Assignment 3 – Math 205 Linear Algebra

November 7, 2019

1 Instructions

This assignment will contribute 6.67% towards the final marks. The deadline to submit the assignment is 18:30 hours November 22, 2019. The assignment must be submitted online via LMS. You should include both the tex and pdf files, and any code files you may have. Please note that this is a group assignment. The **size of the group must be either 2 or 3**. You are responsible for creating your own group. You can't have group members other than your own section. There should be one submission per group. Hence, please nominate one person from your group to submit the assignment on LMS. Finally, **please clearly mention all the group members in file tex/pdf file**

You must form a group and write it in on this spreasheet by November 13, 18:30 hours. If you fail to find a group, you will have to do the assignment alone.

1.1 Marking scheme

Assignment will be marked out of 100. All questions carry equal marks

- 20 marks will be for the presentation.
- 80 for the assignment

A good presentation would contain, but not limited to the following:

- Document typed in LaTeX. The basic template for LaTeXand the associated makefile is available to download from the course website on LMS. You are welcome to use your own LaTeXtemplate. *Please make sure that you submit your pdf and tex files (and any other related file).*

Your submission must be a single zip file on LMS

- A document free of typing errors
- Using figures/diagrams/set diagam, where possible, to explain your answers. As the cliché goes – a picture is worth a thousand words
- Concise and thorough answers. A long report doesn't necessarily mean a good report
- Comments in the code
- List of references

1.2 Late submission policy

Late submissions, unless approved beforehand, will be penalised according to the following.

# hours past the deadline	Percentage penalty on assignment
< 1 hour	5%
1-2 hours	15%
2-3 hours	30%
3-4 hours	45%
>4 hours	0% (not accepted)

1.3 Plagiarism and collusion

There is a zero tolerance policy towards plagiarism and/or collusion. If a student(s) is found to have plagiarised and/or colluded, or the work submitted is not their own, (s)he will be given a **zero** in this assignment. Furthermore, they will most likely be reported to the academic code of conduct committee which would affect your academic standing in the university. If you are unsure whether you are plagiarising, please ask.

Please note that even if you understand everything, copying someone else's work is still plagiarism.

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In the event that something is not clear from the question, you are strongly encouraged to use the discussion forum on workplace. **Individual enquiries to instructors and TA will not be entertained**

2 Questions

1. Does the following set of polynomials constitute a basis for subspace of polynomial of degree less than equal to 2

$$\left\{x^2 + x + 1, x^2 - x + 1, 2x^2, 1\right\}$$

2. Find a basis for the space of polynomials $p(x)$ of degree ≤ 2? Then, find the basis for the subspace with $p(7) = 0$ and sketch the bases.

3. Let P_3 be the real number vector space over $\mathbb{R}$ of cubic polynomials. W is defined as

$$W = \{p(x) \in P_3 \mid p'(-1) = 0 \text{ and } p''(1) = 0\}$$

   Determine whether W forms a subspace of V

4. Find the bases for the four subspaces associated with **A** and **B**:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix} \quad , B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 8 \end{bmatrix}$$

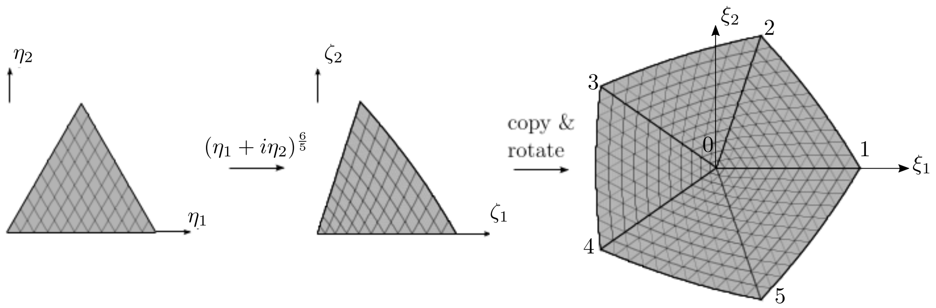
   Sketch the four subspaces if an arbitrary matrix **C** is

   - invertible
   - a zero matrix

5. Find the complete solution for the following equations and describe the solution space:

$$\begin{aligned} x + 3y + 3z &= 1 \\ 2x + 6y + 9z &= 5 \\ -x - 3y + 3z &= 5. \end{aligned} \tag{1}$$

6. You are given an equilateral triangle whose sides are of unit length. You are given a complex function $f(\eta_1, \eta_2)$ which transforms the equilateral triangle as shown in the following figure (left and middle). We can then copy and rotate the transformed triangle to form a complete patch as shown in the following figure on the right. Note that we don't have any duplicate points on the final patch



(a) Calculate the coordinates of the 6 points on the patch.

(b) An engineer measured the charge distrbution q on the 6 points of the patch as follows:

Point #	0	1	2	3	4	5
q	1	3	1	0	1	2

An electrical engineer is interested in finding out the charge q at $\vec{\xi} = (0.5, 0.5)^T$. He also tells you that the charge inside the patch can be approximated using a bilinear polynomial $(1, \xi_1, \xi_2, \xi_1\xi_2)$. Compute q at $\vec{\xi} = (0.5, 0.5)^T$.

(c) Sketch the charge distribution surface over the $\xi_1 - \xi_2$ patch.

7. We have learned that for underdetermined systems, it is impossible to find unique solutions in the absence of some extra constraints. One way of obtaining the unique solution is to impose a minimum L_2 norm constraint i.e.

$$\text{Solve:} \quad \mathbf{A}\vec{x} = \vec{b} \tag{2}$$

$$\text{such that:} \quad \|\vec{x}\|_2 \text{ is minimum} \tag{3}$$

In this case, $\vec{x}_r = \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}\vec{b}$. By way of example explain the relationship between $\vec{x}_p$ and $\vec{x}_n$, and when $\vec{x}_p = \vec{x}_r$. Sketch the four fundamental subspaces

8. Explain, with reason, whether the following statements are true or false

(a) The complete solution is any linear combination of x_p and x_n

(b) A system $\mathbf{A}\vec{x} = \vec{b}$ has at most one particular solution

(c) The solution x_p with all free variables set to zero can be the shortest solution (minimum length $\|x\|$)

(d) If $\mathbf{A}$ is invertible there is no solution x_n in the nullspace

9. True or false (with a reason or a counterexample)

   (a) $\mathbf{A}$ and $\mathbf{A}^T$ have the same number of pivots

   (b) $\mathbf{A}$ and $\mathbf{A}^T$ have the same left nullspace

   (c) If the row space equals the column space then $\mathbf{A} = \mathbf{A}^T$

   (d) If $-\mathbf{A} = \mathbf{A}^T$, then the row space of $\mathbf{A}$ equals the column space

10. For the space $\mathbb{R}^4$, let $w_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, w_2 = \begin{bmatrix} 3 \\ 3 \\ -1 \\ -1 \end{bmatrix}, y = \begin{bmatrix} 6 \\ 0 \\ 2 \\ 0 \end{bmatrix}$, and let $W = \mathrm{sp}\{w_1, w_2\}$.

    (a) Find a basis for W consisting of two orthogonal vectors using Gram-Schmidt process.

    (b) Explain the Gram-Schmidt process intuitively.

    (c) Express y as the sum of a vector in W and a vector in $W^\perp$

    Please refer to this link for this question