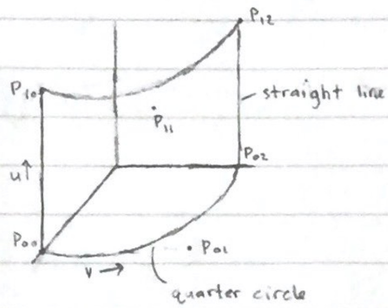
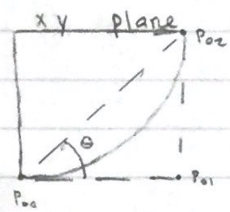
Homework 4

3) quarter circle w/ center @ (0,0,0)
2D profile on xy-plane w/ unit radius $(r=1)$
swept +4 units in +z-axis direction $(z \in [0, 4])$



$P_{00} = (1, 0, 0)$ $h = 1$
$P_{01} = (1, 1, 0)$ $h = 1/\sqrt{2}$
$P_{02} = (0, 1, 0)$ $h = 1$
$P_{10} = (1, 0, 4)$ $h = 1$
$P_{11} = (1, 1, 4)$ $h = 1/\sqrt{2}$
$P_{12} = (0, 1, 4)$ $h = 1$



$\cos\theta = 1/\sqrt{2}$
$\llcorner 45°$

a) $l = 3$ v
$m = 2$
knots $= [0, 0, 0, 1, 1, 1]$
$H = [1, 1/\sqrt{2}, 1]$
$P_{00} = (1, 0, 0)$
$P_{01} = (1, 1, 0)$
$P_{02} = (0, 1, 0)$

b) $P_{10} = (1, 0, 4)$
$P_{11} = (1, 1, 4)$
$P_{12} = (0, 1, 4)$

c) $k = 2$ u
$n = 1$
knots $= [0, 0, 1, 1]$
$H = [1, 1]$
$P_{00} = (1, 0, 0)$
$P_{10} = (1, 0, 4)$

d) $$P(u,v) = \frac{\sum_{i=0}^{n} \sum_{j=0}^{3} h_{ij} P_{ij} N_{i,k}(u) N_{j,\ell}(v)}{\sum_{i=0}^{n} \sum_{j=0}^{3} h_{ij} N_{i,k}(u) N_{j,\ell}(v)}$$

$$= \frac{\sum_{i=0}^{1} \sum_{j=0}^{2} h_{ij} P_{ij} N_{i,2}(u) N_{j,3}(v)}{\sum_{i=0}^{1} \sum_{j=0}^{2} h_{ij} N_{i,2}(u) N_{j,3}(v)}$$

$$N_{i,k}(u) = \frac{(u-u_i) N_{i,k-1}(u)}{u_{i+k-1} - u_i} + \frac{(u_{i+k}-u) N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

$$N_{i,1}(u) = \begin{cases} 1 & \text{if } u_i \le u \le u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$N_{0,2}(u)$ $$N_{0,2}(u) = (u-u_0) \frac{N_{0,1}(u)}{u_1 - u_0} + (u_2 - u) \frac{N_{1,1}(u)}{u_2 - u_1}$$

$$= \begin{cases} (1-u) & 0 \le u \le 1 \\ 0 \end{cases}$$

$$N_{1,1}(u) = \begin{cases} 1 & u_1 \le u \le u_2 \\ 0 \end{cases} = \begin{cases} 1 & 0 \le u \le 1 \\ 0 \end{cases}$$

$N_{1,2}(u)$ $$N_{1,2}(u) = (u-u_1) \frac{N_{1,1}(u)}{u_2 - u_1} + (u_3 - u) \frac{N_{2,1}(u)}{u_3 - u_2}$$

$$= \begin{cases} u & 0 \le u \le 1 \\ 0 \end{cases}$$

$$N_{2,1}(u) = \begin{cases} 1 & u_2 \le u \le u_3 \\ 0 \end{cases} = \begin{cases} 1 & u = 1 \\ 0 \end{cases}$$

$N_{0,3}(v)$ $$N_{0,3}(v) = (v - v_0) \frac{N_{0,2}(v)}{v_2 - v_0} + (v_3 - v) \frac{N_{1,2}(v)}{v_3 - v_1}$$

$$= \begin{cases} (1-v)^2 & 0 \le v \le 1 \\ 0 \end{cases}$$

$$N_{1,2}(v) = (v - v_1) \frac{N_{1,1}(v)}{v_2 - v_1} + (v_3 - v) \frac{N_{2,1}(v)}{v_3 - v_2}$$

$$= \begin{cases} (1-v) & 0 \le v \le 1 \\ 0 \end{cases}$$

$$N_{2,1}(v) = \begin{cases} 1 & v_2 \le v \le v_3 \\ 0 \end{cases} = \begin{cases} 1 & 0 \le v \le 1 \\ 0 \end{cases}$$

$N_{1,3}(v)$ $\quad N_{1,3}(v) = (v - v_1)\, N_{1,2}(v) / v_3 - v_1 + (v_4 - v)\, N_{2,2}(v) / v_4 - v_2$

$$= \begin{cases} v(1-v) & 0 \le v \le 1 \\ 0 & \end{cases} + \begin{cases} v(1-v) & 0 \le v \le 1 \\ 0 & \end{cases} = \begin{cases} [v(1-v)]^2 & 0 \le v \le 1 \\ 0 & \end{cases}$$

$N_{2,2}(v) = (v - v_2)\, N_{2,1}(v) / v_3 - v_2 + (v_4 - v)\, N_{3,1}(v) / v_4 - v_3$

$$= \begin{cases} v & 0 \le v \le 1 \\ 0 & \end{cases}$$

$N_{2,3}(v)$ $\quad N_{2,3}(v) = (v - v_2)\, N_{2,2}(v) / v_5 - v_2 + (v_5 - v)\, N_{3,2}(v) / v_5 - v_3$

$$= \begin{cases} v^2 & 0 \le v \le 1 \\ 0 & \end{cases}$$

$P(.5, .5) = (0.6306, .6306, 2)$

$r_{err} = 10.82\%$?