Erik Sundblad

Homework 5 (1, 2, 3)

1) Calculate the determinant of each

$$\det\begin{bmatrix}1 & 2 & 3\\ -2 & 3 & 1\\ 0 & 4 & 1\end{bmatrix} = 1\times\det\begin{bmatrix}3 & 1\\ 4 & 1\end{bmatrix} - 2\times\det\begin{bmatrix}-2 & 1\\ 0 & 1\end{bmatrix} + 3\times\det\begin{bmatrix}-2 & 3\\ 0 & 4\end{bmatrix}$$

$$= 1\times(3-4) - 2(-2-0) + 3(-8-0)$$

$$= -1 + 4 - 24 = \boxed{-21}$$

$\boxed{21 \neq 0 \text{ so it is invertible}}$

$$\det\begin{bmatrix}-2 & 0 & 1\\ 1 & 3 & 0\\ 3 & 4 & 2\end{bmatrix} = -2\times\det\begin{bmatrix}3 & 0\\ 4 & 2\end{bmatrix} - 0\begin{bmatrix} & \\ & \end{bmatrix} + 1\times\det\begin{bmatrix}1 & 3\\ 3 & 4\end{bmatrix}$$

$$= -2(6-0) - 0 + (4-9)$$

$$-12 - 5 = \boxed{-17}$$

$\boxed{-17 \neq 0 \text{ so we say the matrix is invertible}}$

$$\det\begin{bmatrix}1 & -1 & 2\\ -1 & 0 & 3\\ 2 & 3 & 1\end{bmatrix} = 1\times\det\begin{bmatrix}0 & 3\\ 3 & 1\end{bmatrix} + \det\begin{bmatrix}-1 & 3\\ 2 & 1\end{bmatrix} + 2\times\det\begin{bmatrix}-1 & 0\\ 2 & 3\end{bmatrix}$$

$$= (0-9) + (-1-6) + 2(-3-0)$$

$$= -9 - 7 - 6 = \boxed{-22}$$

$\boxed{-22 \neq 0 \text{ so it is invertible}}$

Homework 5 (1, 2, 3) 1 cont.

$$\det \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 4 & 1 & 4 \end{bmatrix} = 2\det\begin{bmatrix} 0 & 1 \\ 1 & 4 \end{bmatrix} - \det\begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix} + 2\times\det\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$2(-1) - (4-4) + 2(1)$$

$$-2 - 0 + 2 = \boxed{0}$$

$\boxed{0 = 0 \text{ so the matrix is not invertible}}$

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2) find <u>eigenvalus</u> and <u>eigenvectors</u>

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 1 \\ 0 & 4 & 1 \end{bmatrix} \quad \det\begin{bmatrix} 1-\lambda & 2 & 3 \\ -2 & 3-\lambda & 1 \\ 0 & 4 & 1-\lambda \end{bmatrix} = (1-\lambda)((3-\lambda)(1-\lambda)-4) - 2(-2-2\lambda-0) + 3(-8)$$

$$(1-\lambda)(\lambda^2 - 4\lambda + 3) - 4) + 4 + 4\lambda - 24$$

$$(\lambda^2 - 4\lambda - 1 - \lambda^3 + 4\lambda^2 - 3\lambda) + 4\lambda - 20$$

$$-\lambda^3 + 5\lambda^2 - 11\lambda - 1 + 4\lambda - 20$$

$$0 = -\lambda^3 + 5\lambda^2 - 7\lambda - 21$$

$$21 = \lambda(\lambda^2 + 5\lambda - 7) - 21$$ <u>Imaginary Values</u>

Homework 5 (2,3)

2 contined...

$$\begin{bmatrix} -2 & 0 & 1 \\ 1 & 3 & 0 \\ 3 & 4 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \det \begin{bmatrix} -2-\lambda & 0 & 1 \\ 1 & 3-\lambda & 0 \\ 3 & 4 & 2-\lambda \end{bmatrix}$$

$$(-2-\lambda)((3-\lambda)(2-\lambda)-0) - 0 + 4-(9-3\lambda) = 0$$

$$(-2-\lambda)(6-5\lambda+\lambda^2) - 5 + 3\lambda = 0$$

$$-12 + \cancel{10\lambda} - 2\lambda^2 - \cancel{6\lambda} + 5\lambda^2 - \lambda^3 - 5 + \cancel{3\lambda} = 0$$

$$-\lambda^3 + 3\lambda^2 + 7\lambda - 17 = 0$$

λ = 3 values then we do $A\lambda = V_1\lambda$

so $(A-\lambda)v = 0$ then

$$\left[\begin{array}{ccc|c} a_{11}-\lambda & a_{12} & a_{13} & V_{11} \\ a_{21} & a_{22}-\lambda & a_{23} & V_{12} \\ a_{31} & a_{32} & a_{33}-\lambda & V_{31} \end{array}\right] = 0$$

rref we get our eigenvectors

HomeWork 5 Q2, 3

$$\det\left(\begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 3 \\ 2 & 3 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right) = \det\begin{bmatrix} 1-\lambda & -1 & 2 \\ -1 & -\lambda & 3 \\ 2 & 3 & 1-\lambda \end{bmatrix} =$$

$$(1-\lambda)(-\lambda(1-\lambda)-9) + (-1+\lambda-6) + 2\underbrace{(-3+2\lambda)}_{-6+4\lambda} =$$

$$(1-\lambda)(-\lambda+\lambda^2-9) - 13 + 5\lambda = -\lambda^3+\lambda^2+\lambda^2-9\lambda-\lambda-6+4\lambda =$$

$-\lambda^3+2\lambda^2+3\lambda-22$ we get $3\ \lambda$ values
then we input them in the equation $(A-\lambda)v_0 = 0$
and get the augmented matrix:

$$\left[\begin{array}{ccc|c} 1-\lambda & -1 & 2 & v_1 \\ -1 & -\lambda & 3 & v_2 \\ 2 & 3 & 1 & v_3 \end{array}\right] = 0$$ and solve to get our eigenvectors.

Continued

Homework 5 Q2 (3)

eigenvalues/eigenvectors

$$\det\left(\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 4 & 1 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right) = \det\begin{bmatrix} 2-\lambda & 1 & 2 \\ 1 & -\lambda & 1 \\ 4 & 1 & 4-\lambda \end{bmatrix} =$$

$$(2-\lambda)(-\lambda(4-\lambda)-1) - (4-\lambda-4) + 2(1+4\lambda) =$$

$$(2-\lambda)(-4\lambda+\lambda^2-1) \quad -2 \quad -\lambda \quad 2 + 8\lambda$$

$$-\lambda^3+4\lambda^2+2\lambda^2 \cancel{+\lambda-8\lambda} \cancel{-\lambda} +2 \cancel{+8\lambda} =$$

$$0 = -\lambda^3 + 6\lambda^2 + 2\lambda$$

$$(\lambda^2 + 6\lambda + 2) \qquad \frac{6 \pm \sqrt{2-4(6)(2}}{2}$$

$\lambda_1 = 0$

$\lambda_2 =$

$\lambda_3 =$

we get three eigenvalues and then use those to generate our eigenvectors using the formula

$$(A-\lambda)v_0 = 0$$

So we get the augmented mtx

$$\left[\begin{array}{ccc|c} 2-\lambda & 1 & 2 & v_1 \\ 1 & -\lambda & 1 & v_2 \\ 4 & 1 & 4-\lambda & v_3 \end{array}\right] = 0$$

an solve to get our eigen vectors

Homework 5 (3) Find

3) Find LU decomp for invertible Matrixes

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 1 \\ 0 & 4 & 1 \end{bmatrix} = L \cdot U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \quad U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \times \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$a_{11} = U_{11}$	$a_{21} = L_{21}U_{11}$	$a_{31} = L_{31}U_{11}$
$a_{12} = U_{12}$	$a_{22} = L_{21}U_{12} + U_{22}$	$a_{32} = L_{31}U_{12} + L_{32}U_{22}$
$a_{13} = U_{13}$	$a_{23} = L_{21}U_{13} + U_{23}$	$a_{33} = L_{31}U_{13} + L_{32}U_{23} + U_{33}$

So

$1 = 1$	$-2 = L_{21}(1)$	$0 = 1(L_{31})$
$2 = 2$	$3 = -2(2) + U_{22}$	$4 = 0 + L_{32}(7)$
$3 = 3$	$1 = -2(3) + U_{23}$	$1 = 0 + 4/7(7) + U_{33}$

So we get

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 4/7 & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 7 & 7 \\ 0 & 0 & -3 \end{bmatrix}$$

Cont ↓

Homework 5 3
Continued Fin UL Factorizaton

$$A = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 3 & 0 \\ 3 & 4 & 2 \end{bmatrix} = L \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \cdot U \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$a_{11} = U_{11}$	$a_{21} = L_{21}U_{11}$	$a_{31} = L_{31}U_{11}$
$a_{12} = U_{12}$	$a_{22} = L_{21}U_{12} + U_{22}$	$a_{32} = L_{31}U_{12} + L_{32}U_{22}$
$a_{13} = U_{13}$	$a_{23} = L_{21}U_{13} + U_{23}$	$a_{33} = L_{31}U_{13} + L_{32}U_{23} + U_{33}$

$-2 = -2$	$1 = L_{21}(-2)$	$3 = L_{31}(-2)$
$0 = 0$	$3 = (-1/2)(0) + U_{22}$	$4 = \frac{-3}{2}(0) + 3L_{32}$
$1 = 1$	$0 = (-1/2)(1) + U_{23}$	$2 = -3/2(1) + (4/3)(1/2) + U_{33}$

then

$\frac{12}{6} + 5/6 = 7/6 \quad \frac{4}{6} - \frac{9}{6} = -5/6$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -3/2 & 4/3 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & 1/2 \\ 0 & 0 & 17/6 \end{bmatrix}$$

cont. ↓

HomeWork 5 Q3
Continued...

Find UC Decomposition of A

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 3 \\ 2 & 3 & 1 \end{bmatrix} = L \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \cdot U \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$a_{11} = U_{11}$	$a_{21} = l_{21} U_{11}$	$a_{31} = l_{31} U_{11}$
$a_{12} = U_{12}$	$a_{22} = l_{21} U_{12} + U_{22}$	$a_{32} = l_{31} U_{12} + l_{32} U_{22}$
$a_{13} = U_{13}$	$a_{23} = l_{21} U_{13} + U_{23}$	$a_{33} = l_{31} U_{13} + l_{32} U_{23} + U_{33}$

$1 = 1$	$-1 = l_{21}(1)$	$2 = l_{31}(1)$
$-1 = -1$	$0 = (-1)(-1) + U_{22}$	$3 = 2(-1) + l_{32}(-1)$
$2 = 2$	$3 = (-1)(2) + U_{23}$	$1 = 2(2) + (-5)(5) + U_{33}$

So then

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 5 \\ 0 & 0 & 22 \end{bmatrix}$$

Done!