Erik Sundblad

Exam 2 (1.5.4, 2.1.7, 2.2.4a, 2.3.18, 2.4.9a) +Retleton

Sec 1.5

problem(4) looking for 10° at depth X. At 8° at f(9) and 15° at f(5). What is x by secant method?

So we are approximating $f(x) - 10 = 0 = g(x)$

$$\boxed{\begin{matrix} X_1 = 9 \\ X_0 = 5 \end{matrix}}$$

$$X_{n+1} = X_n - \frac{g(x_n)(x_n - x_{n-1})}{g(x_i) - g(x_{i-1})} = \text{Secant Method}$$

$$X_{n+1} = 9 - \frac{-2(9-5)}{-2-5} = 9 - \frac{-8}{-7} = \frac{55}{7} \approx 7.857m$$

By one pass of the secant method and without being able to know our actual temp at 7.857m. I conclude our best approximation is $\underline{55/7 \text{ meters}}$.

Sec 2.1

Problem(7) A given computation require 0.002s to complete 4000x 4000 mtx upper triangular Matrix equation. Estimate time needed for a 9,000 Equations and 9,000 unknowns.

Our computer does $(4000)^2$ opperation in 0.002s on 8×10^9 opperation/second. A general 9000 x 9000 set of equations and unknowns takes $\frac{2(9000)^3}{3} = 4.86\times10^{11}$ opperations

So we get $\frac{4.86\times10^{11}}{8\times10^9} = \boxed{60.75s}$

Erik Sundblad

Exam 2 (2.2.4a, 2.3.18, 2.4.9a) + Reflection

Section 2.2

Problem (4/a) Solve the system by LU factorization and 2 step back substitution.

a)

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$$\underset{A}{\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix}} \xrightarrow{R_3 - R_1} \underset{E_1 A}{\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}} \xrightarrow{R_2 - 2R_1} \underset{E_2 E_1 A = U}{\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}} \qquad A = E_1^{-1} E_2^{-1} U =$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \qquad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A = LU$

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}}$$

Now we use back substitution

$Lc = b$

$Ux = c$

L

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{matrix} c_1 + 0 + 0 = 0 \\ 2c_1 + c_2 + 0 = 1 \\ c_1 + 0 + c_3 = 3 \end{matrix} \quad \text{so } c = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

Then

$$U \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{matrix} 3x_1 + x_2 + 2x_3 = 0 \\ 0 + x_2 + 0 = 1 \\ 0 + 0 + 3x_3 = 3 \end{matrix} \quad \text{So } X = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Erik Sundblad

Exam 2 (2.3.18, 2.4.9a) + Reflecton

Section 2.3 Problem (18)

a) Show the system of equations

$$\begin{bmatrix} 811802 & 810901 \\ 810901 & 810001 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 901 \\ 900 \end{bmatrix}$$ has the

Solution $[1, -1]$:

$$811802(1) + 810901(-1) = 901$$
$$810901(1) + 810001(-1) = 900$$

b) Solve using double precision arithmetic using guassian elimination (in tableau or any other form). How many correct decimal places. Explain using Cond(A)

$$\left[\begin{array}{cc|c} 811802 & 810901 & 901 \\ 810901 & 810001 & 900 \end{array}\right] R_2 - \frac{810901}{811802} R_1 \text{ becomes}$$

$$\left[\begin{array}{cc|c} 811802 & 810901 & 901 \\ 0 & 1.23e^{-6} & -1.23183e^{-6} \end{array}\right] \underset{R_2\left(\frac{1}{1.23e^{-6}}\right)}{\sim} \left[\begin{array}{cc|c} 811802 & 810901 & \\ 0 & 1 & -1.001487805 \end{array}\right]$$

$$\underset{R_1 - 810901(R_2)}{\sim} \left[\begin{array}{cc|c} 811802 & 0 & 813007.4626 \\ 0 & 1 & -1.001487805 \end{array}\right] \underset{R_1/\frac{1}{811802}}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 1.001484922 \\ 0 & 1 & -1.001487805 \end{array}\right]$$

So by double precision we get <u>2 decimal places</u> of accuracy. We can find Cond(A) =

$$\|A\|_\infty \times \|A^{-1}\|$$

<u>Next Page!</u>

Exam 2 (2.3.18, 2.4.9a) + Reflection

Section 2.3
Problem (18) contined

$$A = \begin{bmatrix} 811802 & 810901 \\ 810901 & 810001 \end{bmatrix}$$

$$Cond(A) = ||A||_\infty \times ||A^{-1}||_\infty$$

$$||A||_\infty = 1622703$$

$$A^{-1} = \left[\begin{array}{cc|cc} 811802 & 810901 & 1 & 0 \\ 810901 & 810001 & 0 & 1 \end{array}\right] \quad R_2 - \frac{810901}{811802}$$

$$\left[\begin{array}{cc|cc} 811802 & 810901 & 1 & 0 \\ 0 & 1.23e^{-6} & -\frac{810901}{811802} & 1 \end{array}\right] \quad R_2/1.23e^{-6}$$

$$\left[\begin{array}{cc|cc} 811802 & -810901 & 1 & 0 \\ 0 & 1 & -812105.7914 & 813008.1301 \end{array}\right] \quad R_1 - 810901(R_2)$$

$$\left[\begin{array}{cc|cc} 811802 & 0 & 6.585374e^{11} & -6.592691e^{11} \\ 0 & 1 & -812105.8 & 813008.1301 \end{array}\right] \quad R_1/811802$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 811204.4542 & -812105.7914 \\ 0 & 1 & -812105.8 & 813008.1301 \end{array}\right] = A^{-1} = \begin{bmatrix} 811204.4542 & -812105.79 \\ -812105.7914 & 813008.1301 \end{bmatrix}$$

$$||A^{-1}||_\infty \approx 1625113.922$$

So our $Cond(A) = 1622703 \times 1625113.922 = 2.637e^{12}$

Our error magnification must be less than or equal too $\underline{2.637e^{12}}$

Cont ↓

Erik Sundblad

Exam 2 (2.3.18, 2.4.9a) + Retetori

(18) continued. Cond A = $2.637e^{12}$

Our error magnification is $\frac{\text{forward}}{\text{backward}} = \frac{\frac{\|x - x_a\|_\infty}{\|x\|_\infty}}{\frac{\|r\|_\infty}{\|b\|_\infty}}$

For our system this becomes:

$$\frac{\begin{pmatrix} 1 - 1.001484922 \\ -1 - -1.001487805 \end{pmatrix}}{\frac{1}{901}} \leq 2.637e^{12}$$

Which I argue is less than our Cond(A) of $2.637e^{12}$ by alot therefore our results are within predicted error!

Section 2.4 Find the PA=LU Factorization of Amtx

(9a)

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix} \underset{R_4+R_1}{\sim} \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 2 \end{bmatrix} \underset{R_1+R_3}{\sim} \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 2 \\ -1 & -1 & -1 & 2 \end{bmatrix}$$

$$\underset{R_1+R_2}{\sim} \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 2 \\ -1 & -1 & 1 & 2 \\ -1 & -1 & -1 & 2 \end{bmatrix} \underset{R_2+R_4}{\sim} \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 2 \\ -1 & -1 & 1 & 2 \\ -1 & -1 & -1 & 4 \end{bmatrix} \underset{R_2+R_3}{\sim} \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 2 \\ -1 & -1 & 1 & 4 \\ -1 & -1 & -1 & 4 \end{bmatrix} \underset{R_3+R_4}{\sim}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 2 \\ -1 & -1 & 1 & 4 \\ -1 & -1 & -1 & 8 \end{bmatrix}$$

So our PA=LU equals =

$$\underset{P}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 8 \end{bmatrix}}$$