# Statistics for the Sciences

## Qualitative Predictors and Interaction Effects

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# Outline

- Qualitative Predictors
- Interactions in Regression Models
- Lab 1
- Lab 2

# Qualitative Predictors

- Qualitative or categorical predictor variables can be used in regression models. Many predictor variables of interest in business, economics, and the social and biological sciences are categorical. Examples of categorical predictor variables are gender (male, female), purchase status (purchase, no purchase), and disability status (not disabled, partly disabled, fully disabled).
- **Example.** Suppose we want to model $Y$ (person's weight) as a function of $X_1$ (person's height) and a dummy variable $X_2$ (Gender), where

$$X_2 = \begin{cases} 1 & \text{Male} \\ 0 & \text{Female} \end{cases}$$

Consider the model

$$E(Y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2.$$

# Qualitative Predictors

For males, it becomes

$$E(Y) = (\beta_0 + \beta_2) + \beta_1 x_1.$$

For females, it becomes

$$E(Y) = \beta_0 + \beta_1 x_1.$$

- Why do we combine the data among men and women? Why do not we just model them separately?

Suppose we observe $n_1$ males and $n_2$ females.

$$\begin{aligned} \text{males SSE has df } &= n_1 - 2 \\ \text{females SSE has df } &= n_2 - 2 \end{aligned}$$

But when we combine men and women using the model with interaction effect,

$$\text{SSE has df } = n_1 + n_2 - 3.$$

The larger df is an advantage as long as $\sigma_M^2 = \sigma_F^2$.

# Qualitative Predictors

- With more than two levels, we create additional **dummy variables**. If there are $c$ categories, we need $c-1$ dummy variables.

$$Z_1 = \begin{cases} 1 & \text{category level 1} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_2 = \begin{cases} 1 & \text{category level 2} \\ 0 & \text{otherwise} \end{cases}$$

$$\vdots$$

$$Z_{c-1} = \begin{cases} 1 & \text{category level } c-1 \\ 0 & \text{otherwise} \end{cases}$$

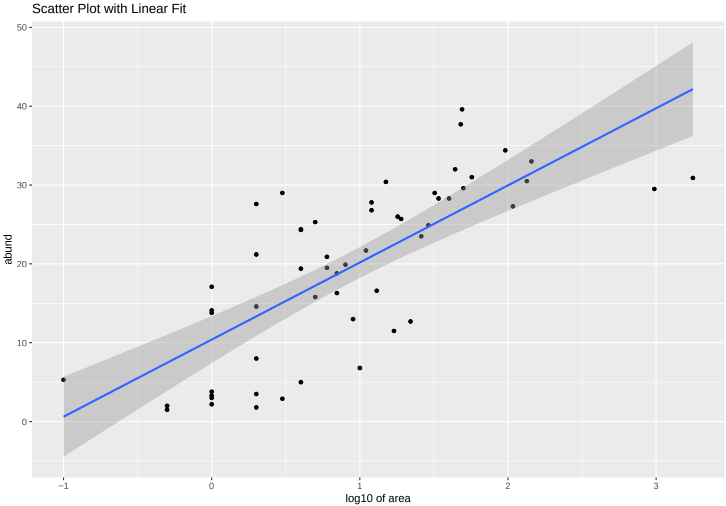
# Qualitative Predictors

- Example (`loyncat.csv`): We re-analysed the data from Loyn (1987) by fitting a simpler model that just included grazing and log patch area.

```
##    abund area graze grazecat  logarea
## 1    5.3  0.1     2      low -1.00000
## 2    2.0  0.5     5  intense -0.30103
## 3    1.5  0.5     5  intense -0.30103
## 4   17.1  1.0     3   medium  0.00000
## 5   13.8  1.0     5  intense  0.00000
## 6   14.1  1.0     3   medium  0.00000
## 7    3.8  1.0     5  intense  0.00000
## 8    2.2  1.0     5  intense  0.00000
## 9    3.3  1.0     4     high  0.00000
## 10   3.0  1.0     5  intense  0.00000
```

# Qualitative Predictors

- `graze` was treated as a numerical predictor
- The following is a SLR fit



# Qualitative Predictors

- The levels of the categorical predictor grazecat

```
## [1] "zero"    "high"    "intense" "low"     "medium"
```

- We fit the MLR model abund~logarea+grazecat with 95% confidence intervals

```
## # A tibble: 6 x 7
##   term            estimate std.error statistic     p.value conf.low conf.high
##   <chr>              <dbl>     <dbl>     <dbl>       <dbl>    <dbl>     <dbl>
## 1 (Intercept)       15.7        2.77    5.68   0.000000687    10.2      21.3
## 2 logarea            7.25       1.26    5.77   0.000000490     4.73      9.77
## 3 grazecathigh      -1.59       2.98   -0.535  0.595          -7.57      4.39
## 4 grazecatintense  -11.9        2.93   -4.06   0.000174      -17.8      -6.01
## 5 grazecatlow        0.383      2.91    0.131  0.896          -5.47      6.23
## 6 grazecatmedium    -0.189      2.55   -0.0742 0.941          -5.31      4.93
```

# Qualitative Predictors

- In the above model, 4 Qualitative Predictors are added so the MLR model is actually is `abund~logarea+Z1+Z2+Z3+Z4`

$$Z_1 = \begin{cases} 1 & \text{graze level} = \text{high} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_2 = \begin{cases} 1 & \text{graze level} = \text{intense} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_3 = \begin{cases} 1 & \text{graze level} = \text{low} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_4 = \begin{cases} 1 & \text{graze level} = \text{medium} \\ 0 & \text{otherwise} \end{cases}$$

# Qualitative Predictors

- After we fit the model, when `graze level = zero`,

$$E(abund) = 15.7 + 7.25 \times logarea$$

- When `graze level = high`,

$$E(abund) = 15.7 + 7.25 \times logarea - 1.59$$

# Interactions in Regression Models

- Example (`paruelo.csv`): Paruelo and Lauenroth (1996) analyzed the geographic distribution and the effects of climate variables on the relative abundance of a number of plant functional types (PFTs) including shrubs, forbs, succulents (e.g. cacti), C3 grasses and C4 grasses. There were 73 sites across North America. The response variable we will focus on is the relative abundance of C3 plants and there were six potential predictors: the latitude in centesimal degrees (LAT), the longitude in centesimal degrees (LONG), the mean annual precipitation in mm (MAP), the mean annual temperature in $^{o}$C (MAT), the proportion of MAP that fell in June, July and August (JJAMAP) and the proportion of MAP that fell in December, January and February (DJFMAP).

```
##      c3   lat   long map  mat jjamap djfmap
## 1  0.65 46.40 119.55 199 12.4   0.12   0.45
## 2  0.65 47.32 114.27 469  7.5   0.24   0.29
## 3  0.76 45.78 110.78 536  7.2   0.24   0.20
## 4  0.75 43.95 101.87 476  8.2   0.35   0.15
## 5  0.33 46.90 102.82 484  4.8   0.40   0.14
## 6  0.03 38.87  99.38 623 12.0   0.40   0.11
## 7  0.00 32.62 106.75 259 14.5   0.47   0.17
## 8  0.02 36.95  96.55 969 15.3   0.30   0.14
## 9  0.05 35.30 101.53 542 13.9   0.44   0.13
## 10 0.05 40.82 104.60 421  8.5   0.31   0.14
```

# Interactions in Regression Models

- We standardized the variables `lat` and `long` and consider these three variables only
  - See https://mjkeough.github.io/examples/paruelo.nb.html check why we standardize the variables

```
##      c3       slat       slong
## 1  0.65  1.1872051  2.04335832
## 2  0.65  1.3606917  1.22289867
## 3  0.76  1.0702902  0.68058728
## 4  0.75  0.7252028 -0.70393837
## 5  0.33  1.2814913 -0.55631779
## 6  0.03 -0.2327448 -1.09085967
## 7  0.00 -1.4113220  0.05436524
## 8  0.02 -0.5948037 -1.53061361
## 9  0.05 -0.9059481 -0.75677099
## 10 0.05  0.1349713 -0.27972344
```

# Interactions in Regression Models

- Now consider the MLR model `c3~slat+slong`

```
##
## Call:
## lm(formula = c3 ~ slat + slong, data = paruelo)
##
## Coefficients:
## (Intercept)         slat        slong
##    0.271370     0.174743    -0.006027
```

- Note that the average effect on `c3` of a one-unit increase in `slat` is always 0.174743, regardless of the value of `slong`.

# Interactions in Regression Models

- But suppose that increasing `slong` actually increases the effectiveness of `slat`, so that the slope term for `slat` should increase as `slong` increases.
- Model takes the form

$$
\begin{aligned}
c_3 =& \beta_0 + \beta_1 slat + \beta_2 slong + \beta_3 (slat \times slong) + \varepsilon \\
=& \beta_0 + (\beta_1 + \beta_3 \times slong) \times slat + \beta_2 slong + \varepsilon
\end{aligned}
$$

```
##
## Call:
## lm(formula = c3 ~ slat + slong + slat * slong, data = paruelo)
##
## Coefficients:
## (Intercept)         slat        slong   slat:slong
##   0.2652689    0.2011131    0.0001836    0.0640657
```

# Interactions in Regression Models

```
##
## Call:
## lm(formula = c3 ~ slat + slong + slat * slong, data = paruelo)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.39563 -0.14722 -0.01491  0.11837  0.40268
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.2652689  0.0222747  11.909  < 2e-16 ***
## slat        0.2011131  0.0245323   8.198 8.69e-12 ***
## slong       0.0001836  0.0225357   0.008   0.9935
## slat:slong  0.0640657  0.0242342   2.644   0.0101 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1893 on 69 degrees of freedom
## Multiple R-squared:  0.4964, Adjusted R-squared:  0.4745
## F-statistic: 22.67 on 3 and 69 DF,  p-value: 2.525e-10
```

# Interactions in Regression Models

## Interpretation

- The results suggests that interactions are important.
- The p-value for the interaction term $slat \times slong$ is low (0.01), indicating that there is strong evidence for $H_a : \beta_3 \neq 0$.
- Adjusted $R^2$ for model without interaction is

```
## [1] 0.4295385
```

- Adjusted $R^2$ for model with interaction is

```
## [1] 0.4744966
```

# Interactions in Regression Models

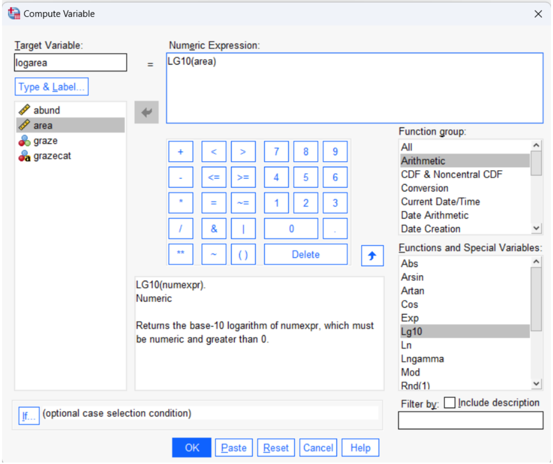
- Sometimes it is the case that an interaction term has a very small p-value, but the associated main effects (in this case, `slat` and `slong`) do not.
- The **hierarchy principle**: If we include an interaction in a model, we should also include the main effects, even if the p-values associated with their coefficients are not significant.
  - The rationale for this principle is that interactions are hard to interpret in a model without main effects - their meaning is changed.

# Interactions in Regression Models

- The concept of interactions applies just as well to qualitative variables, or to a combination of quantitative and qualitative variables.
- Especially if there is an interaction between a qualitative predictor and a quantitative predictor, we can interpret the interaction effects following the interpretation of the qualitative predictors. interpretation.

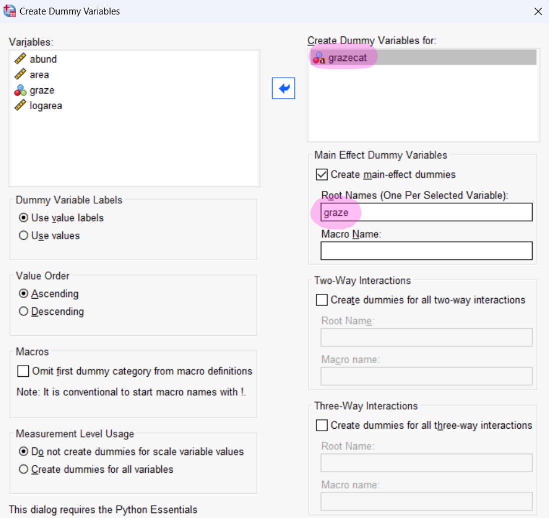
# Lab 1

- Example (`loyncat.csv`): We re-analysed the data from Loyn (1987) by fitting a simpler model that just included grazing and log patch area.
- After importing the data to SPSS, we first add the new variable `logarea`, log10 transformation of `area`



# Lab 1

- In SPSS when we fit a regression model, the indepedent variables must be numerical (scale). So we need to create dummy variables for a `factor` first.
- Click on `Transform` → `Create Dummy Variables`



# Lab 1

- From our discussions, we only need the first 4 dummy variables.

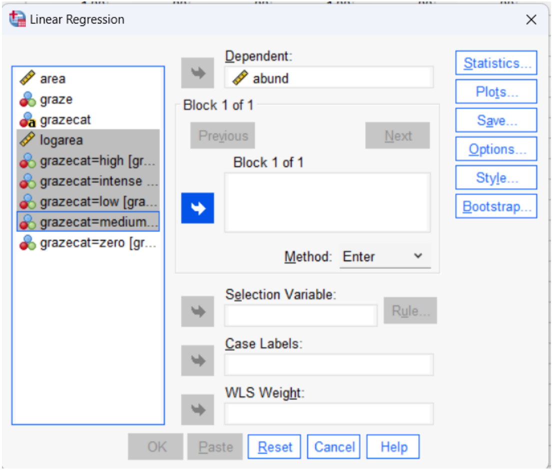
➡ **Create dummy variables**

*Variable Creation*

| | Label |
|---|---|
| graze_1 | grazecat=high |
| graze_2 | grazecat=intense |
| graze_3 | grazecat=low |
| graze_4 | grazecat=medium |
| graze_5 | grazecat=zero |

# Lab 1

- We now fit the regression model with `logarea` and the four dummy variables as independent variables.



# Lab 1

*Model Summary*

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .853[a] | .727 | .700 | 5.8829 |

a. Predictors: (Constant), graze_4 grazecat=medium, logarea, graze_1 grazecat=high, graze_3 grazecat=low, graze_2 grazecat=intense

*ANOVA*[a]

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 4607.530 | 5 | 921.506 | 26.627 | <.001[b] |
| | Residual | 1730.399 | 50 | 34.608 | | |
| | Total | 6337.929 | 55 | | | |

a. Dependent Variable: abund

b. Predictors: (Constant), graze_4 grazecat=medium, logarea, graze_1 grazecat=high, graze_3 grazecat=low, graze_2 grazecat=intense

# Lab 1

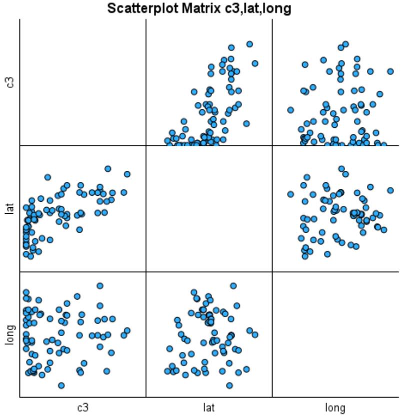
*Coefficients*[a]

| Model | | Unstandardized Coefficients B | Unstandardized Coefficients Std. Error | Standardized Coefficients Beta | t | Sig. |
|---|---|---|---|---|---|---|
| 1 | (Constant) | 15.716 | 2.767 | | 5.679 | <.001 |
| | logarea | 7.247 | 1.255 | .548 | 5.774 | <.001 |
| | graze_1 grazecat=high | -1.592 | 2.976 | -.049 | -.535 | .595 |
| | graze_2 grazecat=intense | -11.894 | 2.931 | -.472 | -4.058 | <.001 |
| | graze_3 grazecat=low | .383 | 2.912 | .013 | .131 | .896 |
| | graze_4 grazecat=medium | -.189 | 2.550 | -.008 | -.074 | .941 |

a. Dependent Variable: abund

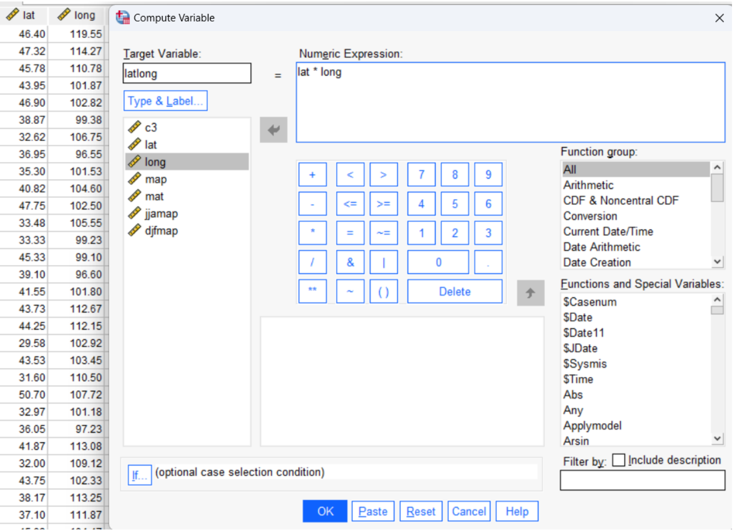
# Lab 2

- Consider data `paruelo.csv`
- Import data, then create a scatter plot matrix of the variables `c3`, `lat` and `long`



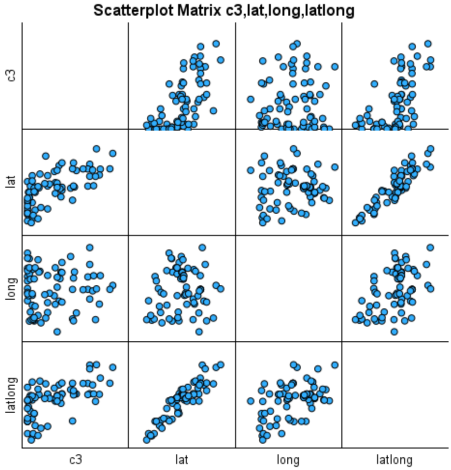
# Lab 2

- Add a new column `latlong` which is `lat*long`



# Lab 2

- Scatter plot matrix again



# Lab 2

- And correlation matrix as well

**Correlations**

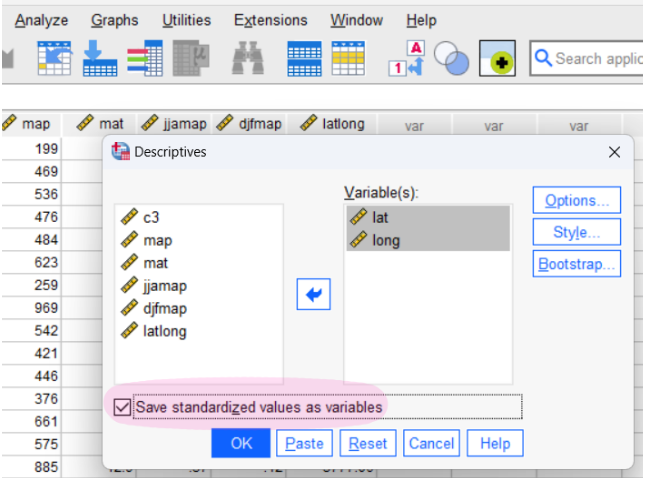
*Correlations*

| | | c3 | lat | long | latlong |
|---|---|---|---|---|---|
| c3 | Pearson Correlation | 1 | .667** | .042 | .611** |
| | Sig. (2-tailed) | | <.001 | .727 | <.001 |
| | N | 73 | 73 | 73 | 73 |
| lat | Pearson Correlation | .667** | 1 | .097 | .914** |
| | Sig. (2-tailed) | <.001 | | .416 | <.001 |
| | N | 73 | 73 | 73 | 73 |
| long | Pearson Correlation | .042 | .097 | 1 | .489** |
| | Sig. (2-tailed) | .727 | .416 | | <.001 |
| | N | 73 | 73 | 73 | 73 |
| latlong | Pearson Correlation | .611** | .914** | .489** | 1 |
| | Sig. (2-tailed) | <.001 | <.001 | <.001 | |
| | N | 73 | 73 | 73 | 73 |

**. Correlation is significant at the 0.01 level (2-tailed).

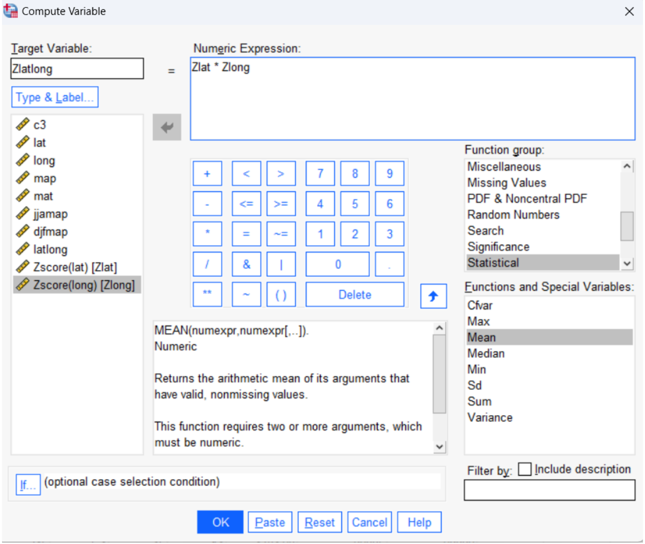
# Lab 2

- Let's standardise the predictors to Zlat and Zlong by clicking on Analyze → Descriptive Statistics → Descriptives...



# Lab 2

- Then add a new column `zlatlong` which is `Zlat*Zlong`



# Lab 2

- Check the correlation matrix

**Correlations**

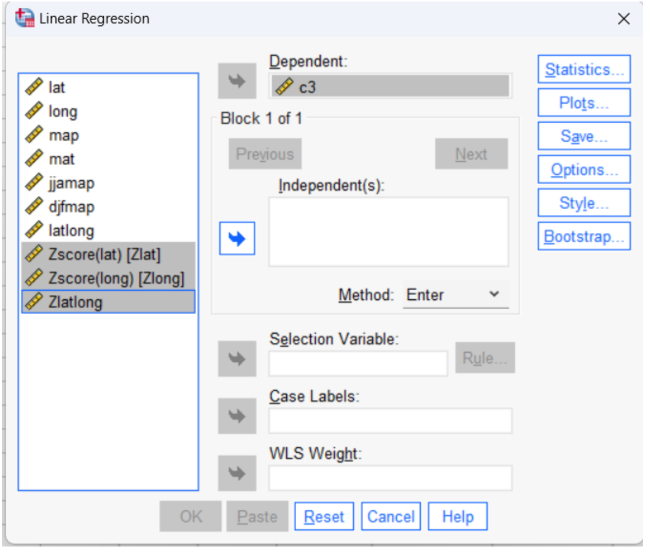
*Correlations*

| | | c3 | Zlat Zscore(lat) | Zlong Zscore (long) | Zlatlong |
|---|---|---|---|---|---|
| c3 | Pearson Correlation | 1 | .667** | .042 | -.069 |
| | Sig. (2-tailed) | | <.001 | .727 | .559 |
| | N | 73 | 73 | 73 | 73 |
| Zlat Zscore(lat) | Pearson Correlation | .667** | 1 | .097 | -.414** |
| | Sig. (2-tailed) | <.001 | | .416 | <.001 |
| | N | 73 | 73 | 73 | 73 |
| Zlong Zscore(long) | Pearson Correlation | .042 | .097 | 1 | -.134 |
| | Sig. (2-tailed) | .727 | .416 | | .257 |
| | N | 73 | 73 | 73 | 73 |
| Zlatlong | Pearson Correlation | -.069 | -.414** | -.134 | 1 |
| | Sig. (2-tailed) | .559 | <.001 | .257 | |
| | N | 73 | 73 | 73 | 73 |

**. Correlation is significant at the 0.01 level (2-tailed).

# Lab 2

- Finally, we fit the MLR model with interaction



# Lab 2

*Model Summary*

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .705[a] | .496 | .474 | .18929 |

a. Predictors: (Constant), Zlatlong, Zlong Zscore(long), Zlat Zscore(lat)

*ANOVA[a]*

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 2.437 | 3 | .812 | 22.670 | <.001[b] |
| | Residual | 2.472 | 69 | .036 | | |
| | Total | 4.909 | 72 | | | |

a. Dependent Variable: c3

b. Predictors: (Constant), Zlatlong, Zlong Zscore(long), Zlat Zscore(lat)

# Lab 2

*Coefficients*[a]

| Model | | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. |
|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | |
| 1 | (Constant) | .265 | .022 | | 11.909 | <.001 |
| | Zlat Zscore(lat) | .201 | .025 | .770 | 8.198 | <.001 |
| | Zlong Zscore(long) | .000 | .023 | .001 | .008 | .994 |
| | Zlatlong | .064 | .024 | .249 | 2.644 | .010 |

a. Dependent Variable: c3

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