Exercise 1.1 (2,4,5,6)

2) Use intermediate Value Theorem to find an interval of length 1 that contains a root of the eq.

a) $x^5 + x = 1$ $[a, b]$ where $a - b = 1$ and $f(a) \cdot f(b) < 0$

$f(x) = x^5 + x - 1$ $b = 0$ $a = 1$

root between $[0, 1]$ $f(b) = -1$ $f(a) = 1$

$f(b) f(a) < 0$

b) $\sin x = 6x + 5$ $[-1, 0]$

$f(x) = 6x + 5 - \sin x$

$f(-1) = -6 + 5 + (\sin(-1) < 1) \approx -0.158529$

$f(0) = 0 + 5 + 0 = 5$

5×-0.158529

So there is a root between $[-1, 0]$

c) $\ln x + x^2 = 3$ $f(x) = \ln(x) + x^2 - 3$

$f(1) = \ln(1) + 1 - 3$

$0 + 1 - 3 = -2$

$f(2) = \ln(2) + 4 - 3$

$= (\ln(2) + 1)$ and $-2 \times (\ln(2) + 1) < 0$

therefore

There is a root between $[1, 2]$

↓ Cont.

Exercise 1.1 4,5,6

4) Consider exercis 2.   Apply 2 steps of Bisect
to find approximate root within ⅛ true Root

$f(x)=x^5+x-1$	$f(x)=6x+5-\sin x$	$f(x)=\ln(x)+x^2-3$
$[0,1]$	$[-1,0]$	$[1,2]$

Column 1:

$f(x)=x^5+x-1$
$[0,1]$

Step 1

$\frac{0+1}{2}=0.5$

$(0.5)^5+0.5-1=0.46875$
So then
$[0.5,1]$

Step 2

So $\frac{1+0.5}{2}=0.75$

$f(0.75)=0.0126$

So $[0.75,1]$.

$\frac{1}{8}<\left|\left(\frac{0.75+1}{2}\right)-r\right|$

$\boxed{r-0.875<\tfrac{1}{8}}$

Column 2:

$f(x)=6x+5-\sin x$
$[-1,0]$

Step 1

$f(-0.5)=2.4794126$

So $[-1,-0.5]$

-0.75

Step 2

$f(-0.75)=1.18164$

So $[-1,-0.75]$

$\tfrac{1}{8}>\left|\left(\frac{-1+-0.75}{2}\right)-r\right|$

$\boxed{\frac{1}{8}>\left|-0.875-r\right|}$

Column 3:

$f(x)=\ln(x)+x^2-3$
$[1,2]$

Step 1

1.5

$f(1.5)=-0.349535$

So $[1.5,1]$

Step 2 $\frac{1.5+2}{2}=1.75$

$f(1.75)=0.622$

So $[1.5,1.75]$

$\tfrac{1}{8}<\left|\frac{1.5+1.75}{2}-r\right|$

$\boxed{\frac{1}{8}<\left|1.625-r\right|}$

Cont ↓

Exercise 1.1 (5,6)

5) Consider $x^4 = x^3 + 10$

    a) Find interval $[a,b]$ of length 1 inside which the solution exists

$f(x) = -x^4 + x^3 + 10$

So $[2,3]$

$f(2) = 2$

$f(3) = -44$

b) Starting with $[a,b] = [2,3]$ how many steps to get an answer within 10^{-10} answer with an integer!

$$\frac{1}{2^{n+1}} < 2.5 \times 10^{-10}$$

$$> 1 < 2.5 \times 10^{-10} \, 2 \, (2^n)$$

$$\log_2 \left(\frac{1}{5 \times 10^{-10}} \right) < n = \frac{\ln\left(\frac{1}{5}e^{-10}\right)}{\ln(2)} \approx 30.897$$

$\boxed{31}$ steps

6) $[-2,1]$ $f(x) = \frac{1}{x}$ Step1

-0.5

- It does not converge to a real number
- That number is not the root

Exercise

1.2: 1, 2, 4, 8, 16

1) Find all fixpoints

a) $\frac{3}{x} = x \Rightarrow 3 = x^2 = \sqrt{3}$

$\boxed{-\sqrt{3}, \sqrt{3}}$

b) $x^2 - 2x + 2 = x$ $x^2 - 3x + 2 = (x-1)(x-2) = 0$

$\boxed{x = 1, 2}$

c) $x^2 - 4x + 2 = x \rightarrow x^2 - 5x + 2 = 0$

$\frac{5 \pm \sqrt{5^2 - 4(2)}}{2}$ $\frac{5 \pm \sqrt{17}}{2} \approx \rightarrow$ $\boxed{\begin{array}{l} x = 4.561552813 \\ 0.438447187 \end{array}}$

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2) Find Fix Points

a) $\frac{x+6}{3x-2} = x \rightarrow x + 6 = 3x^2 - 2x \rightarrow 0 = 3x^2 - 3x - 6$

$3(x^2 - x - 2)$

$3(x-2)(x+1) = 0$

$\boxed{x = 2, -1}$

b) $\frac{8 + 2x}{2 + x^2} = x \rightarrow 8 + 2x = 2x + x^3 \rightarrow 0 = x^3 - 8$

$\boxed{x = 2}$

c) $x^5 = x$

$x^5 - x = 0$

$\boxed{x = 1, 0, -1}$

1.2 Exercises 4, 8, 16

4) Show that $-1, 0, 1$ are fixed points of

a) $\dfrac{4y}{x^2+3}=x$ $4x = x^3 + 3x = 0 = x^3 + 3x - 4x = x^3 - x = 0$

$$\boxed{\begin{array}{l} 0^3 - 0 = 0 \\ 1^3 - 1 = 0 \\ (-1)^3 + 1 = 0 \end{array}}$$

b) $\dfrac{x^2 - 5x}{x^2 + x - 6} = x \rightarrow x^2 - 5x = x^3 + x^2 - 6x$

$\rightarrow 0 = x^3 - x$

$$\boxed{\begin{array}{l} 0 = 0^3 = 0 \\ 0 = 1^3 - 1 \\ 0 = (-1)^3 + 1 \end{array}}$$

8) Use theorem to determine if if fixed point iteration is locally convergent to r.

a) $\dfrac{2y-1}{x^2} = g(x) \quad r = 1$

$g(r) = \dfrac{2(1)-1}{1} = 1 \checkmark$

$g'(r) = \dfrac{d}{dx}\left(\dfrac{2x-1}{x^2}\right) \quad \dfrac{(x^2)(2) - (2x-1)(2x)}{x^4} = \dfrac{2 - (1)(2)}{1} = 0 < 1 \quad r=1$

$\therefore$ Theorem 1.6 implies that yes $g(x)$ will converge to $1 = r$

b) $g(x) = \cos(x) + \pi + 1; \quad r = \pi$ $\cos(\pi) + \pi + 1 = \pi \checkmark$

$g'(x) = -\sin(x) \checkmark$

$\quad\quad -\sin(\pi) = 0 < 1$

$\therefore$ Theorem 1.6 implies that yes $g(x)$ fixed point iteration converges on Pi π!

con't $\downarrow$

Exercise 1.2 8 cont.. , 16

8) cont.... Determin if fixed Point itteration is convergent
to r?

c) $g(x) = e^{2x} - 1$ ⤶ $r=0$ $e^0 - 1 = 0$ ✓

$g'(x) = 2e^{2x}$ $g'(r) = 2e^{2(0)} = \underline{2 > 1}$!

∴ Beccaus $g'(r) > 1$ theorem 1.6 implies $g(x)$'s fixed point
itteration diverges arround 0!

16) Which converge to $\sqrt[3]{4}$ and which is fastest?

(a)

$g(x) = \dfrac{2}{\sqrt{x}} =$

$\left((4)^{1/3}\right)^{1/2}$

$\dfrac{2}{(4)^{1/6}} = \sqrt[3]{4}$ ✓

$2x^{-1/2} = g(x)$
$g'(x) = -x^{-3/2}$

$g'(x) = \dfrac{-1}{2\sqrt{x^3}}$

$g'(r) = \dfrac{1}{\sqrt{4}} = |{}^1/2| < 1$

$\underline{S = 1/2}$

$\underline{Slowest!}$

b) $g(x) = \dfrac{3x}{4} + \dfrac{1}{x^2}$

$\dfrac{3\sqrt[3]{4}}{4} + \dfrac{1}{\sqrt[3]{4} \cdot \sqrt[3]{4}} = \sqrt[3]{4}$ ✓

x^{-2}

$g'(x) = \dfrac{3}{4} - \dfrac{2}{x^3}$

$g'(r) = \dfrac{3}{4} - \dfrac{2}{4} = \dfrac{1}{4} < 1$

yes

$\underline{S = 1/4}$

$\underline{2nd \; Fastest}$

c) $g(x) = \dfrac{2}{3}x + \dfrac{4}{3x^2}$

$\dfrac{2}{3}(\sqrt[3]{4}) + \dfrac{4}{3\sqrt[3]{16}} = \sqrt[3]{4}$ ✓

$g'(x) = \dfrac{2}{3} - \dfrac{8}{3}(x)^{-3} \to \sqrt[3]{4}$
$\dfrac{-8}{3} \cdot \dfrac{1}{4}$

$g'(r) = \dfrac{2}{3} - \dfrac{2}{3} = 0 < 1$ ✓

$\underline{S = 0}$

$\underline{Fastest!}$