Statistics for the Sciences

Repeated Measures - Part I

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Outline

- Repeated measures
- Single-factor experiments with repeated measures
- Lab

Repeated Measures

- In a **repeated measures** design, the responses of the experimental units (or subjects) are recorded for a number of trials/times (under different treatment conditions).
- Investigators evaluate the performance of each subject under several experimental conditions.
- Repeated measurements on experimental units can occur in any type of design. We consider repeated measures under several types of experimental designs.
  - Single-factor experiments with repeated measures

Single-factor experiments with repeated measures

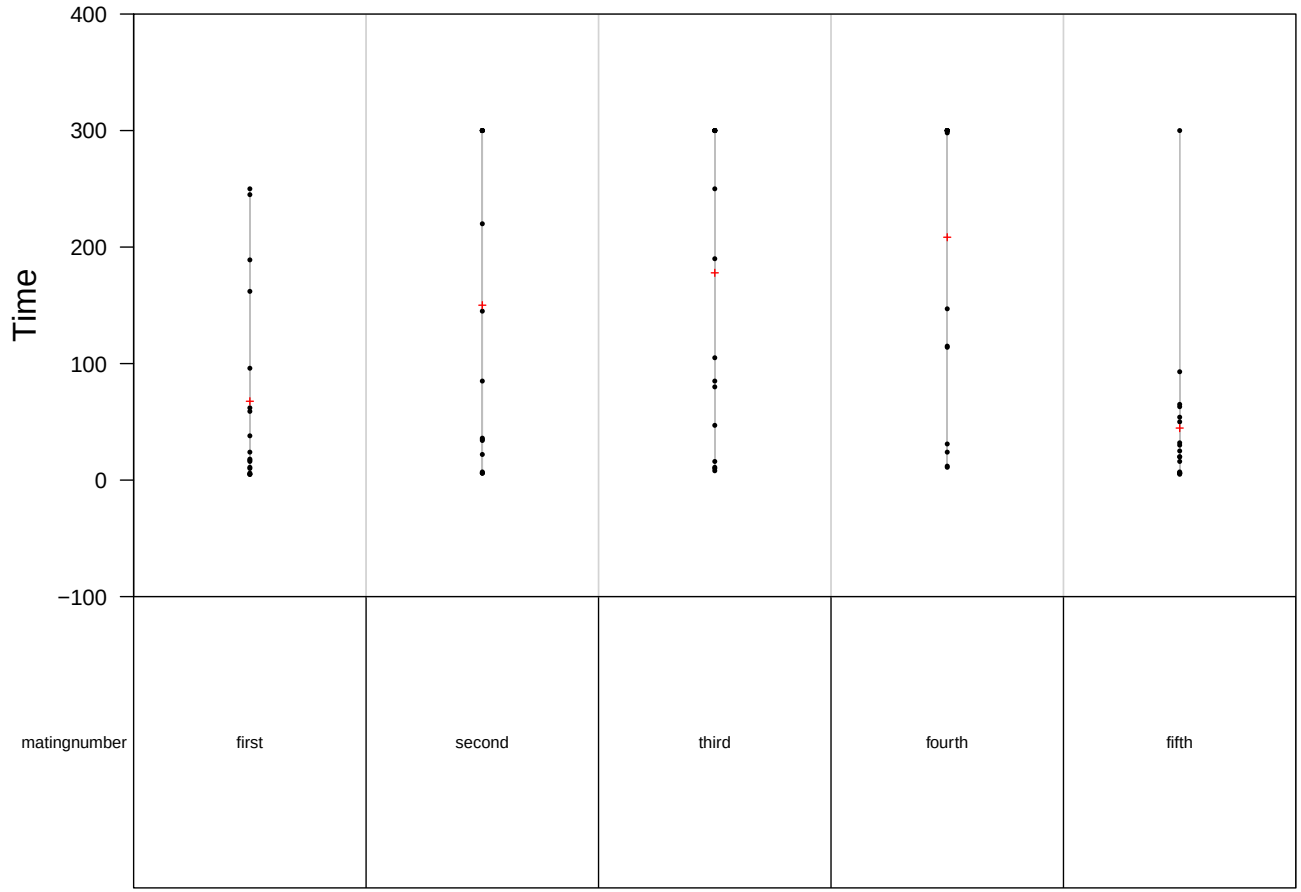
- Example (`steiger.csv`): Steiger et al. (2008) studied the Coolidge effect, the decline in males' interest in mating with the same female compared to novel females, using the burying beetle Nicrophorus vespilloides. Eighteen male beetles were presented with the same female beetle four times, and then a novel female on the fifth occasion.
  - This was a repeated measures design as the same individual males were repeatedly presented with females. There was no evidence that physical exhaustion affected time to mating as a separate control group of males were presented with novel, unmated females five times in succession, and there was no change in time to mating.
  - The within-subjects factor was the order of presented females, and while this could have been treated as a continuous covariate, we treated it as a fixed factor with five groups. The response variable recorded on each occasion was time to mating.

Single-factor experiments with repeated measures

```
##    matingnumber individual time
## 1         first          1  250
## 2         first          2   96
## 3         first          3  162
## 4         first          4  245
## 5         first          5   11
## 6         first          6    6
## 7         first          7   17
## 8         first          8   38
## 9         first          9   16
## 10        first         10   59
## 11        first         11   24
## 12        first         12   18
## 13        first         13    5
## 14        first         14    5
## 15        first         15    5
## 16        first         16   10
## 17        first         17  189
## 18        first         18   62
## 19       second          1  220
## 20       second          2  300
## 21       second          3  300
## 22       second          4   22
## 23       second          5    6
## 24       second          6   35
## 25       second          7  300
## 26       second          8    7
## 27       second          9  145
## 28       second         10    6
## 29       second         11   34
## 30       second         12  300
## 31       second         13  300
## 32       second         14    6
## 33       second         15   85
## 34       second         16   36
## 35       second         17  300
## 36       second         18  300
## 37        third          1  300
## 38        third          2   16
## 39        third          3    8
```

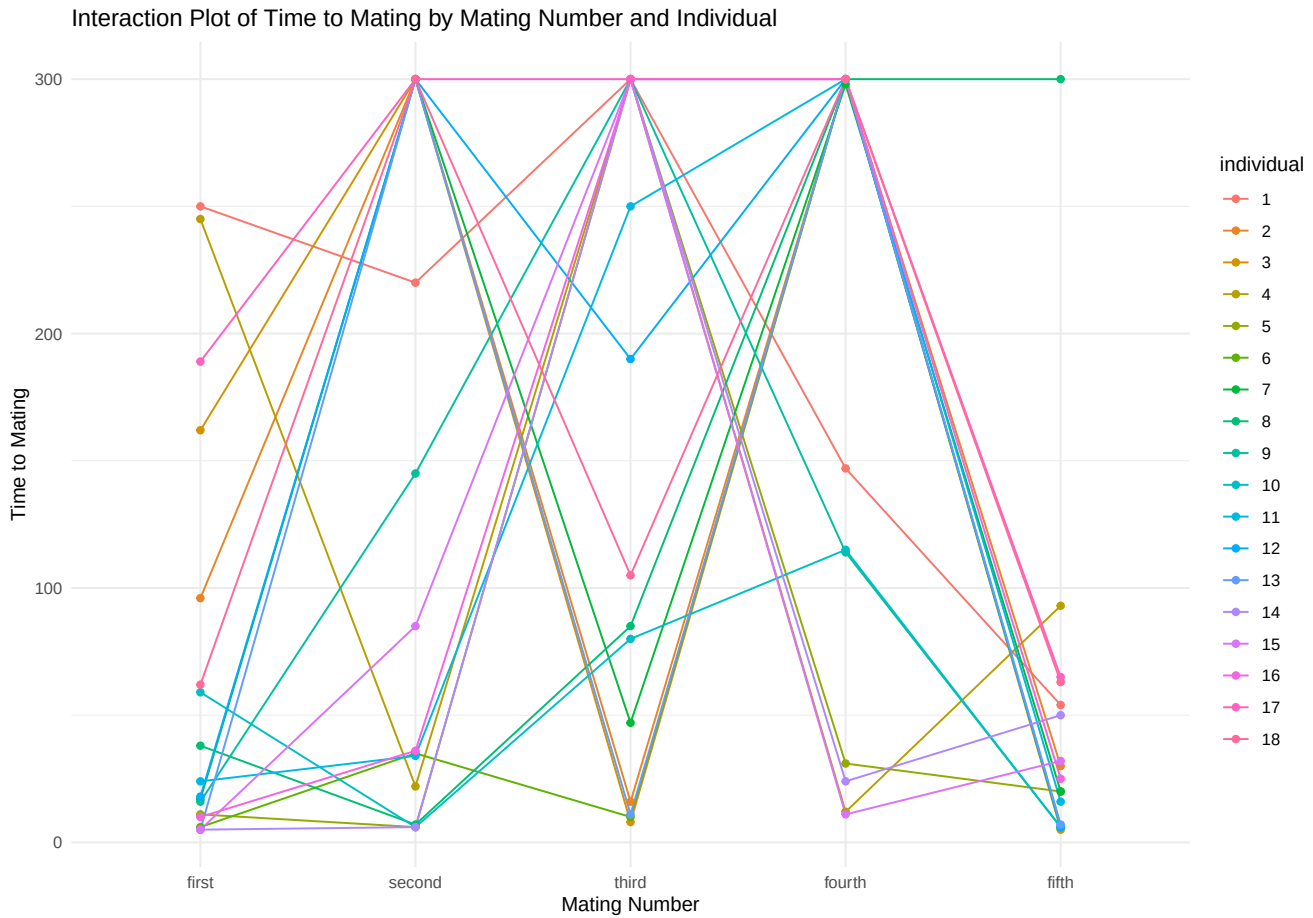
Single-factor experiments with repeated measures

- Dot plot to check variances for different mating numbers



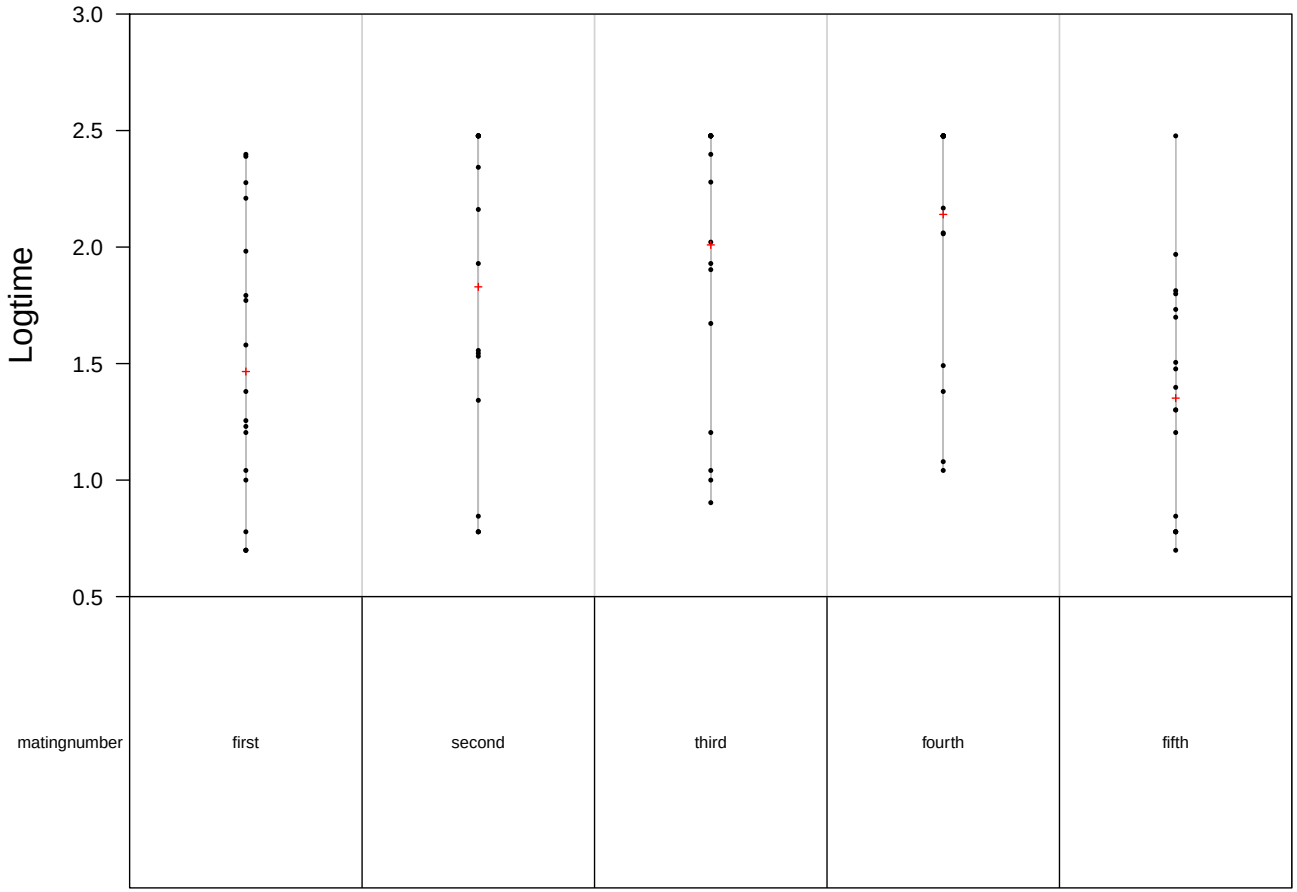
Single-factor experiments with repeated measures

- Interaction plot
  - The rank order of treatments consistent but the sizes of the differences vary greatly between individuals.



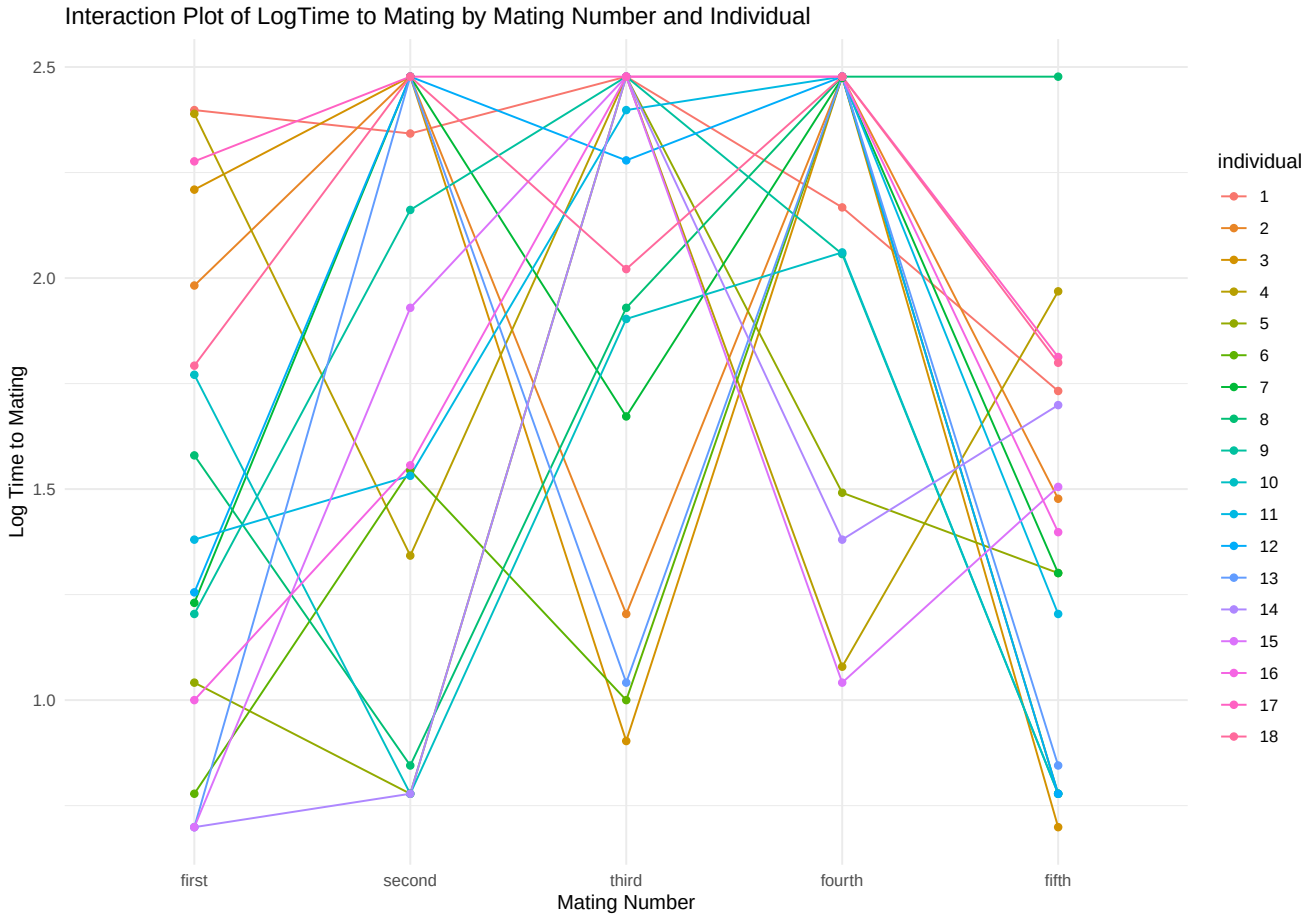
Single-factor experiments with repeated measures

- Try log-transformed response



Single-factor experiments with repeated measures

- Interaction plot again



Single-factor experiments with repeated measures

- **Subject is a factor**; it is random in general
- We are interested in a comparison across treatment conditions *within each subject*
  - It is not of interest to look at averaged group performance at each condition
- Statistically, each subject is considered a unique block in the design.
- ANOVA is just like **two-way block design**.

Single-factor experiments with repeated measures

- Suppose there are s subjects and k treatments
  - Subject is treated as random
- Statistical model:

$$Y_{ij} = \mu + \rho_i + \beta_j + \varepsilon_{ij}, i = 1, 2, \ldots, s, j = 1, 2, \ldots, k,$$

  - $\rho_i \sim N(0, \sigma_\rho^2)$, $i = 1, 2, \ldots, s$ are independent
  - $\sum \beta_j = 0$
  - $\varepsilon_{ij} \sim N(0, \sigma^2)$ are independent
  - ρ_i and ε_{ij} are independent

Single-factor experiments with repeated measures

- ANOVA Table

Table 1: ANOVA Table for 1-factor repeated measures

Source	df	SS	MS	F
Subjects	$s-1$	SS_S	$SS_S/(s-1)$	
Treatments	$k-1$	SS_T	$SS_T/(k-1)$	MS_T/MSE
Error	$(s-1)(k-1)$	SSE	$SSE/(s-1)(k-1)$	
Total	$sk-1$	SS_{total}		

- $E(MS_S) = \sigma^2 + k\sigma^2_\rho$
- $E(MS_T) = \sigma^2 + s\frac{\sum \beta_j^2}{k-1}$
- $E(MS_E) = \sigma^2$

Single-factor experiments with repeated measures

- REML fit

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: logtime ~ matingnumber + (1 | individual)
##    Data: steiger
##
## REML criterion at convergence: 164.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.9213 -0.8145  0.2051  0.6980  1.9271
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  individual (Intercept) 0.01289  0.1135
##  Residual               0.33094  0.5753
## Number of obs: 90, groups:  individual, 18
##
## Fixed effects:
##                   Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)          1.4658     0.1382 84.5246  10.606  < 2e-16 ***
## matingnumbersecond   0.3634     0.1918 68.0000   1.895 0.062300 .
## matingnumberthird    0.5435     0.1918 68.0000   2.834 0.006040 **
## matingnumberfourth   0.6743     0.1918 68.0000   3.516 0.000784 ***
## matingnumberfifth   -0.1140     0.1918 68.0000  -0.595 0.554034
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##             (Intr) mtngnmbrs mtngnmbrt mtngnmbrfr
## mtngnmbrscn -0.694
## mtngnmbrthr -0.694  0.500
## mtngnmbrfrt -0.694  0.500     0.500
## mtngnmbrfft -0.694  0.500     0.500     0.500
```

Single-factor experiments with repeated measures

- ANOVA table

```
## Type III Analysis of Variance Table with Satterthwaite's method
##                Sum Sq Mean Sq NumDF DenDF F value    Pr(>F)
## matingnumber 8.3635  2.0909     4    68   6.318 0.0002202 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

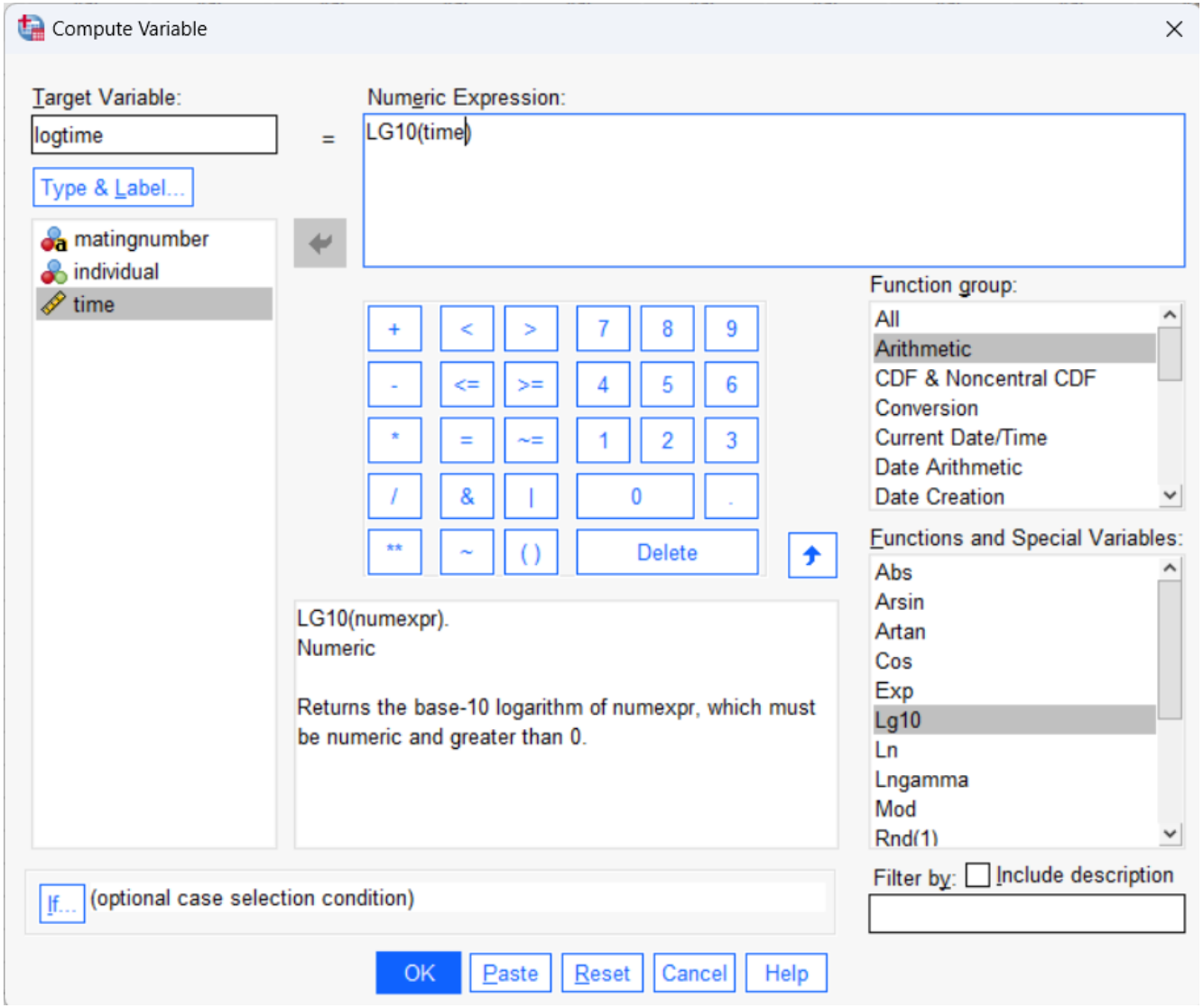
Single-factor experiments with repeated measures

- Variance components

```
##
##
## ANOVA-Type Estimation of Mixed Model:
## -------------------------------------
##
##  [Fixed Effects]
##
##                int  matingnumberfifth  matingnumberfirst matingnumberfourth
##           2.009337          -0.657543          -0.543510           0.130802
## matingnumbersecond  matingnumberthird
##          -0.180066           0.000000
##
##
##  [Variance Components]
##
##   Name       DF        SS        MS       VC       %Total    SD       CV[%]
## 1 total      84.524647                     0.34383  100       0.58637  33.330258
## 2 individual 17        6.721791  0.395399 0.012892 3.749614  0.113544 6.454044
## 3 error      68        22.503779 0.330938 0.330938 96.250386 0.575272 32.69941
##
## Mean: 1.759273 (N = 90)
##
## Experimental Design: balanced  |  Method: ANOVA
```

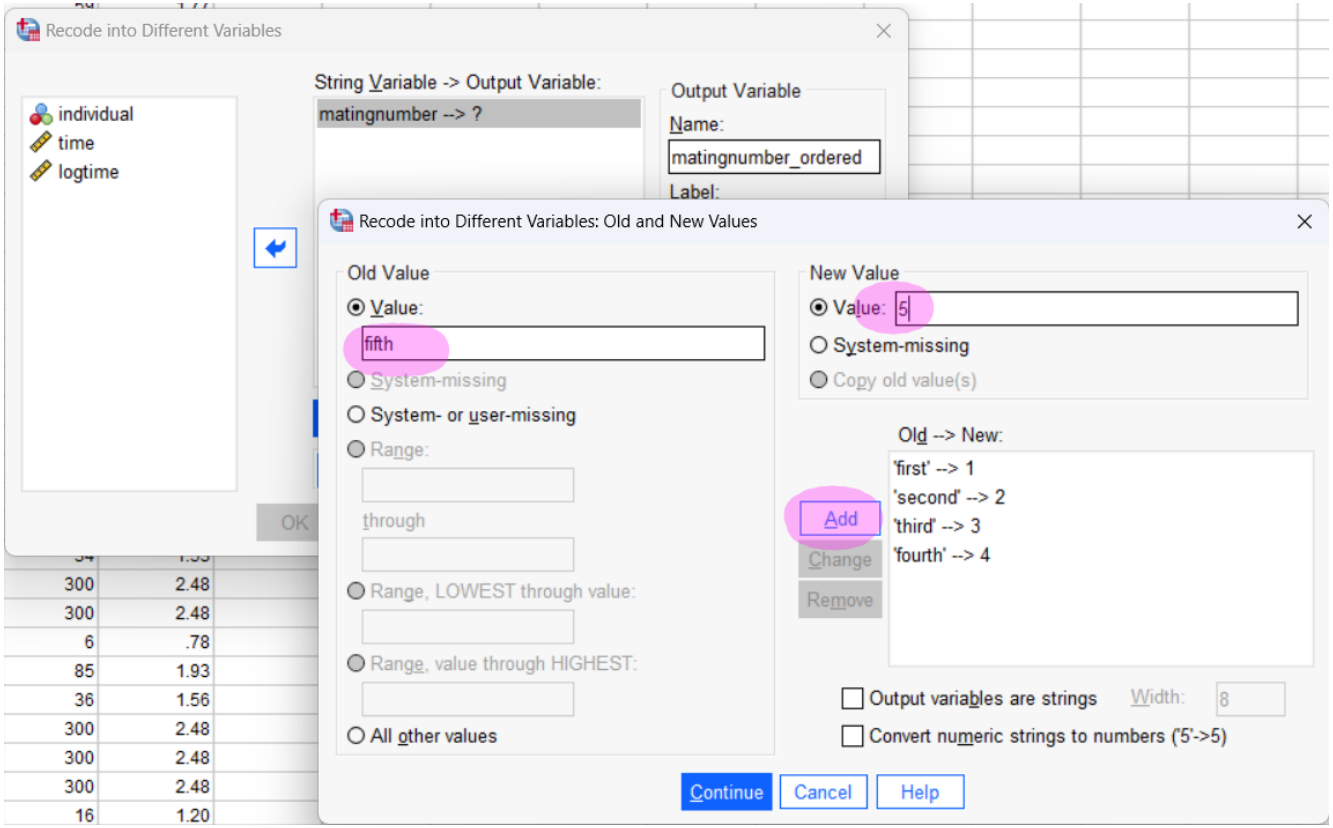
Lab

- After importing data `steiger.csv`, we add a variable of log10 transformation of the response `time`

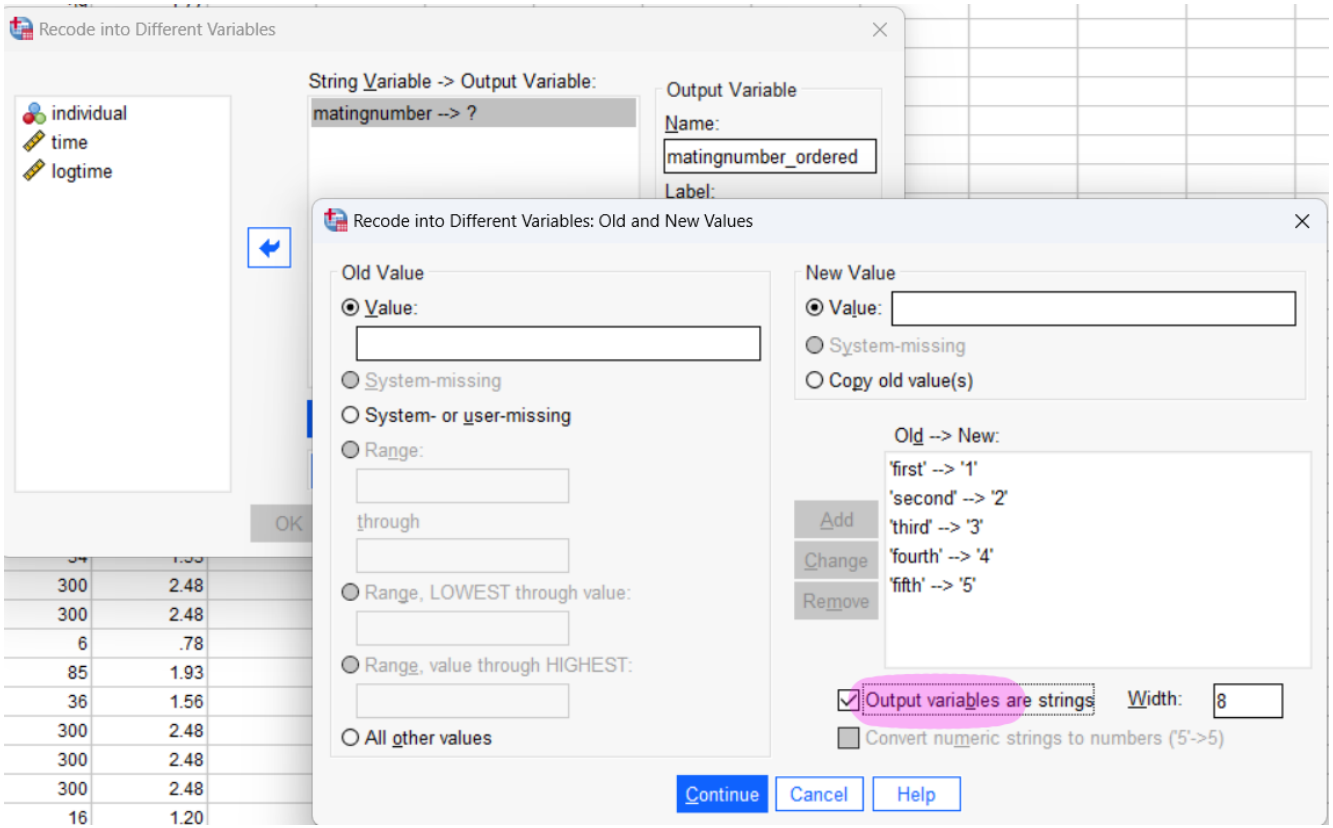


Lab

- Also, we order the `matingnumber` levels by adding a new variable
  - Click on `Transform` → `Recode into Different Variables...`

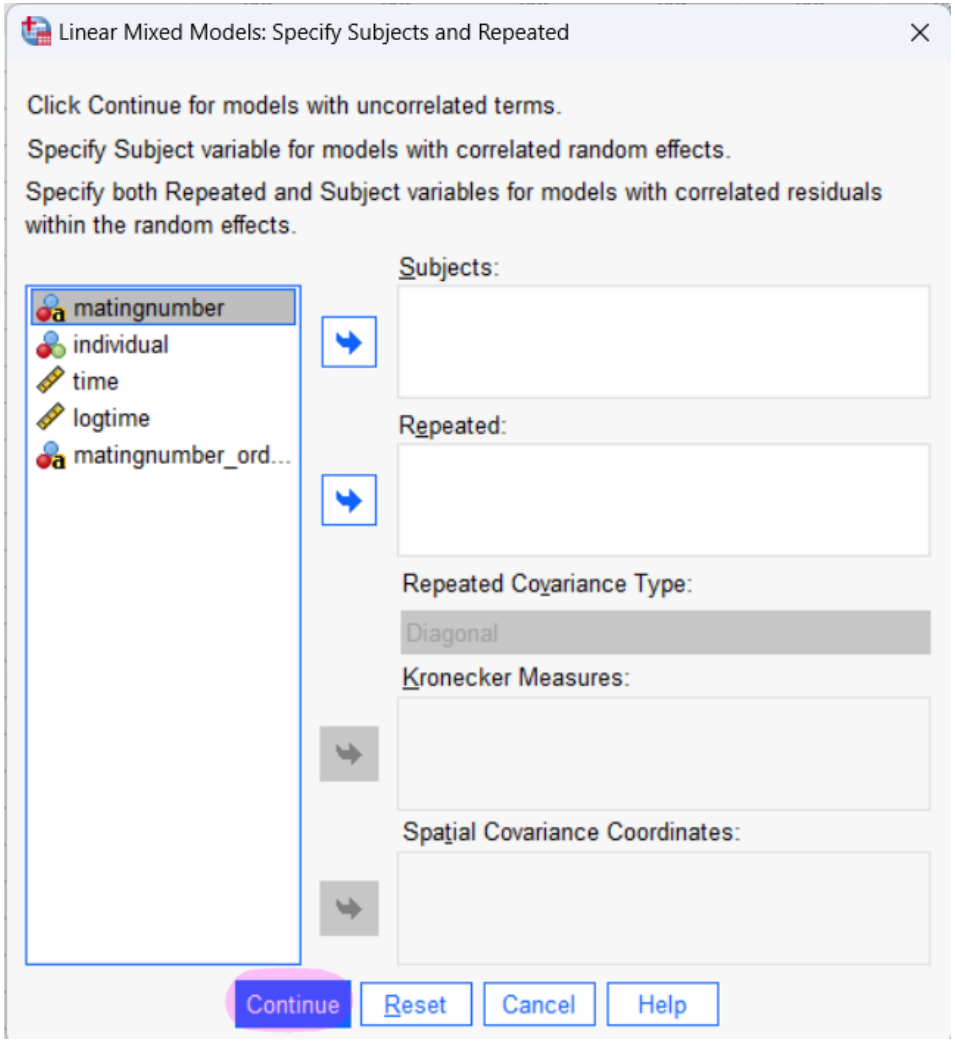


Lab


Recode into Different Variables
individual
time
logtime
String Variable -> Output Variable:
matingnumber --> ?
Output Variable
Name:
matingnumber_ordered
Label:
OK
Recode into Different Variables: Old and New Values
Old Value
Value:
System-missing
System- or user-missing
Range:
through
Range, LOWEST through value:
Range, value through HIGHEST:
All other values
New Value
Value:
System-missing
Copy old value(s)
Old --> New:
'first' --> '1'
'second' --> '2'
'third' --> '3'
'fourth' --> '4'
'fifth' --> '5'
Add
Change
Remove
Output variables are strings
Width:
8
Convert numeric strings to numbers ('5'->5)
Continue
Cancel
Help
300 2.48
300 2.48
6 .78
85 1.93
36 1.56
300 2.48
300 2.48
300 2.48
16 1.20

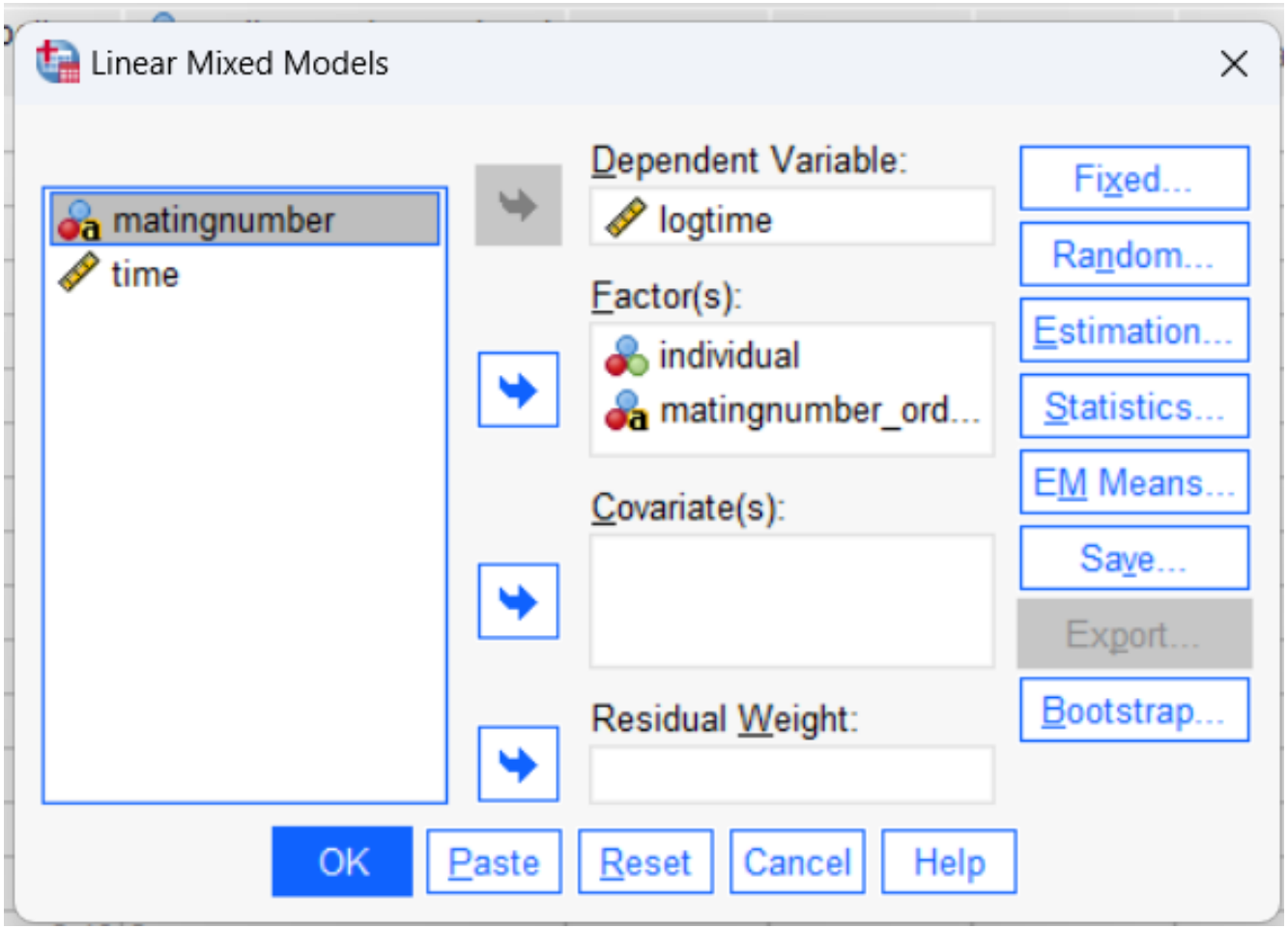
Lab

- In the `Linear Mixed Models` dialog, click on `Continue`



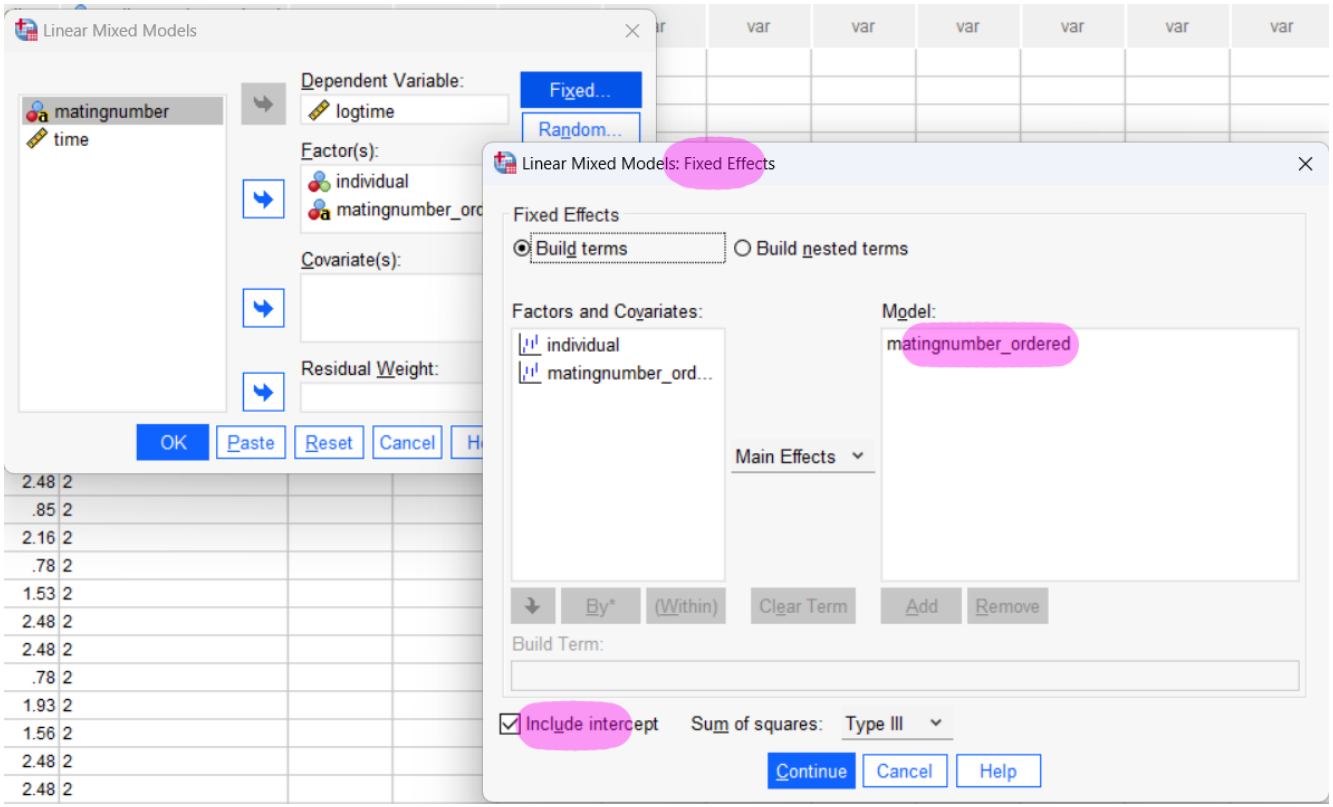
Lab

- Specify the dependent variable and factors



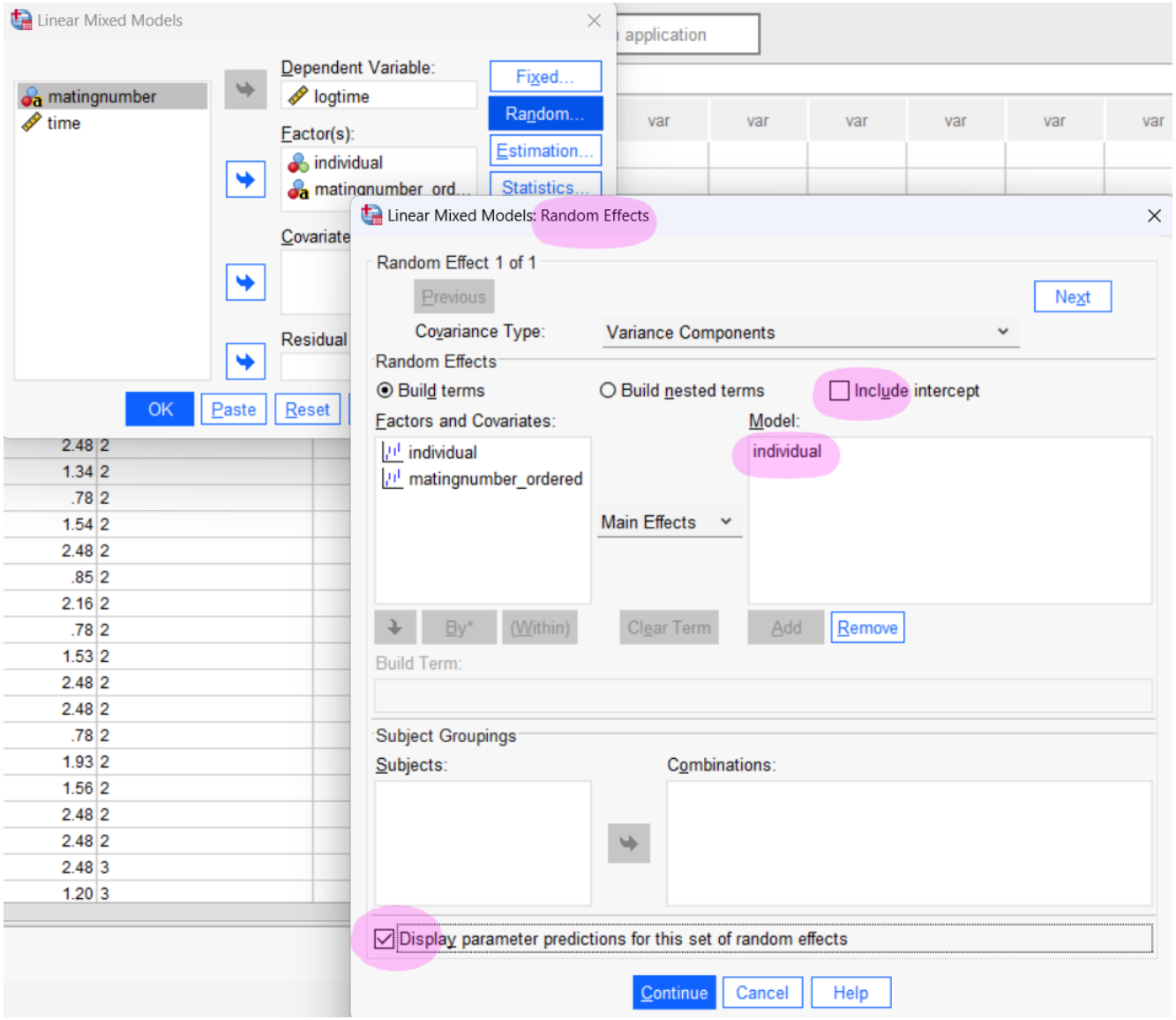
Lab

- Define the fixed-effect factor

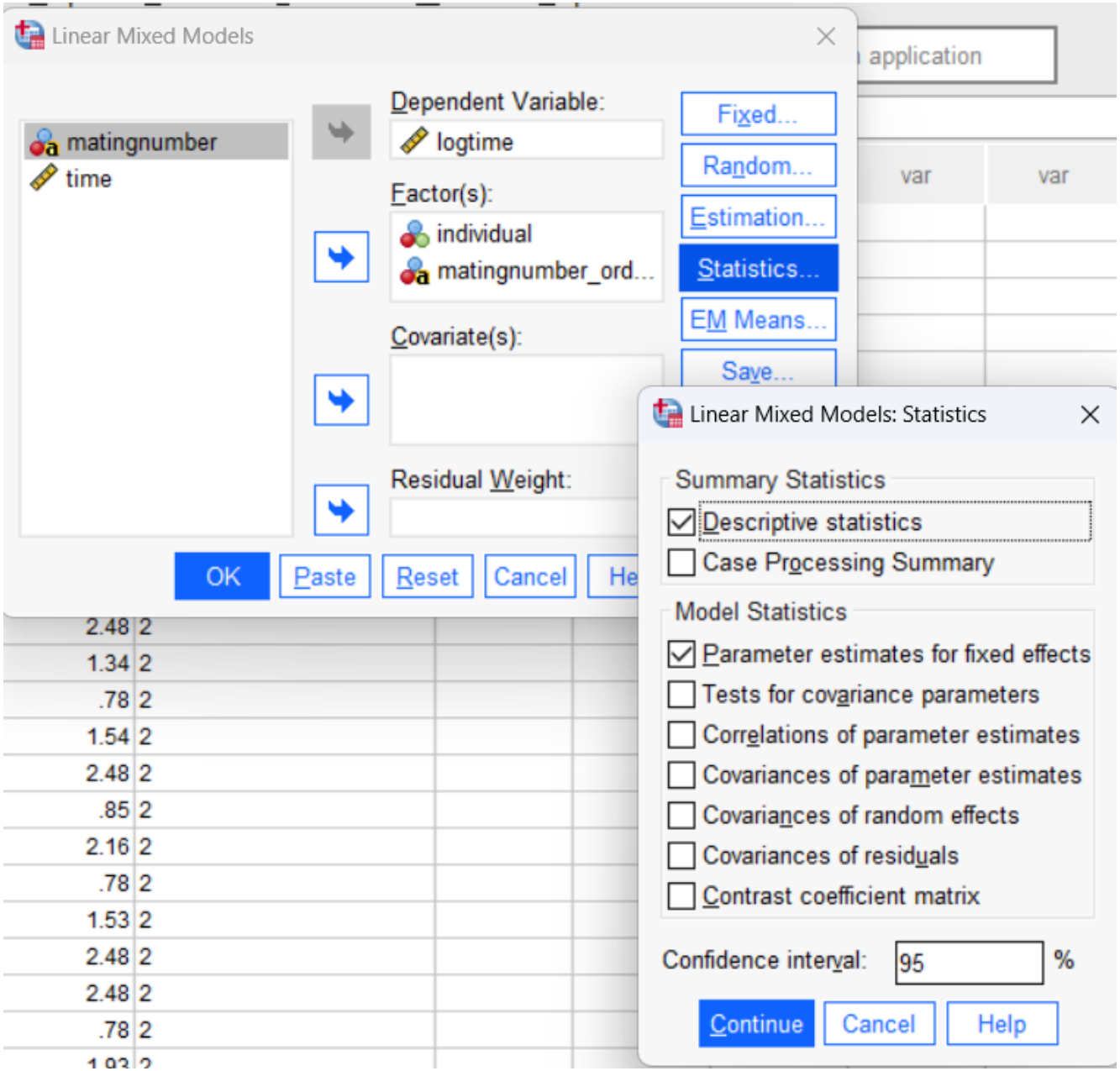


Lab

- Define the random factor

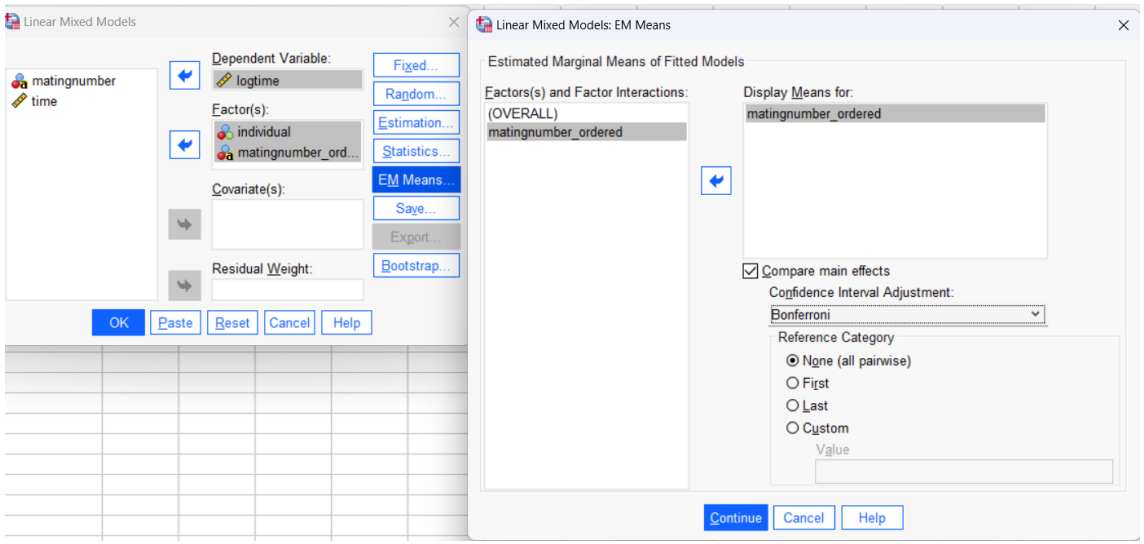


Lab


Linear Mixed Models
matingnumber
time
Dependent Variable:
logtime
Factor(s):
individual
matingnumber_ord...
Covariate(s):
Residual Weight:
Fixed...
Random...
Estimation...
Statistics...
EM Means...
Save...
OK
Paste
Reset
Cancel
application
var
var
Linear Mixed Models: Statistics
Summary Statistics
Descriptive statistics
Case Processing Summary
Model Statistics
Parameter estimates for fixed effects
Tests for covariance parameters
Correlations of parameter estimates
Covariances of parameter estimates
Covariances of random effects
Covariances of residuals
Contrast coefficient matrix
Confidence interval:
95
%
Continue
Cancel
Help
2.48 2
1.34 2
.78 2
1.54 2
2.48 2
.85 2
2.16 2
.78 2
1.53 2
2.48 2
2.48 2
.78 2

Lab

- We may want to compare the means of the fixed effects



Lab

Model Dimension[a]

		Number of Levels	Covariance Structure	Number of Parameters
Fixed Effects	Intercept	1		1
	matingnumber_ordered	5		4
Random Effects	individual	18	Variance Components	1
Residual				1
Total		24		7

a. Dependent Variable: logtime.

Lab

Fixed Effects

Type III Tests of Fixed Effects[a]

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	17.000	704.488	<.001
matingnumber_ordered	4	68.000	6.318	<.001

a. Dependent Variable: logtime.

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	1.352	.138	84.525	9.781	<.001	1.077	1.627
[matingnumber_ordered=1]	.114	.192	68.000	.595	.554	-.269	.497
[matingnumber_ordered=2]	.477	.192	68.000	2.490	.015	.095	.860
[matingnumber_ordered=3]	.658	.192	68.000	3.429	.001	.275	1.040
[matingnumber_ordered=4]	.788	.192	68.000	4.111	<.001	.406	1.171
[matingnumber_ordered=5]	0[b]	0	.	.	.	.	.

a. Dependent Variable: logtime.

b. This parameter is set to zero because it is redundant.

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