# Assignment 4 – Math 205 Linear Algebra

November 25, 2019

## 1 Instructions

This assignment will contribute 6.67% towards the final marks. The deadline to submit the assignment is 18:30 hours December 6, 2019. The assignment must be submitted online via LMS. You should include both the tex and pdf files, and any code files you may have. Please note that this is a group assignment. The **size of the group must be either 2 or 3**. You are responsible for creating your own group. You can't have group members other than your own section. There should be one submission per group. Hence, please nominate one person from your group to submit the assignment on LMS. Finally, **please clearly mention all the group members in file tex/pdf file**

*You must form a group and write it in on this spreasheet by November 27, 18:30 hours. If you fail to find a group, you will have to do the assignment alone.*

### 1.1 Marking scheme

Assignment will be marked out of 100. All questions carry equal marks

- 20 marks will be for the presentation.
- 80 for the assignment

A good presentation would contain, but not limited to the following:

- Document typed in LaTeX. The basic template for LaTeXand the associated makefile is available to download from the course website on LMS. You are welcome to use your own LaTeXtemplate. *Please make sure that you submit your pdf and tex files (and any other related file).*

*Your submission must be a single zip file on LMS*

- A document free of typing errors
- Using figures/diagrams/set diagam, where possible, to explain your answers. As the cliché goes – a picture is worth a thousand words
- Concise and thorough answers. A long report doesn't necessarily mean a good report
- Comments in the code
- List of references

### 1.2 Late submission policy

Late submissions, unless approved beforehand, will be penalised according to the following.

| # hours past the deadline | Percentage penalty on assignment |
|---|---|
| $<$ 1 hour | 5% |
| 1-2 hours | 15% |
| 2-3 hours | 30% |
| 3-4 hours | 45% |
| $>$4 hours | 0% (not accepted) |

### 1.3 Plagiarism and collusion

There is a zero tolerance policy towards plagiarism and/or collusion. If a student(s) is found to have plagiarised and/or colluded, or the work submitted is not their own, (s)he will be given a **zero** in this assignment. Furthermore, they will most likely be reported to the academic code of conduct committee which would affect your academic standing in the university. If you are unsure whether you are plagiarising, please ask.

Please note that even if you understand everything, copying someone else's work is still plagiarism.

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In the event that something is not clear from the question, you are strongly encouraged to use the discussion forum on workplace. **Individual enquiries to instructors and TA will not be entertained**

## 2 Questions

1. You are given two functions:

$$f(x,y) = -1 + 4(e^x - x) - 5x\sin y + 6y^2 \tag{1}$$
$$g(x,y) = (x^2 - 2x)\cos y \tag{2}$$

   Determine whether $(0,0)$ and $(1,\pi)$ is the stationary point for $f(x,y)$ and $g(x,y)$ respectively. If they are the stationary points, then determine the nature of the stationary points

2. Using the quadratic form $\vec{x}^T\mathbf{A}\vec{x}$, show that $f(x,y) = x^2 + 4xy + 3y^2$ does not have a minimum at $(0,0)$

3. Let $\mathbf{A} = \begin{bmatrix} 1 & b & 0 \\ c & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix}$. For what values of $b$ and $c$, the matrix will be positive definite and positive semi-definite?

4. Determine whether the following statements are true or false. You must give counterexamples and/or justifications for your choice

   (a) Given $\mathbf{A} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$, then the following equation holds $\mathbf{AB} - \mathbf{BA} = \mathbf{I}$ (where $\mathbf{I}$ is the identity matrix).

   (b) The matrix $\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$ is diagonalizable

5. Let $\vec{p}, \vec{q} \in \mathbb{R}^n$. If $\mathbf{A} = \vec{p}\vec{q}^T$

   a.) Show that $\vec{p}$ is an eigenvector and find it's corresponding eigenvalue $\lambda$.

   b.) Determine all the other eigenvalues of $\mathbf{A}$ and explain how you obtained them.

   c.) What can you say about the trace of $\mathbf{A}$?

6. The great barrier reef has been known to include over 1500 species of fish. One particular fish that is indigenous to that area is the clown fish. Data shows that in the first year 4000 clown fish inside the reef migrated to other areas, while 2250 clown fish migrated to the reef. It is estimated that the ratio of clown fish leaving the reef to the population inside the reef is constant. Similarly, the ratio of clown fish entering the reef to the population outside the reef is also constant.

With an initial population of 22500 clown fish outside the reef and 20000 clown fish inside the reef, construct a system that is indicative of the number of fish inside (as well as outside) after the end of the first year.

Will the system ever reach a steady state? If so, approximately how long would it take to reach such a state? How will this state be affected if the initial population values were doubled?

You will be given points for clear and concise explanations.

7. It just so happens that we have lost all the data that you had submitted for assignment 3. The question now arises as to how we are supposed to grade this assignment. The TAs have suggested that we use a formula where the grade for each assignment (from the third onwards) will be calculated by taking the average of the previous two (i.e. $A_3 = \frac{1}{2} * (A_2 + A_1)$.

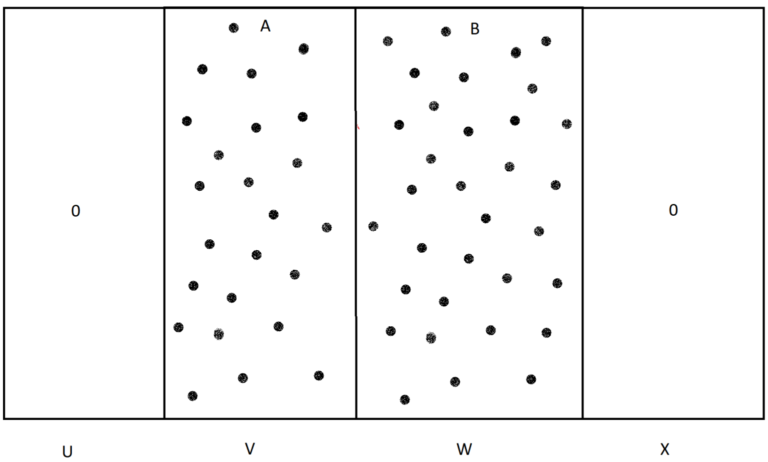
   a.) Find the matrix **B** such that the following system is satisfied:

   $$\begin{bmatrix} A_{k+2} \\ A_{k+1} \end{bmatrix} = \mathbf{B} \begin{bmatrix} A_{k+1} \\ A_k \end{bmatrix} \text{ where } k \geq 1$$

   b.) Find the eigenvalues and eigenvectors of **B**.

   c.) Find the limit as $n \to \infty$ of the matrices $\mathbf{B^n} = S\Lambda^n S^{-1}$.

8.




The figure above shows a room that has been divided into 4 parts u,v,w and x. Initially, sections v and w contain gas with concentrations $A$ and $B$ respectively. At each unit of time, the gas in sections $v$ and

$w$ diffuses to it's adjacent sections at a rate that is equal to the difference in concentrations of the respective sections. You may assume that within a section the gas is uniformly spread.

a.) Construct a system that involves the rate of flow into $v$ and the rate of flow into $w$.

b.) Determine the rate at which the concentrations decay.

c.) What happens to the concentrations as $t \to \infty$