51.012752

$\frac{3}{81}$ $\frac{45}{81}$ $\frac{27}{81}$ $\quad 27 \times 3 = 81$

$\frac{6}{81} + \frac{1}{27} + \frac{5}{9} + \frac{1}{3} + \frac{81}{81} = \boxed{2}$

0.1 Excersices!

1) $x = \frac{1}{3}$

a) $6x^4 + x^3 + 5x^2 + x + 1 = 6(\frac{1}{3})^4 + (\frac{1}{3})^3 + 5(\frac{1}{3})^2 + \frac{1}{3} + 1$

Nested

$= 1 + x(6x^3 + x^2 + 5x + 1)$

$= 1 + x(1 + x(6x^2 + x + 5))$

$= 1 + x(1 + x(5 + x(6x + 1)))$

$1 + \frac{1}{3}(1 + \frac{1}{3}(5 + \frac{1}{3}(\frac{6}{3} + 1)))$

$1 + \frac{1}{3}(1 + \frac{1}{3}(5 + \frac{1}{3}(\frac{9}{3})))$

$1 + \frac{1}{3}(1 + \frac{1}{3}(\frac{45}{9} + \frac{9}{9}))$

$1 + \frac{1}{3}(1 + \frac{1}{3}(\frac{54}{9}))$

$1 + \frac{1}{3}(\frac{27}{27} + \frac{54}{27})$

$1 + \frac{1}{3}(\frac{81}{27})$

$1 + \frac{81}{81} = \boxed{2}$

b) $x = \frac{1}{3}$

$-3x^4 + 4x^3 + 5x^2 - 5x + 1$

Nested

$1 + x(-3x^3 + 4x^2 + 5x - 5)$

$1 + x(-5 + x(-3x^2 + 4x + 5))$

$1 + x(-5 + x(5 + x(-3x + 4)))$

$1 + x(-5 + x(5 + x(4 - 3x)))$

$x = \frac{1}{3}$

$1 + \frac{1}{3}(-5 + \frac{1}{3}(5 + \frac{1}{3}(4 - 1)))$

$1 + \frac{1}{3}(-5 + \frac{1}{3}(5 + 1))$

$1 + \frac{1}{3}(-3)$

$1 - 1 = \boxed{0}$

or

$-3(\frac{1}{3})^4 + 4(\frac{1}{3})^3 + 5(\frac{1}{3})^2 - 5(\frac{1}{3}) + 1$

$-\frac{3}{81} + \frac{12}{81} + \frac{45}{81} - \frac{135}{81} + \frac{81}{81} = \boxed{0}$

0.1 Excercise

1. $x = 1/3$

c) $2x^4 + x^3 - x^2 + 1$

$1 + x(2x^3 + x^2 - x)$

$1 + x^2(2x^2 + x - 1)$

$1 + x^2(-1 + x(2x + 1))$

$x = 1/3$

$1 + (1/3)^2(-1 + 1/3(2/3 + \frac{3}{3}))$

$1 + 1/9(-\frac{9}{9} + 5/9)$

$1 + 1/9(\frac{-4}{9}) = \frac{-4}{81} + \frac{81}{81} = \boxed{\frac{77}{81}}$

$x = 1/3$

$2(1/3)^4 + (1/3)^3 - (1/3)^2 + \frac{3}{3}$

$\frac{2}{81} + \frac{9}{81} + \frac{-9}{81} + \frac{81}{81}$

$\frac{-4}{81} + \frac{81}{81} = \boxed{\frac{77}{81}}$

2) $x = -1/2$

a) $6x^3 - 2x^2 - 3x + 7$

$7 + x(6x^2 - 2x - 3)$

$7 + x(-3 + x(6x - 2)$

$x = -1/2$

$-3 - 2$

$7 - 1/2(-3 - 1/2(-5)$

$7 - 1/2(-3 + 5/2)$

$7 - 1/2(-1/2)$

$\boxed{7.25 \text{ or } 29/4}$

b) $8x^5 - x^4 - 3x^3 + x^2 - 3x + 1$

$1 + x(8x^4 - x^3 - 3x^2 + x - 3)$

$1 + x(-3 + x(8x^3 - x^2 - 3x + 1))$

$1 + x(-3 + x(1 + x(8x^2 - x - 3)))$

$1 + x(-3 + x(1 + x(-3 + x(8x - 1))))$

$x = 1/2$

$1 + 1/2(-3 + 1/2(1 + -1/2(-3 + 1/2(4 - 1))))$

$1 + -1/2(-3 + -1/2(1 + -1/2(-3 + 5/2))$

con't

Erik S

Continued ch 0.1

Q2), $[3,4,5,6]\ 6/2$

b) $1+{-1/2}(-3+{-1/2}(1+{-1/2}(-3+5/2))) = 1+{-1/2}(-3+{-1/2}(1+1/4$

$\rightarrow 1+{-1/2}(-3-5/8) = 1+{-1/2}\left(\frac{-29}{8}\right)\ 1+{-29/16} = \boxed{-13/16}$

$x=-1/2$

c) $4x^6-2x^4-2x+4 = 4+x(4x^5-2x^3-2) = 4+2x(2x$

$\rightarrow 4+2x(-1+x^3(2x^2-1)))\ x=-1/2 =$

$4+{-2/2}(-1+(-1/2)^3(2(-1/2)^2-1))) = 4-1(-1+(-1/8)(-1/2))$

$-1/2$

$\rightarrow 4-1\left(\frac{-16}{16}+1/16\right)\ 4+15/16 = \frac{64}{16}+15/16 = \boxed{79/16}$

3) $x^2=1/4 \quad P(x^2) = 6x^3-4x^2+2x+1$
$x=1/2$

$1+x(6x^2-4x+2) = 1+x(2+x(6x-4)))$

$1+1/4(2+1/4(\frac{6}{4}-\frac{16}{4}))\ (2-10/16)1/4+1\ \left(\frac{22}{16}\right)1/4+1\ \frac{22}{64}+\frac{64}{64} =$

$\frac{86}{64} = \boxed{\frac{43}{32}}$

4) a) $x=5 \quad P(x) = 1+x(1/2+(x-2)(1/2+(x-3)(-1/2)))$

$1+5(1/2+3(1/2+2(-1/2))) =$
$1+5(1/2+3(1/2-1)) = 1+5(1/2-3/2)$
$1-5 = \boxed{-4}$

part B

Ch 0.1 Cont 4,5,6,7

4) b) $x=-1$

$P(x) = 1 + x(1/2 + (x-2)(1/2 + (x-3)(-1/2)))$

$1 + -1(1/2 + -3(1/2 + -4(-1/2))) = 1 - 1(1/2 - 3(5/2)) = 1 - 1(-\frac{14}{2}) \rightarrow$

$1 + 7 = \boxed{8}$

---

5) $P(x) = 4 + x(4 + (x-1)(1 + (x-2)(3 + (x-3)(2))))$

a) $x = 1/2$ $\quad 4 + 1/2(4 + (-1/2)(1 + (-3/2)(3 + (-5/2)(2))))$

$4 + 1/2(4 + (-1/2)(1 + (-\frac{3}{2})(-2))) = 4 + 1/2(4 + -1/2(4)) =$

$4 + 1/2(2) = \boxed{5}$

b) $x = -1/2 \Rightarrow 4 + -1/2(4 + (-3/2)(1 + (-5/2)(3 + (-7/2)(2))))$

$4 + -1/2(4 + (-3/2)(1 + (-5/2)(-4)) = 4 + -1/2(4 + (-3/2)(1 + (10))) = 4 + -1/2(4 + \frac{-33}{2})$

$4 - 1/2(\frac{-25}{2}) \quad 4 + 25/4 = \boxed{31/4}$

---

6) a) We can substitute x^5's value in so the problem becomes $P(x^5) = a_0 + a_5x + a_{10}x^2 + a_{15}x^3$ Then splitting it using horners method we get $a_0 + x_5(a_5 + x(a_{10} + a_{15}x))$ Then doing 3 additions and 2 Multiplications!

b)

0.1 Excercise Cont

6, B) [7]

$P(x) = a_7x^7 + a_{12}x^{12} + a_{17}x^{17} + a_{22}x^{22} + a_{27}x^{27}$

$\cancel{x}^7(a_7 + a_{12}x^5 + a_{17}x^{10} + a_{22}x^{15} + a_{27}x^{20}$

For this problem first we pull the value of x^7 to get a nice polynomial we can then set as factors of x^5. thus giving: $x^7 p(x5) = (a_7 + a_{12}x + a_{17}x^2 + a_{22}x^3 + a_{27}x^4)$

Which can be solved with 5 multiplications and 5 additions (! Not counting x^5 and x^7)

---

7) with the general nested algorithm a polynomial of degree n will take n multiplications, and n+n+1 Additions ~~additions~~ with base points!

~~Ch 0.2 Excercise (2-8)~~ End 0.1 Written Problems

~~4)~~

Erik Sundst...

Excercise 0.2 1-8

128 256

1) Binary of these decimal #'s

a) 64 = 1000000

c) 79 = 1001111

b) 17 10001

d) 227 = 11100011

128 99-7 35-3

2) Binary of these decimal #'s

a) $1/8 = 0.001$

c) $35/16 = 2\,3/16 = 10.0011$

b) $7/8 = 0.111$

$4/8 + 2/8 + 1/8$

d) $31/64 = 0.011111$

32 16+8+4+2+1

$\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{16}\ \frac{1}{32}\ \frac{1}{64}$

3) Convert Decim. to bin

a) $10.5 = 1010.1$

b) $1/3 = 0.010101$

$\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{16}\ \frac{1}{32}\ \frac{1}{64}$

$\frac{4}{12} - \frac{3}{12} = \frac{1}{12} - \frac{1}{16}\quad \frac{4}{48} - \frac{3}{48}\ \frac{1}{48}$

e) $55.4 = 110111.\overline{0110}$

32 23 16 7

f) $0.1 = 0.0\overline{0011}$

$\frac{1}{10}\ \frac{2}{10}\ \frac{4}{10}\ \frac{8}{10}\ \frac{16}{10}\ \frac{12}{10}\ \frac{4}{10}\ 8$

c) $5/7 = 0.101\,101\,1$ So $= (0.\overline{1011})$

$\frac{10}{7} - \frac{7}{7}\quad \frac{3}{7}\times 2\quad \frac{6}{7}\times 2\quad \frac{12}{7} - \frac{7}{7}\quad \frac{5}{7}\times 2$

d) $12.8 = (1100.\overline{1100})_2$

$\frac{4}{5}\times 2 = \frac{8}{5}\ .5\quad \frac{3}{5}\times \frac{6}{5}\quad \frac{5}{5} = \frac{1}{5}\times 2\quad \frac{2}{5}\times \frac{4}{5}$

Continued 4-8

$$\begin{array}{r} 13 \\ \times\ 7 \\ \hline 91 \end{array}$$

Excercise 0.2 4-8

4) Convert to Binary

a) $11.25 = 1011.01$

b) $2/3 = 0.\overline{10}$

c) $3/5 = 0.\overline{1001}$

d) $3.2 = 11.\overline{0011}$

e) $30.6 = 11110.\overline{1001}$

f) $99.9 = 1100011.11100$ 1

---

5) $Pi = 3.14159265$ $-3 = \text{decimal} \times 2^{-1}$

11.0010010000111

---

6) $e = 2.718281828$

10.1011011111000

---

7) Convert Bin to DeC

a) $1010101 = \boxed{85}$

64 16 4 1

b) $1011.101 =$ $\frac{1}{2} + \frac{1}{8}$

$\boxed{11.75}$

c) $10111.\overline{01} = 23\frac{1}{3} = \frac{70}{3}$

16 8 4 2 1 $\frac{1}{3}$

d) $110.\overline{10} = 6\frac{2}{3} = \frac{20}{3}$

$\frac{2}{3}$

e) $10.\overline{110} = 2\frac{6}{7} = \frac{20}{7}$

$2^3 - 1 = 7$

f) $\underbrace{110}_{6}.1\overline{101} = 6 + \frac{1}{2} + \frac{1}{2}\frac{5}{7}$

0.5

$\frac{13}{2} + \frac{5}{14}$ $\frac{96}{14} = \frac{48}{7}$

g) $10.01\underbrace{0}\overline{101}$

$\frac{1}{4}$ $\frac{1}{8}$

$2 + \frac{1}{4} + \left(\frac{9}{4} + \frac{1}{8}\frac{13}{15}\right) = \frac{270}{120} + \frac{13}{120}$

$\boxed{283/120}$

---

h) $111.\overline{1}$ $7 + (1 \times 1) = \boxed{8}$

(Cont

$$\begin{array}{r} {}^{4}55 \\ \times\ 8 \\ \hline 440 \end{array}$$

$$\begin{array}{r} {}^{3}47 \\ \times\ 15 \\ 235 \\ 470 \\ \hline 705 \end{array}$$

Excercise 0.2 8

8) Convert Bin to Dec

a) $11011 = 27$

b) $11011.00\overline{1}$

$55 + 1/8 = \boxed{\frac{441}{8}}$

c) $111.\overline{001}$

$7 + \frac{1}{7} = \boxed{50/7}$

d) $1010.\overline{01}$

$10 + 1/3$ $\boxed{31/3}$

e) $10111.\overline{10101}$

$23 + 1/2 + 1/2^{*} \frac{5}{15}$

$\frac{47}{2} + \frac{5}{30} = \frac{710}{30} = \boxed{71/3}$

f) $1111.010\overline{001}$

$15 + 1/4 + 1/8 \, 1/7 = \frac{61}{4} + \frac{1}{56}$ $\boxed{\frac{855}{56}}$

$$\begin{array}{r} 61 \\ \times\ 14 \\ \hline 244 \\ 610 \\ 854 \end{array}$$

Ch 0.3

Excercise 2-16 Evens

2)

a) $9.5_{10} = (\underbrace{1001}.1)_2 = 1.00110\ldots0 \times 2^3$

b) $9.6_{10} = \underbrace{1001}.\overline{1001} = \boxed{1.0011001100110011\;0011001100110011\;0011001100110011\;001100} \times 2^3$

64 32 16 8 4 2 1

c) $100.2_{10} = \underbrace{1100100}.\overline{0011} = \boxed{1.1001000011\;1100110011\;0011001100\;1100110011\;0011001100\;1100} \times 2^6$

-64

36

d) $\frac{44}{7} = \underbrace{110}.\overline{010}$

$6\,2/7$

$\boxed{1.1001001001001001\;0010010010010010\;0100100100100100\;1001001001001001\;001001100} \times 2^2$

4) $fl(19+2^{k}) > fl(19)$

$fl(19) =$

$19_{10} = 10011 = 1.0011 \times 2^4$

$1.0011 \times 2^{4} + 2^{-k}$ (smallest $= 2^{-52} \times 2^{0}$

$\overset{52}{1.0011\ldots00}$

$+\ 1.0000$

$\underbrace{2^{-52}} \times 2^{4} = \boxed{2^{-48}}$

Excrise 0.3 (6-16) evens

6) a.) $(1+(2^{-51}+2^{-53}))-1$

$$1.0\ldots\overset{51}{1}\overset{52}{0}\overset{53}{1}0000\ldots \doteq 1.0\ldots\overset{51}{1}0 - 1 = \boxed{0.\underbrace{0\ldots}_{50}\overset{51}{1}\overset{52}{0}}$$

By rounding rule if 53 is 1 but all following are 0's we only add 1 if the 52 is a 1

b) $(1+(2^{-51}+2^{-52}+2^{-60}))-1 =$

$$1.0\ldots+(0.0\ldots\overset{51}{1}\overset{52}{1}\overset{53}{0}\ldots\overset{60}{0})-1 = \boxed{0.\underbrace{0\ldots}_{50}11}$$

---

8) $fl(1/3)+fl(2/3) =$

$0.\overline{01}\times 2^{0}$ $\quad$ $0.\overline{10}\times 2^{0}$

$fl(1/3) = 1.\overline{01}\ldots\overset{51}{1}\overset{52}{0}\overset{53}{1}\times 2^{-2}$

$fl(2/3) = 10.10\ldots 0101\times 2^{-2}$

$11.1\ldots\ldots 1001\times 2^{2}$

I think we are wrong about padding if the binary sequence is known to repete forever we wouldn't pad with 0's!

With Rounding the 53 bit will round up and as a result we get 1.0 for the final addition if it was just cut we would lose that value

10) a) no by rounding Rule

b) Yes by having a known 1 past the 53 rounding we round UP

---

12 ↓ Cont

Exercise 0.3 12, 14, 16

12) Find for each that $|fl(x) - x| \le \frac{\epsilon_{mach}}{2} = 2^{-52}$

a) $x = 1/3$ $\quad 0.\overline{01} = 1.01 \ldots \overset{51\,52}{01} \times 2^{-2}$

$$\begin{array}{l} \overset{51\,52}{1.01 \ldots 0101} \times 2^{-2} \\ - \underline{1.01 \ldots 01} \\ 0.00\,0 \ldots \overline{01} \times 2^{-2} \end{array} = \boxed{2^{-56} < \epsilon_{mach}}$$

53 54

55, 56

b) $x = 3.3$

$3.3 \quad 11.\overline{0100}11 \ldots 0011\overset{52\,53}{0011} \times 2^{-1}$

$fl(3.3) \quad 11.0100 11 \ldots 0011 0010 \times 2^{-1}$

$= 0.000 \ldots 10 \; 2 \times 2^{-1}$

-53

$= \boxed{2^{-54} < \epsilon_{mach}}$

14) a) $(4.3 - 3.3) - 1$

$100.0\overline{1100} - 011.0\overline{1100}$

$1.000\overline{1100} \times 2^{2} - 1.10\overline{1100} \ldots 011 \times 2^{1}$

$1.000\overline{1100} \ldots 01 \times 2^{2} = 0.110\,1100 \ldots$

$$\begin{array}{l} - 1.000110011001100 11 \ldots 01 \times 2^{-2} \\ 0.110110011001100 11 \ldots 01 \\ 0.010 \ldots 0 \times 2^{-2} = 1.00 \times 2^{-2} - 1 = \boxed{0} \end{array}$$

b, c, d

$\boxed{\text{are the same!}}$

Exercise 0.3

16)

$\epsilon_{mach} = 2^{-52}$

$|f(x) - x| > \epsilon_{mach}$

a) 2.75

$$\begin{array}{r} 10.110 \ldots 0 \\ - \; 10.110 \ldots 0 \\ \hline \end{array} = \boxed{0.0000 < \epsilon_{mach}!}$$

b, c Are te Same

Execise 0.4 (1, abc, 2, 4)

1) alternate forms of X

a)

$$\frac{1-\sec x}{\tan^2 x} \quad x \text{ at } 0, 2\pi, -2\pi$$

$$\frac{(1-\sec x)}{\tan^2 x} \times \frac{(1+\sec x)}{1+\sec x} = \frac{1-\sec^2 x}{\tan^2 x+\tan^2 \sec x}$$

$$\frac{-\tan^2 x}{\tan^2 x+\tan^2 x \sec} = \boxed{\frac{1}{1+\sec x} \overset{x=0}{=} \boxed{1/2}}$$

b)

$$\frac{1-(1-x)^3}{x} \quad \text{as } x \text{ approches } 0$$

$$(1-x)^2 = (1-2x+x^2)(1-x)$$

$$\frac{1-(1-2x+x^2-x+2x^2+x^3)}{x} = \frac{2x-x^2+x-2x^2-x^3}{x} =$$

$$\frac{3x-3x^2-x^3}{x} = \boxed{3-3x-x^2 \quad x=0} = 3$$

c)

$$\frac{1}{1+x} - \frac{1}{1-x} \quad \text{as } x \text{ approches } 0$$

$$\frac{(1-x)-(1+x)}{(1-x)(1+x)} = \boxed{\frac{2x}{1-x^2} \overset{x=0}{=} 0}$$

Exercise 0.4 2,4

2) Find Roots of $x^2 + 3x - 8^{-14} = 0$

Since $b = 3 > 0$ and $4(-8)^{-14} \ll 3^2$

We use

$$x_1 = \frac{b \mp \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{2c}{(b + \sqrt{b^2 - 4ac})}$$

$$x_1 = \frac{3 + \sqrt{9 - 4(-8)^{-14}}}{2} = \frac{3 + 3}{2} = \boxed{3}$$

$$x_2 = \frac{2(-8^{-14})}{3 + \sqrt{3 - 4(-8)^{-14}}} = \boxed{\frac{-2}{6}}$$

---

4) Evaluate to at least 3 decimal places

$x\sqrt{x^2 + 17} - x^2$ where $x = 9^{10}$

$$9^{10}\sqrt{9^{20} + 0} - 9^{10}$$

$$9^{20} - 9^{10} = 9^{10}(9^2 - 1) = 9^{20}(80) \approx \boxed{9.726e^{20}}$$