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SUBJECT Computational Methods in Additive Manufacturing - I

CLASS ____________

SECTION ____________

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1) $K = 4$ [4^{th} order]
$n = K-1 = 3$ [cubic curve]

.) control points
B_1 (1, 2)
B_2 (2, 7)
B_3 (5, 12)
B_4 (3, 2)
non- periodic B-spline curve,

a) Knot count $= n+k+1 = 3+4+1 \Rightarrow 8$
Knot Values $= 0$ to $(n-k+2) \Rightarrow [0-1]$
Knot Vectors = It repeats K times i.e 4 times

$$[u_0 \ u_1 \ u_2 \ u_3 \ u_4 \ u_5 \ u_6 \ u_7] = [0\ 0\ 0\ 0\ 1\ 1\ 1\ 1]$$

b) B-Spline curve is composed of $\underline{\underline{1}}$ curve segment

c) Each segment is controlled by K control points i.e 4 control points

d) 4 control points control each each segment.
$B_1 = (1,2)$
$B_2 = (2,7)$
$B_3 = (5,12)$
$B_4 = (3,2)$

e) convex hull property ensures that a parametric curve will never pass outside of the convex hull formed by four control points.] For this case

Convex hull property of B-splines applies locally, such that a span lies within the convex hull of the control points that affect it.

→ Each curve segment must lie within the convex hull of 'k' consecutive control points

f) 4 control points contribute to the convex hull

$B_1 = (1,2)$

$B_2 = (2,7)$

$B_3 = (5,12)$

$B_4 = (3,2)$

•) To obtain the equation of the curve

Equation of B-spline curve,

$p(u) = P_0 N_{0,4} + P_1 N_{1,4} + P_2 N_{2,4} + P_3 N_{3,4}$

where,

$$\left.\begin{aligned} P_0 &= B_1 \\ P_1 &= B_2 \\ P_2 &= B_3 \\ P_3 &= B_4 \end{aligned}\right\} \text{Control Points}$$

Basis functions,

$N_{0,4}$
$N_{1,4}$
$N_{2,4}$
$N_{3,4}$

It can be calculated by performing exact same procedure used for problem 3.

However,

This is the special type of B-spline curve where the order of the curve equals to the number of control points and curve transforms to a Bezier curve.

To simplify the calculations,
Blending functions for Bezier curve will be used.

For $n = 3$ i.e 4 control points,

$$p(u) = \sum_{i=0}^{3} \vec{P}_i \, B_{i,3}(u) \qquad u \in [0,1]$$

$$B_{i,n}(u) = C(n,i) \, u^i (1-u)^{n-i}$$

where,

$$C(n,i) = \frac{n!}{i!(n-i)!}$$ = The binomial coefficient.

Need to calculate,

$$\left.\begin{matrix} B_{0,3} \\ B_{1,3} \\ B_{2,3} \\ B_{3,3} \end{matrix}\right\} \text{Basis functions}$$

$$B_{0,3} = C(3,0)\, u^0 (1-u)^{3-0} = \frac{3!}{0!(3-0)!} (1)(1-u)^3$$

$$B_{0,3} = (1-u)^3 \quad \text{— I}$$

similarly,

$$B_{1,3} = C(3,1)\, u^1 (1-u)^{3-1} = \frac{3!}{1!(3-1)!} \times u (1-u)^2$$

$$= \frac{3!}{2!} u (1-u)^2$$

$$B_{1,3} = 3u(1-u)^2 \quad \text{— II}$$

similarly,

$$B_{2,3} = C(3,2)\, u^2 (1-u)^{3-2} = \frac{3!}{2!\,1!} u^2 (1-u)$$

$$B_{2,3} = 3u^2(1-u) \quad \text{— III}$$

$$B_{3,3} = C(3,3)\, u^3 (1-u)^{3-3} = \frac{3!}{3!\,0!} u^3 (1-u)^0$$

$$B_{3,3} = u^3 \quad \text{— IV}$$

$$P(u) = P_0 B_{03} + P_1 B_{13} + P_2 B_{23} + P_3 B_{33} \quad \text{— V}$$

Substituting control points and eq^n I, II, III, IV in V,

$$P(u) = (1-u)^3 \vec{P_0} + 3u(1-u)^2 \vec{P_1} + 3u^2(1-u)\vec{P_2} + u^3 \vec{P_3} \quad \text{— VI}$$

Equation VI represents the equation of the curve.

solving Equation VI on matlab to calculate the actual co-ordinates of curve at $u=0.3$ and $u=0.6$

- At $u = 0.3$,
  $[x, y] \longrightarrow [2.251, 6.095]$

- At $u = 0.6$

  $[x, y] \longrightarrow [3.448, 7.760]$

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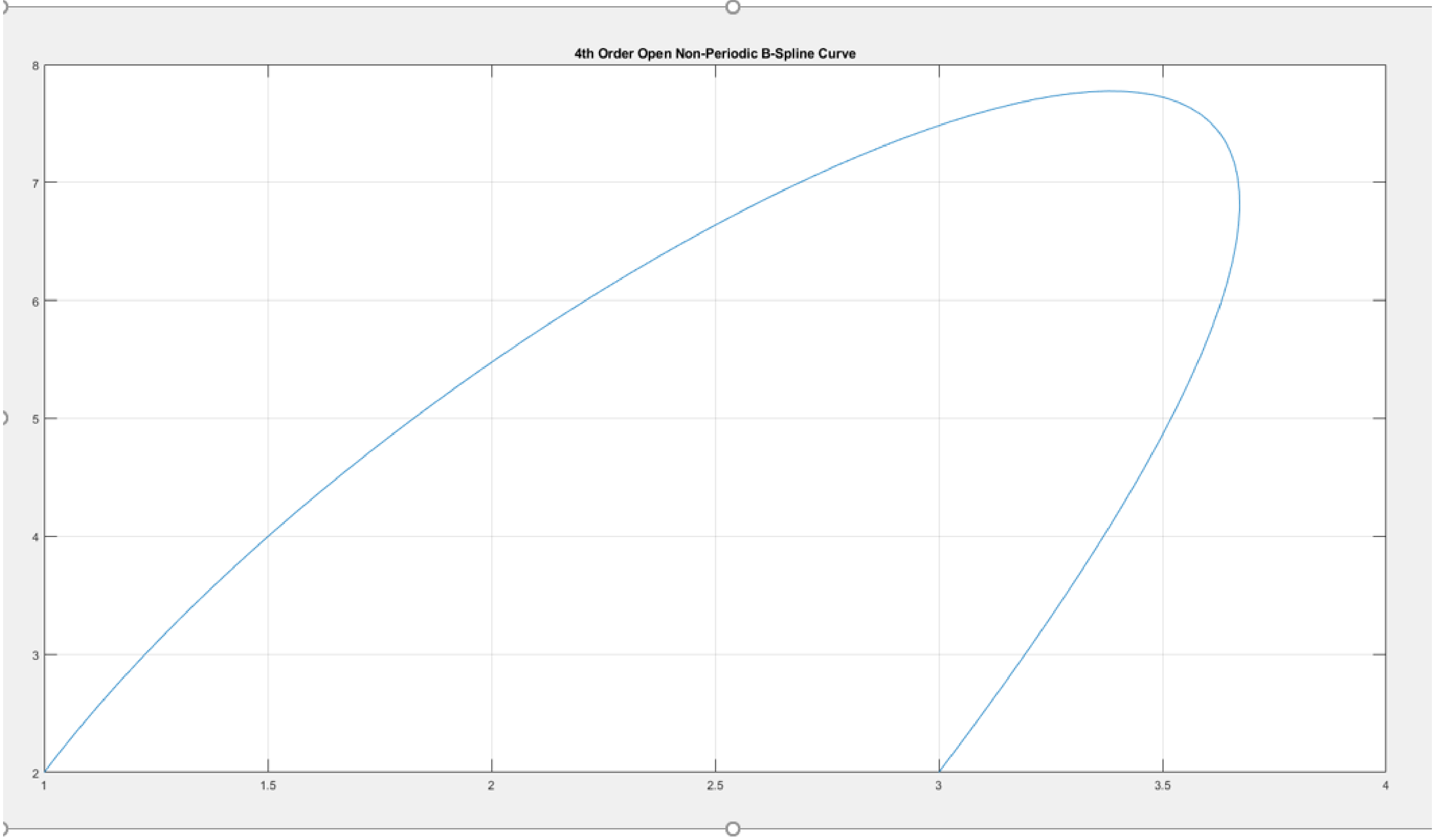


Problem 1- 4^{th} Order Open Non-Periodic B-Spline Curve

2) For $K = 4$ [order = 4]

Degree = 3 → cubic curve

$$P_i(u) = U_4 M_4 \begin{bmatrix} P_{(i-1)} \bmod (n+1) \\ P_i \bmod (n+1) \\ P_{(i+1)} \bmod (n+1) \\ P_{(i+2)} \bmod (n+1) \end{bmatrix}$$

$$U_4 = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix}$$

$$M_4 = \frac{1}{6}\begin{bmatrix} 1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

This is a 3rd degree curve defined by $(n+1)$ control points

$\therefore n+1 = 7$

$\therefore n = 6$

Range of $u \Rightarrow 0 \le u \le n+1 \Rightarrow 0 \le u \le 7$

$i \Rightarrow$ segment number $\Rightarrow i \in [1 : n+1]$

$i \in [1 : 7]$

For each segment of the curve, (i), the parameter 'u' varies from 0-1

For i=1,

$$P_1(u) = U_4 M_4 \begin{bmatrix} P_0 \bmod 7 \\ P_1 \bmod 7 \\ P_2 \bmod 7 \\ P_3 \bmod 7 \end{bmatrix}$$

$$P_1(u) = U_4 M_4 \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$\Rightarrow P_1(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} (-1,2) \\ (1.75,4) \\ (2,1) \\ (2.25,4) \end{bmatrix}$$

— I

Similarly,

i=2,

$$P_2(u) = U_4 M_4 \begin{bmatrix} P_1 \bmod 7 \\ P_2 \bmod 7 \\ P_3 \bmod 7 \\ P_4 \bmod 7 \end{bmatrix}$$

$$P_2(u) = U_4 M_4 \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

$$P_2(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} (1.75, 4) \\ (2, 1) \\ (2.25, 4) \\ (5, 2) \end{bmatrix} \quad \text{— (II)}$$

Similarly,

$i = 3$

$$P_3(u) = U_4 \; M_4 \begin{bmatrix} P_2 \bmod 7 \\ P_3 \bmod 7 \\ P_4 \bmod 7 \\ P_5 \bmod 7 \end{bmatrix}$$

$$P_3(u) = U_4 \; M_4 \begin{bmatrix} P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

$$P_3(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} (2, 1) \\ (2.25, 4) \\ (5, 2) \\ (2, -1) \end{bmatrix} \quad \text{— (III)}$$

Similarly,

$i = 4$

$$P_4(u) = U_4 M_4 \begin{bmatrix} P_3 \bmod 7 \\ P_4 \bmod 7 \\ P_5 \bmod 7 \\ P_6 \bmod 7 \end{bmatrix}$$

$$P_4(u) = U_4 M_4 \begin{bmatrix} P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix}$$

$$\Rightarrow P_4(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} (2.25, 4) \\ (5, 2) \\ (2, -1) \\ (2, -1) \end{bmatrix} \quad \text{— IV}$$

similarly,

For $i=5$

$$P_5(u) = U_4 M_4 \begin{bmatrix} P_4 \bmod 7 \\ P_5 \bmod 7 \\ P_6 \bmod 7 \\ P_7 \bmod 7 \end{bmatrix}$$

$$P_5(u) = U_4 M_4 \begin{bmatrix} P_4 \\ P_5 \\ P_6 \\ P_0 \end{bmatrix}$$

$$P_5(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} (5,2) \\ (2,-1) \\ (2,-1) \\ (-1,2) \end{bmatrix} \quad \text{— V}$$

Similarly,

For $i = 6$

$$P_6(u) = U_4 \; M_4 \begin{bmatrix} p_5 \bmod 7 \\ p_6 \bmod 7 \\ p_7 \bmod 7 \\ p_8 \bmod 7 \end{bmatrix}$$

$$P_6(u) = U_4 \; M_4 \begin{bmatrix} p_5 \\ p_6 \\ p_0 \\ p_1 \end{bmatrix}$$

$$P_6(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} (2,-1) \\ (2,-1) \\ (-1,2) \\ (1.75,4) \end{bmatrix} \quad \text{— VI}$$

Similarly,

For i = 7

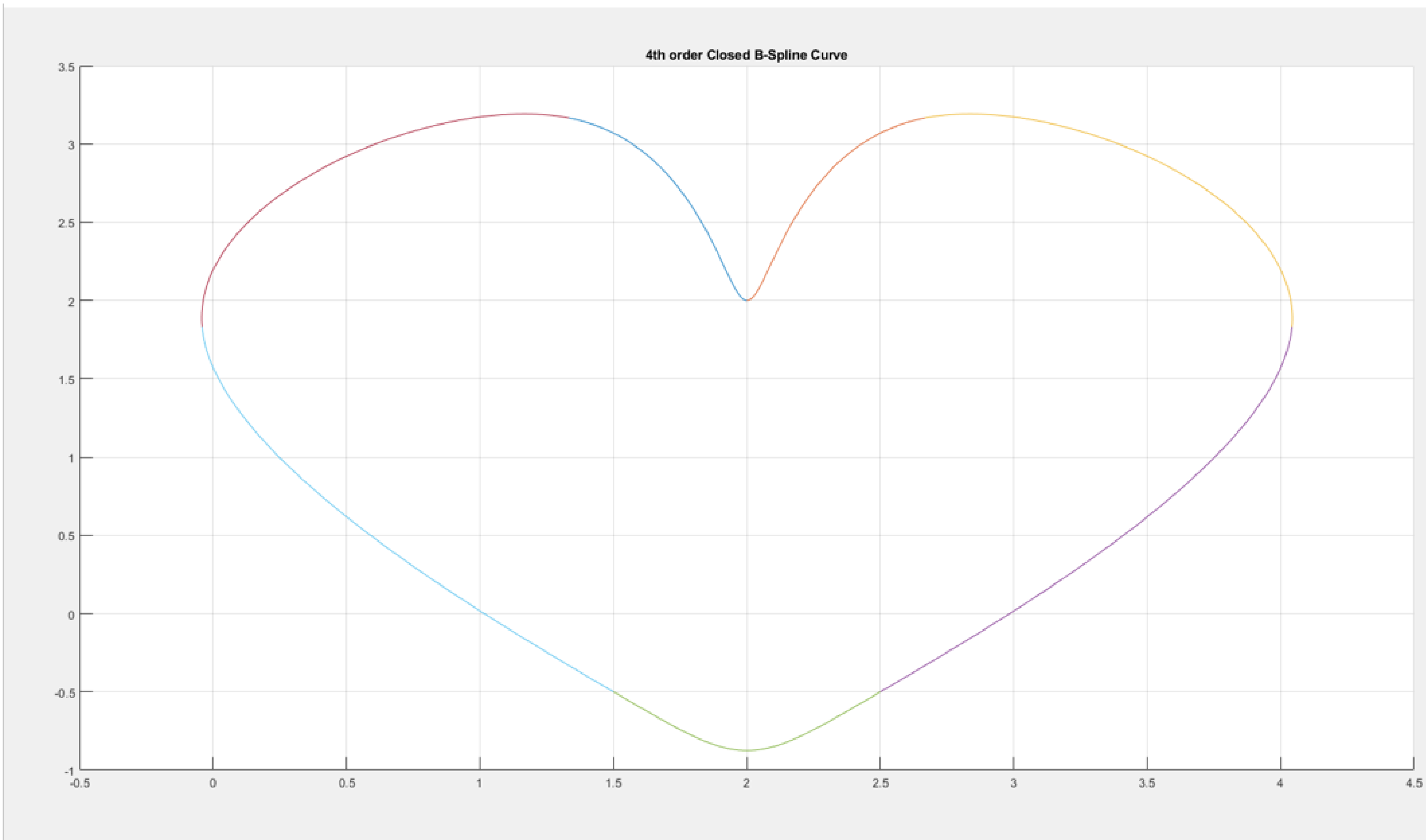
$$P_7(u) = U_4 M_4 \begin{bmatrix} P_6 \bmod 7 \\ P_7 \bmod 7 \\ P_8 \bmod 7 \\ P_9 \bmod 7 \end{bmatrix}$$

$$= U_4 M_4 \begin{bmatrix} P_6 \\ P_0 \\ P_1 \\ P_2 \end{bmatrix}$$

$$P_7(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_6 \\ P_0 \\ P_1 \\ P_2 \end{bmatrix}$$

$$\Rightarrow P_7(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} (2,-1) \\ (-1,2) \\ (1.75,4) \\ (2,1) \end{bmatrix}$$

— VII



Problem 2- 4th Order Closed Periodic B-Spline Curve

3) First curve $(K=3)$ [Quadratic NURBS curve]

3 control points

$B_0 = (0,0)$

$B_1 = (0,2)$

$B_2 = (2,2)$

① Knot Vector (U) = (0 to n-k+2)

where,

K= order of the curve

n= degree of the curve

Knot Vector (U) = 0 to (2-3+2)

Knot Vector (U) = [0-1] ⟹ Knot Vector range for curve 1

② Number of knots in the knot vector must always be equal to no. of control points + order of the curve i.e (N+K) where, N is no. of control points

j ⇒ 0 to N+K ⇒ 0 to 6

Knot Vector consists of 6 points and it goes from 0-1

Knot Vector for the curve 1 = [0 0 0 1 1 1]

— (I)

③ Range of knot vector for curve 2, since its a 3rd order NURBS curve with 4 control points

$B_2 = (2,2)$

$B_3 = (4,2)$

$B_4 = (8,5)$

$B_5 = (3,7)$

Knot Vector (U) = (0 to n-k+2)

Knot Vector (U) = [0 to 2]

However, since the two curves are connected at a common point B_2, knot vector (U) for the second curve will start at 1 and end at 3

Knot Vector (U) = [1-3] ⇒ Knot Vector range for curve 2.

④ No. of knots in a knot vector for the second curve will be [N+k] i.e 4+3 ⇒ 7

j ⇒ 0 - N+K → (0-7)

∴ Knot Vector consists of 7 points and it goes from 1-3

Knot Vector for the curve 2 = [1 1 1 2 3 3 3]

— II

(5) Composite Knot Vector for the combined curve,

$$U = [0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 2 \quad 3 \quad 3 \quad 3] \quad - \quad \text{III}$$

$$\begin{array}{ccccccccc} u_0 & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 \end{array}$$

It ranges from 0-3 with 9 points.

Equations I, II and III shows the individual knot vectors for each curve as well as composite knot vector for the composite curve respectively.

(6) curve (1) is obtained by smoothly blending a 90° circular arc defined by polygon vertices.

$\therefore \ \theta = 45^\circ$,

$h_0 = 1$

$h_1 = \cos\theta = \cos(45) = 1/\sqrt{2}$

$h_2 = 1$

For the first curve,

weighing Factor $[H] = [h_0 \quad h_1 \quad h_2]$

weighing Factor $= [0, 1/\sqrt{2}, 1] \quad - \quad \text{IV}$

(7) Given,

$h_3 = h_4 = h_5 = 1$

For the second curve,

weighing Factor $\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$ — (V)

$\quad\quad\quad\quad h_2 \; h_3 \; h_4 \; h_5$

(8) combined weighing factor will be,

$$h = \begin{bmatrix} h_0 & h_1 & h_2 & h_3 & h_4 & h_5 \end{bmatrix}$$

$$h = \begin{bmatrix} 1 & 1/\sqrt{2} & 1 & 1 & 1 & 1 \end{bmatrix} \quad - \text{(VI)}$$

Equation (IV), (V) and (VI) represent the Individual weighing factors as well as the combined weighing factor respectively.

(9) To find the NURBS equation for the entire composite curve.

we have, 6 control points from $[B_0, B_1, B_2, B_3, B_4$ and $B_5]$

- $n+1 \Rightarrow 6$
- $\underline{\underline{n=5}}$ i.e $i = 5 \Rightarrow i \in (0, 5)$

Need,

$N_{i,k}$ for $k=3$ with $i \in [0, 5]$

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To calculate,

$N_{0,3}$, $N_{1,3}$, $N_{2,3}$, $N_{3,3}$, $N_{4,3}$, $N_{5,3}$

Use posted basis functions in B-spline example problem ppt,

$$N_{0,3} = (u-u_0)\frac{N_{0,2}}{u_2-u_0} + (u_3-u)\frac{N_{1,2}}{u_3-u_1}$$

where,

$$\left.\begin{aligned} u_0 &= 0 \\ u_1 &= 0 \\ u_2 &= 0 \\ u_3 &= 1 \end{aligned}\right\} \text{From } \boxed{III}$$

$$N_{0,3} = \cancel{(u)\frac{N_{0,2}}{0-0}} + (1-u)\frac{N_{1,2}}{1}$$

$$N_{0,3} = (1-u)\,N_{1,2} \quad - \text{(i)}$$

$$N_{1,2} = (u-u_1)\frac{N_{1,1}}{u_2-u_1} + (u_3-u)\frac{N_{2,1}}{u_3-u_2}$$

$$N_{1,2} = \cancel{(u)\frac{N_{1,1}}{0-0}} + (1-u)\frac{N_{2,1}}{1}$$

$$N_{1,2} = (1-u)\,N_{2,1} \quad - \text{(ii)}$$

$$N_{2,1} = \begin{cases} 1, & 0 \le u \le 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{— (iii)}$$

$$N_{1,2} = (1-u)\, N_{2,1} = \begin{cases} (1-u), & 0 \le u \le 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{— (iv)}$$

$$\rightarrow N_{0,3} = (1-u)\, N_{1,2} = \begin{cases} (1-u)^2, & 0 \le u \le 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{— (v)}$$

$$N_{1,3} = (u-u_1)\frac{N_{1,2}}{u_3-u_1} + (u_4-u)\frac{N_{2,2}}{u_4-u_2}$$

$$N_{1,3} = (u)\, N_{1,2} + (1-u)\, N_{2,2}$$

$$N_{1,3} = (u)(1-u) + (1-u)\, N_{2,2}$$ From (iii) and (iv)

$$N_{2,2} = (u-u_2)\frac{N_{2,1}}{u_3-u_2} + (u_4-u)\frac{N_{3,1}}{u_4-u_3}$$

$$= (u-u_2)(1) + \cancel{\frac{(1)\, N_{3,1}}{0-0}}$$

$$N_{2,2} = u$$

$$\rightarrow N_{1,3} = u(1-u) + u(1-u) \quad \text{— (v)} = \begin{cases} 2[u(1-u)] & 0 \le u \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$N_{2,3} = \frac{u-u_2}{u_4-u_2} N_{2,2} + \frac{u_5-u}{u_5-u_3} N_{3,2}$$

$$N_{2,3} = (u) N_{2,2} + (2-u) N_{3,2}$$

$$N_{3,2} = \frac{u-u_3}{u_4-u_3} N_{3,1} + \frac{u_5-u}{u_5-u_4} N_{4,1}$$

$$= \cancel{\frac{u-1}{0-0} N_{3,1}} + \frac{2-u}{2-1} N_{4,1}$$

$$N_{3,2} = (2-u) N_{4,1}$$

$$N_{4,1} = \begin{cases} 1 & 1 \leq u \leq 2 \\ 0 & \text{otherwise} \end{cases} \quad - \text{(vi)}$$

$$N_{3,2} = (2-u) = \begin{cases} (2-u) & 1 \leq u \leq 2 \\ 0 & \text{otherwise} \end{cases} \quad - \text{(vii)}$$

$$N_{2,3} = (u) N_{2,2} + (2-u) N_{3,2} = \begin{cases} u^2 & 0 \leq u \leq 1 \\ (2-u)^2 & 1 \leq u \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$- \text{(viii)}$$

$$N_{3,3} = \frac{u-u_3}{u_5-u_3} N_{3,2} + \frac{u_6-u}{u_6-u_4} N_{4,2}$$

$$N_{3,3} = \frac{u-1}{1} N_{3,2} + \frac{3-u}{2} N_{4,2}$$

$$N_{4,2} = \frac{u-u_4}{u_5-u_4} N_{4,1} + \frac{u_6-u}{u_6-u_5} N_{5,1}$$

$$N_{4,2} = \frac{u-1}{1} N_{4,1} + \frac{3-u}{1} N_{5,1}$$

$$N_{4,2} = \begin{cases} u-1 & 1 \le u \le 2 \\ 3-u & 2 \le u \le 3 \\ 0 & \text{otherwise} \end{cases} \quad - \text{(ix)}$$

$$\Longrightarrow \quad N_{3,3} = \begin{cases} (2-u)(u-1) + \frac{1}{2}(3-u)(u-1) & 1 \le u \le 2 \\ \frac{1}{2}(3-u)^2 & 2 \le u \le 3 \\ 0 & \text{otherwise} \end{cases} \quad - \text{(x)}$$

$$N_{5,1} = \begin{cases} 1 & 2 \le u \le 3 \\ 0 & \text{otherwise} \end{cases} \quad - \text{(xi)}$$

$$N_{4,3} = \frac{u-u_4}{u_6-u_4} N_{4,2} + \frac{u_7-u}{u_7-u_5} N_{5,2}$$

$$N_{4,3} = \frac{u-1}{2} N_{4,2} + \frac{3-u}{1} N_{5,2}$$

$$N_{5,2} = \frac{u-u_5}{u_6-u_5} N_{5,1} + \frac{u_7-u}{u_7-u_6} N_{6,1}$$

$$= \frac{u-2}{1} N_{5,1} + \cancel{\frac{3-u}{0-0} N_{6,1}}$$

$$N_{5,2} = (u-2) N_{5,1} = \begin{cases} (u-2) & 2 \le u \le 3 \\ 0 & \text{otherwise} \end{cases} \quad - \text{xii}$$

$$= (u-2)$$

$$N_{6,1} = \begin{cases} 1 & 3 \le u \le 3 \Rightarrow u = 3 \\ 0 & \text{otherwise} \end{cases} \quad - \text{xiii}$$

$$N_{5,2} = \begin{cases} (u-2) & 2 \le u \le 3 \\ 0 & \text{otherwise} \end{cases} \quad - \text{xiv}$$

$$\rightarrow N_{4,3} = \begin{cases} \frac{1}{2}(u-1)^2 & 1 \le u \le 2 \\ \frac{1}{2}(u-1)(3-u) + (3-u)(u-2) & 2 \le u \le 3 \\ 0, & \text{otherwise} \end{cases} \quad - \text{xv}$$

$$N_{5,3} = \frac{u-u_5}{u_7-u_5} N_{5,2} + \frac{u_8-u}{u_8-u_6} N_{6,2}$$

$$= \frac{u-2}{1} N_{5,2} + \cancel{\frac{3-u}{0}} N_{6,2}$$

$$N_{5,3} = (u-2) N_{5,2}$$

$$N_{5,3} = \begin{cases} (u-2)^2 & 2 \le u \le 3 \\ 0, & \text{otherwise} \end{cases} \quad - (xvi)$$

(10) $B_0 = (0,0)$
$B_1 = (0,2)$
$B_2 = (2,2)$
$B_3 = (4,2)$
$B_4 = (8,5)$
$B_5 = (3,7)$

Given control points

(10) $B(u) = B_0 N_{0,3} + B_1 N_{1,3} + B_2 N_{2,3} + B_3 N_{3,3} + B_4 N_{4,3} + B_5 N_{5,3}$

•) $B_0 N_{0,3} = \begin{cases} \overset{x\;\;y}{(0,0)} (1-u)^2 & 0 \le u \le 1 \\ 0 & \text{otherwise} \end{cases}$

•) $B_1 N_{1,3} = \begin{cases} \overset{x\;\;y}{(0,2)} \; 2u(1-u) & 0 \le u \le 1 \\ 0 & \text{otherwise} \end{cases}$

• $$B_2 N_{2,3} = \begin{cases} \overset{x\ y}{(2,2)}\ u^2 & 0 \le u \le 1 \\ (2,2)\ (2-u)^2 & 1 \le u \le 2 \\ 0 & \text{Otherwise} \end{cases}$$

• $$B_3, N_{3,3} = \begin{cases} \overset{x\ y}{(4,2)} \left[(2-u)(u-1) + \frac{1}{2}(3-u)(u-1)\right] & 1 \le u \le 2 \\ \overset{x\ y}{(4,2)} \left[\frac{1}{2}(3-u)^2\right] & 2 \le u \le 3 \\ 0 & \text{otherwise} \end{cases}$$

• $$B_4 N_{4,3} = \begin{cases} \overset{x\ y}{(8,5)} \left[\frac{1}{2}(u-1)^2\right] & 1 \le u \le 2 \\ \overset{x\ y}{(8,5)} \left[\frac{1}{2}(u-1)(3-u) + (3-u)(u-2)\right] & 2 \le u \le 3 \\ 0 & \text{Otherwise} \end{cases}$$

• $$B_5 N_{5,3} = \begin{cases} \overset{x\ y}{(3,7)}\ (u-2)^2 & 2 \le u \le 3 \\ 0 & \text{otherwise} \end{cases}$$

(12) Final NURBS equation for the composite curve,

$$B(u) = \begin{cases} (0,0)(1-u)^2 + (0,2)\,2u(1-u) + (2,2)\,u^2 \Longrightarrow 0 \le u \le 1 \\[2ex] (2,2)(2-u)^2 + (4,2)\left[(2-u)(u-1) + \frac{1}{2}(3-u)(u-1)\right] \\ \quad + (8,5)\left[\frac{1}{2}(u-1)^2\right] \Longrightarrow 1 \le u \le 2 \\[2ex] (4,2)\left(\frac{1}{2}(3-u)^2\right) + (8,5)\left[\frac{1}{2}(u-1)(3-u) + (3-u)(u-2)\right] \\ \quad + (3,7)(u-2)^2 \longrightarrow 2 \le u \le 3 \end{cases}$$

(13) Using matlab,
Values of the composite curve at,

$u = 1.5 \Rightarrow [\ 4,\ 2.375\ ]$

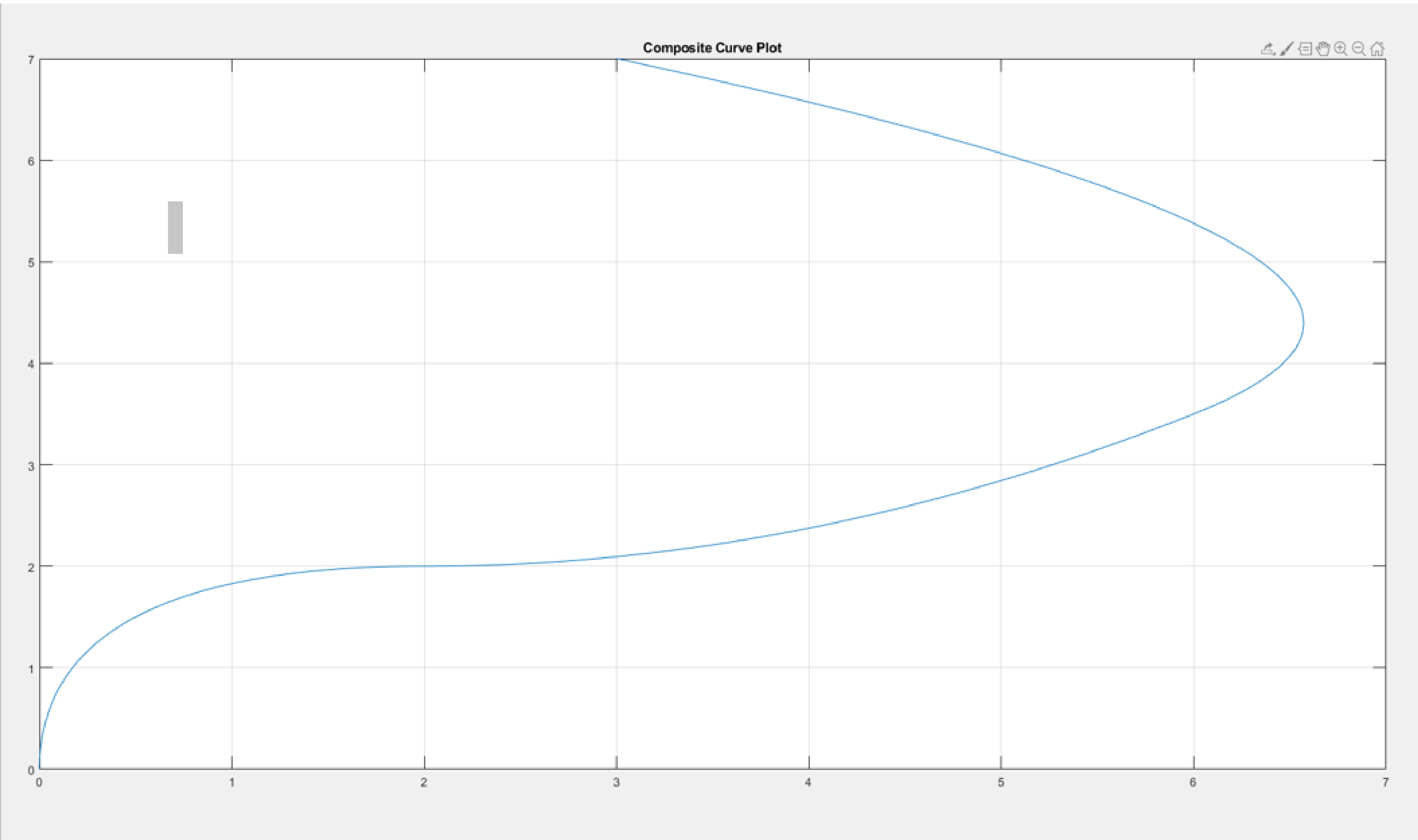
and

$u = 2 \rightarrow [\ 6,\ 3.5\ ]$

Final Answers

(14) Matlab code has been attached to show the plot of the composite curve

Scanned with CamScanner



Problem 3- Composite Curve Plot