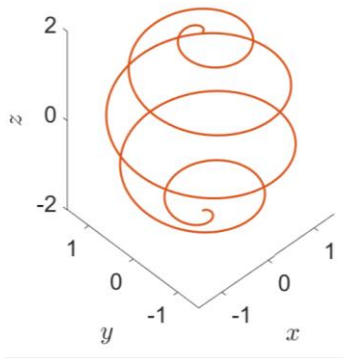

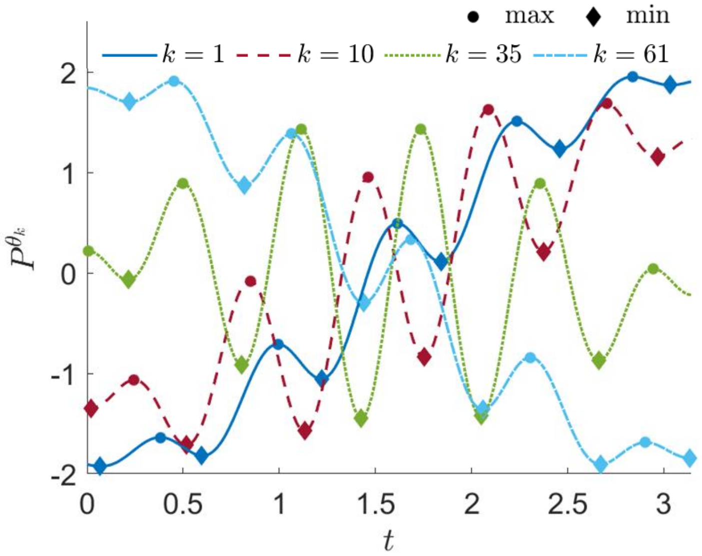

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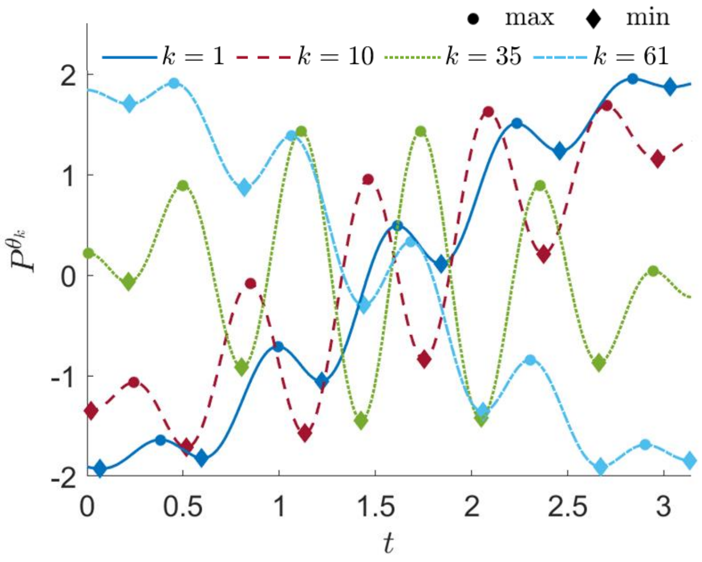

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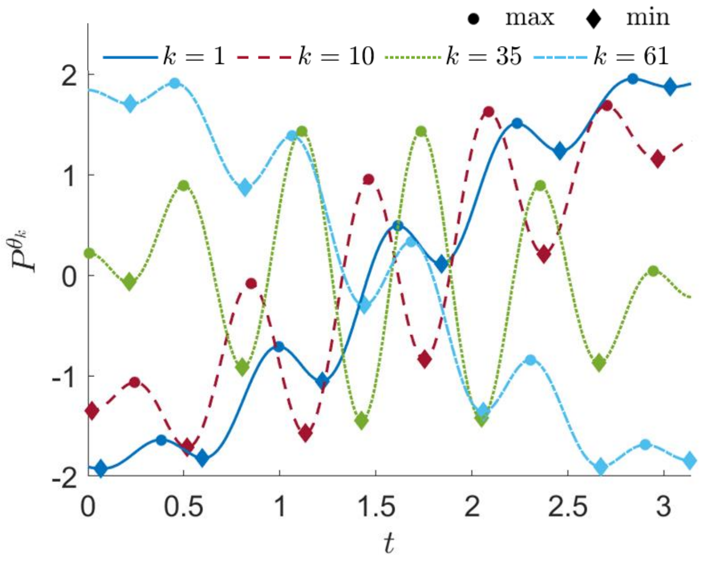

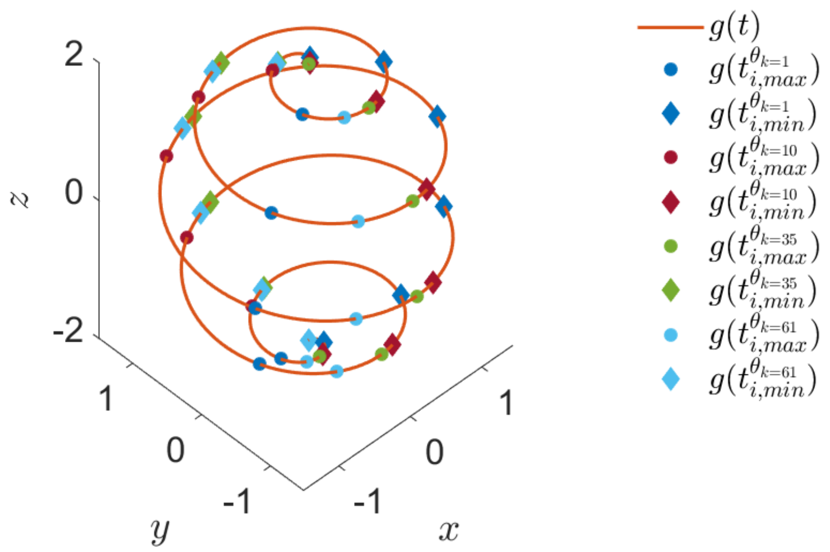

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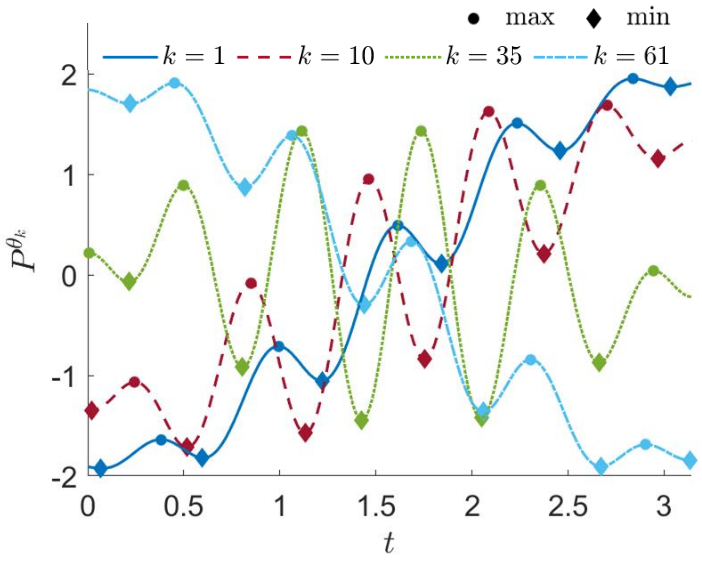

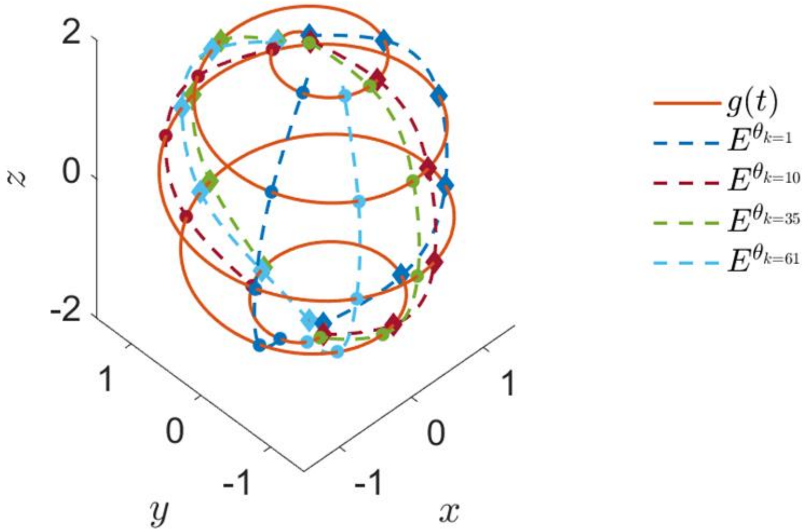

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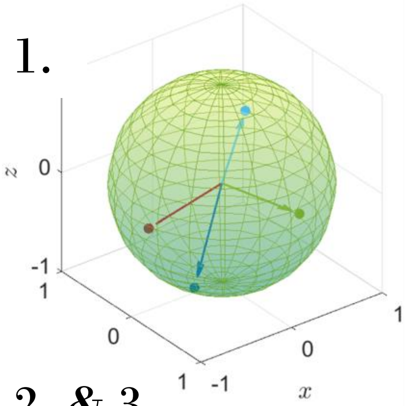
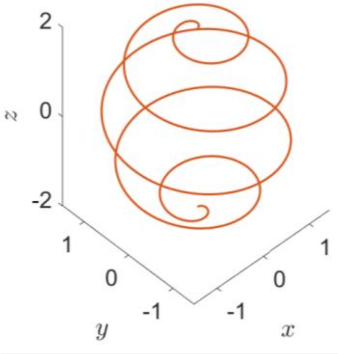
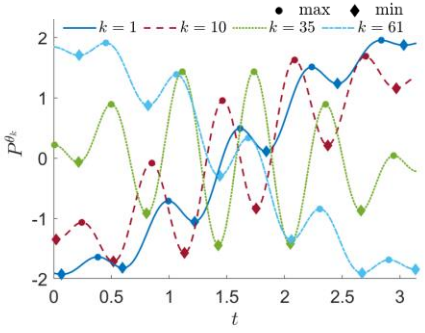
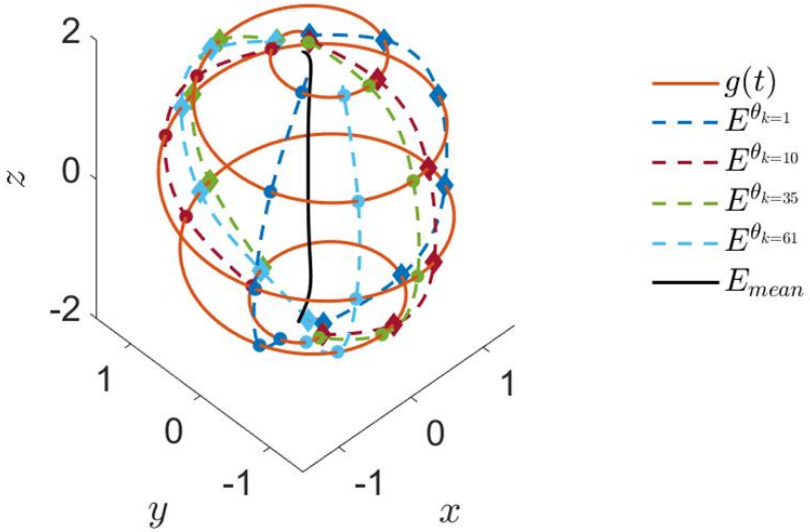




2. & 3.





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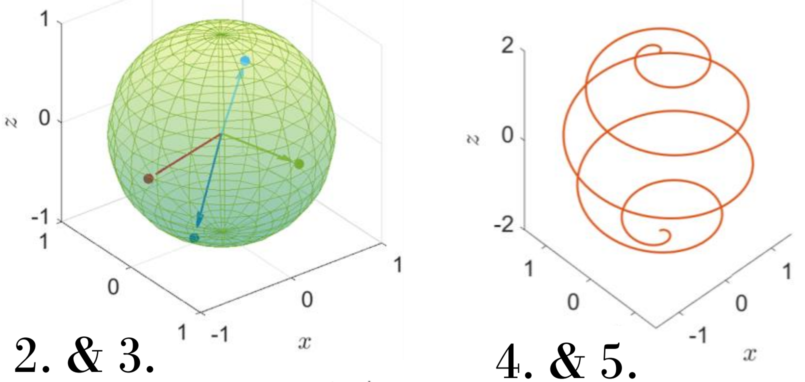

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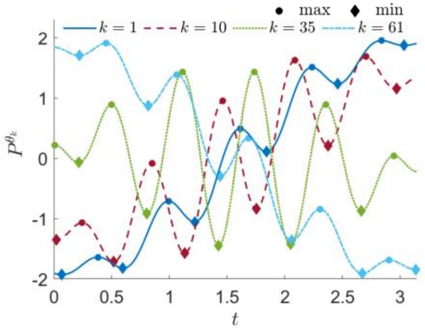

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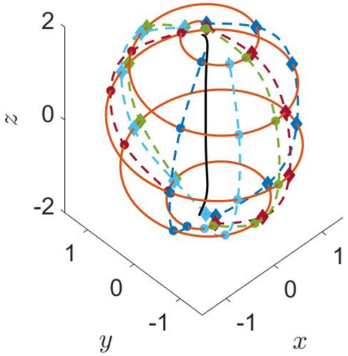

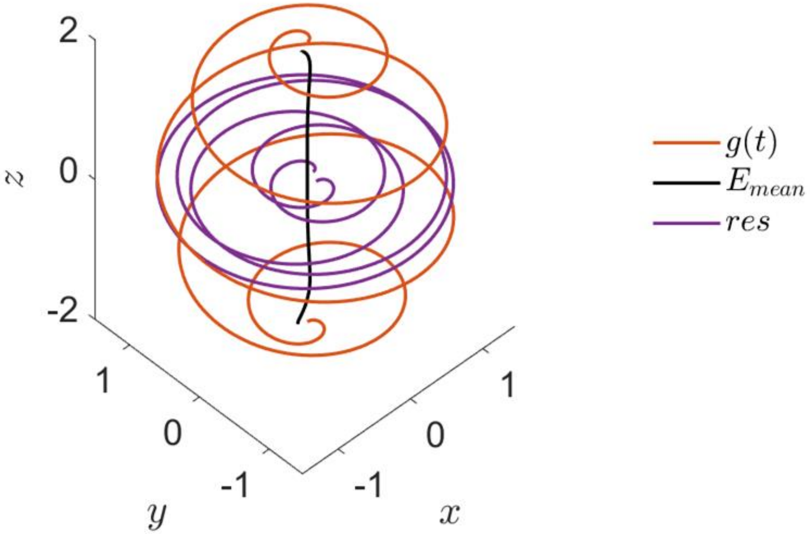

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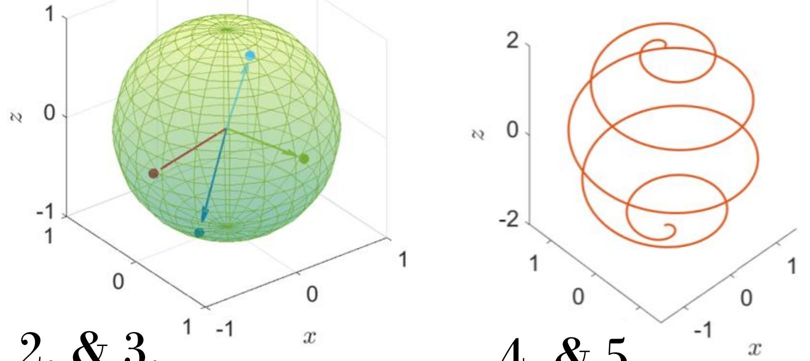
6. Determine residual

$$\boldsymbol{res}\,(t) = \boldsymbol{g}(t) - E_{mean}(t)$$

1.



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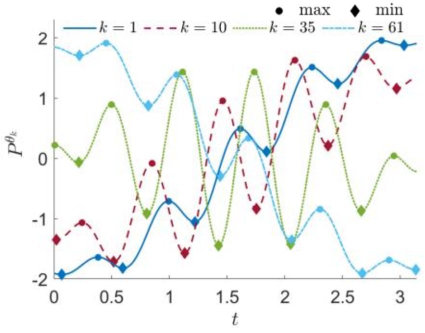

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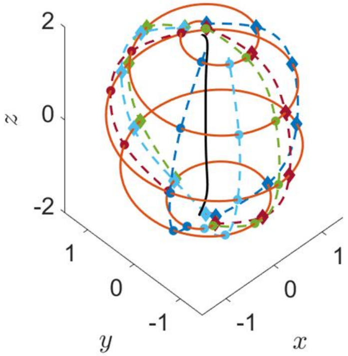

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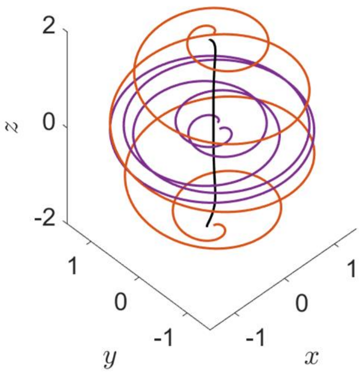

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6. Determine residual

$$\boldsymbol{res}\ (t) = \boldsymbol{g}(t) - E_{mean}(t)$$

7. Check stopping criterion

The stopping criterion

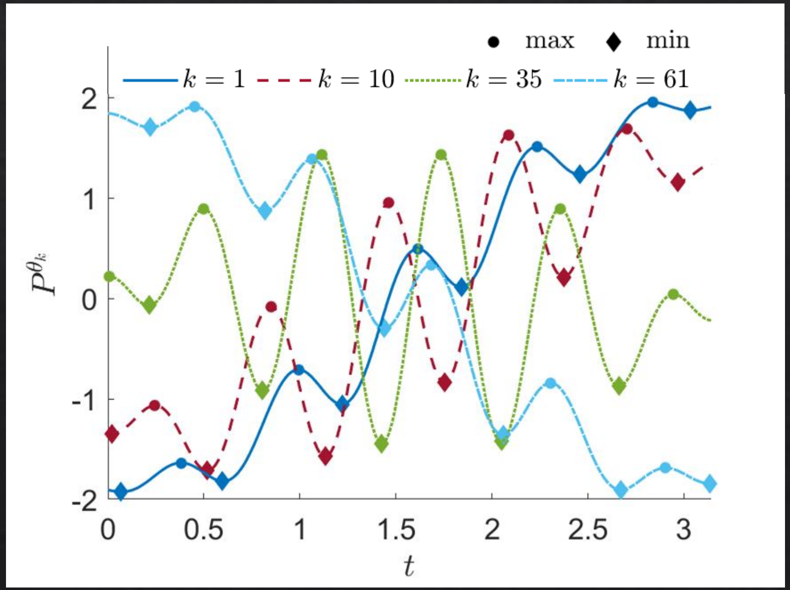
both criteria true to continue sifting

The stopping criterion

both criteria true to continue sifting

Number of extrema:
at least three extrema along at least one projection direction

$$any\left(\#\left\{t_i^{\theta_k}\right\} > 2\right)$$



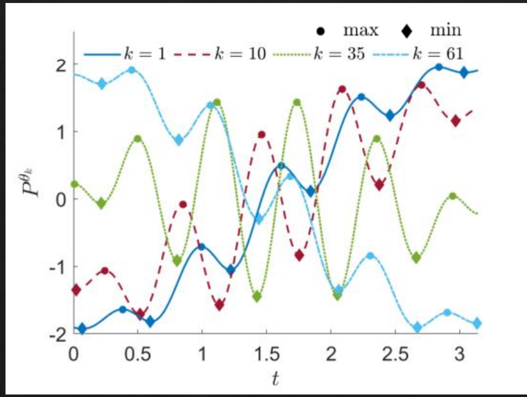
The stopping criterion

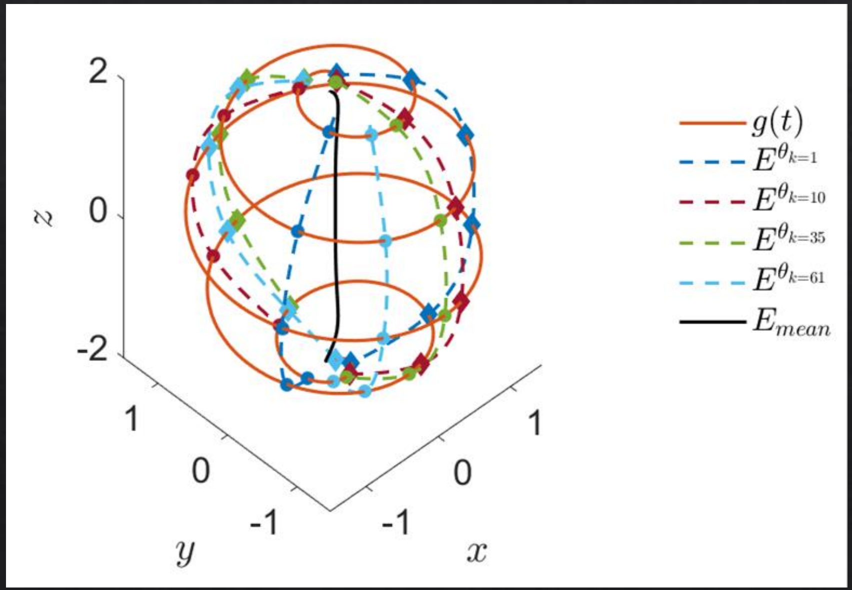
both criteria true to continue sifting

<u>Number of extrema:</u>
at least three extrema along at least one projection direction

$$any\left(\#\left\{t_i^{\theta_k}\right\} > 2\right)$$

<u>Mean envelope characteristics:</u>
relation of maxima and minima envelopes to mean envelope





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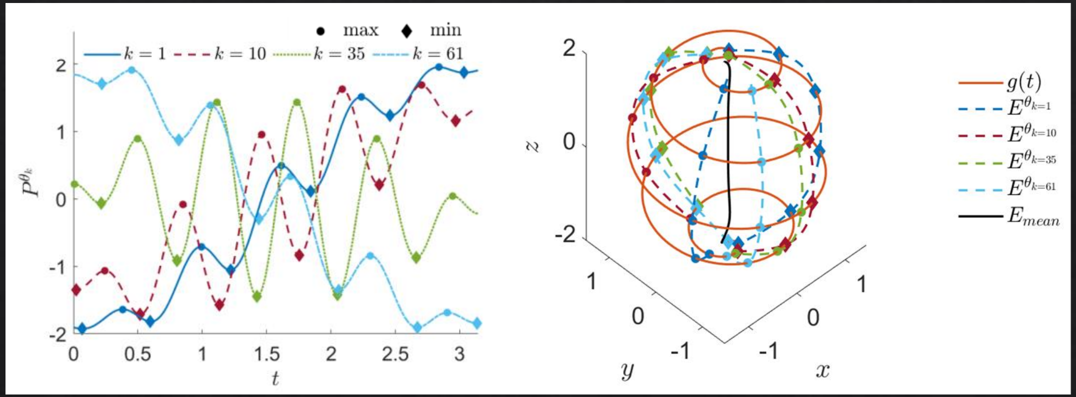

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Mean envelope characteristics:
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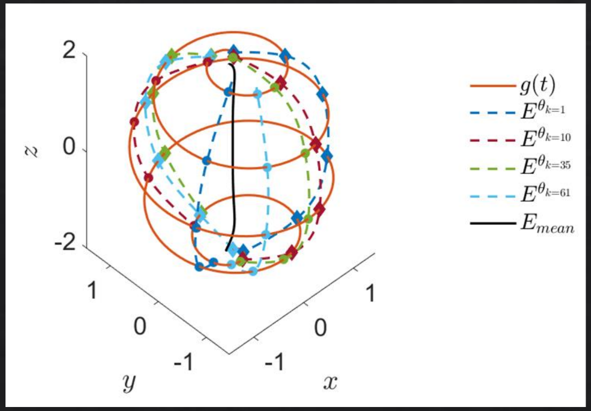
at least one true to continue

$$\Lambda = \frac{\sqrt{\sum_{n=1}^{N} E_{mean}^2(t)}}{\frac{1}{2K}\sum_{k=1}^{K}\sqrt{\sum_{n=1}^{N}\left(E_{max}^{\theta_k}(t) - E_{min}^{\theta_k}(t)\right)^2}}$$



The stopping criterion

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Local zero mean:
optimize mean envelope

$$\left(\frac{1}{G}\sum_{g=1}^{G}(\Lambda > \epsilon_1)\right) > \epsilon_2$$

The stopping criterion

both criteria true to continue sifting

<u>Number of extrema:</u>
at least three extrema along at least one projection direction

$$any\left(\#\left\{t_i^{\theta_k}\right\} > 2\right)$$

<u>Mean envelope characteristics:</u>
relation of maxima and minima envelopes to mean envelope

at least one true to continue

<u>Local zero mean:</u>
optimize mean envelope

$$\left(\frac{1}{G}\sum_{g=1}^{G}(\Lambda > \epsilon_1)\right) > \epsilon_2$$

<u>Outlier detection:</u>

$$any(\Lambda > \epsilon_3)$$

$$\Lambda = \frac{\sqrt{\sum_{n=1}^{N} E_{mean}^2(t)}}{\frac{1}{2K}\sum_{k=1}^{K}\sqrt{\sum_{n=1}^{N}\left(E_{max}^{\theta_k}(t) - E_{min}^{\theta_k}(t)\right)^2}}$$

The algorithm

Inner loop: iterate until residual possesses IMF characteristics

1. Choose a suitable pointset and determine direction vectors $\boldsymbol{x}^{\theta_k} = \{x_1^k, x_2^k, \dots, x_N^k\}$

   $res(t)$ $res(t)$

2. Project N-variate signal $\cancel{\boldsymbol{g}(t)}$ onto direction vectors $\boldsymbol{x}^{\theta_k}$ yielding projections $P^{\theta_k}(t)\}_{k=1}^{K}$
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   res

4. Interpolate signal $\cancel{\boldsymbol{g}}\left(t_i^{\theta_k}\right)$ at $\{t_i^{\theta_k}\}$ to obtain multivariate envelopes $E^{\theta_k}(t)\}_{k=1}^{K}$
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6. Determine residual

   $res(t)$

$$\boldsymbol{res}\,(t) = \cancel{\boldsymbol{g}(t)} - E_{mean}(t)$$

if false

7. Check stopping criterion

The algorithm

Inner loop: iterate until residual possesses IMF characteristics

1. Choose a suitable pointset and determine direction vectors $\boldsymbol{x}^{\theta_k} = \{x_1^k, x_2^k, \dots, x_N^k\}$

   $res(t)$

2. Project N-variate signal $\overset{res(t)}{\cancel{\boldsymbol{g}(t)}}$ onto direction vectors $\boldsymbol{x}^{\theta_k}$ yielding projections $P^{\theta_k}(t)\}_{k=1}^{K}$
3. Determine extrema locations $\{t_i^{\theta_k}\}$ of projections
4. Interpolate signal $\overset{res}{\cancel{\boldsymbol{g}}}\left(t_i^{\theta_k}\right)$ at $\{t_i^{\theta_k}\}$ to obtain multivariate envelopes $E^{\theta_k}(t)\}_{k=1}^{K}$
5. Average over all projection directions to get mean envelopes

$$E_{mean}(t) = \frac{1}{2K}\sum_{k=1}^{K} E_{min}^{\theta_k}(t) + E_{max}^{\theta_k}(t)$$

6. Determine residual

$$\boldsymbol{res}\,(t) = \overset{res(t)}{\cancel{\boldsymbol{g}(t)}} - E_{mean}(t)$$

7. Check stopping criterion

if false

$res(t)$

The algorithm

<u>Inner loop</u>: iterate until residual possesses IMF characteristics

<u>Outer loop</u>: iterate until all IMFs are extracted from the signal

1. Choose a suitable pointset and determine direction vectors $= \{x_1^k, x_2^k, \dots, x_N^k\}$

$g(t) \leftarrow g(t) - imf_j(t)$

$res(t)$

2. Project N-variate signal $\boldsymbol{g}(t)$ onto direction vectors $\boldsymbol{x}^{\theta_k}$ yielding projections $P^{\theta_k}(t)\}_{k=1}^{K}$
3. Determine extrema locations $\{t_i^{\theta_k}\}$ of projections
4. Interpolate signal $\boldsymbol{g}\left(t_i^{\theta_k}\right)$ at $\{t_i^{\theta_k}\}$ to obtain multivariate envelopes $E^{\theta_k}(t)\}_{k=1}^{K}$
5. Average over all projection directions to get mean envelopes

$$E_{mean}(t) = \frac{1}{2K}\sum_{k=1}^{K} E_{min}^{\theta_k}(t) + E_{max}^{\theta_k}(t)$$

6. Determine residual

$$\boldsymbol{res}\,(t) = \boldsymbol{g}(t) - E_{mean}(t)$$

7. Check stopping criterion

if true

$imf_j(t) = res\,(t)$

if false