Combining EMD with NOISE

Addition of noise to data

Objective: enhance the EMD filtering process

by reducing mode mixing

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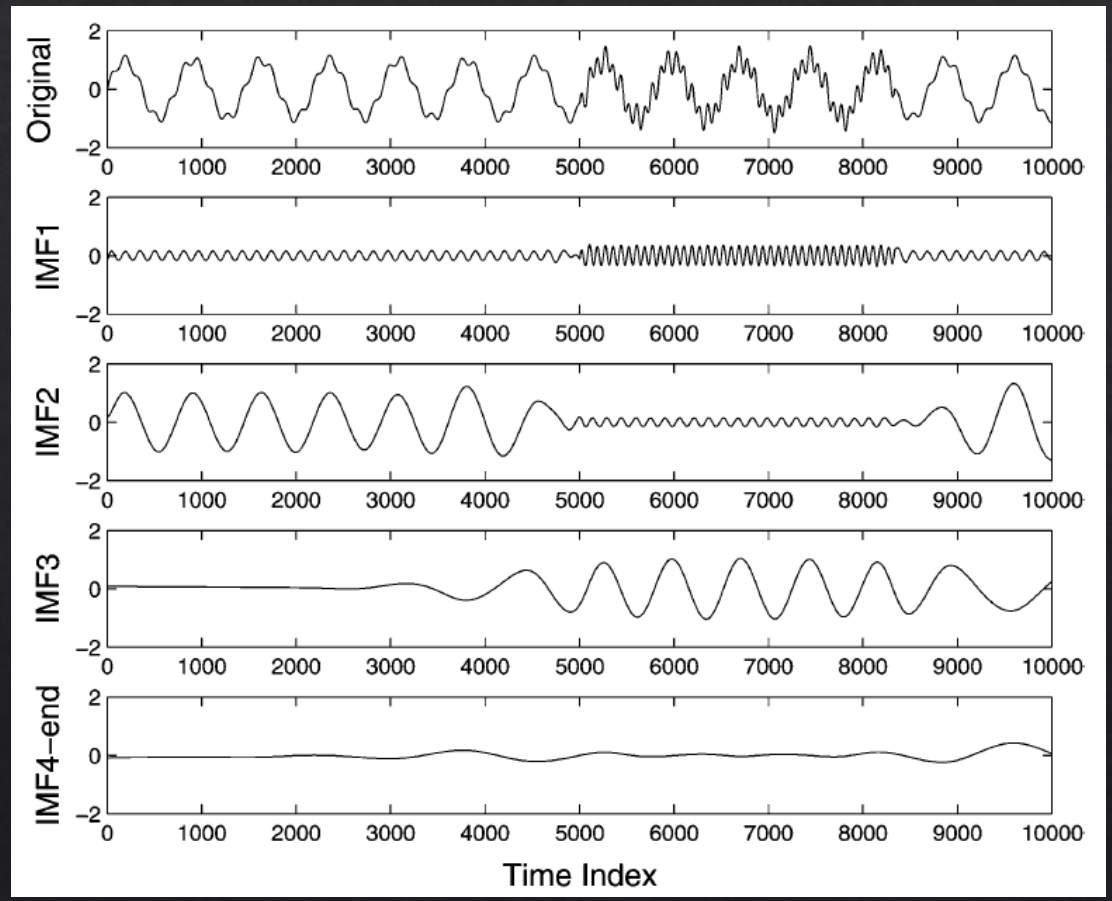
Mode mixing:

- single IMF contains multiple oscillatory modes
- single mode residing in multiple IMFs

→ physical meaning of IMFs may be compromised



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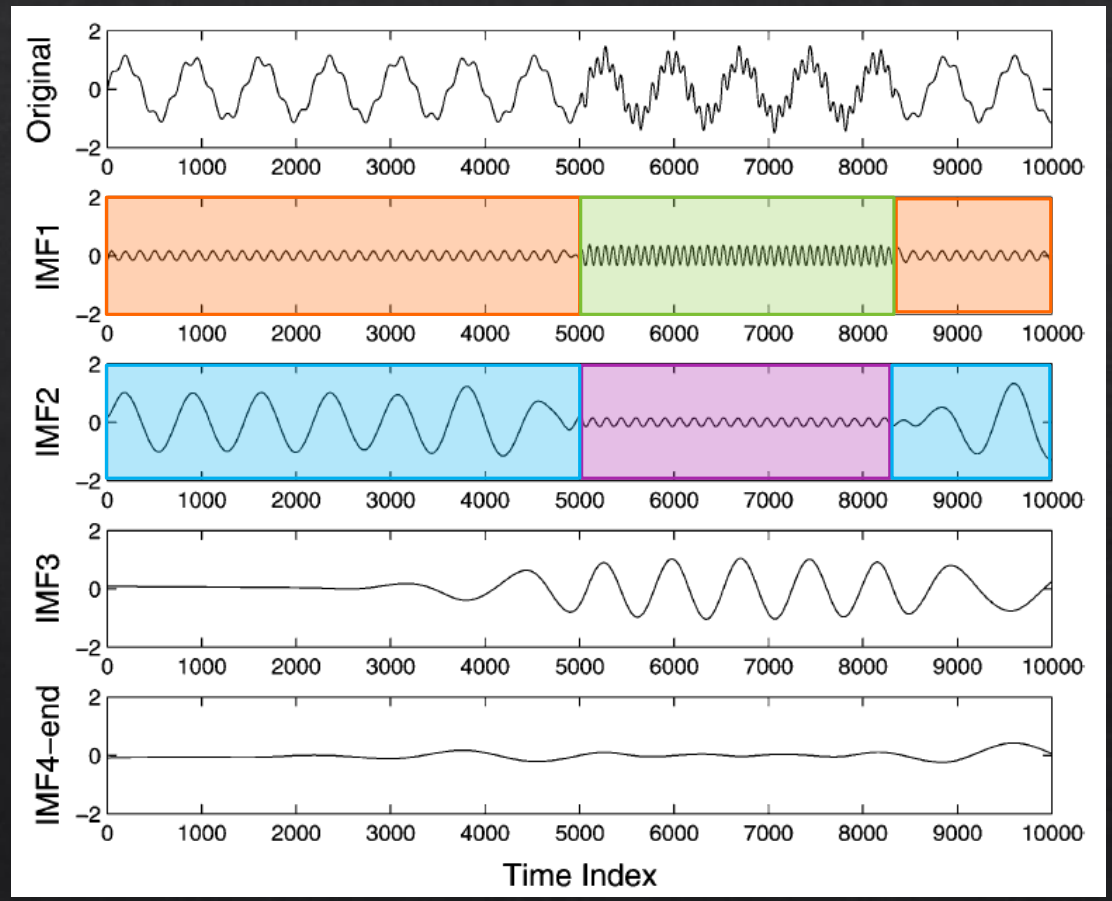

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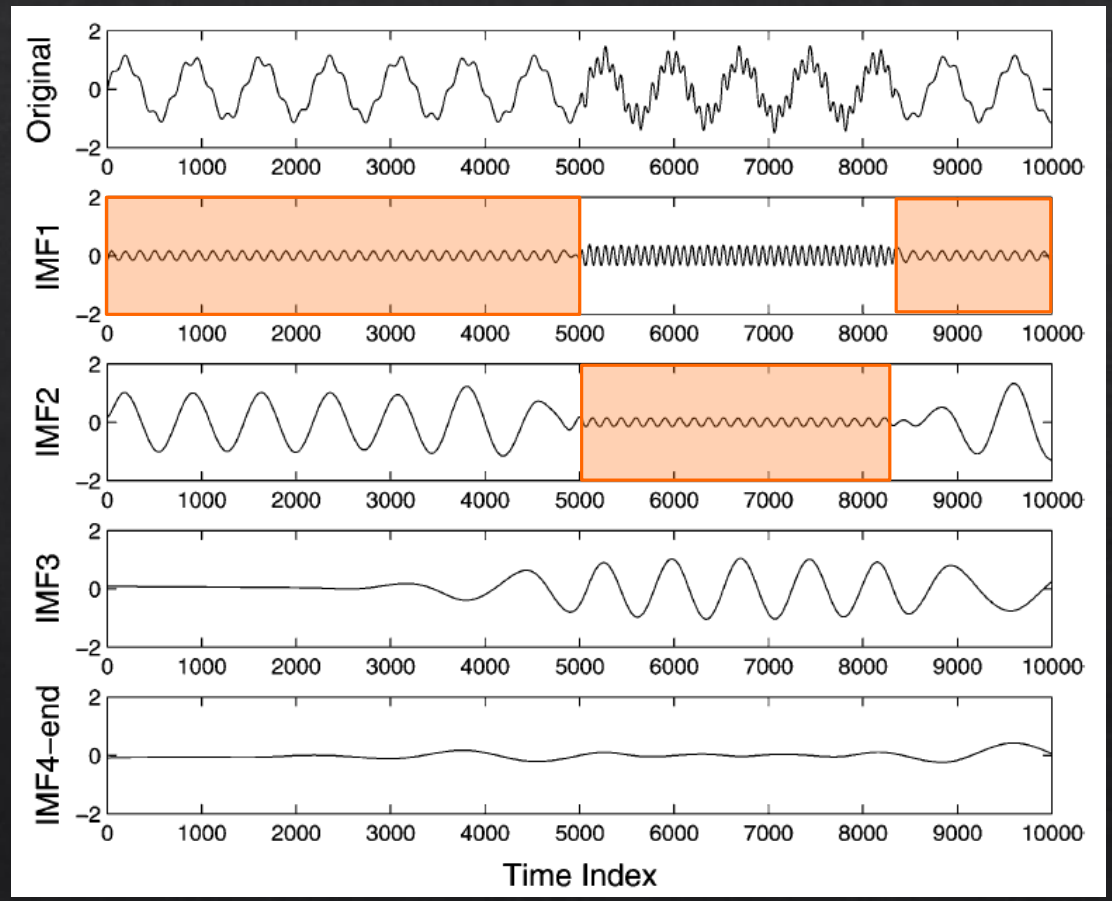

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How can we reduce mode mixing?

Using dyadic filter bank property of EMD

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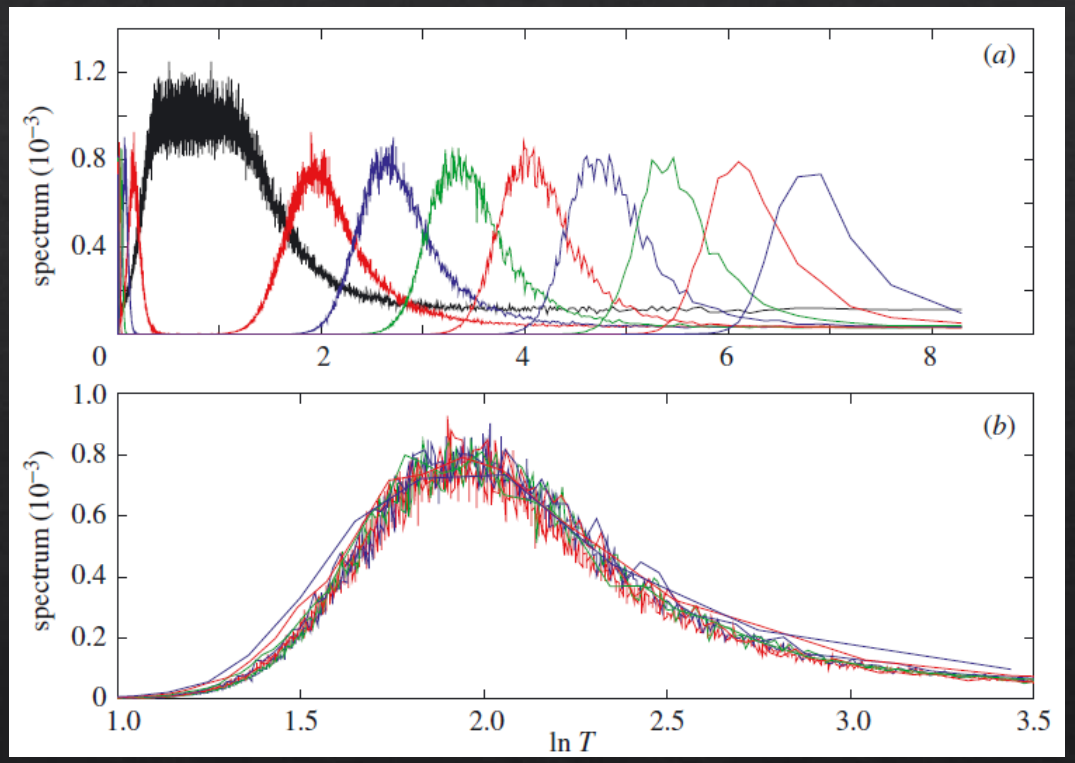
- doubled mean periods of neighboring IMFs



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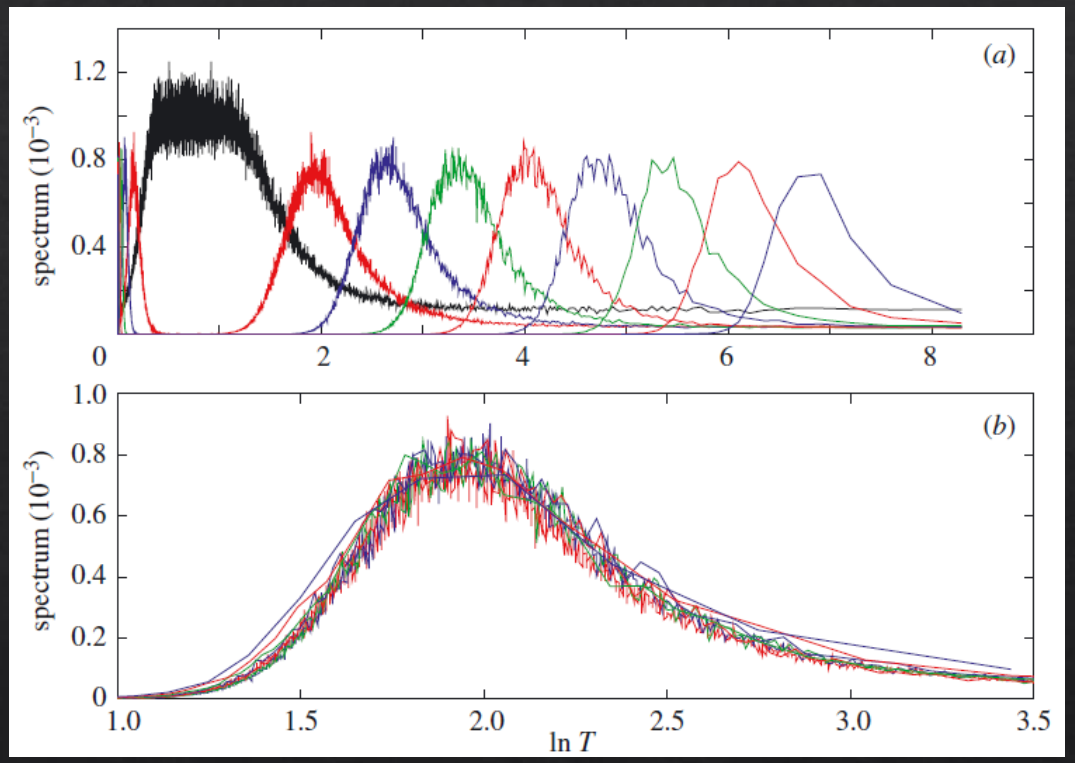

[1]

→ Enforce this distinct mode separation to arbitrary data by using noise in the decomposition

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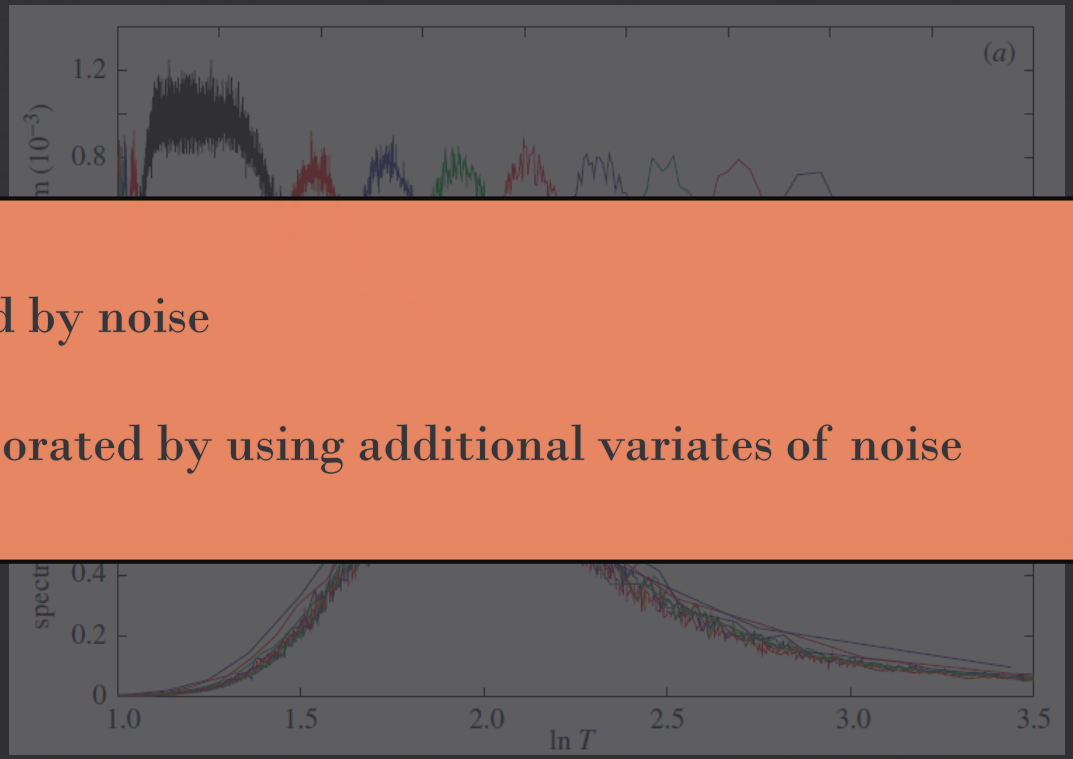


→ **Ensemble EMD**: input data is directly disturbed by noise

→ **Noise-assisted MEMD**: noise is indirectly incorporated by using additional variates of noise

- IMFs have identical Fourier spectra
- doubled mean periods of neighboring IMFs

[1]

→ Enforce this distinct mode separation to arbitrary data by using noise in the decomposition

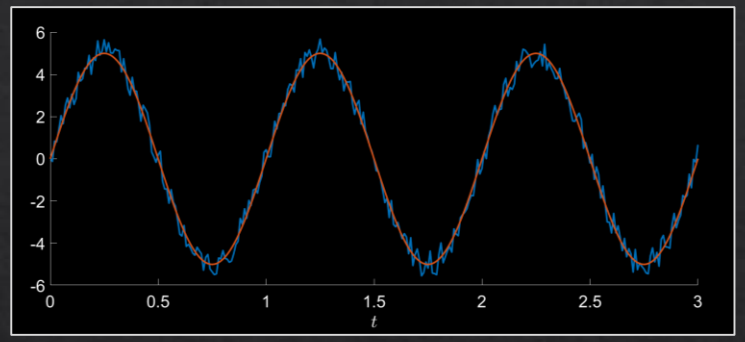
Ensemble EMD

Approach:

1. Add ensembles of white Gaussian noise $w_n(t)$ to input data $f(t)$

$$f_n(t) = f(t) + w_n(t)$$

2. Decomposition of the noisy data $f_n(t)$ with the standard EMD



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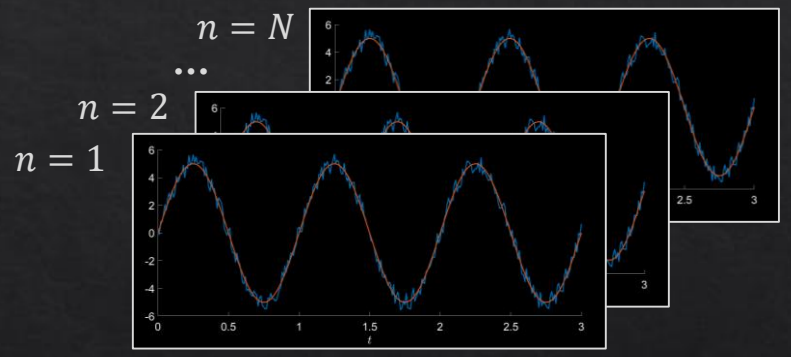

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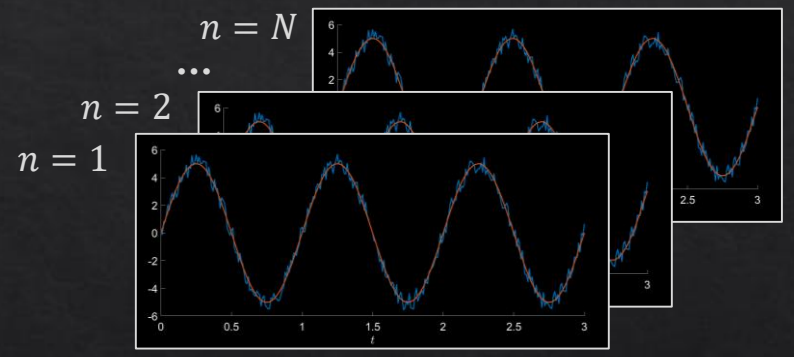

3. Taking ensemble mean of all resulting IMFs

$$imf(t) = \frac{1}{N}\sum_{n=1}^{N} imf_n(t)$$

→ statistically, the added noise cancels out in the ensemble result

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Ensemble EMD

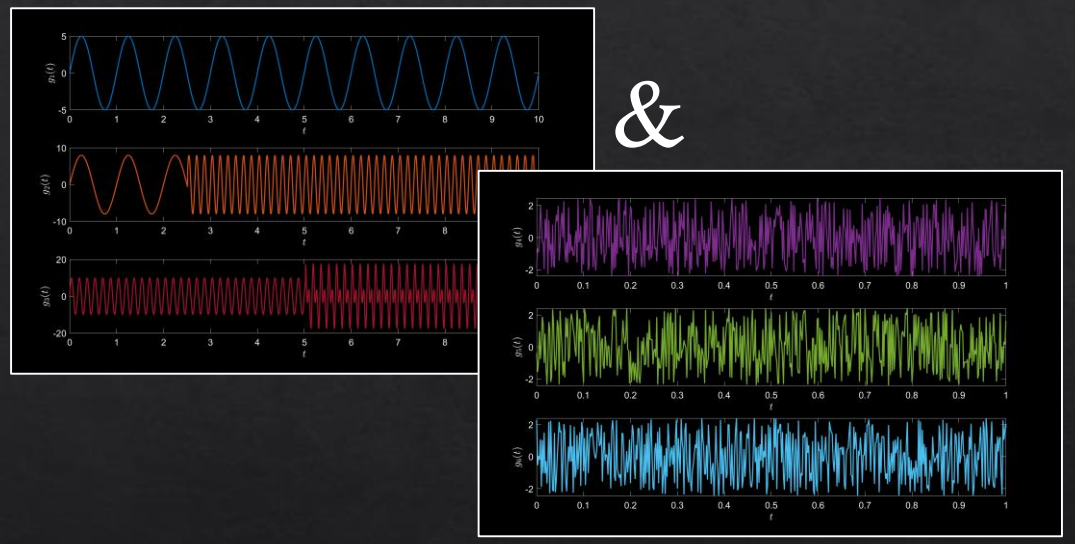
Drawbacks:

- Infinitely many ensemble members are necessary to completely remove the effect of noise addition
  - → residual noise is mostly present in the result
  - → High accuracy requires high computational effort
- Ensemble mean can spoil the IMF characteristics
- A varying number of IMFs could be obtained in different trials, which makes it impossible to adequately perform the ensemble averaging

Noise-assisted MEMD

Approach:

White Gaussian noise is appended as adjacent, independent realizations in separate channels, i.e., additional variates



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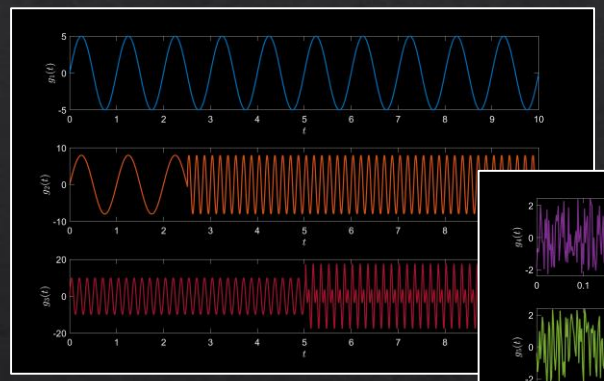
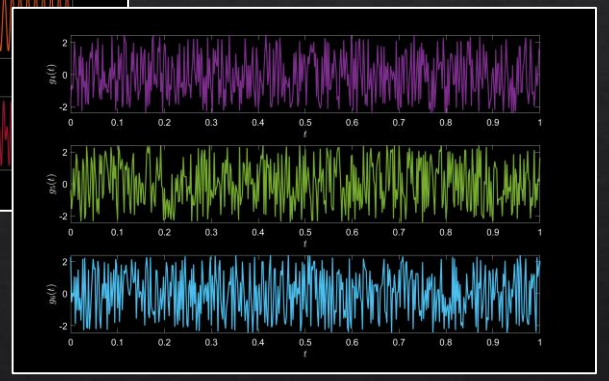


&



Advantages:

- No noise artefacts in original data
- Generally, less computational effort compared to EEMD

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Example of NA-MEMD

Frequencies: $f_1 = 1\ Hz$
$f_2 = 6\ Hz$
$f_3 = 12\ Hz$

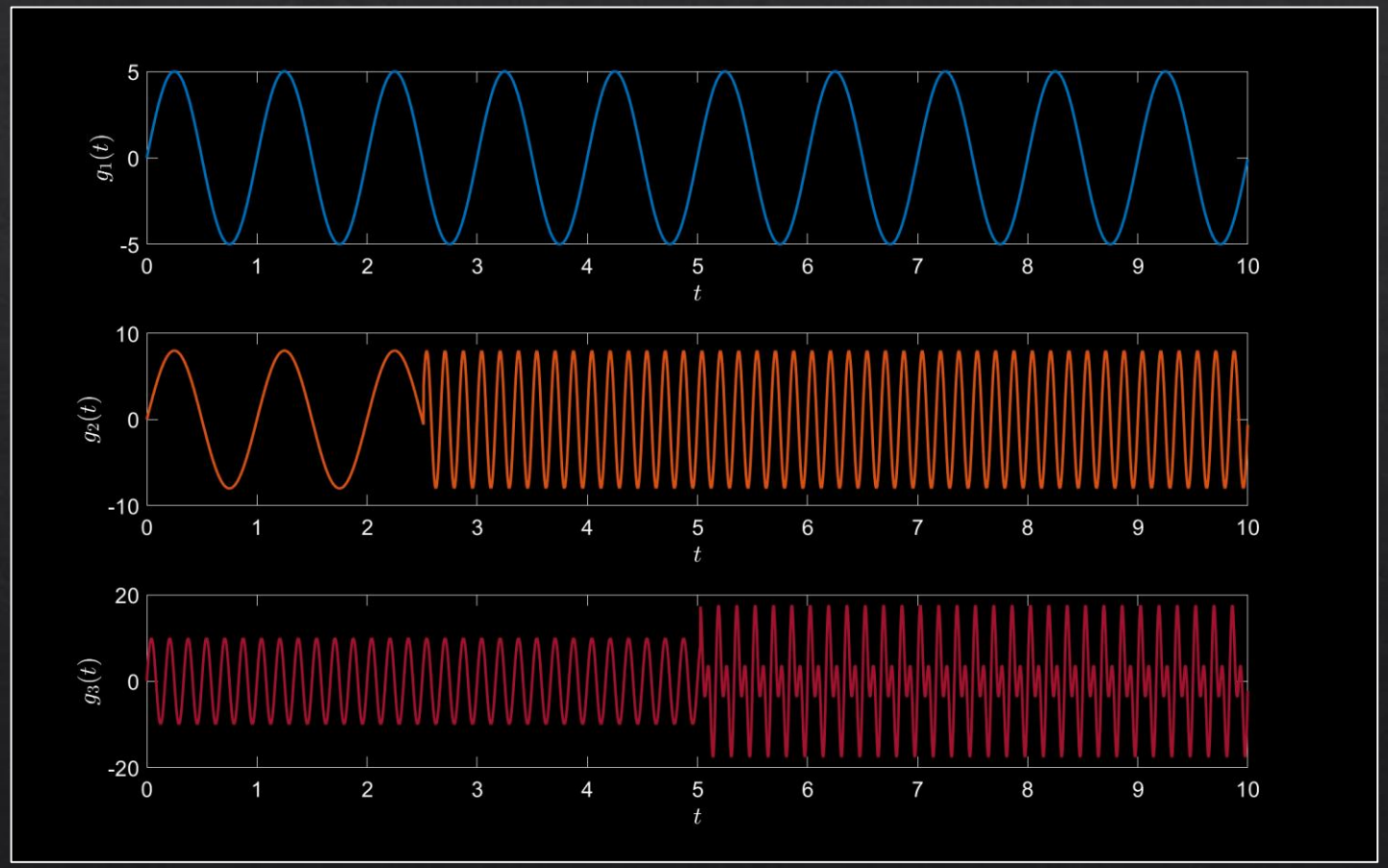
Amplitudes: $A_1 = 5$
$A_2 = 8$
$A_3 = 10$

Data:

$g_1(t) = A_1 \sin(2\pi f_1 t)$

$g_2(t) = A_2 \sin(2\pi f_1 t), t \leq 2.5$
$g_2(t) = A_2 \sin(2\pi f_2 t), t > 2.5$

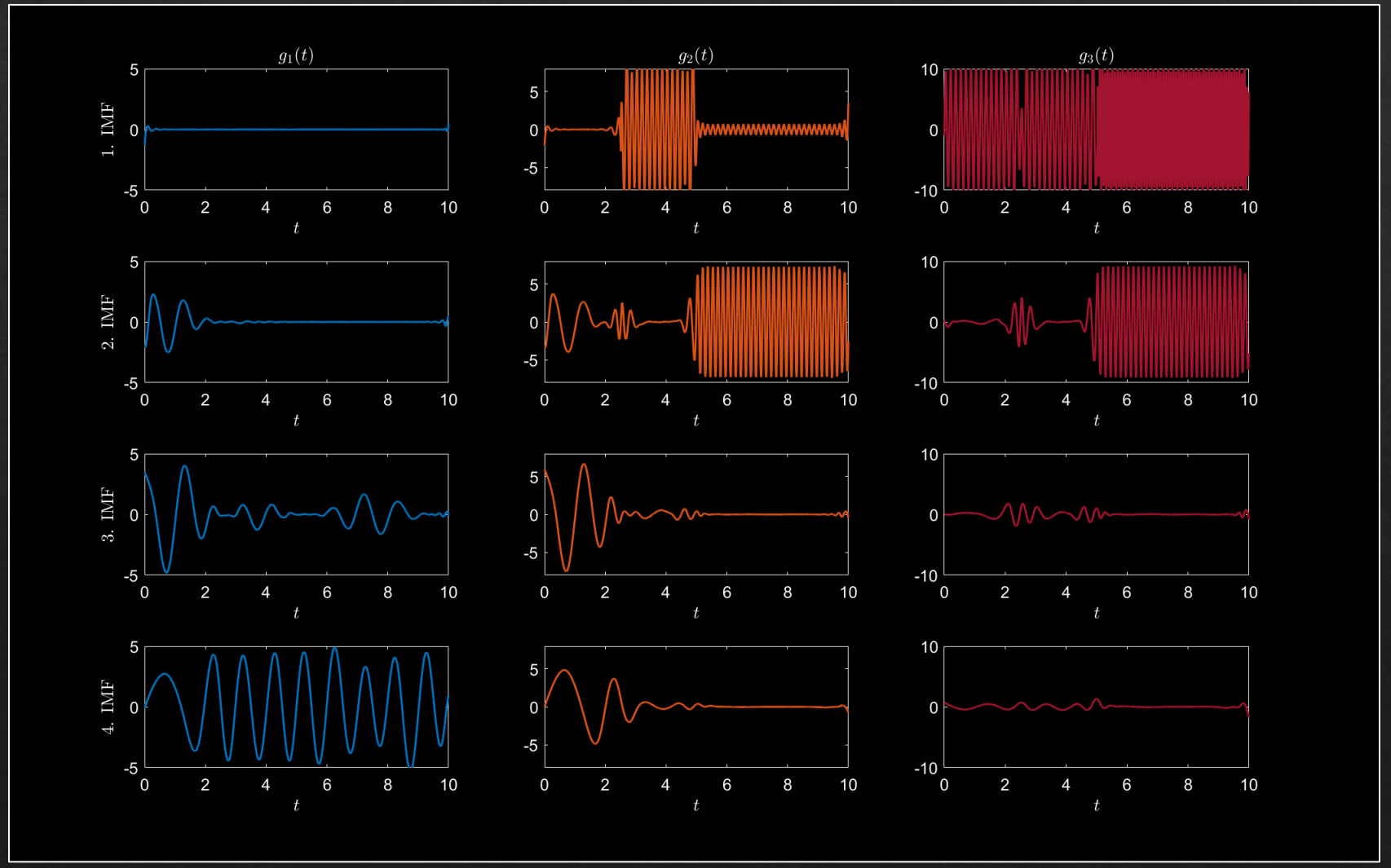
$g_3(t) = A_2 \sin(2\pi f_2 t), \quad t \leq 5$
$g_3(t) = A_2 \sin(2\pi f_2 t) + A_3 \sin(2\pi f_3 t), t > 5$



Example of NA-MEMD

Evaluation without noise assistance

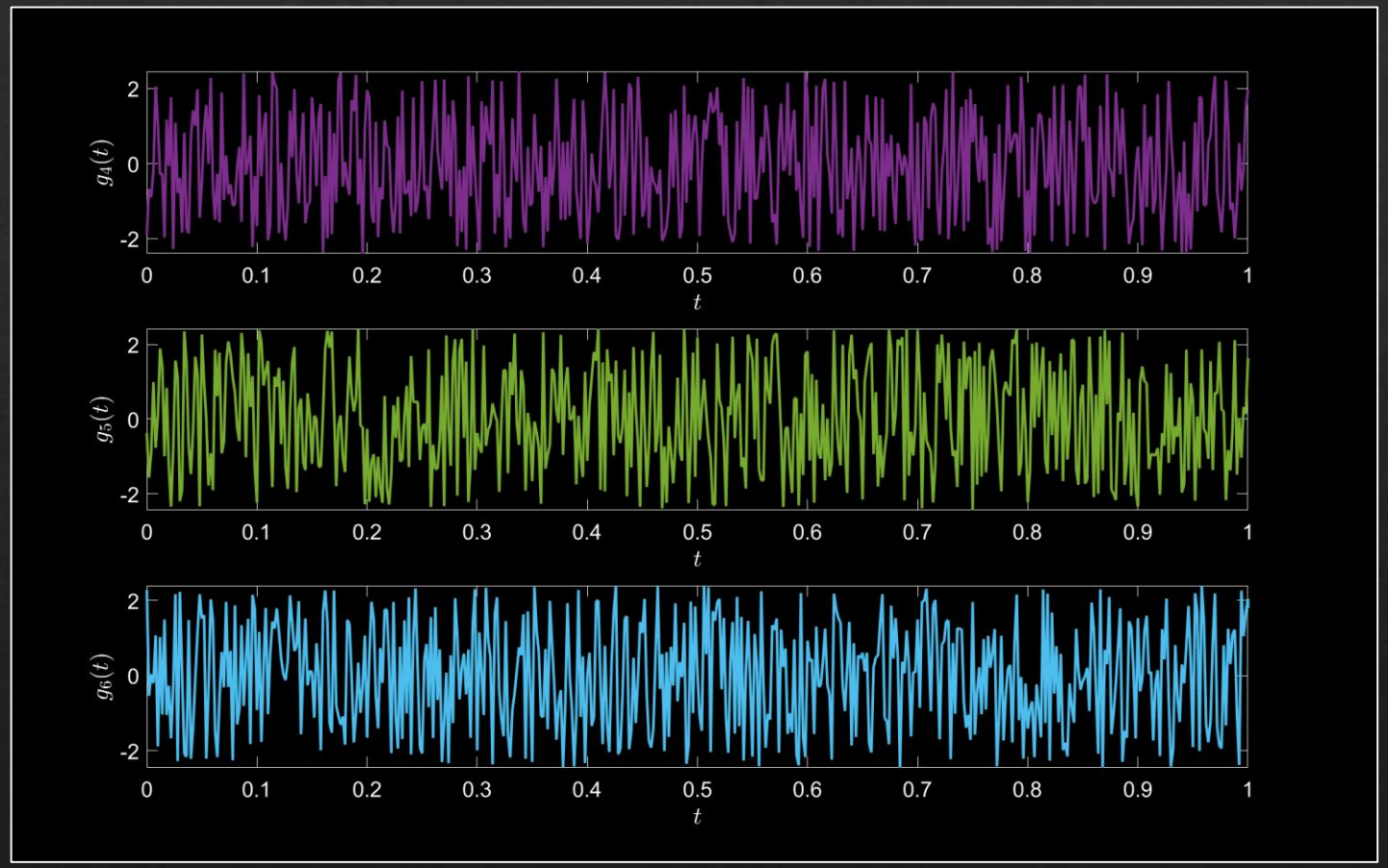
- Severe mode mixing
  - Expect only a single IMF for g_1
  - Single-frequency for $t > 2.5$ in g_2 is split into two modes
  - Frequencies in g_3 are not correctly separated



Example of NA-MEMD

Noise properties:

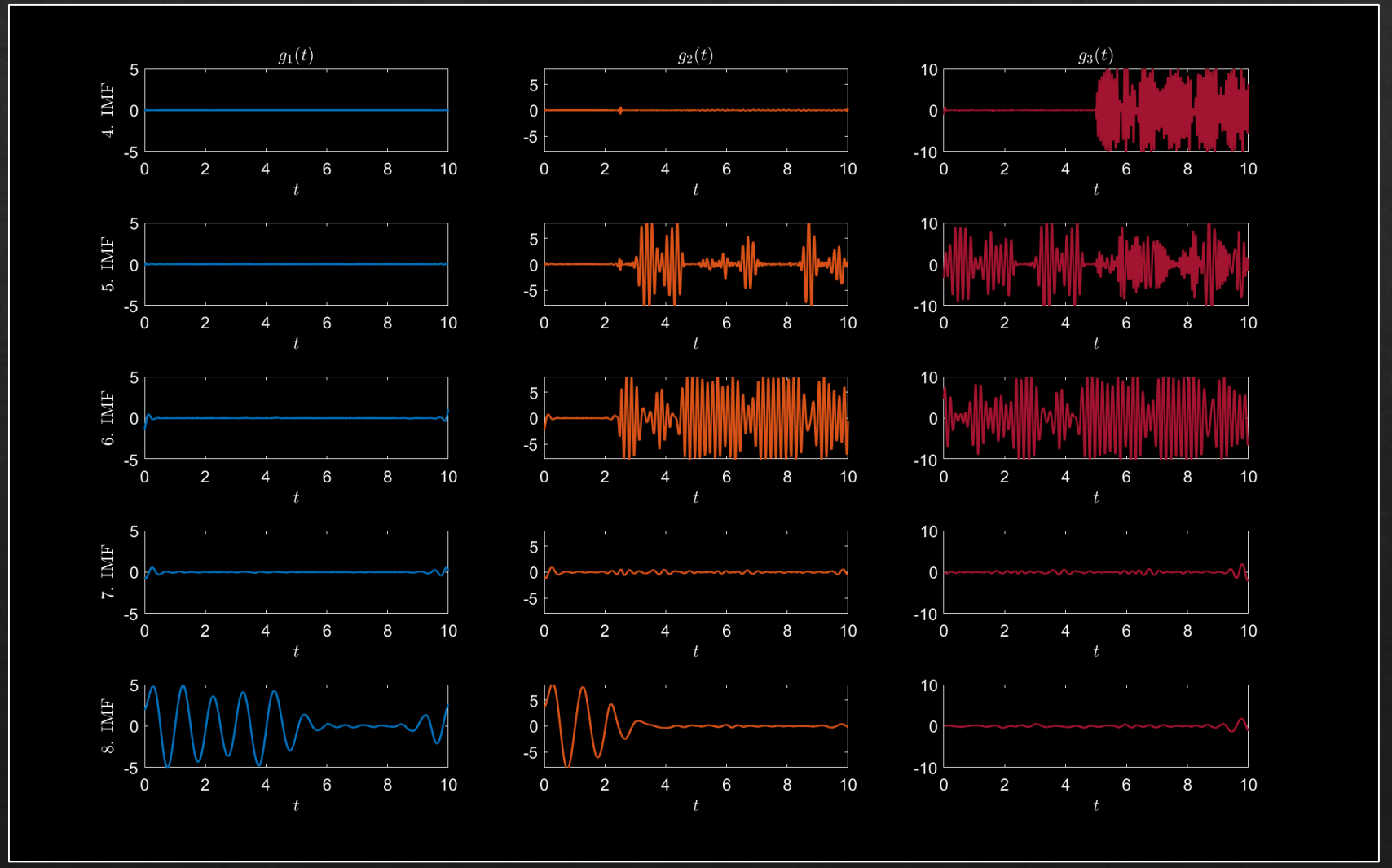
- Randomly generated Gaussian noise
- Variance: $\sigma = 0.05 \cdot \sigma_{data}$
- Zero mean
- Increasing number of noise channels



Example of NA-MEMD

Evaluation with one noise channel

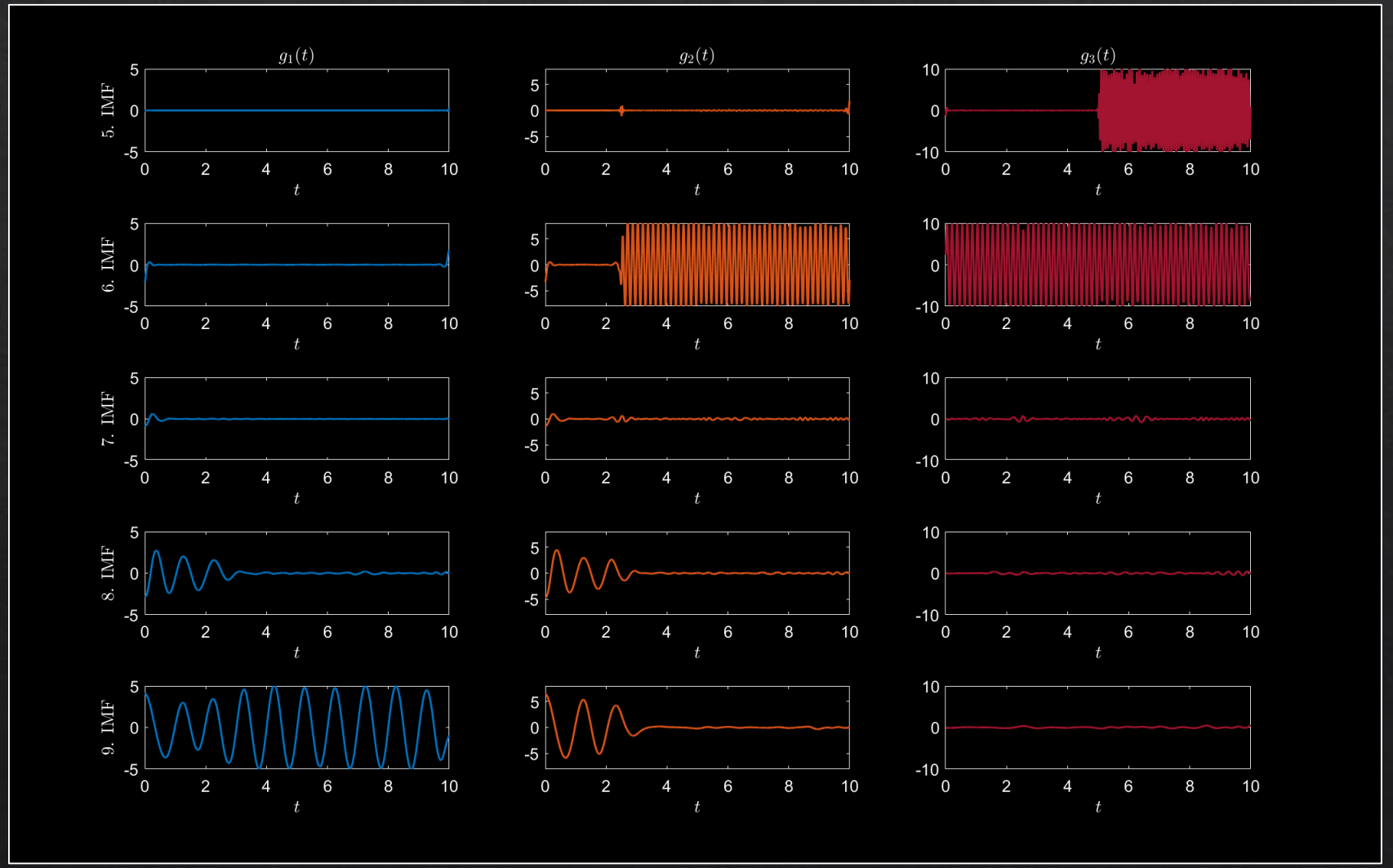
- Smallest modes contain only noise (not shown)
- Still distinct mode mixing
  - 4th IMF of g_3 discontinuous for $t > 5$
  - 5th and 6th IMF of g_2, g_3 should not be separated



Example of NA-MEMD

Evaluation with two noise channels

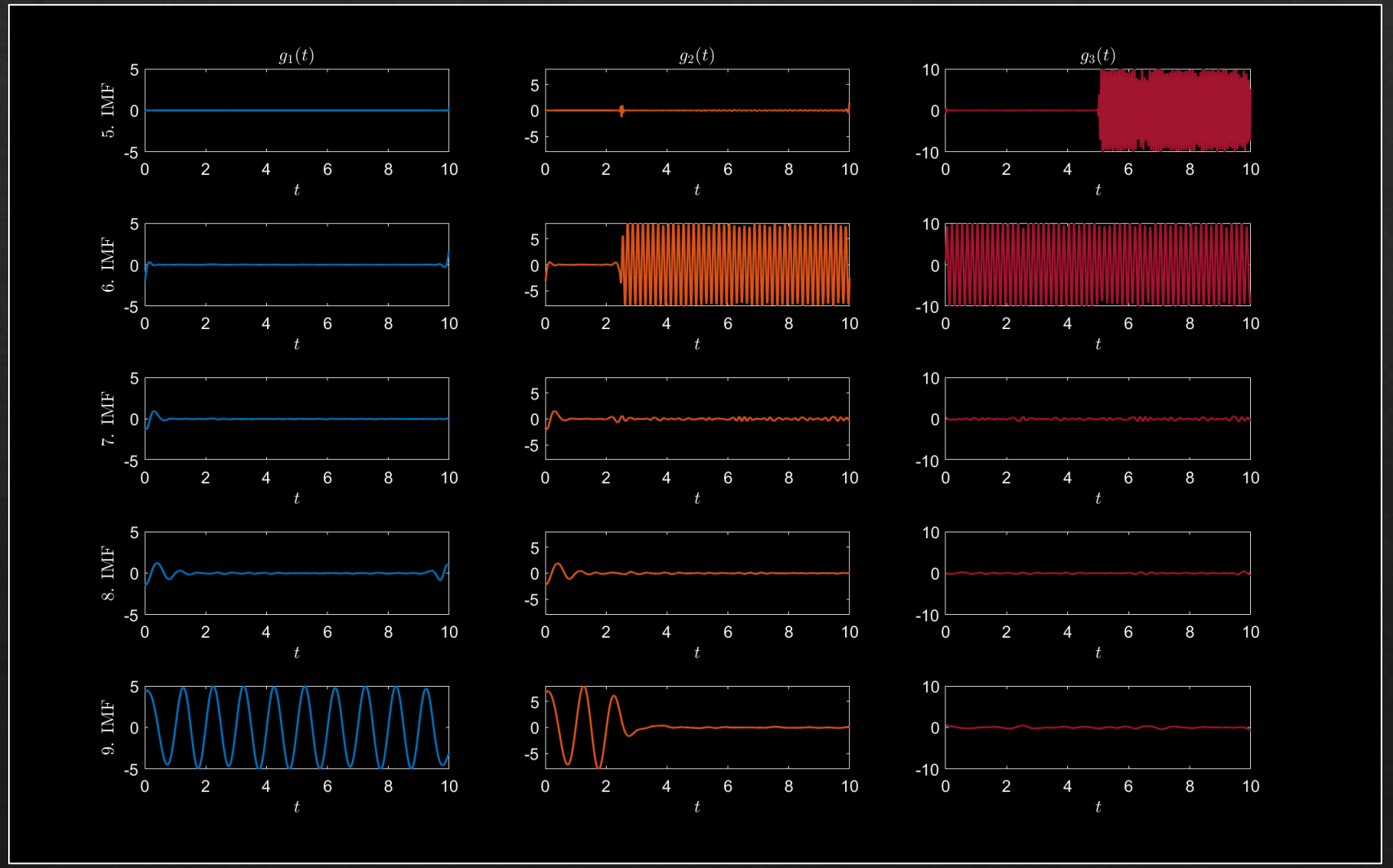
- Smallest modes contain only noise (not shown)
- Mode mixing phenomenon is massively reduced
- Still, the lowest frequency (contained in g_1, g_2) is not restricted to a single mode



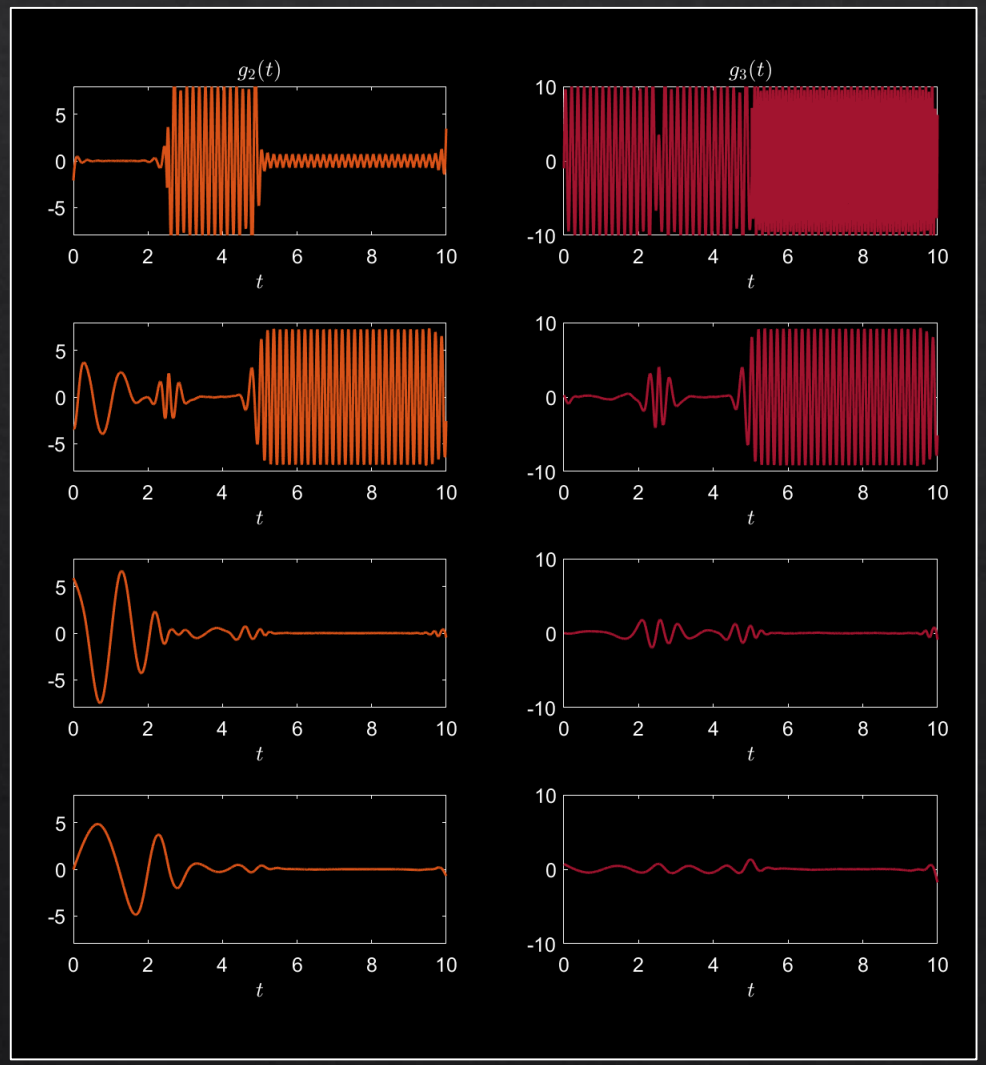
Example of NA-MEMD

Evaluation with three noise channels

- Smallest modes contain only noise (not shown)
- No mode mixing, i.e., lowest frequency is now only contained within the 9th mode



Example of NA-MEMD



Standard MEMD

Noise-assisted MEMD