hw4

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1 Homework 4

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CMAM
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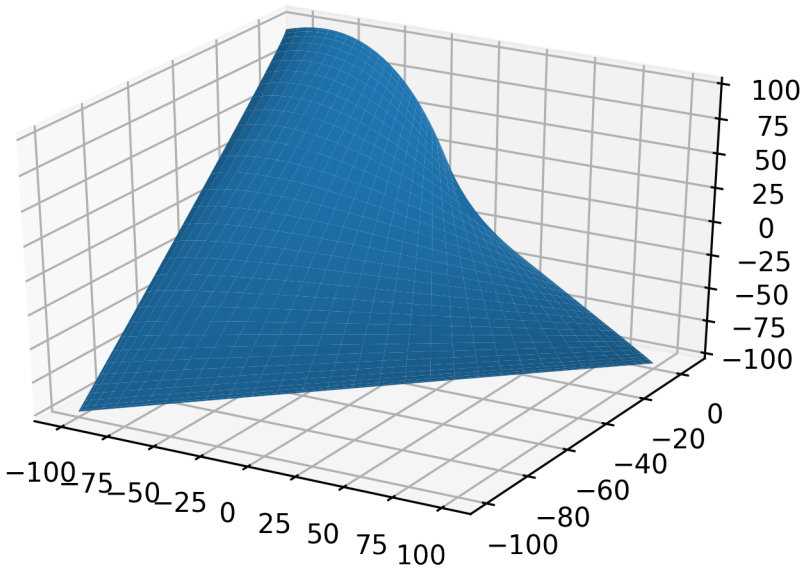
[1]:
```
# libs
from functools import reduce
import numpy as np
import matplotlib.pyplot as plt

# interactive images
#%matplotlib widget

# static inline images
%matplotlib inline
# %config InlineBackend.figure_format = 'svg'
import matplotlib as mpl
mpl.rcParams['figure.dpi']= 300
```

1.1 Problem 1

[3]:
```
# P, P^u, P^w, P^(uw)
P = np.array([
    [[-100, 0, 100], [-100, -100, -100]],
    [[100, -100, 100], [100, 0, -100]],
]) # indexable as P[0, 1] = P_sub01
PSupU = np.array([
    [[100, 100, 0], [1, 1, 0]],
    [[1, -1, 0], [1, -1, 0]],
])
PSupW = np.array([
    [[0, 10, -10], [0, -1, -1]],
    [[0, 1, -1], [0, -1, -1]],
])
PSupUW = np.array([
    [[0, 0, 0], [0.1, 0.1, 0.1]],
    [[0.1, -0.1, -0.1], [0, 0, 0]],
```

```
])

# M_H, B
MSubH = np.array([
    [2, -2, 1, 1],
    [-3, 3, -2, -1],
    [0, 0, 1, 0],
    [1, 0, 0, 0],
])
B = np.array([
    [P[0, 0], P[0, 1], PSupW[0, 0], PSupW[0, 1]],
    [P[1, 0], P[1, 1], PSupW[1, 0], PSupW[1, 1]],
    [PSupU[0, 0], PSupU[0, 1], PSupUW[0, 0], PSupUW[0, 1]],
    [PSupU[1, 0], PSupU[1, 1], PSupUW[1, 0], PSupUW[1, 1]],
])

# U, W
start = 0
stop = 1
step = .01
u = np.arange(start, stop + step, step)
w = u.copy()
U = np.array([u**3, u**2, u**1, u**0]).transpose()
W = np.array([w**3, w**2, w**1, w**0]).transpose()

# P(u, w) = U * M_H * B * M_H^T * W^T
POut = np.zeros((len(u), len(w), 3))
for iu, uu in enumerate(u):
    for iw, ww in enumerate(w):
        for coord in range(3): # (x, y, z)
            POut[iu, iw, coord] = reduce(np.dot, [U[iu], MSubH, B[:, :, coord],␣
 ↪MSubH.transpose(), W.transpose()[:, iw]])

# plot surface
fig = plt.figure(1)
ax = plt.axes(projection='3d')
ax.plot_surface(POut[:, :, 0], POut[:, :, 1], POut[:, :, 2])
plt.savefig(f"./problem{fig.number}.png", dpi=300)
plt.show()

# U^u, W^w
USupU = np.array([3*(u**2), 2*(u**1), 1*(u**0), 0*u]).transpose()
WSupW = np.array([3*(w**2), 2*(w**1), 1*(w**0), 0*w]).transpose()

# P^u, P^w
POutSupU = np.zeros((len(u), len(w), 3))
POutSupW = np.zeros((len(u), len(w), 3))
```

```
for iu, uu in enumerate(u):
    for iw, ww in enumerate(w):
        for coord in range(3): # (x, y, z)
            POutSupU[iu, iw, coord] = reduce(np.dot, [USupU[iu], MSubH, B[:, :,␣
 ↪coord], MSubH.transpose(), W.transpose()[:, iw]])
            POutSupW[iu, iw, coord] = reduce(np.dot, [U[iu], MSubH, B[:, :,␣
 ↪coord], MSubH.transpose(), WSupW.transpose()[:, iw]])

#        find point,     find point,     find unit normal vector
uu = (    0.35,           0.65,           0.85)
ww = (    0.45,           0.75,           0.95)

for uuu, www in zip(uu[:-1], ww[:-1]):
    u_indx, w_indx = [int(x*100) for x in (uuu, www)]
    print(f"P[{u_indx}, {w_indx}] = {POut[u_indx, w_indx]}")

u_indx, w_indx = [int(x*100) for x in (uu[-1], ww[-1])]
n = np.cross(POutSupU[u_indx, w_indx], POutSupW[u_indx, w_indx])
u = n / np.linalg.norm(n)
print(f"u[{u_indx}, {w_indx}] = {u}")
```

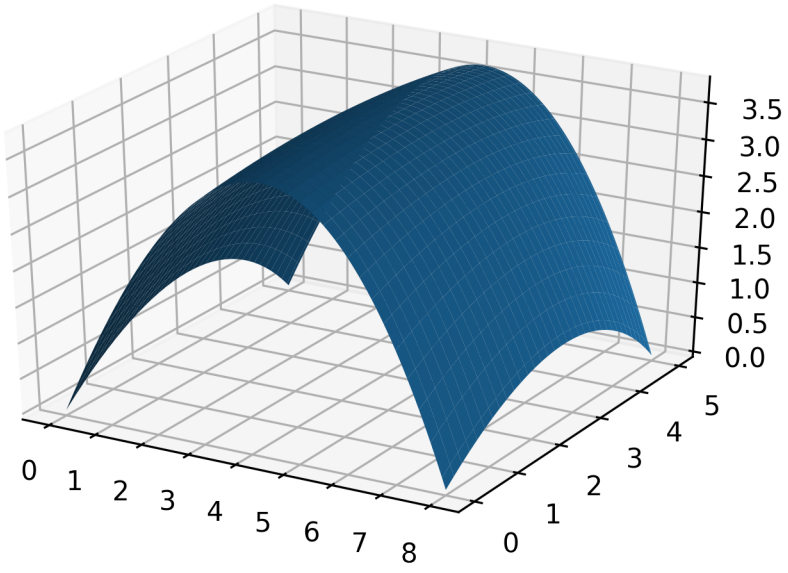



```
P[35, 45] = [-35.17035638 -36.96865506  14.04474091]
P[65, 75] = [ 44.81163633 -33.23017656 -68.77553984]
u[85, 95] = [-0.41033671  0.84291004  0.34803224]
```

1.2 Problem 2

[4]:
```
# P, M_(BU), M_(BW)
P = np.array([
    [[0, 0, 0], [0, 3, 2], [0, 5, 0]],
    [[2, 0, 3], [2, 3, 3], [2, 5, 3]],
    [[4, 0, 5], [4, 3, 5], [4, 5, 5]],
    [[6, 0, 4], [6, 3, 4], [6, 5, 4]],
    [[8, 0, 0], [8, 3, 2], [8, 5, 0]],
]) # indexable as P[0, 2] = P_sub02
MSubBSubU = np.array([
    [1, -4, 6, -4, 1],
    [-4, 12, -12, 4, 0],
    [6, -12, 6, 0, 0],
    [-4, 4, 0, 0, 0],
    [1, 0, 0, 0, 0],
])
MSubBSubW = np.array([
    [1, -2, 1],
    [-2, 2, 0],
    [1, 0, 0],
])

# U, W
start = 0
stop = 1
step = .01
u = np.arange(start, stop + step, step)
U = np.array([u**4, u**3, u**2, u**1, u**0]).transpose()
w = u.copy()
W = np.array([w**2, w**1, w**0]).transpose()

# P(u, w) = U * M_(BU) * P * M_(BW)^T * W^T
POut = np.zeros((len(u), len(w), 3))
for iu, uu in enumerate(u):
    for iw, ww in enumerate(w):
        for coord in range(3): # (x, y, z)
            POut[iu, iw, coord] = reduce(np.dot, [U[iu], MSubBSubU, P[:, :,␣
 ↪coord], MSubBSubW.transpose(), W.transpose()[:, iw]])

# plot surface
fig = plt.figure(2)
ax = plt.axes(projection='3d')
ax.plot_surface(POut[:, :, 0], POut[:, :, 1], POut[:, :, 2])
plt.savefig(f"./problem{fig.number}.png", dpi=300)
plt.show()
```

```
# U^u, W^w
USupU = np.array([4*(u**3), 3*(u**2), 2*(u**1), 1*(u**0), 0*u]).transpose()
WSupW = np.array([2*(w**1), 1*(w**0), 0*w]).transpose()

# P^u, P^w
POutSupU = np.zeros((len(u), len(w), 3))
POutSupW = np.zeros((len(u), len(w), 3))
for iu, uu in enumerate(u):
    for iw, ww in enumerate(w):
        for coord in range(3): # (x, y, z)
            POutSupU[iu, iw, coord] = reduce(np.dot, [USupU[iu], MSubBSubU, P[:
 ↪, :, coord], MSubBSubW.transpose(), W.transpose()[:, iw]])
            POutSupW[iu, iw, coord] = reduce(np.dot, [U[iu], MSubBSubU, P[:, :,␣
 ↪coord], MSubBSubW.transpose(), WSupW.transpose()[:, iw]])

#   find unit surface normal
uu = 0.35
ww = 0.45

u_indx, w_indx = [int(x*100) for x in (uu, ww)]
n = np.cross(POutSupU[u_indx, w_indx], POutSupW[u_indx, w_indx])
u = n / np.linalg.norm(n)
print(f"u[{u_indx}, {w_indx}] = {u}")
```



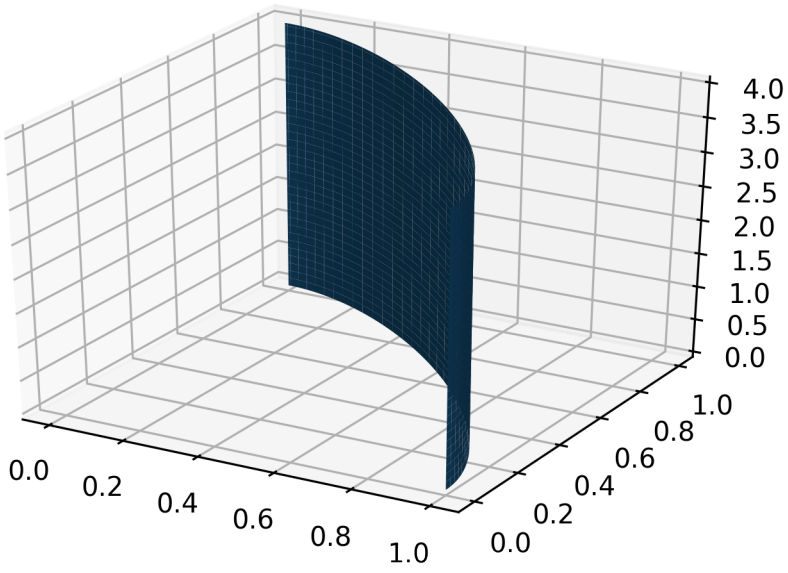

```
u[35, 45] = [-0.47214112 -0.01337773  0.88142146]
```

1.3 Problem 3

[5]:
```
def NSubISub2(i, u):
    if i == 0:
        return (1-u if u >= 0 and u <= 1 else 0)
    elif i == 1:
        return (u if u >= 0 and u <= 1 else 0)
    else:
        raise IndexError(f"i = {i} is not within the bounds of this basis␣
 ↪function")

def NSubJSub3(j, v):
    if j == 0:
        return ((1-v)**2 if v >= 0 and v <= 1 else 0)
    elif j == 1:
        return (v*(1-v) if v >= 0 and v <= 1 else 0)
    elif j == 2:
        return (v**2 if v >= 0 and v <= 1 else 0)
    else:
        raise IndexError(f"i = {i} is not within the bounds of this basis␣
 ↪function")

def HSubISubJ(i, j):
    HSubI = [1, 1]
    HSubJ = [1, 1/np.sqrt(2), 1]
    return HSubI[i]*HSubJ[j]

P = np.array([
    [[1, 0, 0], [1, 1, 0], [0, 1, 0]],
    [[1, 0, 4], [1, 1, 4], [0, 1, 4]],
])

def POutFunc(u, v):
    num, den = 0, 0
    for i in range(2):
        for j in range(3):
            num += HSubISubJ(i, j)*P[i, j]*NSubISub2(i, u)*NSubJSub3(j, v)
            den += HSubISubJ(i, j)*NSubISub2(i, u)*NSubJSub3(j, v)
    return num / den

# find point @ u = v = .5
uu, ww = .5, .5
POut = POutFunc(uu, ww)
print(f"P[{uu}, {ww}] = {POut}")
rAct = np.sqrt(POut[0]**2 + POut[1]**2) # hypotenuse of x and y coord is the␣
 ↪radius from z-axis
err = 1-rAct/1
```

```
print(f"Radius error = {err}")

# U, W
start = 0
stop = 1
step = .01
u = np.arange(start, stop + step, step)
w = u.copy()

POut = np.zeros((len(u), len(w), 3))
for iu, uu in enumerate(u):
    for iw, ww in enumerate(w):
        POut[iu, iw] = POutFunc(uu, ww)

# plot surface
fig = plt.figure(3)
ax = plt.axes(projection='3d')
ax.plot_surface(POut[:, :, 0], POut[:, :, 1], POut[:, :, 2])
plt.savefig(f"./problem{fig.number}.png", dpi=300)
plt.show()
```

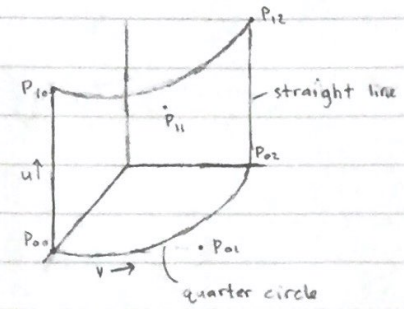
```
P[0.5, 0.5] = [0.63060194 0.63060194 2.        ]
Radius error = 0.10819418755438781
```



[ ]:

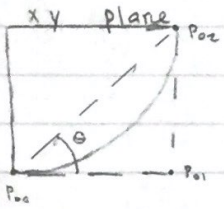
Homework 4

3) quarter circle w/ center @ (0,0,0)
2D profile on xy-plane w/ unit radius ($r=1$)
swept +4 units in +z-axis direction ($z \in [0, 4]$)



$P_{00} = (1, 0, 0)$, $h = 1$
$P_{01} = (1, 1, 0)$, $h = 1/\sqrt{2}$
$P_{02} = (0, 1, 0)$, $h = 1$
$P_{10} = (1, 0, 4)$, $h = 1$
$P_{11} = (1, 1, 4)$, $h = 1/\sqrt{2}$
$P_{12} = (0, 1, 4)$, $h = 1$

a) $\ell = 3$ (v)
$m = 2$
knots $= [0, 0, 0, 1, 1, 1]$
$H = [1, 1/\sqrt{2}, 1]$
$P_{00} = (1, 0, 0)$
$P_{01} = (1, 1, 0)$
$P_{02} = (0, 1, 0)$



$\cos\theta = 1/\sqrt{2}$
$\theta = 45°$

b) $P_{10} = (1, 0, 4)$
$P_{11} = (1, 1, 4)$
$P_{12} = (0, 1, 4)$

c) $k = 2$ (u)
$n = 1$
knots $= [0, 0, 1, 1]$
$H = [1, 1]$
$P_{00} = (1, 0, 0)$
$P_{10} = (1, 0, 4)$

d) $$P(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} h_{ij} P_{ij} N_{i,k}(u) N_{j,\ell}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m} h_{ij} N_{i,k}(u) N_{j,\ell}(v)}$$

$$= \frac{\sum_{i=0}^{1}\sum_{j=0}^{2} h_{ij} P_{ij} N_{i,2}(u) N_{j,3}(v)}{\sum_{i=0}^{1}\sum_{j=0}^{2} h_{ij} N_{i,2}(u) N_{j,3}(v)}$$

$$N_{i,k}(u) = \frac{(u-u_i) N_{i,k-1}(u)}{u_{i+k-1} - u_i} + \frac{(u_{i+k} - u) N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

$$N_{i,1}(u) = \begin{cases} 1 & \text{if } u_i \le u \le u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$N_{0,2}(u)$

$$N_{0,2}(u) = \frac{(u-u_0) N_{0,1}(u)}{u_1 - u_0} + \frac{(u_2 - u) N_{1,1}(u)}{u_2 - u_1}$$

$$= \begin{cases} (1-u) & 0 \le u \le 1 \\ 0 \end{cases}$$

$$N_{1,1}(u) = \begin{cases} 1 & u_1 \le u \le u_2 \\ 0 \end{cases} = \begin{cases} 1 & 0 \le u \le 1 \\ 0 \end{cases}$$

$N_{1,2}(u)$

$$N_{1,2}(u) = \frac{(u-u_1) N_{1,1}(u)}{u_2 - u_1} + \frac{(u_3 - u) N_{2,1}(u)}{u_3 - u_2}$$

$$= \begin{cases} u & 0 \le u \le 1 \\ 0 \end{cases}$$

$$N_{2,1}(u) = \begin{cases} 1 & u_2 \le u \le u_3 \\ 0 \end{cases} = \begin{cases} 1 & u = 1 \\ 0 \end{cases}$$

$N_{0,3}(v)$

$$N_{0,3}(v) = \frac{(v-v_0) N_{0,2}(v)}{v_2 - v_0} + \frac{(v_3 - v) N_{1,2}(v)}{v_3 - v_1}$$

$$= \begin{cases} (1-v)^2 & 0 \le v \le 1 \\ 0 \end{cases}$$

$$N_{1,2}(v) = \frac{(v-v_1) N_{1,1}(v)}{v_2 - v_1} + \frac{(v_3 - v) N_{2,1}(v)}{v_3 - v_2}$$

$$= \begin{cases} (1-v) & 0 \le v \le 1 \\ 0 \end{cases}$$

$$N_{2,1}(v) = \begin{cases} 1 & v_2 \le v \le v_3 \\ 0 \end{cases} = \begin{cases} 1 & 0 \le v \le 1 \\ 0 \end{cases}$$

$N_{1,3}(v)$ $N_{1,3}(v) = (v - v_1)\, N_{1,2}(v)/(v_3 - v_1) + (v_4 - v)\, N_{2,2}(v)/(v_4 - v_2)$

$$= \begin{cases} v(1-v) & 0 \le v \le 1 \\ 0 \end{cases} + \begin{cases} v(1-v) & 0 \le v \le 1 \\ 0 \end{cases} = \begin{cases} [v(1-v)]^2 & 0 \le v \le 1 \\ 0 \end{cases}$$

$N_{2,2}(v) = (v - v_2)\, N_{2,1}(v)/(v_3 - v_2) + (v_4 - v)\, N_{3,1}(v)/(v_4 - v_3)$

$$= \begin{cases} v & 0 \le v \le 1 \\ 0 \end{cases}$$

$N_{2,3}(v)$ $N_{2,3}(v) = (v - v_2)\, N_{2,2}(v)/(v_5 - v_2) + (v_5 - v)\, N_{3,2}(v)/(v_5 - v_3)$

$$= \begin{cases} v^2 & 0 \le v \le 1 \\ 0 \end{cases}$$

$P(.5, .5) = (0.6306, .6306, 2)$

$r_{err} = 10.82\%$?