Tyler Estes
Utkarsh Wani

ASSIGNMENT NO-2

- **Problem 1**

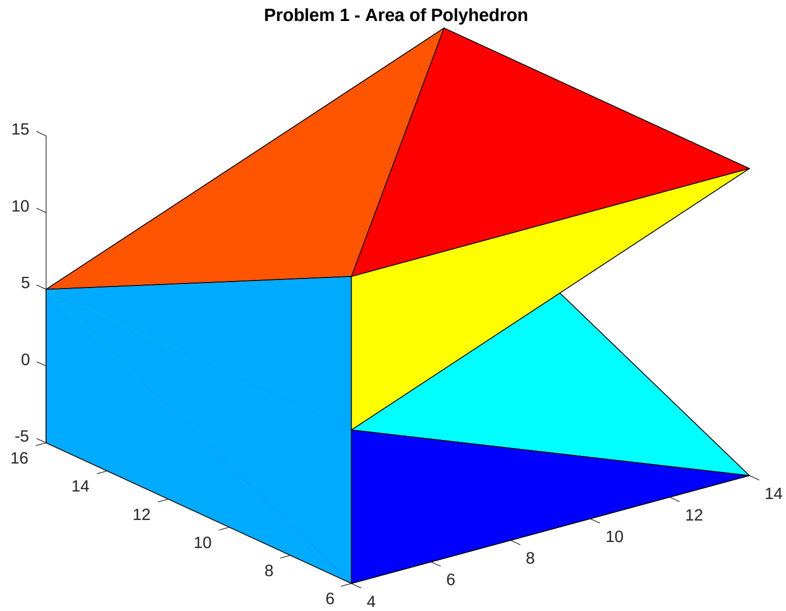

Figure 1.1 Polyhedron having 11 faces composing of 9 unique vertices

- Volume (from code): 666.67 units cubed
- Volume (from NX): 1333.33 units cubed
- Our value is different than the NX evaluation; however, if we take the absolute value of all the normal during the calculation and do nothing else with the normal' s directions then our volume is 1322.22 units cubed (aka closer to NX).

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- **Problem 2**

  - **Projected Points**

	X	Y	Z
V1	5	1	0
V2	5.6667	1.6667	1.3333
V3	6	1	1
V4	4.6667	1.6667	0.3333
V5	5.6667	0.6667	0.3333
V6	5	2	1

Table 2.1 Final coordinates of all vertices after projection

  - **Image of projected points on plane of V1, V3, V6**

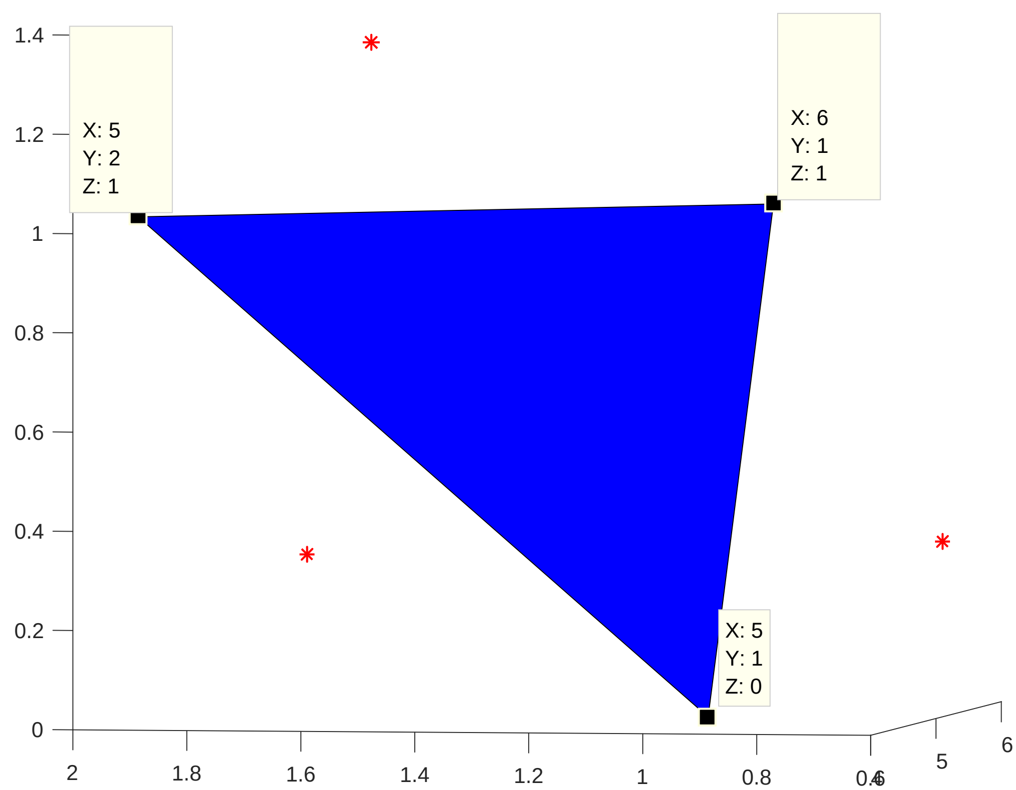

Figure 2.1 Projected view of the object onto the plane

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- **Problem 3**

a) Generate and plot a Hermite cubic curve for the following parameters: Start Point P1= (4, 2, 6), Start Tangent M1= (3, 1, -1), End Point P2= (2, 8, 4), End Tangent M2= (-1, 1, -1). Also calculate the tangent vector and the unit tangent vector for this curve at u=0.6.

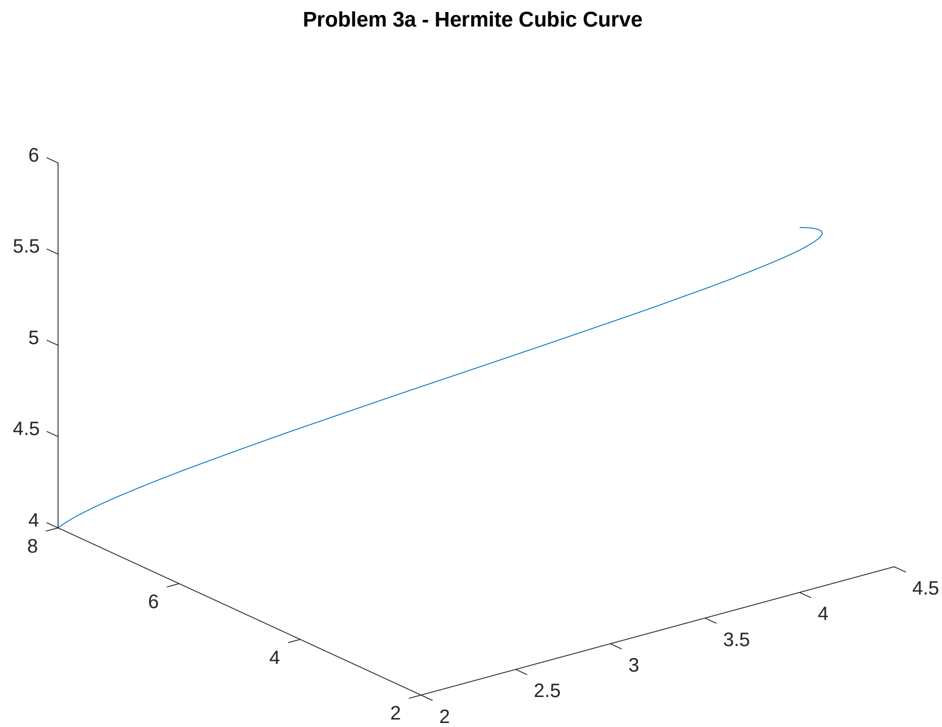

Figure 3.1 Hermite Curve plot

- **Tangent Vector (@ u = 0.60) = [-3.72, 8.20, -2.44]**
- **Unit Tangent Vector = [-0.40, 0.88, -0.26]**

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(b) Generate and plot a Hermite cubic curve having C1 continuity with the end point of the curve in (a). For this second curve, use End Point P2= (-2, 5, 4), End Tangent M2= (1, 2, -1).

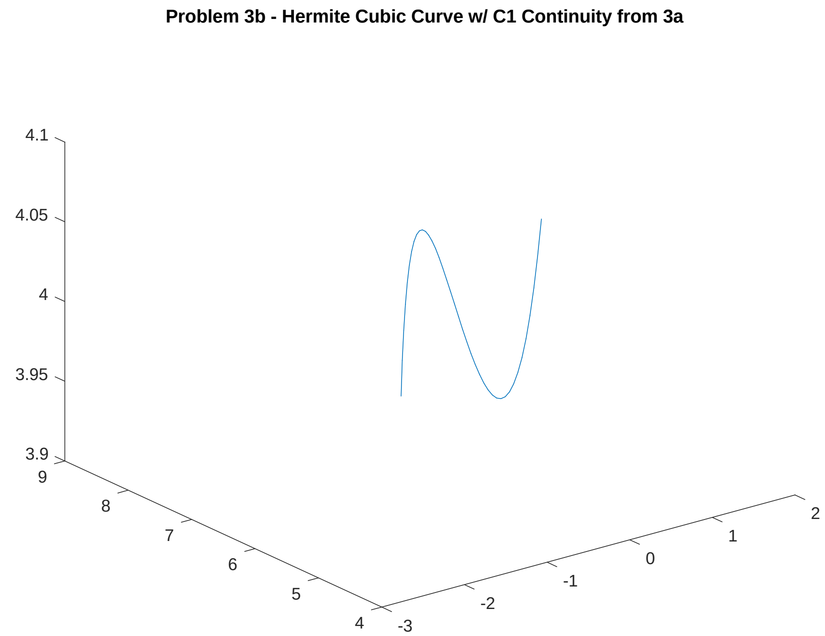

Figure 3.2 Hermite Curve plot with C_1 Continuity

(c) Plot both curves in a single MATLAB plot

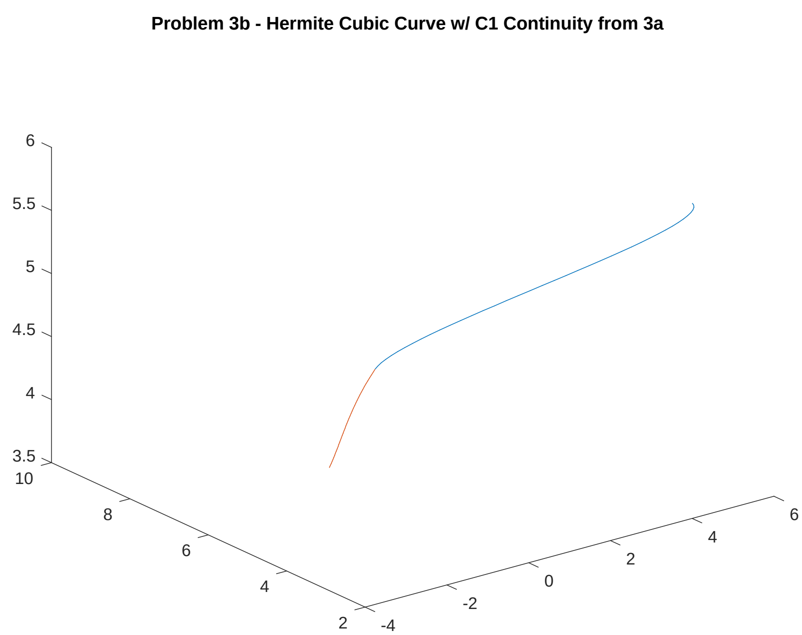

Figure 3.3 Combined Plot

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- **Problem 4**

(a) For the Bezier control points given below, generate and plot the curve using MATLAB: A= (1, 1, 1), B= (1, 3, 3), C= (3, 2, 1), E= (2, 4, 2)

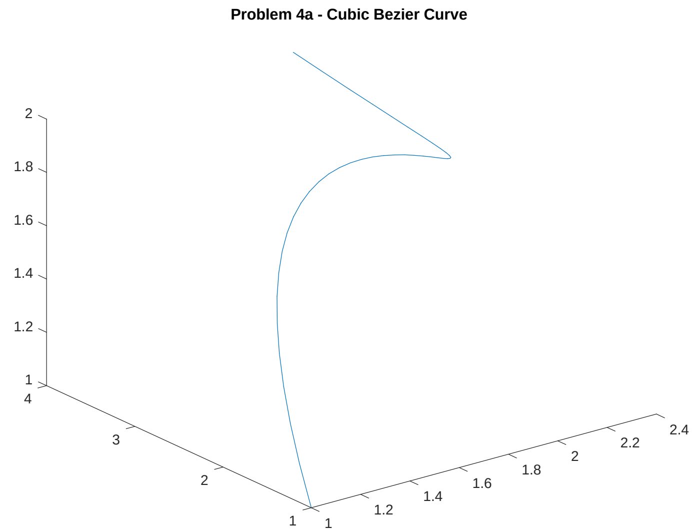

Figure 4.1 Bezier Curve Plot (n=3)

(b) For the curve in (a), add a duplicate point D in the same location as point C, resulting in the Bezier control points given below: A= (1, 1, 1), B= (1, 3, 3), C= (3, 2, 1), D= (3, 2, 1), E= (2, 4, 2).
Generate and plot the curve using MATLAB. What is the degree of the new curve? Calculate the value of the tangent vector for this curve at u=0, u=0.5, u=0.75, and u=1.

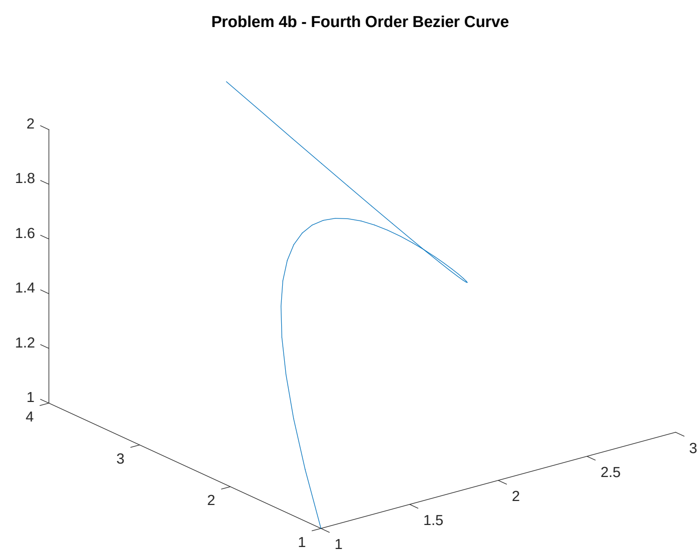

Figure 4.2 Bezier Curve Plot (n=4)

- **Degree of the new curve is 4.**

- **Problem 4b - Tangent Vector (@ u = 0.00) = [0.00, 8.00, 8.00]**
- **Problem 4b - Tangent Vector (@ u = 0.50) = [2.50, 0.50, -1.50]**
- **Problem 4b - Tangent Vector (@ u = 0.75) = [-0.56, 2.94, 0.69]**
- **Problem 4b - Tangent Vector (@ u = 1.00) = [-4.00, 8.00, 4.00]**

(c) For the curve in (b), change the coordinates of point B to (0, 5, -2) and plot this curve along with the curve from (b) using MATLAB. Explain whether this change affects the shape of the curve locally or globally.

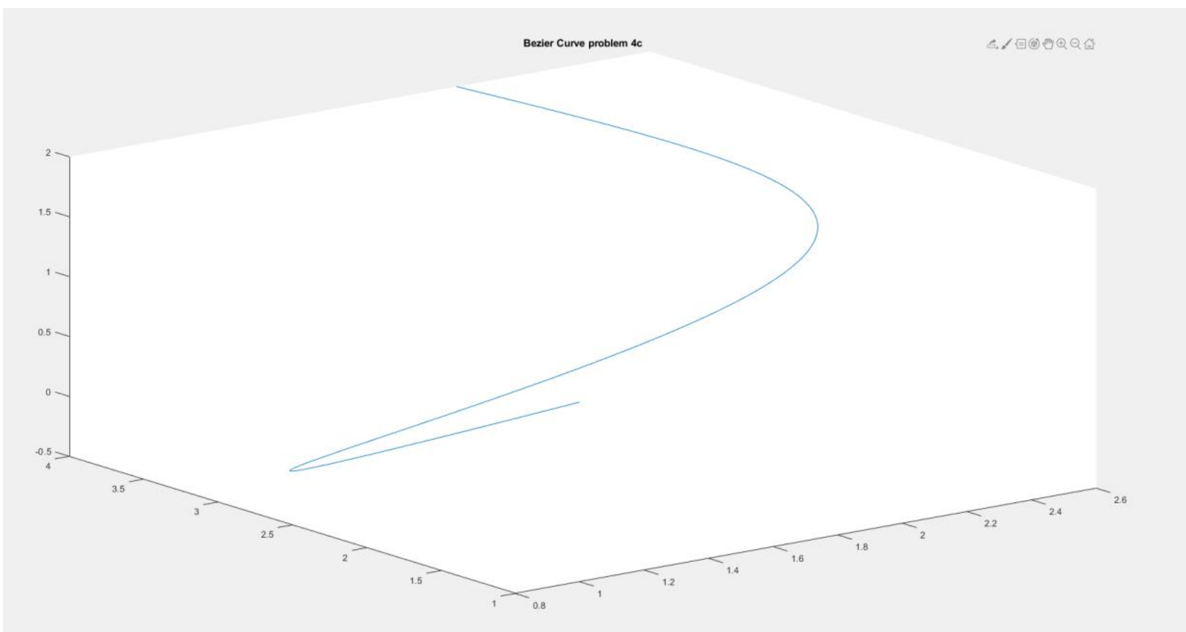

Figure 4.3 Bezier Curve Plot (n=4)

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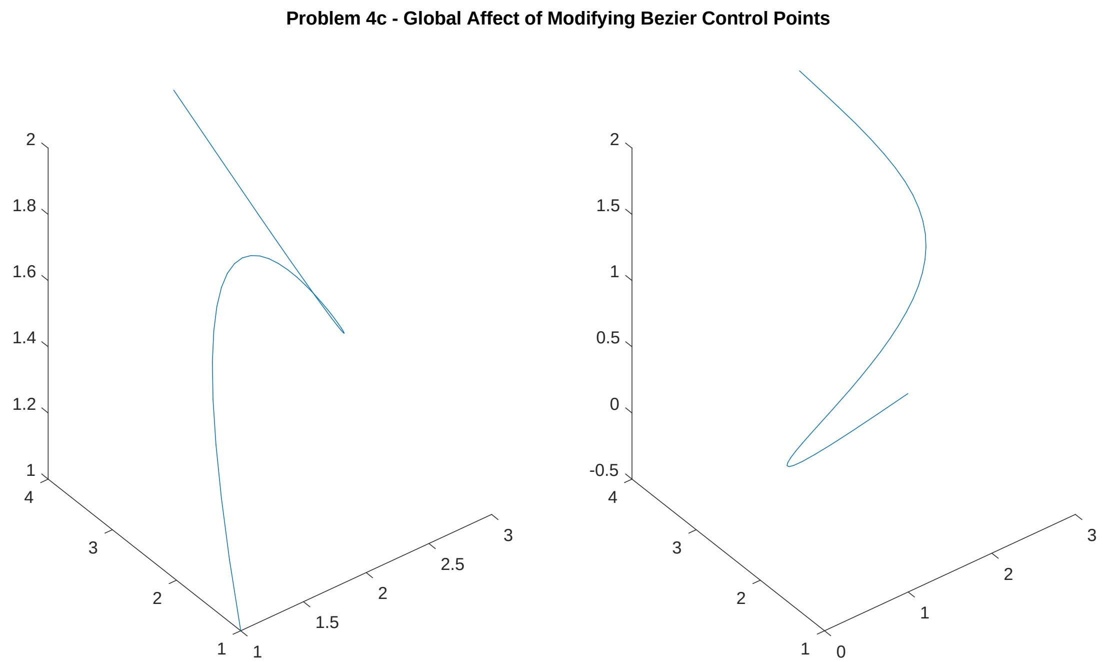

Figure 4.3 Bezier Curve Plot (n=4)

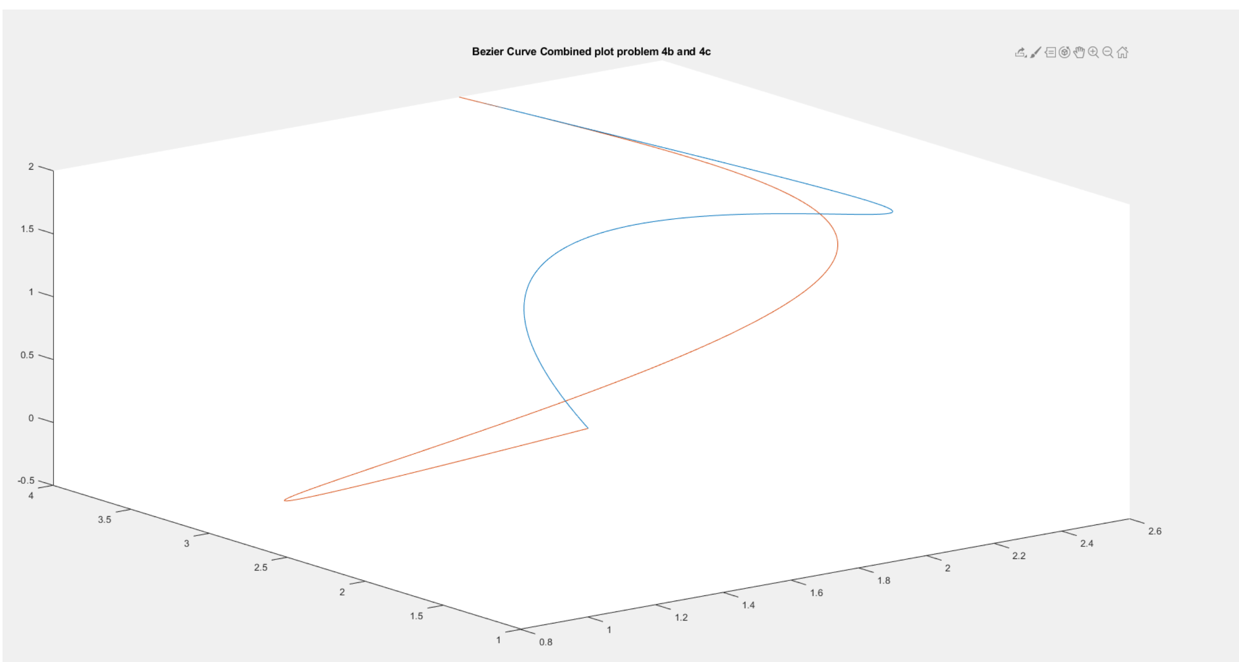

Figure 4.4 Combined Plot

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- **Problem 4c – This curve affects globally, as it is a Bezier Curve. There is *no local control* of this shape modification. Every point on the curve move whenever any interior control point is moved. This property can also be observed in the images shown above.**

- **Problem 5**

Determine a Bezier curve of degree 3 that approximates a quarter circle centered at (0, 0). The end points of the quarter are (0, 5) at u=0 and (-5, 0) at u=1. Ensure that the curve passes through the exact quarter circle at u=0.5. Plot the generated Bezier curve using MATLAB. Calculate the radial error of the Bezier curve at u=0.25 and u=0.75. Also determine the maximum radial deviation and the corresponding u value of the curve from the ideal circle.

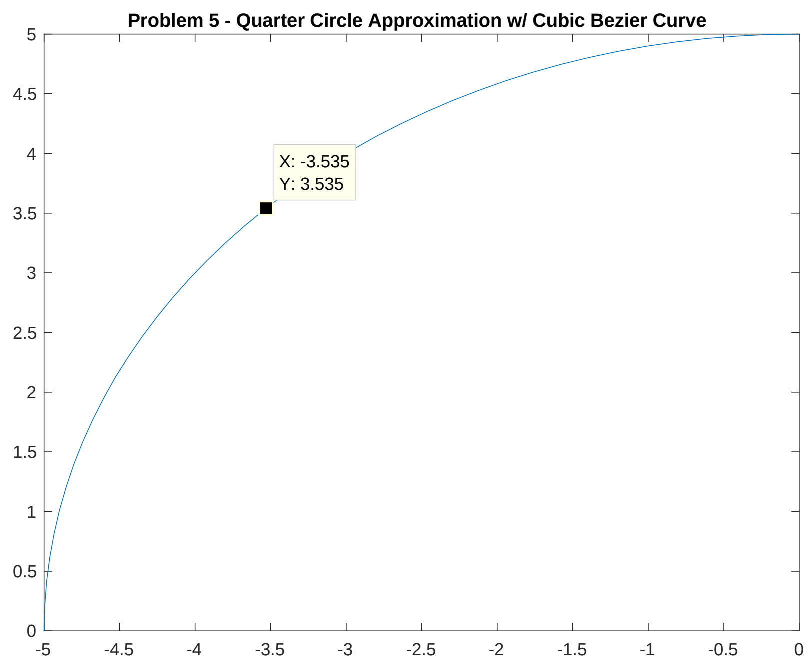

Figure 5.1 Bezier Curve plot

- **Using MATLAB, radial error of Bezier Curve at u=0.25 and u=0.75.**
    - **Problem 5 - Radial Error (@ u = 0.25) = 0.00124**
    - **Problem 5 - Radial Error (@ u = 0.75) = 0.00108**

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- Determine the maximum radial deviation and the corresponding u value of the curve from the ideal circle.

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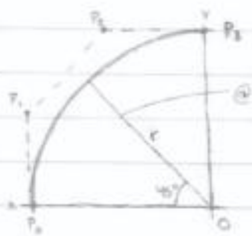
5)



$P_0 = (-5, 0)$ $P_3 = (0, 5)$ $P_1 = (-5, k)$ $P_2 = (-k, 5)$

$@\, u = .5 \rightarrow \underset{\text{(exactly)}}{r = 5} \rightarrow (5\cos(\pi/4), 5\sin(\pi/4)) = (-5/\sqrt{2}, 5/\sqrt{2})$

Cubic Bezier Curve

$P(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u) P_2 + u^3 P_3$

$@\, u = .5$ this curve should lie exactly on quarter-circle $\therefore$

$@\, u = .5 \quad P = (-5/\sqrt{2}, 5/\sqrt{2})$

$P(.5) = (1-.5)^3(-5,0) + 3(.5)(1-.5)(-5,k) + 3(.5)^2(1-.5)(-k,5) + (.5)^3(0,5)$

$(-5/\sqrt{2}, 5/\sqrt{2}) = .1250(-5,0) + .3750(-5,k) + .3750(k,5) + .1250(0,5)$

can be solved in either x or y component, but we will solve in x

$x: -5/\sqrt{2} = -.625 - 1.875 + .3750k$

$k = -2.7613$

$P_0 = (-5, 0)$ $P_1 = (-5, -2.7613)$ $P_2 = (2.7613, 5)$ $P_3 = (0, 5)$

Max Deviation

critical pts of slope/derivative of dist formula

$D(u) = \sqrt{P_x^2(u) + P_y^2(u)}$

$dD(u)/du = \frac{1}{2\sqrt{P_x^2(u) + P_y^2(u)}} \left[2P_x(u) P_x'(u) + 2P_y(u) P_y'(u) \right]$

term I: $(1-u)^3 P_0$

$(1-u)^3 P_0 = (1 - u^3 - 3u + 3u^2) P_0$

$x: (1-u)^3 P_{0_x} = -5 + 5u^3 + 15t - 15t^2$

$y: (1-u)^3 P_{0_y} = 0$

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term II : $3u(1-u)^2 P_1$

$$3u(1-u)^2 P_1 = (3u - 6u^2 + 3u^3) P_1$$

$$x: 3u(1-u)^2 P_{1_x} = -15u + 30u^2 - 15u^3$$

$$y: 3u(1-u)^2 P_{1_y} = 8.2839u - 16.5678u^2 + 8.2839u^3$$

term III : $3u^2(1-u) P_2$

$$3u^2(1-u) P_2 = (3u^2 - 3u^3) P_2$$

$$x: 3u^2(1-u) P_{2_x} = -8.2839u^2 + 8.2839u^3$$

$$y: 3u^2(1-u) P_{2_y} = 15u^2 - 15u^3$$

term IV : $u^3 P_3$

$$x: u^3 P_{3_x} = 0$$

$$y: u^3 P_{3_y} = 5u^3$$

$$P_x(u) = -1.7161u^3 + 6.7161u^2 - 5$$

$$P_y(u) = -1.7161u^3 - 1.5678u^2 + 8.2839u$$

$$P_x'(u) = -5.1483u^2 + 13.4322u$$

$$P_y'(u) = -5.1483u^2 - 3.1356u + 8.2839$$

$$P_x^2(u) = (-1.7161u^3 + 6.7161u^2 - 5)^2$$

$$= 2.9450u^6 - 23.051u^5 + 45.1060u^4 + 17.1610u^3 - 67.1610u^2 + 25$$

$$P_y^2(u) = (-1.7161u^3 - 1.5678u^2 + 8.2839u)^2$$

$$= 5.89u^6 - 17.67u^5 + 19.1320u^4 - 8.840u^3 + 1.4620u^2 + 25$$

$$2(P_x(u) \cdot P_x'(u)) = 2[(-1.7161u^3 + 6.7161u^2 - 5)(-5.1483u^2 + 13.4322u)]$$

$$= 17.67u^5 - 115u^4 + 180.42u^3 + 51.48u^2 - 134.32u$$

$$2(P_y(u) \cdot P_y'(u)) = 2[(-1.7161u^3 - 1.5678u^2 + 8.2839u)(-5.1483u^2 - 3.1356u + 8.2839)]$$

$$= 17.67u^5 + 26.9u^4 - 103.88u^3 - 77.9u^2 + 137.24u$$

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$$\left[2P_xP_x' + 2P_yP_y'\right] = 35.34u^5 - 88u^4 + 76.54u^3 - 26.42u^2 + 2.92u$$

$$\tfrac{1}{2}\sqrt{P_x^2 + P_y^2} = \sqrt{2498u^6 - 8836u^5 + 9566u^4 - 4401u^3 + 731u^2 + 12500}\,/\,5\sqrt{5}$$

$$dD(u)/du = 35.34u^4 - 88u^3 + 76.54u^2 - 26.42u + 2.92 = 0$$

$u = .9045 + .1289j$

$u = .9045 - .1289j$

$u = .4709 + .000j$

$u = .2102 + .000j$

$$\boxed{@\ u = .2102 \quad r = 5.0013 \rightarrow \text{deviation} = 5.0013 - 5 = .0013 = .13\%}$$

- **Answer: Maximum radial deviation occurs at u= 0.2102 and the corresponding value of r is 5.0013. Value of maximum radial deviation is 0.013%**