1.3 Exercises 2, 4, 6, 7

2) find forward and backward error where root $= \frac{1}{3}$ and $x_a = 0.3333$

a) $f(x) = 3x - 1$ Forward Error $|r - x_a| = |\frac{1}{3} - 0.3333| \approx 0.0000\overline{3}$

Backward Error $f(x_a) = 3(0.3333) - 1 = -0.0001$

b) $(3x-1)^2 = f(x)$

Forward Error $|r - x_a| = |\frac{1}{3} - 0.3333| \approx 0.0000\overline{3}$

Backward Error $f(x_a) = (3(0.3333)-1)^2 = 1 \times 10^{-8}$

c) $(3x-1)^3 = f(x)$

Forward Error $|r - x_a| = |\frac{1}{3} - 0.3333| \approx 0.0000\overline{3}$

Backward Error $f(x_a) = (3(0.3333)-1)^3 = -1 \times 10^{-12}$

d) $f(x) = (3x-1)^{1/3}$

Forward Error $|r - x_a| = |\frac{1}{3} - 0.3333| = 0.0000\overline{3}$

Backward Error $f(x_a) = (3(0.3333)-1)^{1/3} = \sqrt[3]{-0.0001}$

4) Find Multiplicity of $x^2 \sin x^2$ $r = 0$ | Find Error $(\overleftarrow{B}, \overrightarrow{F})$ $x_a = 0.01$

a) $f(x) = x^2 \sin x^2 : f(0) = 0$; $f'(x) = 2x \sin 2x$: $f'(0) = 0$;

$f''(x) = 4\cos(2x)$ $f''(0) = 4$ So $\boxed{\text{Multiplicity of } 2}$

b) Forward Error $|r - x_a| = |0 - 0.01| = 0.01$

Backward Error $f(x_a) = (0.01)^2 (\sin(0.01))^2 \approx 9.999666671 \times 10^{-9}$

1.3 Exercise 6, 7

6) Let n be a positive integer $x^n - A = 0$

Backward = $f(x_a) \approx nA^{\frac{(n-1)}{n}} |r - x_a|$

for a small value of $|r - x_a|$

we need a small $r - x_a$ or we may diverge

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7) $W(x)$ = Wilkinson a) Prove that $W'(16) = 15!4!$

b) find analogous formula for $W'(x)$ when $1 \le x \le 20$

a) $W(x) = (x-1)(x-2)\ldots(x-20)$ $W(16) = (16-1)(16-2)\ldots(16-16$

$w'(x)$ On Nex Page

b) When we take the derivative we always lose half of our chain rule containing value of x leading to a result of x

$(x-1)!(20-x)!$

For even values of x for odd values the negatives stay negative giving! $(x-1)!(x-20)!$

1.3 Excercise 7

7) Prove that the willson polynomial $w(x)$' derivative at $x=16$ evaluates to $15!4!$

$$w(x) = \underbrace{(x-1)(x-2)(x-3)(x-4)(x-5)(x-5)(x-6)(x-7)(x-8)(x-9)(x-10)}_{w_1(x)}\underbrace{(x-11)(x-12)(x-13)(x-14)(x-15)(x-16)(x-\ldots}_{w_2(x)}$$

$$w_2'(x) \cdot w_1(x) + w_1'(x) w_2(x)$$ at $x=16$ goes to 0

$$\downarrow$$

$$\underbrace{(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7)(x-8)(x-9)(x-10)}_{x=16\ =\ 15!/5!} \times \frac{d}{dx} w_2(x) =$$

$$\frac{d}{dx} w_2(x) = \underbrace{(x-11)(x-12)(x-13)(x-14)(x-15)}_{w_{21}(x)}\underbrace{(x-16)(x-17)(x-18)(x-19)(x-20)}_{w_{22}(x)}$$

$$\frac{15!}{10!}\left(w_{21}(x) w_{22}'(x) + w_{21}'(x) w_{22}(x) \to \text{goes to } 0 \text{ at } x=16\right)$$

$$w_{21} = (5)(4)(3)(2)(1) = 5!$$

$$w_{22}(x) = \underbrace{(x-16)(x-17)}_{w_{221}(x)}\underbrace{(x-18)(x-19)(x-20)}_{w_{222}(x)}$$

$w_{221}(x) w_{222}'(x)$ → goes to 0 $\times$ $$w_{22}' w_{222} = \overset{1}{(x-17)}\overset{2}{(x-18)}\overset{3}{(x-19)}\overset{4}{(x-20)} =$$

$$\frac{15!}{\cancel{5!}} \times \cancel{5!} \times 4! = \boxed{15!4!}$$

Exercise 1.4 2, 6, 8, 10, 11

1) Apply two steps of newtons Method wit $x_0 = 1$

a) $x^3 + x^2 - 1 = 0$ Newtons : $x_n = x_{n-1} + \frac{f(x)}{f'(x)}$

1st

$$x = 1 - \frac{1^3 + 1^2 - 1}{3(1)^2 + 2(1)} \quad 1 - \frac{1}{5} = 4/5$$

2nd

$$x = 4/5 - \frac{(4/5)^3 + (4/5)^2 - 1}{3(4/5)^2 + 2(4/5)} = 4/5 - \frac{\frac{64}{125} + \frac{16^{\times 5}}{25_{\times 5}} - \frac{125}{125}}{\frac{48}{25} + \frac{8^{\times 5}}{5_{\times 5}}}$$

$$= \frac{64}{125} + \frac{80}{125} - \frac{125}{125} = \frac{19}{_5\cancel{125}} \times \frac{\cancel{25}^1}{98} = \frac{19}{490} \text{ so } \frac{4}{5} - \frac{19}{490}$$

$$\frac{392}{490} - \frac{19}{490} = \boxed{\frac{373}{490}}$$

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b)

$$f(x) = \frac{x^2+1}{x+1} - 3x = 0 \qquad f'(x) = \frac{2x(x+1) + (x^2+1)}{(x+1)^2} - 3$$

1st

$$x = 1 - \left(\frac{1^2+1}{1+1} - 3(1)\right) \times \frac{1}{\frac{2(1)(1+1) + (1^2+1)}{(1+1)^2} - 3}$$

$x =$ $\frac{6}{4} - 3$

$$x = 1 - (-2)\left(\frac{-1}{6/4}\right)$$

$$x = 1 - 8/6$$

$x = -1/3$

Cont

Exercise 1.4 2, 6, 8, 10, 11

2) 2nd Step of Newtons Method

b) $$x = \frac{-1}{3} - \left(\frac{(-1/3)^2+1}{-1/3+1} + \frac{-3}{3}\right)\left(\frac{1}{\frac{(2)(-1/3)(-1/3+1)+(-1/3^2+1)}{(-1/3+1)^2}} - 3\right)$$

$$x = -1/3 - \left(\frac{1/9+\frac{9}{9}}{-1/3+\frac{3}{3}} - 1\right)\left(\frac{\left(\frac{2}{3}\right)^2}{-4/9+\frac{10}{9}} - 3\right)$$

$$x = -1/3 - \left(\frac{10}{9}\times\frac{3}{2} - 1\right)\left(\frac{4}{9}\times\frac{9}{6} - 3\right) = -1/3 - \left(\frac{12}{18}\right)\left(\frac{36}{53} - \frac{159}{53}\right)$$

$$x = -1/3 - \left(\frac{12}{18}\right)\left(-\frac{123}{53}\right) = +\frac{1476}{954} - 1/3 = \frac{1476}{954} - \frac{318}{954} =$$

$$\boxed{\frac{1158}{954} = x}$$

c) $5x - 10 = 0$ $\qquad f'(x) = 5$

1st Step

$$x = 7 - \frac{5-10}{5} = \boxed{x = 2}$$

2nd Step

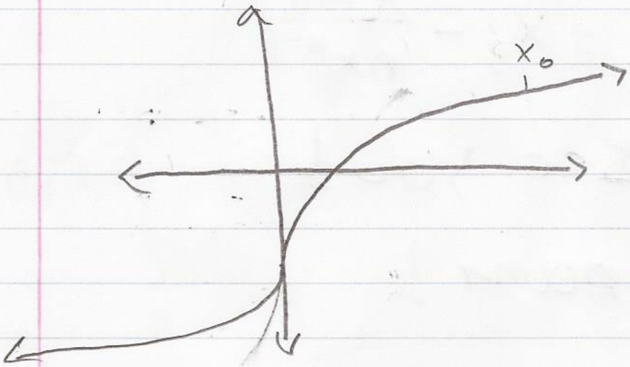
$$x = 2 - \frac{5(2)-10}{5}$$

$$\boxed{x = 2}$$

Next Page

Exercise 1.4 6, 8, 10, 11

6) Sketch a function f and an initial geouss for which newtons method diverges!



$f'(x_0)$ will be very small making our $\frac{f(x)}{f'(x)}$ beocme very large and negative then larger and positive and so on

8) $f(x) = ax + b$ converges in one step we can obsere at any arbitray x_0 that our formula beomes

$$a\left(x_0 - \frac{ax_0 + b}{a}\right) + b = 0$$

So $0 = ax_0 - (ax_0 + b) + b$

or $0 = ax_0 - ax_0 - b + b$ $\therefore$ We can conclude that for all x_0 the function $f(x)$ will converg in one step of newtons method ∎

10) $f(x) = x^3 - A$ Newtons as Fixed Point Itteraton

$$x = x - \frac{x^3 - A}{3x^2}$$ for a given A

Exercise 1.4, 11

11) Use Newton to Calculate the n^{th} root of A of n

$$x^n - A = 0 \qquad x = x - \frac{x^n - A}{n x^{n-1}}$$

I'll come in and see you!

Missing some theorem!