Erik Sandblad

1.5 Exercise 1, 3, 6, 7ABC

1) Apply two Steps to each equation with $x_0 = 1$ $x_1 = 2$

a) $x^3 = 2x + 2$ $\quad x = \frac{x^3 - 2}{2}$ $\quad x_{i+1} = 2 - \frac{f(2)(2-1)}{f(2) - f(1)} \Rightarrow$

Step 1

$$\Rightarrow 2 - \frac{\frac{2^3 - 2}{2}(1)}{\frac{2^3-2}{2} - \frac{1^3 - 2}{2}} = \frac{3(1)}{3 + {}^{+}\frac{1}{2}} \quad 2 - \frac{3}{7/2} = 2 - 6/7 = \boxed{8/7}$$

$$\frac{6}{2} \ \frac{1}{2}$$

Step 2

$$x = 8/7 - \frac{f(8/7)(8/7 - 2)}{f(8/7) - f(2)} = \frac{\left(\frac{(8/7)^3 - 2}{2}\right)\left(-6/7\right)}{\frac{(8/7)^3 - 2}{2} - \frac{2^3 - 2}{2}} = 8/7 - \frac{\left(\frac{512}{343} - \frac{686}{343}\right)\left(\frac{-6}{7}\right)\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{512}{343} - \frac{686}{343}\right) - 3}$$

$$= \frac{8}{7} - \left(\frac{\frac{1044}{4802}}{\frac{-174}{686} - 3}\right) = \left(\frac{1044}{4802}\right)\left(\frac{686}{2232}\right) = \frac{+29}{434} + 8/7$$

$$\frac{29}{434} + \frac{496}{434} = \boxed{\frac{75}{62}}$$

Exercise 1.5 1, 3, 6, 7ABC

1) Use Secant with $x_0 = 1$ $x_1 = 2$ on the following eq's.

a) $0 = x^3 = 2x + 2 = x^3 - 2x - 2$

Step 1:

$$2 - \frac{(2^3 - 2(2) - 2)(1)}{(2^3 - 2(2) - 2) - (1^3 - 2 - 2)} = 2 - \frac{2}{5} = \boxed{\frac{8}{5}}$$

$2 \qquad +3$

Step 2

$$\frac{8}{5} - \frac{(8/5^3 - 2(8/5) - 2)(8/5 - 2)}{(8/5^3 - 2(8/5) - 2) - (2^3 - 2(2) - 2)} = \frac{\left(\frac{512}{125} - \frac{16}{5} - \frac{256}{125}\right)\left(\frac{-2}{10}\right)}{\left(\frac{512}{125} - \frac{16}{5} - \frac{256}{125}\right) - 2} =$$

$$\frac{8}{5} - \frac{\left(\frac{512}{125} - \frac{400}{125} - \frac{250}{125}\right)\left(\frac{-2}{10}\right)}{\left(\frac{512}{125} - \frac{400}{125} - \frac{250}{125}\right) - \frac{500}{250}} \approx \boxed{1.742268}$$

$x_0 = 1$ $x_1 = 2$

Step 1 b) $e^x + x - 7 = 0$

$$x = 2 - \frac{(e^2 + 2 - 7)(1)}{(e^2 + 2 - 7) - (e^1 + 1 - 7)} = 2 - \frac{2.38906}{5.67077}$$

$\boxed{x \approx 1.57870}$

Cont...

Exercise 1.5 1, 3, 6, 7 Cont

$$\downarrow \quad x = 1.57870$$

1.b

Step 2

$$x = 1.57870 - \frac{\left(e^{1.57870} + 1.57870 - 7\right)(1.57870 - 2)}{\left(e^{1.57870} + 1.57870 - 7\right) - \left(e^{2} + 2 - 7\right)} =$$

$$\boxed{x \approx 1.66016}$$

1.c $\quad e^x + \sin x = 4 \quad f(x) = e^x + \sin x - 4 \quad x_0 = 1 \; x_1 = 2$

Step 1

$$x = 2 - \frac{\left(e^2 + \sin 2 - 4\right)(1)}{\left(e^2 + \sin(2) - 4\right) - \left(e^1 + \sin(1) - 4\right)} \approx \boxed{1.09290658}$$

Step 2

$$x = 1.0929 - \frac{\left(e^{1.0929} + \sin(1.0929) - 4\right)(1.0929 - 2)}{\left(e^{1.0929} + \sin(1.0929) - 4\right) - \left(e^2 + \sin 2 - 4\right)} \approx \boxed{1.119356}$$

3) Same Eq from 1. with $x_0 = 1 \quad x_1 = 2 \quad x_2 = 0$!

a)

$f(x) = x^3 - 2x - 2 \quad$ Step 1.

$$x_3 = x_2 - \frac{\left(\frac{f(x_2)}{f(x_1)}\right)\left(\frac{f(x_2)}{f(x_1)} - \frac{f(x_0)}{f(x_1)}\right)(x_2 - x_1) + \left(1 - \frac{f(x_2)}{f(x_1)}\right)\left(\frac{f(x_2)}{f(x_0)}\right)(x_2 - x_0)}{\left(\frac{f(x_0)}{f(x_1)} - 1\right)\left(\frac{f(x_2)}{f(x_1)} - 1\right)\left(\frac{f(x_2)}{f(x_0)} - 1\right)}$$

Continued...

Exercise 1.5 3, 6, 7ABC

3) $x_0 = 1 \quad x_1 = 2 \quad x_2 = 0 \qquad f(x) = x^3 - 2x - 2$

Part A

Invers Quadratic Interpolation:

$$q = \frac{f(x_0)}{f(x_1)} \qquad r = \frac{f(x_2)}{f(x_1)} \qquad s = \frac{f(x_2)}{f(x_0)}$$

$$x_2 - \frac{r(r-q)(x_2-x_1) + (1-r)s(x_2-x_0)}{(q-1)(r-1)(s-1)} = x_3$$

$$0 - \frac{\frac{f(0)}{f(2)}\left(\frac{f(0)}{f(2)} - \frac{f(1)}{f(2)}\right)(0-2) + \left(1 - \frac{f(0)}{f(2)}\right)\left(\frac{f(0)}{f(1)}\right)(0-1)}{\left(\frac{f(1)}{f(2)} - 1\right)\left(\frac{f(0)}{f(2)} - 1\right)\left(\frac{f(0)}{f(1)} - 1\right)}$$

$$\frac{f(1)}{f(2)} = -3/2 \qquad \frac{f(0)}{f(2)} = -1 \qquad \frac{f(0)}{f(1)} = 2/3$$

$$x_3 = \frac{-1(\overset{0.5}{-1 + {}^{+}3/2})(-2) + (\overset{2}{1 + 1})(\tfrac{2}{3})(-1)}{\left(\frac{-3}{2} - 1\right)(-1-1)(2/3 - 1)}$$

$$(-5/2)(-2) \qquad (-1/3$$

Step 1

$$x_3 = -\frac{1 + \frac{-4}{3}}{-10/6} = -\left(-1/3 \times \frac{-\overset{2}{\cancel{6}}}{10}\right) = \boxed{-1/5}$$

Step 2 next page

Contine

$x_0 = 2,\ x_1 = 0,\ x_2 = 1/5$

Exercise 1.5 3, 6, 7 abc

3) continued... $f(x) = x^3 - 2x - 2$ $\quad x_0 = 2, x_1 = 0\ x_2 = -1/5$

Step 2.

$$x_3 = x_2 - \frac{r(r-q)(x_2 - x_1) + (1-r)s(x_2 - x_0)}{(r-1)(s-1)(q-1)} \qquad q = \frac{f(x_0)}{f(x_1)} = -1$$

$$r = \frac{f(x_2)}{f(x_1)} = \frac{-13/8}{-2} =$$

$$x_3 = -1/5 - \frac{\frac{13}{16}\left(\overset{29/16}{\frac{13}{16}+1}\right)\left(\overset{-1/5}{-1/5 - 0}\right) + \left(\overset{3/16}{1-\frac{13}{16}}\right)\left(\frac{-13}{16}\right)\left(\overset{-11/5}{-1/5 - 2}\right)}{(-1+1)\left(\frac{13}{16}-1\right)\left(\frac{-13}{16}-1\right)} \qquad s = \frac{f(x_2)}{f(x_0)} = \frac{-13/8}{2} = \frac{-13}{16}$$

$$x_3 = -1/5 - \frac{\left(\frac{13}{16}\right)\left(\frac{29}{16}\right)\left(\frac{-1}{5}\right) + \left(\frac{3}{16}\right)\left(\frac{-13}{16}\right)\left(\frac{-11}{5}\right)}{(-2)(-3/16)(-19/16)} \approx \boxed{-0.1199618}$$

Part: B, C will Be similar Steps

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(6)

if the secant Method Converges to r, $f'(r) \neq 0$ and $f''(r) \neq 0$

Then the approximate error is $e_{i+1} \approx \left|\frac{f''(r)}{2f'(r)}\right| e_i e_{i-1}$ holds

Prove that $\lim_{i\to\infty} e_{i+1}/e_i^{\alpha}$ exists and is non zero for $\alpha > 0$

then $\alpha = (1+\sqrt{5})/2$ and $e_{i+1} \approx \left|f''(r)/f'(r)\right|^{\alpha-1} \cdot e_i^{\alpha}$!

Continued

Exercise 1.5 6, 7abc $\qquad f''(r) \neq 0,\ f'(r) \neq 0$

6) Continued... $e_{i+1} \approx \left|\frac{f''(r)}{f'(r)}\right| e_i e_{i-1}$

<u>Show</u>

$\lim_{i \to \infty} e_{i+1}/e_i^{\alpha}$ <u>exists</u> and non zero for $\alpha > 0$

and $\underline{\alpha = (1+\sqrt{5})/2}$, <u>Finally</u> $e_{i+1} = \left|\frac{f''(r)}{f'(r)}\right|^{\alpha-1} e_i^{\alpha}$

$$f = \left|\frac{f''(r)}{2f'(r)}\right|$$

$e_{i+1} = f e_i e_{i-1}$ divide both by e_i^{α} we get

$$e_{i+1}/e^{\alpha} = f e_i e_{i-1} ; \quad e_i^{-\alpha} = \frac{f e_i^{1-\alpha} e_{i-1}}{e_{i+1}} = \alpha = -\log_{e_i}\left(\frac{f e_i^{1-\alpha} e_{i-1}}{f e_i e_{i-1}}\right)$$

$$\alpha = -\log_{e_i}\left(e_i^{-\alpha}\right) = -\log_{e_i}\left(f e_{i-1}^{-\alpha \cdot \alpha} e_{i-2}^{-\alpha}\right) = \log_{e_i}\left(f^{-2\alpha} e_{i-2}^{-2\alpha} e_{i-3}^{\alpha}\right)$$

$$\alpha = \frac{\log_{2}\left(f^{-2\alpha} e_{i-2}^{-2\alpha} e_{i-3}^{-\alpha}\right)}{\log_{\alpha}\left(f^2 e_{i-2}^2 e_{i-3}^2\right)} = \alpha = \frac{\log f^{2\alpha} e_{i-2}^{2\alpha} e_{i-3}^{\alpha}}{\log\left(f^2 e_{i-2}^2 e_{i-3}^2\right)} \approx \frac{a_n}{a_{n-1}} \overset{?}{\approx}$$

therefore $\alpha = \frac{1+\sqrt{5}}{2}$! And we see $\underline{e_{i+1} = f e_i^{\alpha}}$

Exercise 1.5 7 abC

7) Analyze Speed of convergence for
A) Bisection B) Secant C) Fixed Point itteration

A): we find Bisection runs in logrithmic time where we half our error each itteration!

B) Secant Method:
We would say the secant method converges superlinearly as its e reduction is a power greater than One but less than 2.
$\alpha = 1+\sqrt{5}/2 \approx 1.61803\ldots$

C) We observe that fixed point itteration is convergent linearily as its error is a rate S such that $\lim_{i\to\infty} \frac{e_{i+1}}{e_i} = S < 1$! We should note C here itself doesn't converge but fixed point itteration set propurly would!

In Conclusion we can stat secant is the fastest followed by fixed point in second and Bisection being the slowest!