MATLAB Tutorial 9

ENME 303 Computational Methods for Engineers

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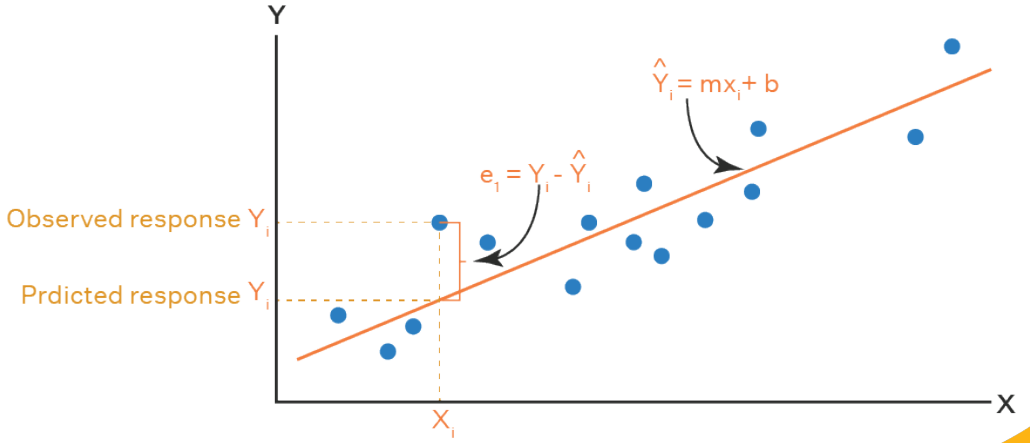
Adapted from Parham Oveissi (2023)

Linear Regression

- Prediction Error:
  - $e_i \triangleq y_i - \hat{y}_i = y_i - mx_i - c$
- Prediction Cost:
  - $J = \sum_{i=1}^{n} e_i^2$

Goal: Find m and c such that J is minimized



- Let's write $y_i = \varphi_i \theta$
- Where $\varphi_i = [x_i \quad 1]$ and $\theta = \begin{bmatrix} m \\ c \end{bmatrix}$

$$\hat{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \qquad \Phi = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{bmatrix} \qquad \Theta = \begin{bmatrix} m \\ c \end{bmatrix}$$

$$\hat{y} = \Phi\Theta$$

This looks like Ax = b!

Least Squares Solution

- $E \triangleq y - \hat{y} = y - \Phi\Theta$
- $J = E^T E = \Theta^T \Phi^T \Phi\Theta - 2\Theta^T \Phi^T y + y^T y$
- Minimization: $\frac{\partial J}{\partial \Theta} = 0 \rightarrow \Theta = \left(\Phi^T \Phi\right)^{-1} \Phi^T y$
- Compare with:
  - $x = (A^T A)^{-1} A^T b$

Practice

- We are going to look at accidents.mat from the MATLAB example data sets
- MATLAB Example Data Sets - MATLAB & Simulink
- Data on US traffic accidents and fatalities in 2004 from the US Department of Transportation. The data covers all 50 states and the District of Columbia.
- We want to find the relationship between the population of a state and the number of fatalities from traffic accidents

Practice

- We want to fit a linear model to the measured data. That is, find an m and c that best fit our x and y data.

$$y = mx + c$$

- Let's code this in MATLAB

Motivation

Why is linear regression useful?

- It allows us to quantify relationships between datasets
- Can be used to predict the value of unmeasured data

Practice Continued

- A new state, 303Land, is joining the United States
- 303Land has a population of 2.5 million
- The Department of Transportation wants to know how many fatalities to expect 303Land to have, but they don’t have any data
- We can use our linear regression to estimate this value for them

Thanks!