Assignment 1 - Math 205 Linear Algebra

September 20, 2019

1 Instructions

This assignment will contribute 6.67% towards the final marks. The deadline to submit the assignment is 18:30 hours September 27, 2019. The assignment must be submitted online via LMS. You should include both the tex and pdf files, and any code files you may have. Please note that this is a group assignment. The **size of the group must be either 2 or 3**. You are responsible for creating your own group. You can't have group members other than your own section. There should one submission per group. Hence, please nominate one person from your group to submit the assignment on LMS. Finally, **please clearly mention all the group members in file tex/pdf file**

1.1 Marking scheme

All questions carry equal marks. Assignment will be marked out of 100.

- 30 marks will be for the presentation.
- 70 for the assignment

A good presentation would contain, but not limited to the following:

- Document typed in LaTeX. The basic template for LaTeXand the associated makefile is available to download from the course website on LMS. You are welcome to use your own LaTeXtemplate.
- A document free of typing errors
- Using figures/diagrams/set diagram, where possible, to explain your answers. As the cliché goes – a picture is worth a thousand words

- Concise and thorough answers. A long report doesn't necessarily mean a good report
- Comments in the code
- List of references

1.2 Late submission policy

Late submissions, unless approved beforehand, will be penalised according to the following.

# hours past the deadline	Percentage penalty on assignment
$<$ 1 hour	5%
1-2 hours	15%
2-3 hours	30%
3-4 hours	45%
>4 hours	0% (not accepted)

1.3 Plagiarism and collusion

There is a zero tolerance policy towards plagiarism and/or collusion. If a student(s) is found to have plagiarised and/or colluded, or the work submitted is not their own, (s)he will be given a **zero** in this assignment. Furthermore, they will most likely be reported to the academic code of conduct committee which would affect your academic standing in the university. If you are unsure whether you are plagiarising, please ask.

Please note that even if you understand everything, copying someone else's work is still plagiarism.

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In the event that something is not clear from the question, you are strongly encouraged to use the discussion forum on workplace.**Individual enquiries to instructors and TA will not be entertained**

2 Questions

1. Implement a basic Gaussian Elimination algorithm, and compare the runtime complexity of your algorithm with that of a library imple-

mentation. You are allowed to use any library implementation. You should plot your results on a graph and comment on interesting features/trends you observe.

2. For a given square matrix $\mathbf{A}$, once you have factorised it using LU decomposition, the cost of solving the system for various $\vec{b}$'s is $\mathcal{O}(kn^2)$. In other words, if you have a system of equations $\mathbf{A}\vec{x}_1 = \vec{b}_1, \mathbf{A}\vec{x}_2 = \vec{b}_2, \ldots, \mathbf{A}\vec{x}_k = \vec{b}_k$, the cost of solving is $\mathcal{O}(n^3 + kn^2)$.

   On the other hand, if you repeatedly apply Gaussian Elimination, the runtime complexity of solving the system is $\mathcal{O}(kn^3)$. Numerically (not mathematically) prove this complexity by plotting it on a graph. You should not implement LU decomposition yourself, instead use a library implementation.

3. Explain linear combination by examples and figures, and how does linear combination show whether a given system has unique, infinite, and no solutions.

4. Given $\mathbf{A}$ is a 4×4 identity matrix except for a vector $\vec{v}$ in column 2:

$$\begin{bmatrix} v_1 & 0 & 0 & 0 \\ v_2 & 1 & 0 & 0 \\ v_3 & 0 & 1 & 0 \\ v_4 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

   (a) Factor $\mathbf{A}$ into $\mathbf{LU}$, assuming $v_1 \neq 0$

   (b) Find $\mathbf{A}^{-1}$

   (c) For a general non-singular matrix $\mathbf{A}$, you are given that $\mathbf{L}_1\mathbf{U}_1 = \mathbf{L}_2\mathbf{U}_2$, where $\mathbf{L}_i$ and $\mathbf{U}_i$ are lower and upper triangular matrices respectively. Prove that for a non-singular matrix $\mathbf{L}_1 = \mathbf{L}_2$ and $\mathbf{U}_1 = \mathbf{U}_2$. Please explain your proof by referring to basic matrix multiplication properties (explaining doesn't mean that you write your equation in words)

   (d) Using your answer from part c), or otherwise, prove that for a general symmetric matrix $\mathbf{A}$ we can write the LU decomposition as $\mathbf{A} = \mathbf{LDL}^T$.