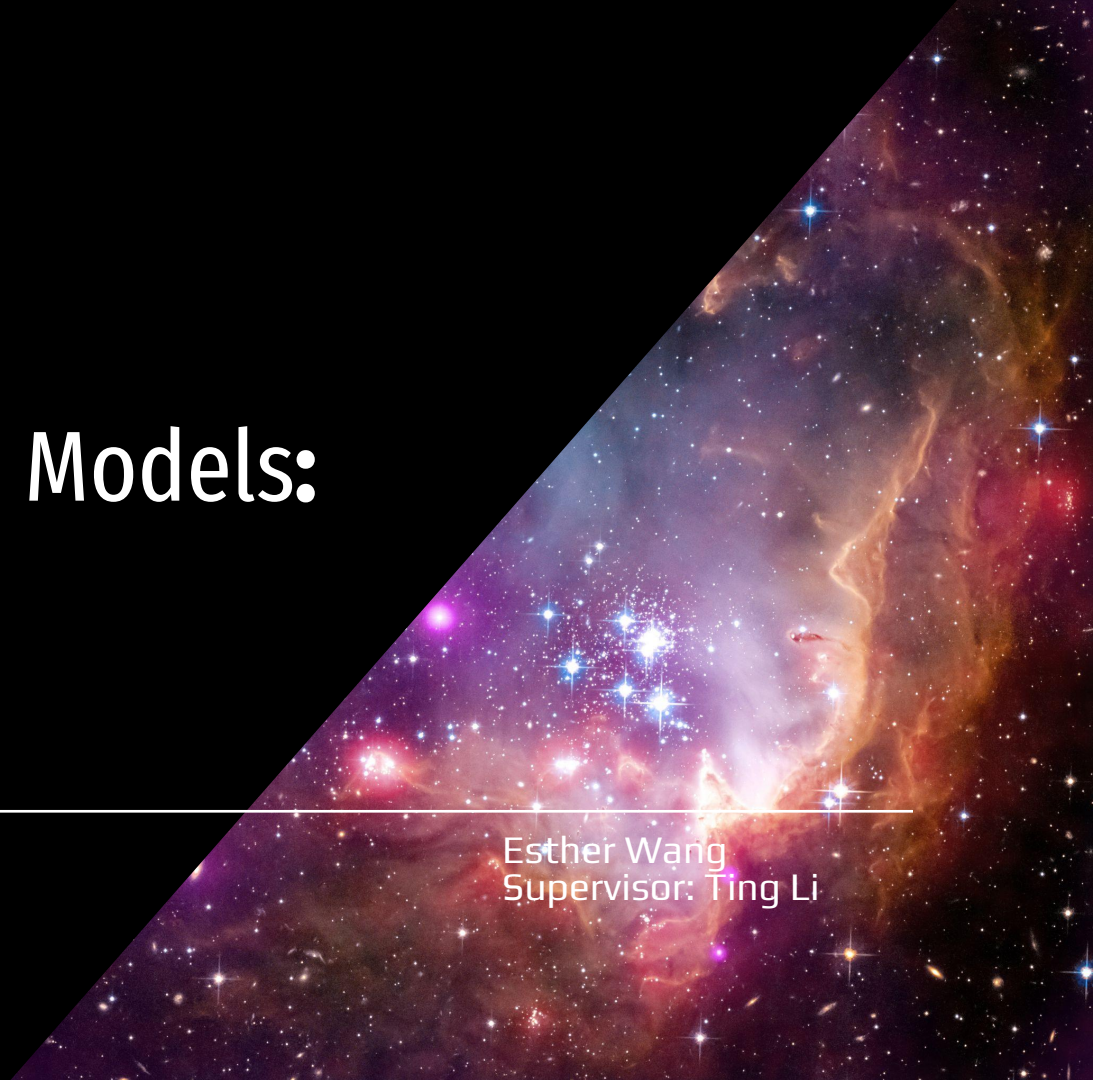
SMCNOD & Gaussian Mixture Models: 1-D to 2-D



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Overview

The Bayesian Approach

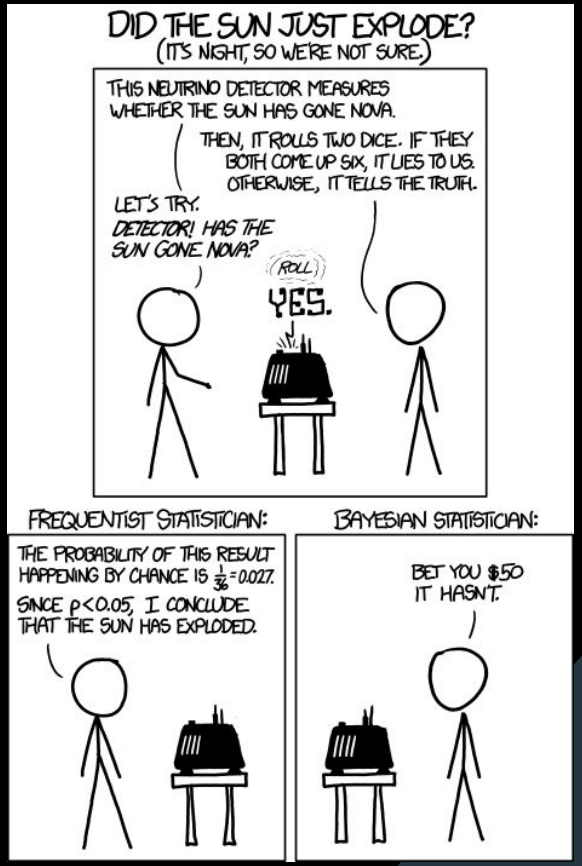
- Mixture Models in Astronomy & GMM

SMCNOD

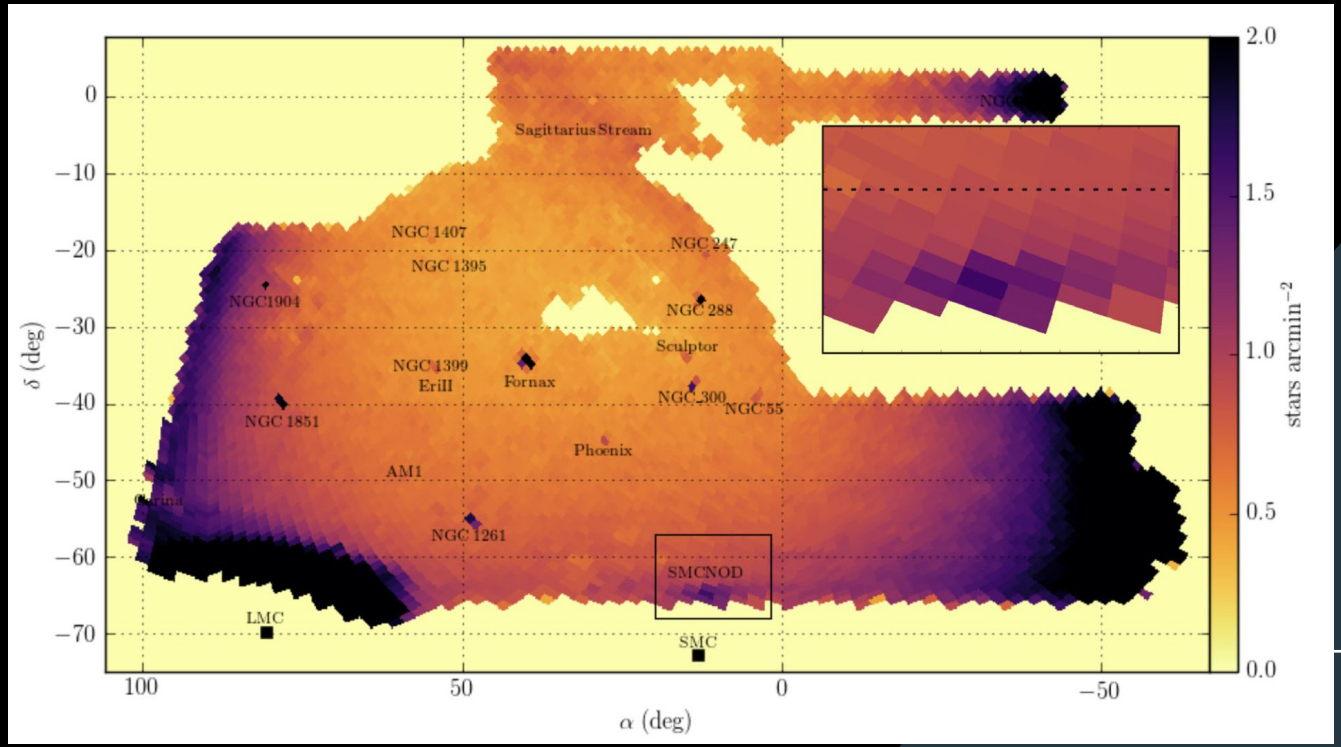
- Kinematics, metallicity and membership probability of SMCNOD

Tutorial

- Fitting GMM to 1-D and 2-D data



SMCNOD: stellar overdensity associated with SMC



- indistinguishable in **age, metallicity** and **distance** from the nearby SMC stars
- Primarily composed of intermediate-age stars
  (6 Gyr, Z = 0.001)
- Small fraction of young stars
  (1 Gyr, Z = 0.01)

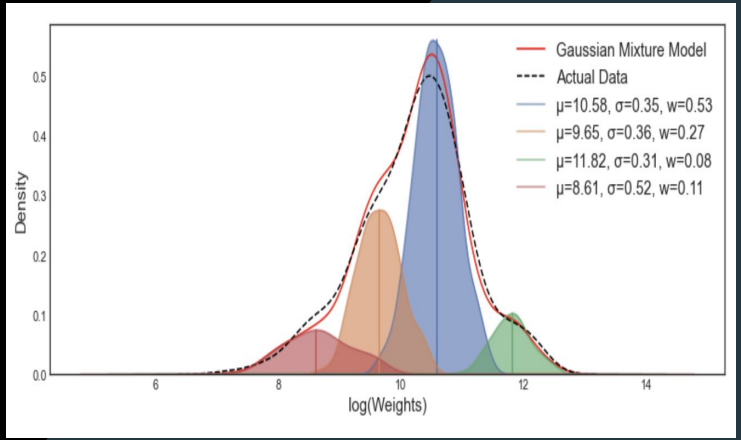
(Pieres et al. 2017)

- (RA, Dec) ~ (12°, -64.8°)
- Half-light radius: ~ 120.4 arcmin

Mixture Models in Astronomy

- Gaussian mixture models (GMMs):
  - Astronomical data: noise and incompleteness.
  - signal vs background
  - multiple sources contributing to an image.

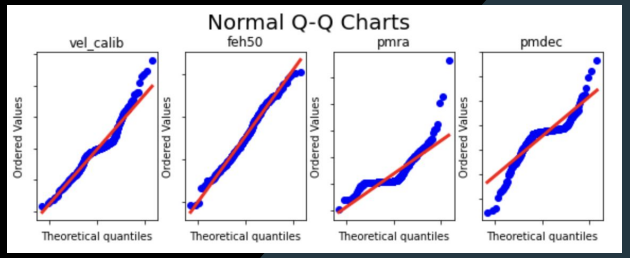
- Trade off in GMM: # of components & likelihood
  - Increading # of components may lead to:
  - 1. better fit & higher likelihood
  - 2. increase complexity, risk of overfit

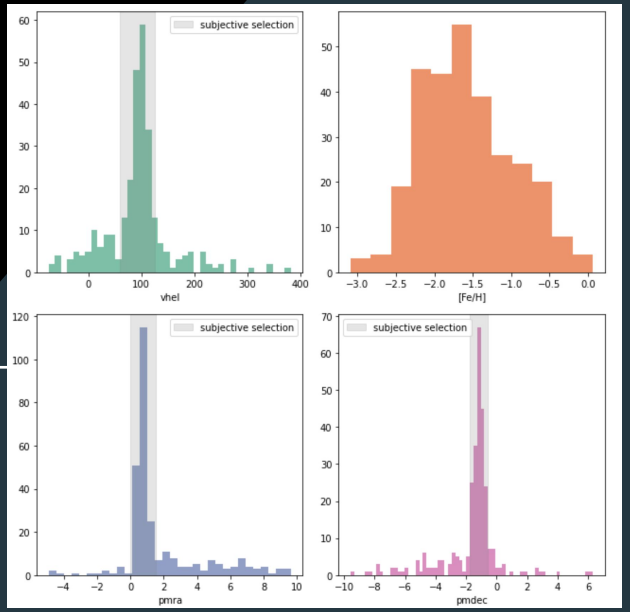


Step 1: Make Assumptions & Prior Beliefs

- Data Assumption:
- i.i.d random variables (same prob distributions)

- Prior Belief:
- what distribution does the data follow?
- what parameters best describe the distribution?
- Gaussian distribution: most cases
- Alternative distributions: e.g Poisson(detection of astronomical objects...)



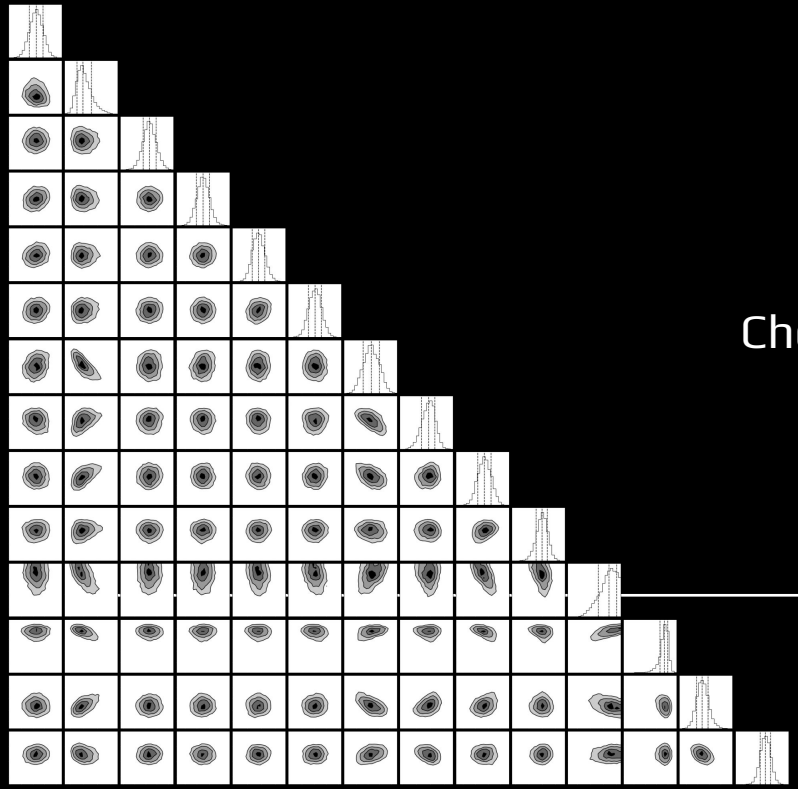


Step 2: Build Likelihood Function

$$\log\text{-}\mathcal{L} = f_{\mathrm{SMCNOD}} \log\text{-}\mathcal{L}_{\mathrm{SMCNOD}} + f_{\mathrm{bg1}} \mathcal{L}og\text{-}\mathcal{L}_{\mathrm{bg1}} + f_{\mathrm{bg2}} \log\text{-}\mathcal{L}_{\mathrm{bg2}}$$

- 16 Parameters, 2 backgrounds
  * pgal = fraction of stars in the galaxy
  * pbg1 = fraction of stars in the 1st background component
  * pmra = Heliocentric proper motion, RA of the galaxy in mas/yr
  * pmdec = Heliocentric proper motion, Dec of the galaxy in mas/yr
  * vhel = mean velocity of the galaxy in km/s
  * lsigv = log10 the velocity dispersion of the galaxy in km/s
  * feh = mean metallicity of the galaxy in dex
  * lsigfeh = log10 the metallicity dispersion of the galaxy in dex
  * vbg1, lsigvbg1, fehbg1, lsigfeh1 = same parameters for 1st background component
  * vbg2, lsigvbg2, fehbg2, lsigfeh2 = same parameters for 2nd background component

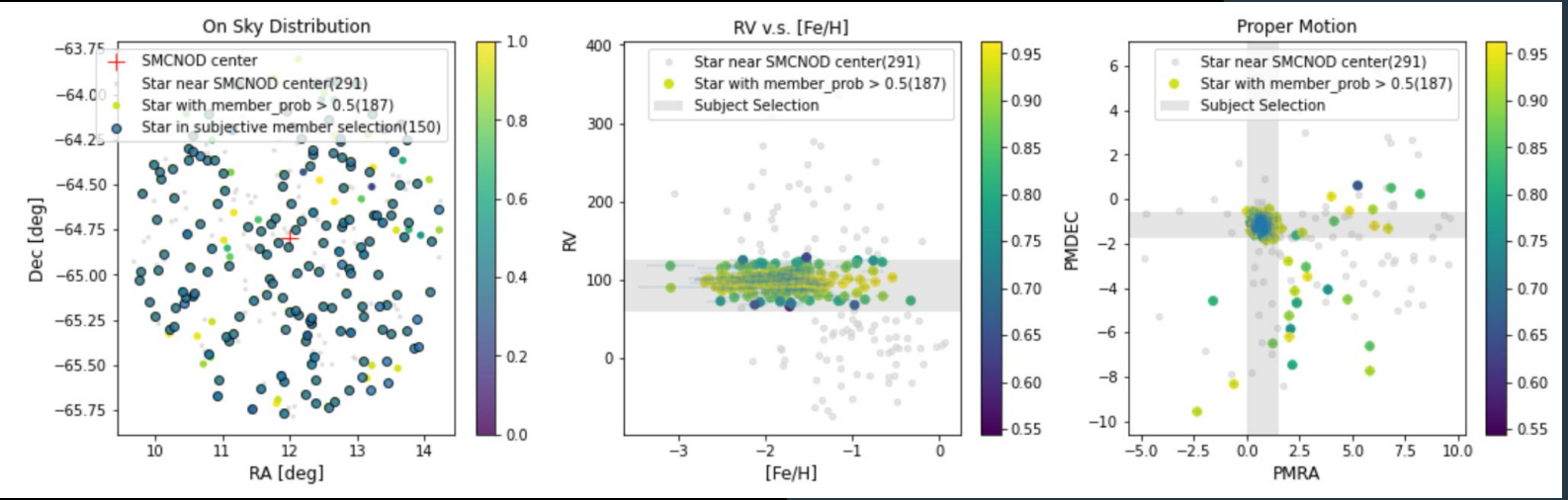
Step 3: Posterior Sampling - MCMC



Check if things are nice and round!

Step 4: Make inference: Membership

$$p = f_{\mathrm{SMCNOD}}\,\mathcal{L}_{\mathrm{SMCNOD}} \,/\, ((1 - f_{\mathrm{SMCNOD}})\,\mathcal{L}_{\mathrm{MW}} + f_{\mathrm{SMCNOD}}\,\mathcal{L}_{\mathrm{SMCNOD}})$$



Normal v.s Multivariate Normal

Normal Distribution:

- 1 variable, univariate

Mulrivariate Normal Distribution:

- > 1 variables, joint distribution, take correlation into account

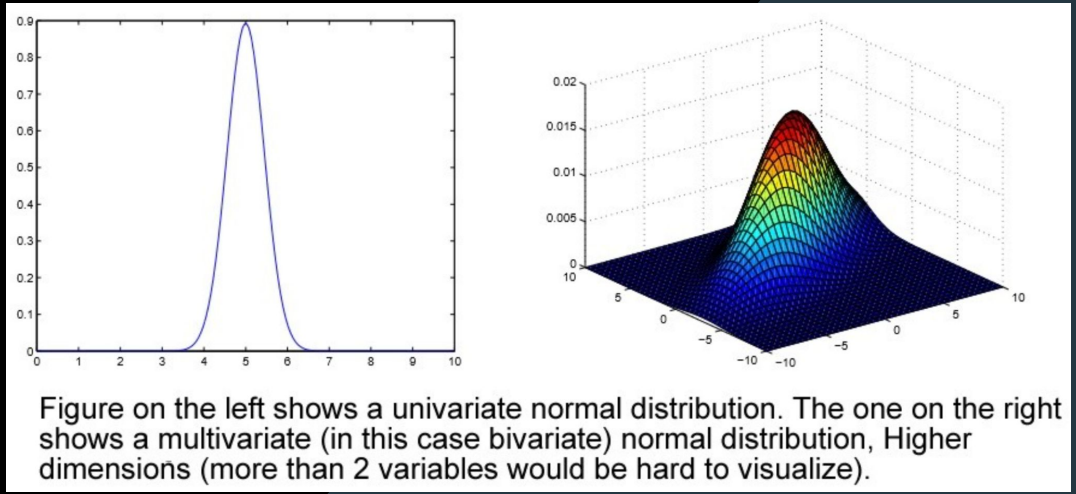
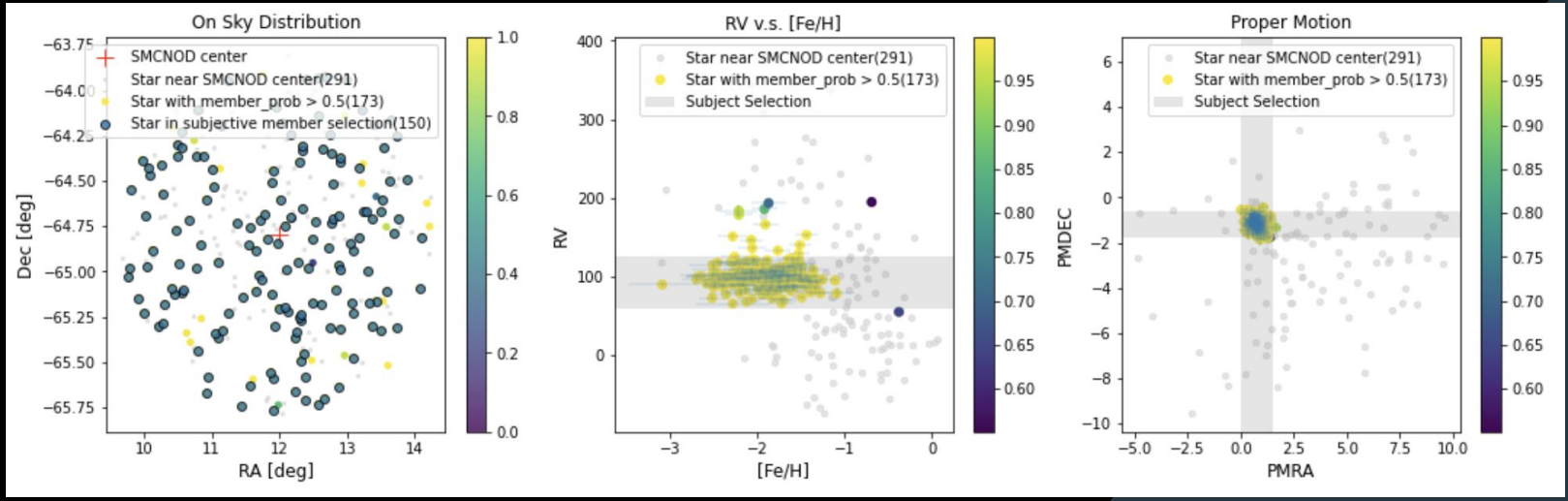


Figure on the left shows a univariate normal distribution. The one on the right shows a multivariate (in this case bivariate) normal distribution, Higher dimensions (more than 2 variables would be hard to visualize).

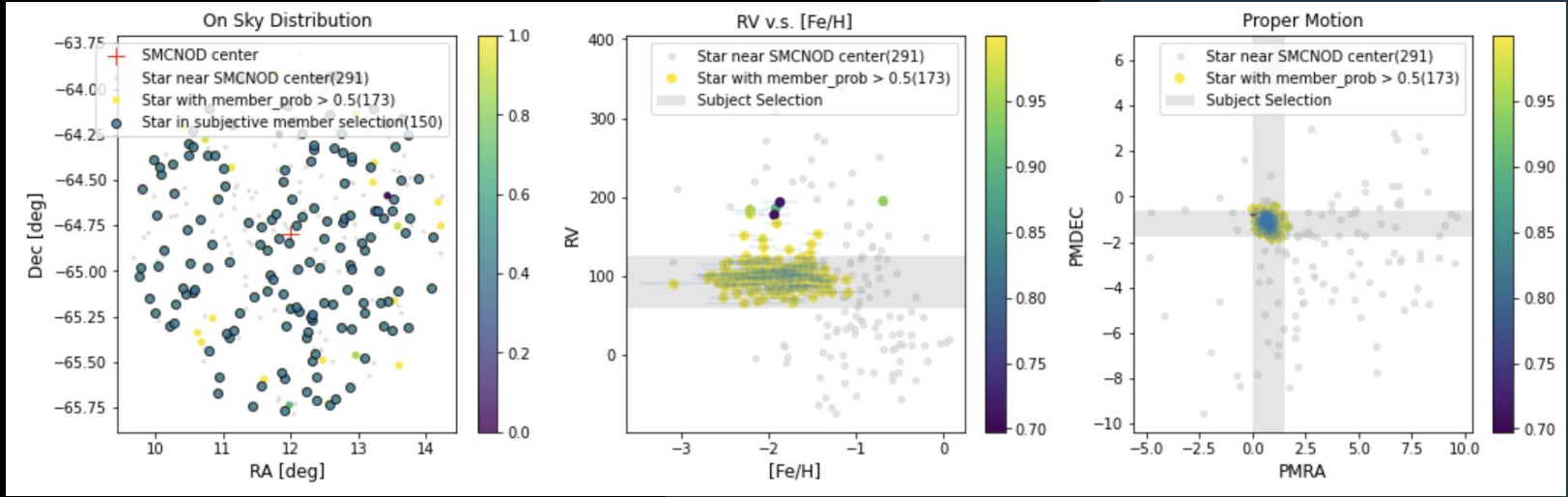
Step 5: Compare Models

- 3 GMM Models:
- Model only rv, feh
- Model rv, feh, pmra, pmdec (all 1-D, no cov matrix)
- Model rv, feh, and 2-D proper motions(multivariate normal)

	Full model (2D pm)	Full model (1D pm)	RV, [Fe/H] only model
vhel	101.836(+/-1.637)	102.188(+/-2.379)	98.656(+/-1.189)
sigv	20.839(+/-1.903)	30.53(+/-2.265)	12.882(+/-0.94)
feh	-1.874(+/-0.028)	-1.864(+/-0.029)	-1.857(+/-0.036)
sigfeh	0.305(+/-0.024)	0.34(+/-0.026)	0.335(+/-0.033)
pmra	0.703(+/-0.017)	0.718(+/-0.016)	-
pmdec	-1.128(+/-0.018)	-1.140(+/-0.020)	-


On Sky Distribution
SMCNOD center
Star near SMCNOD center(291)
Star with member_prob > 0.5(173)
Star in subjective member selection(150)
Dec [deg]
RA [deg]
RV v.s. [Fe/H]
Star near SMCNOD center(291)
Star with member_prob > 0.5(173)
Subject Selection
RV
[Fe/H]
Proper Motion
Star near SMCNOD center(291)
Star with member_prob > 0.5(173)
Subject Selection
PMDEC
PMRA

1-D


On Sky Distribution
SMCNOD center
Star near SMCNOD center(291)
Star with member_prob > 0.5(173)
Star in subjective member selection(150)
Dec [deg]
RA [deg]
RV v.s. [Fe/H]
Star near SMCNOD center(291)
Star with member_prob > 0.5(173)
Subject Selection
RV
[Fe/H]
Proper Motion
Star near SMCNOD center(291)
Star with member_prob > 0.5(173)
Subject Selection
PMDEC
PMRA

2-D

Thank you & Discussions?