Present value

ECON 331

This document summarizes the concept of present value, which shows up frequently in dynamic problems. This is an important concept in asset pricing, corporate finance, and any problem where agents makes a financial decision that has implications across time.

Financial decisions rely on comparing **cash flows** at different periods of time. Cash flows are the arrival of departure of money at a given period of time; depositing a pay check or giving someone money in exchange for a stock are both cash flows.

Money that arrives at different times isn't the same thing – a dollar in a year is not the same as having a dollar today. So to do the combining of cash flows correctly, we need to apply the following three principles:

Three principles of the time value of money:

1. You can only compare or combine values at the same point in time.
2. To move a cash flow forward in time, you compound it. The future value (FV) of a cash flow (CF) in n periods is $FV = CF \times (1 \times i)^n$.
3. To move a cash flow backwards in time, you discount it (find the equivalent value today of a future cash flow). The present value of a cash flow that arrives n periods in the future is $PV = \frac{CF}{(1+i)^n}$.

Any asset has a "stream of cash flows" associated with it – exchanges of cash at different times. To value that stream, we convert all of the payments to the same period of time using the principles outlined above.

Although things can get more complicated (especially in a corporate finance context), we'll focus on relatively simple assets – loans with fixed payments, and bonds that offer periodic payments and a one-time payment for their face value.

These pricing formulas abstract from risk aversion. A risk averse agent prefers a sure payment more than the equivalent expected value of a risky payment. Here, we're focusing only on dollar values and timing.

We are also abstracting from default. We could treat the cash flows as being the expected value (e.g., if a firm has probability p of defaulting before you get your cash, then the expected value is $(1-p) \cdot C + p \cdot 0$). However, we would have to assume that hte agent is risk neutral.

Related to the stream of cash flows is the **yield to maturity** (YTM). The yield to maturity is the (hypothetical) interest rate that equates the present value of cash flow payments a security gives out to the price of the security.

Here are some examples.

Fixed payment loan Fixed payment loads are loans where you make a certain number of payments over the life of the loan; each payment covers interest and a portion of the principal. Such loans are called "fully amortized." Some examples are mortgages and student loan contracts. The present value of the loan when it occurs is the money that is lent. The cash flows are the sizes of the fixed payments.

Annuity An ordinary annuity is a financial instrument hat pays a certain amount of money every period for a fixed number of periods. (This is how most lottery payouts are structured, and certain retirement funds). The present value of these payments is the sum of the present value of each individual payment. Typically we write this as if the payments start one period in the future:

$$PV = \sum_{k=1}^{n} \frac{C}{(1+i)^n} = C\left[\frac{1-\frac{1}{(1+i)^n}}{1-\frac{1}{(1+i)}}\right] = \frac{C}{i}\left[1-\frac{1}{(1+i)}\right]$$

Where to go from the second expression to the 3rd, we used the the fact that this is a convergent geometric sum as long as $i > 0$.

Coupon bonds Coupon bonds pay a given dollar amount C every period. They have a given face value (sometimes called the value at *par*). This coupon is often expressed as a percentage of the face value (So a bond that had a face value of 100 dollars and paid 7 dollar coupons would have a coupon rate of 7%). On the last period (*at maturity*), you get the last coupon payment, plus the face value of the bond.

A coupon bond is basically an annuity with an extra payment (the face value) attached.

There is no particular reason to believe that the coupon rate ant the YTM will be the same. That is only true when the price of the bond is its face value. Otherwise, the **price and yield are negatively related**. A higher YTM implies that future payments are discounted at a higher rate, which means the present value will be lower.

Perpetuities A perpetuity, or a consol, is a coupon bond that never "pays off." It promises a coupon payment every period forever. The present value of these payments can be calculated quite simply because it's an infinite geometric sum:

$$
\begin{aligned}
\text{PV of a perpetuity} = \sum_{k=1}^{\infty} \frac{C}{(1+i)^k} &= C \times \left[\sum_{k=0}^{\infty} \frac{1}{(1+i)^k} - 1 \right] \\
&= C \times \left[\frac{1}{1 - \frac{1}{1+i}} - 1 \right] \\
&= C \times \left[\frac{1}{\frac{1+i-1}{1+i}} - 1 \right] \\
&= C \times \left[\frac{1+i}{i} - 1 \right] \\
&= C \times \left[\frac{1+i}{i} - \frac{i}{i} \right] \\
&= C \times \frac{1}{i}
\end{aligned}
$$

If we call P the price (and treat that as its present value), then the object $i = \frac{C}{P}$ is sometimes called the *current yield* of the security.

Zero coupon bonds Zero coupon bonds are what they sound like: They're bonds that do not pay any coupons, but do give the face value at maturity. As long as yields are positive, that implies their present value is less than the face value:

$$
PV = \frac{\text{Face value}}{(1+i)^n}
$$

which is why zero coupon bonds are sometimes called **discount bonds**.

As an aside: How common are zero coupon bonds in practice? Most short-term Treasury debt are zero coupon bonds. Longer-term Treasury debt pays coupons, but often investment firms will buy those longer-term coupon bonds and break them up into a sequence of zero-coupon bonds (treating the coupon on a particular date as the face value). These assets are called STRIPs.