(1)

$AP \cdot N = 0$

$A = (2, 3, 4)$ $\quad AP = (x-2, y-3, z-4)$

$P = (x, y, z)$

$N = (1, 1, 3)$

$AP \cdot N = x-2+y-3+3z-12 = 0$

$x + y + 3z = 17$

$P' = (0,0,0) + (1,1,3)\lambda$

$\lambda + \lambda + 9\lambda = 17$

$11\lambda = 17$

$\lambda = \frac{17}{11} \rightarrow P' = \left(\frac{17}{11}, \frac{17}{11}, \frac{51}{11}\right)$

$d = \|P'\| = 5.126$

(2) direction of line 1: $\overline{AB} = (1, -1, -1)$

" " " 2: $\overline{CD} = (3, 1, -3)$

Shortest line connecting lines 1 and 2 will be perpendicular to both lines 1 and 2

$$N = \overline{AB} \times \overline{CD} = (4, 0, 4)$$

for each line, assume a plane with normal N upon which the line lies

the shortest distance between the lines will be the distance between planes

$$d = \left|\overline{AC} \cdot \hat{N}\right| = \left|(0,1,1)\cdot(4,0,4)\frac{1}{\sqrt{32}}\right| = \left|\frac{4}{\sqrt{32}}\right| = \frac{4}{\sqrt{32}} = \frac{\sqrt{2}}{2} = .7071$$

③ normal of plane on which triangle lies

$N = \overline{FG} \times \overline{FH} = (5, -3, 7)$

equation of the plane

$P = (x, y, z)$

$\overline{PF} \cdot N = 0 \rightarrow (x, y, z) \cdot (5, -3, 7) = 0 \rightarrow 5x - 3y + 7z = 0$

intersection of line with this plane

$5(2+\lambda) - 3(3+2\lambda) + 7(4+2\lambda) = 0 \rightarrow 10 + 2\lambda - 9 - 6\lambda + 28 + 14\lambda = 0 \rightarrow 13\lambda + 29 = 0$

$\lambda = \frac{-29}{13}$

$A = (2,3,4) - \frac{29}{13}(1,2,2) = (-.23, -1.46, -.46)$

$-\frac{3}{13} \quad \frac{-19}{13} \quad \frac{-6}{13}$

Area of triangle FGH

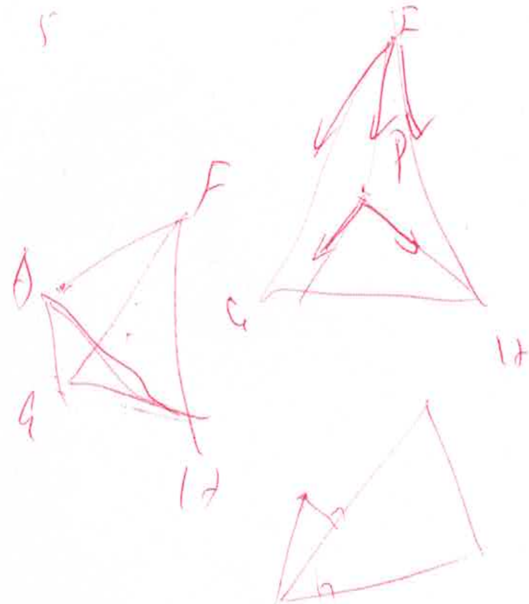
$\Delta_{FGH} = 4.555 \quad \frac{\sqrt{83}}{2} \quad 2$

Area of triangles AFG, AGH, AHF

$\Delta_{AFG} = 1.752 \quad \frac{\sqrt{2073}}{26} \quad 2$

$\Delta_{AGH} = 1.402 \quad \frac{\sqrt{1328}}{26} \quad 2$

$\Delta_{AHF} = 1.402$



$\Delta_{AFG} + \Delta_{AGH} + \Delta_{AHF} = \Delta_{FGH}$, so the line does intersect the triangle

2

④ $A = \dfrac{\|\overline{BC} \times \overline{BD}\|}{2} = \dfrac{\|(-5,-2,2) \times (-3,5,4)\|}{2} = \dfrac{\|(-18,14,-31)\|}{2}$

$A = 19.2419$

5

$\hat{n} = \dfrac{\overline{BC} \times \overline{BD}}{\|\overline{BC} \times \overline{BD}\|} = (-.4677, .3638, -.8055)$

$H = |\overline{BP} \cdot \hat{n}| = |(2,-2,6) \cdot \hat{n}|$

$H = 6.4962$ $\quad \dfrac{-250}{\sqrt{1481}}$ $\quad$ 5

$V = \dfrac{AH}{3} = 41.667$

10

$$V = \dfrac{\dfrac{\|\overline{BC} \times \overline{BD}\|}{2} \left| \overline{BP} \cdot \dfrac{\overline{BC} \times \overline{BD}}{\|\overline{BC} \times \overline{BD}\|} \right|}{3} = \frac{1}{6} \left| \overline{BP} \cdot (\overline{BC} \times \overline{BD}) \right|$$

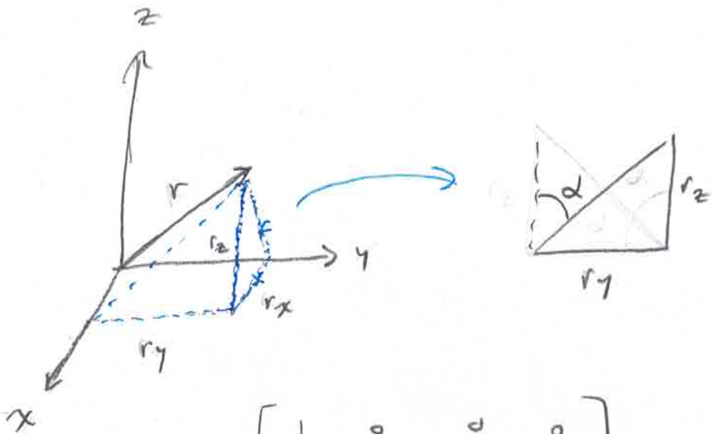
They don't necessarily have to write this part out

(5) has tons of different possible answers

(6) translate point to be coincident with origin

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & -3 & -4 & 1 \end{bmatrix}$$

rotate about x axis to put axis on xz plane

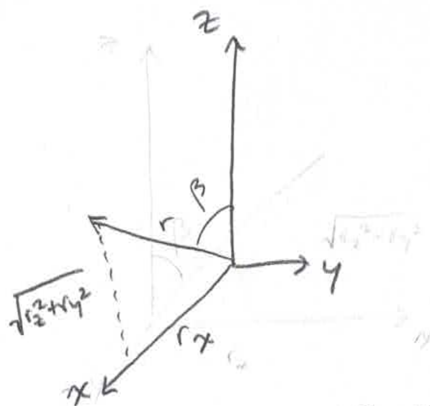


$$\sin\alpha = \frac{r_y}{\sqrt{r_z^2 + r_y^2}} = \frac{2}{\sqrt{8}} = \frac{1}{\sqrt{2}}$$

$$\cos\alpha = \frac{r_z}{\sqrt{r_z^2 + r_y^2}} = \frac{2}{\sqrt{8}} = \frac{1}{\sqrt{2}}$$

$$[R_x] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

rotate about y axis to align with z axis



$$\sin\beta = \frac{-r_x}{\sqrt{r_x^2+r_y^2+r_z^2}} = \frac{-1}{\sqrt{9}} = \frac{-1}{3}$$

$$\cos\beta = \frac{\sqrt{r_z^2+r_y^2}}{\sqrt{r_x^2+r_y^2+r_z^2}} = \frac{\sqrt{8}}{\sqrt{9}} = \frac{2\sqrt{2}}{3}$$

$$[R_y] = \begin{bmatrix} \frac{2\sqrt{2}}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{3} & 0 & \frac{2\sqrt{2}}{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6 cont

rotate about z axis by 30 degrees

$$\sin 30° = \frac{1}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$[R_z] = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

overall transformation

$$[T_R] = [T][R_x][R_y][R_z][R_y]^{-1}[R_x]^{-1}[T]^{-1}$$

$$= \begin{bmatrix} .8809 & .3631 & -.3036 & 0 \\ -.3036 & .9256 & .2262 & 0 \\ .3631 & -.1071 & .9256 & 0 \\ -.3036 & -.0744 & .2262 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' & 1 \\ B' & 1 \\ C' & 1 \end{bmatrix} = \begin{bmatrix} A & 1 \\ B & 1 \\ C & 1 \end{bmatrix} [T_R]$$

$A' = (1.0595 \quad 1.8184 \quad 3.1518)$

$B' = (1.8809 \quad 1.3631 \quad 1.6964)$

$C' = (1.1191 \quad 2.6369 \quad 4.3036)$