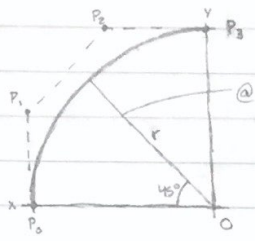
Estes, Wani

5)



$P_0 = (-5, 0)$ $P_3 = (0, 5)$ $P_1 = (-5, k)$ $P_2 = (-k, 5)$

@ $u = .5 \rightarrow r = 5$ (exactly) $\rightarrow (5\cos^{x}(\pi/4), 5\sin^{y}(\pi/4)) = (-5/\sqrt{2}, 5/\sqrt{2})$

Cubic Bezier Curve

$P(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u) P_2 + u^3 P_3$

@ $u = .5$ this curve should lie exactly on quarter-circle ∴

@ $u = .5$ $P = (-5/\sqrt{2}, 5/\sqrt{2})$

$P(.5) = (1-.5)^3(-5, 0) + 3(.5)(1-.5)(-5, k) + 3(.5)^2(1-.5)(-k, 5) + (.5)^3(0, 5)$

$(-5/\sqrt{2}, 5/\sqrt{2}) = .1250(-5, 0) + .3750(-5, k) + .3750(k, 5) + .1250(0, 5)$

can be solved in either x or y component, but we will solve in x

$x: -5/\sqrt{2} = -.625 - 1.875 + .3750k$

$k = -2.7613$

$P_0 = (-5, 0)$ $P_1 = (-5, -2.7613)$ $P_2 = (2.7613, 5)$ $P_3 = (0, 5)$

Max Deviation

critical pts of slope / derivative of dist formula

$D(u) = \sqrt{P_x^2(u) + P_y^2(u)}$

$dD(u)/du = 1/2\sqrt{P_x^2(u) + P_y^2(u)} \left[2P_x(u)P_x'(u) + 2P_y(u)P_y'(u)\right]$

term I : $(1-u)^3 P_0$

$(1-u)^3 P_0 = (1 - u^3 - 3u + 3u^2) P_0$

$x: (1-u)^3 P_{0x} = -5 + 5u^3 + 15t - 15t^2$

$y: (1-u)^3 P_{0y} = 0$

term II : $3u(1-u)^2 P_1$

$3u(1-u)^2 P_1 = (3u - 6u^2 + 3u^3) P_1$

x: $3u(1-u)^2 P_{1x} = -15u + 30u^2 - 15u^3$

y: $3u(1-u)^2 P_{1y} = 8.2839u - 16.5678u^2 + 8.2839u^3$

term III : $3u^2(1-u) P_2$

$3u^2(1-u) P_2 = (3u^2 - 3u^3) P_2$

x: $3u^2(1-u) P_{2x} = -8.2839u^2 + 8.2839u^3$

y: $3u^2(1-u) P_{2y} = 15u^2 - 15u^3$

term IV : $u^3 P_3$

x: $u^3 P_{3x} = 0$

y: $u^3 P_{3y} = 5u^3$

$P_x(u) = -1.7161u^3 + 6.7161u^2 - 5$

$P_y(u) = -1.7161u^3 - 1.5678u^2 + 8.2839u$

$P_x'(u) = -5.1483u^2 + 13.4322u$

$P_y'(u) = -5.1483u^2 - 3.1356u + 8.2839$

$P_x^2(u) = (-1.7161u^3 + 6.7161u^2 - 5)^2$

$= 2.9450u^6 - 23.051u^5 + 45.1060u^4 + 17.1610u^3 - 67.1610u^2 + 25$

$P_y^2(u) = (-1.7161u^3 - 1.5678u^2 + 8.2839u)^2$

$= 5.89u^6 - 17.67u^5 + 19.1320u^4 - 8.840u^3 + 1.4620u^2 + 25$

$2(P_x(u) \cdot P_x'(u)) = 2[(-1.7161u^3 + 6.7161u^2 - 5)(-5.1483u^2 + 13.4322u)]$

$= 17.67u^5 - 115u^4 + 180.42u^3 + 51.48u^2 - 134.32u$

$2(P_y(u) \cdot P_y'(u)) = 2[(-1.7161u^3 - 1.5678u^2 + 8.2839u)(-5.1483u^2 - 3.1356u + 8.2839)]$

$= 17.67u^5 + 26.9u^4 - 103.88u^3 - 77.9u^2 + 137.24u$

$$\left[2P_x P_x' + 2P_y P_y'\right] = 35.34u^5 - 88u^4 + 76.54u^3 - 26.42u^2 + 2.92u$$

$$\tfrac{1}{2}\sqrt{P_x^2 + P_y^2} = \sqrt{2495u^6 - 8836u^5 + 9566u^4 - 4401u^3 + 731u^2 + 1250} \Big/ 5\sqrt{5}$$

$$dD(u)/du = 35.34u^4 - 88u^3 + 76.54u^2 - 26.42u + 2.92 = 0$$

$u = .9045 + .1289j$

$u = .9045 - .1289j$

$u = .4709 + .000j$

$u = .2102 + .000j$

@ $u = .2102$ $r = 5.0013$ → deviation $= 5.0013 - 5 = .0013 = .13\%$