MECH-6024, MECH-5124

Computational Methods in Additive Manufacturing I

Assignment 3 (100 points)

To be worked in groups of two, include both names on the final report. Do not consult other groups.

Due on March 12th, 2020 (In Class)

Note: Show all detailed calculations and workings.

- **Questions should be solved using MATLAB or equivalent programming language and a report should be compiled with all of the necessary solutions and plots.**
- **Clearly indicate the steps followed and comment the code for full grade.**
- **Submit a zip archive of the code and report on blackboard. Zip archives should be submitted as LastName_FirstName.zip for only one of the students per group.**

1. Calculate and plot the 4th order $(\mathbf{k} = \mathbf{4})$ open non – periodic B – Spline curve with following control points: $\mathbf{B_1}(\mathbf{1}, \mathbf{2}), \mathbf{B_2}(\mathbf{2}, \mathbf{7}), \mathbf{B_3}(\mathbf{5}, \mathbf{12}), \mathbf{B_4}(\mathbf{3}, \mathbf{2})$

For this curve, answer the following questions:

a) What are the knot values and the knot vectors

b) How many different physical curve segments is the B-Spline curve composed of

c) How many control points contribute to each curve segment

d) Indicate the control points controlling each segment

e) What is the convex hull property of B-Splines

f) Which control points contribute to the convex hull

Calculate the actual coordinates of the curve at $\mathbf{u} = \mathbf{0.3}$ and $\mathbf{u} = \mathbf{0.6}$

Clearly indicate the steps followed to obtain the equation of curve. (The equations and the basis functions can be hand calculated or calculated using MATLAB)

2. Calculate and plot the 4th order $(\mathbf{k} = \mathbf{4})$ closed periodic B – Spline curve defined by following control polygon vertices: $\mathbf{P_0}(-\mathbf{1}, \mathbf{2}), \mathbf{P_1}(\mathbf{1.75}, \mathbf{4}), \mathbf{P_2}(\mathbf{2}, \mathbf{1}), \mathbf{P_3}(\mathbf{2.25}, \mathbf{4}), \mathbf{P_4}(\mathbf{5}, \mathbf{2}), \mathbf{P_5}(\mathbf{2}, -\mathbf{1}), \mathbf{P_6}(\mathbf{2}, -\mathbf{1})$

Plot the curve along with control points. Clearly indicate the steps followed to generate and plot a curve.

3. Construct a single third order $(\mathbf{k} = \mathbf{3})$ NURBS curve by smoothly blending a $\mathbf{90}°$ circular arc defined by a quadratic NURBS curve having polygon vertices $\mathbf{B_0}(\mathbf{0}, \mathbf{0}), \mathbf{B_1}(\mathbf{0}, \mathbf{2}), \mathbf{B_2}(\mathbf{2}, \mathbf{2})$ with the third order NURBS curve defined by control points $\mathbf{B_2}(\mathbf{2}, \mathbf{2}), \mathbf{B_3}(\mathbf{4}, \mathbf{2}), \mathbf{B_4}(\mathbf{8}, \mathbf{5}), \mathbf{B_5}(\mathbf{3}, \mathbf{7})$ with $\boldsymbol{h_i} = \mathbf{1}$ for $\mathbf{3} \leq \boldsymbol{i} \leq \mathbf{5}$. Specify the individual knot vectors for each curve as well as the composite knot vector for the composite curve. Use the composite knot vector to construct and plot the composite curve.

Calculate the values of the composite curve for $\mathbf{u} = \mathbf{1.5}$ and $\mathbf{u} = \mathbf{2}$.

Clearly show the steps for calculation of knot vectors, weights and the final NURBS equation for the entire composite curve. Plot the composite curve in MATLAB.