Erik Sundblad

85% recipe = 55%

Exam 2 (1.5.4, 2.1.7, 2.2.4a, 2.3.18, 2.4.9a) +Retleton

Sec. 1.5

problem (4) looking for 10° at depth x. At 8° at f(9) and 15° at f(5). What is x by secant method?

So we are approximating $f(x) - 10^\circ = 0 = g(x)$

$$\boxed{\begin{array}{l} x_1 = 9 \\ x_0 = 5 \end{array}} \qquad x_{n+1} = x_n - \frac{g(x_n)(x_n - x_{n-1})}{g(x_i) - g(x_{i-1})} = \text{Secant Method}$$

$$x_{n+1} = 9 - \frac{-2(9-5)}{-2-5} = 9 - \frac{-8}{-7} = \frac{55}{7} \approx 7.857\,m$$

By one pass of the secant method and without being able to know our actual temp at 7.857 m. I conclude our best approximation is $55/7$ meters.

Sec 2.1

Problem (7) A given computation require 0.002 s to complete 4000x 4000 mtx upper triangular Matrix equation. Estimat time needed for a 9,000 Equations and 9,000 unknowns.

Our computer does $(4000)^2$ opperation in 0.002 s on 8×10^9 opperation/second. A general 9000 × 9000 set of equations and unknowns takes $\frac{2(9000)^3}{3} = 4.86 \times 10^{11}$ opperations

So we get $\frac{4.86 \times 10^{11}}{8 \times 10^9} \approx \boxed{60.75\,s}$

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Exam 2 (2.2.4a, 2.3.18, 2.4.9a) + Reflection

Section 2.2
Problem (4/a) Solve the system by LU factorization and 2 step back substitution.

a)
$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$$\underset{A}{\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix}} \xrightarrow{R_3 - R_1} \underset{E_1A}{\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}} \xrightarrow{R_2 - 2R_1} \underset{E_2E_1A = U}{\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}} \quad A = E_1^{-1}E_2^{-1}U =$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \qquad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A = LU$

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}}$$

Now we use back substitution

$Lc = b$

$Ux = c$

$$\underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{matrix} c_1 + 0 + 0 = 0 \\ 2c_1 + c_2 + 0 = 1 \\ c_1 + 0 + c_3 = 3 \end{matrix} \quad \text{So } c = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

Then

$$U \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{matrix} 3x_1 + x_2 + 2x_3 = 0 \\ 0 + x_2 + 0 = 1 \\ 0 + 0 + 3x_3 = 3 \end{matrix} \quad \text{So } X = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

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Exam 2 (2.3.18, 2.4.9a) + Reflection

Section 2.3 Problem (18)

a) Show the system of equations

$$\begin{bmatrix} 811802 & 810901 \\ 810901 & 810001 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 901 \\ 900 \end{bmatrix}$$ has the

Solution $[1, -1]$:

$$811802(1) + 810901(-1) = 901$$
$$810901(1) + 810001(-1) = 900$$

b) Solve using double precision arithmetic using guassian elimination (in tableau or any other form). How many correct decimal places. Explain using Cond(A)

$$\left[\begin{array}{cc|c} 811802 & 810901 & 901 \\ 810901 & 810001 & 900 \end{array}\right] R_2 - \frac{810901}{811802} R_1 \quad \text{becomes}$$

$$\left[\begin{array}{cc|c} 811802 & 810901 & 901 \\ 0 & 1.23e^{-6} & -1.23183e^{-6} \end{array}\right] \underset{R_2\left(\frac{1}{1.23e^{-6}}\right)}{\sim} \left[\begin{array}{cc|c} 811802 & 810901 & \\ 0 & 1 & -1.001487805 \end{array}\right]$$

$$\underset{R_1 - 810901(R_2)}{\sim} \left[\begin{array}{cc|c} 811802 & 0 & 813007.41626 \\ 0 & 1 & -1.001487805 \end{array}\right] \underset{R_1/811802}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 1.001484922 \\ 0 & 1 & -1.001487805 \end{array}\right]$$

So by double precision we get 2 decimal places of accuracy.

We can find Cond(A) =

$\|A\|_\infty \times \|A^{-1}\|$?

Why?

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Exam 2 (2.3.18, 2.4.9a) + Reflection

Section 2.3
Problem (18) contined

$A = \begin{bmatrix} 811802 & 810901 \\ 810901 & 810001 \end{bmatrix}$

$Cond(A) = \|A\|_\infty \times \|A^{-1}\|_\infty$

$\|A\|_\infty = 1622703$

$A^{-1} = \left[\begin{array}{cc|cc} 811802 & 810901 & 1 & 0 \\ 810901 & 810001 & 0 & 1 \end{array}\right] \sim R_2 - \frac{810901}{811802}$

$\left[\begin{array}{cc|cc} 811802 & 810901 & 1 & 0 \\ 0 & 1.23e^{-6} & -\frac{810901}{811802} & 1 \end{array}\right] R_2 / 1.23e^{-6}$

$\frac{1}{\det A} \left[\sim \right]$ A^{-1}

$\left[\begin{array}{cc|cc} 811802 & 810901 & 1 & 0 \\ 0 & 1 & -812105.7914 & 813008.1301 \end{array}\right] \sim R_1 - 810901(R_2)$

$\left[\begin{array}{cc|cc} 811802 & 0 & 6.585374e^{11} & -6.592691e^{11} \\ 0 & 1 & -812105.8 & 813008.1301 \end{array}\right] \sim R_1 / 811802$

$\left[\begin{array}{cc|cc} 1 & 0 & 811204.4542 & -812105.7914 \\ 0 & 1 & -812105.8 & 813008.1301 \end{array}\right] = A^{-1} = \begin{bmatrix} 811204.4542 & -812105.79 \\ -812105.7914 & 813008.1301 \end{bmatrix}$

$\|A^{-1}\|_\infty \approx 1625113.922$

So our $Cond(A) = 1622703 \times 1625113.922 = 2.637e^{12}$

Our error magnification must be less than or equal too $2.637e^{12}$

Cont ↓

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Exam 2 (2.3.18, 2.4.9a) + Retake

(18) continued. $\text{Cond } A = 2.637e^{12}$

Our error magnification is $\frac{\text{forward}}{\text{backward}} = \dfrac{\frac{\|x - x_a\|_\infty}{\|x\|_\infty}}{\frac{\|r\|_\infty}{\|b\|_\infty}}$

For our system this becomes:

$$\cancel{\dfrac{\begin{pmatrix} 1 - 1.0011484922 \\ -1 - -1.0011487805 \end{pmatrix}}{\frac{1}{901}}} \leq 2.637e^{12}$$

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~~which I argue is less than our Cond(A) of $2.637e^{12}$ by alot therefore our results are within predicted error!~~

Section 2.4 Find the $PA = LU$ Factorization of A_{mtx}

(9a)

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix} \underset{R_4+R_1}{\sim} \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 2 \end{bmatrix} \underset{R_1+R_3}{\sim} \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 2 \\ -1 & -1 & -1 & 2 \end{bmatrix}$$

$$\underset{R_1+R_2}{\sim} \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 2 \\ -1 & -1 & 1 & 2 \\ -1 & -1 & -1 & 2 \end{bmatrix} \underset{R_2+R_4}{\sim} \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 2 \\ -1 & -1 & 1 & 2 \\ -1 & -1 & -1 & 4 \end{bmatrix} \underset{R_2+R_3}{\sim} \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 2 \\ -1 & -1 & 1 & 4 \\ -1 & -1 & -1 & 4 \end{bmatrix} \underset{R_3+R_4}{\sim}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 2 \\ -1 & -1 & 1 & 4 \\ -1 & -1 & -1 & 8 \end{bmatrix}$$

So our $PA = LU$ equals =

$$\overset{P}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \overset{A}{\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}} = \overset{L}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{bmatrix}} \overset{U}{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 8 \end{bmatrix}}$$

Exam 2 Reflection

Reflecting on my learning. I would say there are three areas I have increased either my knowledge base or improved my reasoning. I believe I have increased my comfort with matrix operations and understanding. I believe I have gained some insight into what I imagine graduate-level work may look like. Lastly, I think my mathematical reasoning has improved because of the work in this class. I have grown and gained from being here.

When it comes to my work with matrices and utilizing matrix algebra, I have grown. I took my initial matrix understanding class during covid as an async class (as it conflicted with the core curriculum). This resulted in less than maximum focus. I would skim video lectures and pass quizzes. The knowledge gave me fundamentals but ultimately left me lacking in the work in this class. I spent time this semester really refining and widening my understanding of matrix algebra and operations. I am happy to fill this known knowledge gap, and potentially investigate further.

This class has opened me up to a few key new ideas when it comes to academic study. As a senior experience, I realize the class was going to be more group-focused and learning driven. This has absolutely been the case. A novel concept never before broached was the literal action of emailing the writer of a textbook. Let alone the concept of an errata to discover all the mistakes. This was illuminating and honestly empowering. To feel like we are competent enough to find mistakes and really address those ~~even while~~ learning new concepts. This felt like a victory and what I imagine involved graduate-level work is like.

Excellent!

My math reasoning has been refined through this course. We ask questions of the book of the proofs and of the theories constantly. Before I may debate whether I had the tools or resources to definitively come to results. Now I feel like with every problem posed I need to tackle, tools or not with reasoning I can come to conclusions or I can try and learn continually. This has been ratified by pursuits with matrix logic and all the gaps I have filled and tried to champion. It has been a captivating course.

I like a challenge and this course has had those challenges, I've had to fill my own knowledge gaps, learned to trust myself more, and improved my personal reasoning. Keep the challenges comming the man in the arena is always ready!

A-/A. More clarity and careful use of theorems on the final $\Rightarrow$ A

otherwise A- right now.

Same performance $\Rightarrow$ A-