[5] Bezier Curve for degree 3 quarter circle

W2

Blending function

$$M = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

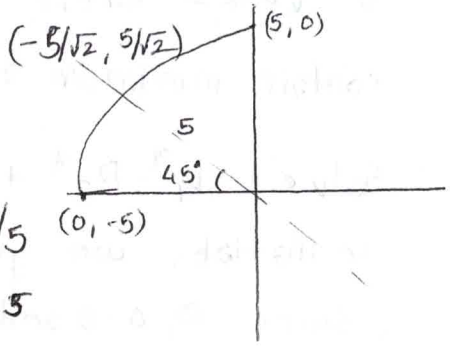
Since points 2 and 3 lie on the tangents of the start and end points respectively, $P_2 = (K, 5)$ and $P_3 = (-5, l)$

$$B = \begin{bmatrix} 0 & 5 \\ K & 5 \\ -5 & l \\ -5 & 0 \end{bmatrix}$$

Since the curve is identical to the circle at $U = 1/2$

$$G = [U][M][B]$$

$$[x \quad y] = [U^3 \quad U^2 \quad U \quad 1] \begin{bmatrix} 0 & 5 \\ K & 5 \\ -5 & l \\ -5 & 0 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$



where $[x \quad y] = -\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}$ since $\cos 45 = -x/5$

$\sin 45 = x/5$

Solving equations for K and l,

Values of $P_2 = (-2.761, 5)$

Values of $P_3 = (-5, 2.761)$

Calculation of deviation at $u = 0.25$ and $u = 0.75$

$t = 1/4$ and $t = 3/4$

$P_{new} = [t^3 \; t^2 \; t \; 1] [M] [B]$ ← Calculated in previous step

Solving equations in Matlab,

Radius at $u = 1/4$ and $u = 3/4$ are approximately 5.00129

and deviation = 0.00129 units

For maximum deviation,

$$U_{diff} = [3u^2 \; 2u \; 1 \; 0]$$

$$D_1 = [u^3 \; u^2 \; u \; 1][M][B]$$

$$D_2 = [3u^2 \; 2u \; 1 \; 0][M][B]$$

Equating maximum deviation with 0

The basic idea is to get an equation of the radial distance using derivatives. By equating it to zero, we are finding the u values where its slope is zero. These 'u' values will contain minimum and maximum

Solve $D_1^x \cdot D_2^x + D_1^y D_2^y = 0$

In Matlab, we get five values of $u = 0, 0.5, 1, 0.21$ and 0.79

Since 0, 0.5 and 1 are on circle they are neglected.

Deviation at $u = 0.21$ and $u = 0.79$

Value = 0.00135