

$\langle 2, 1 \rangle$ $(0, 0)$

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Worksheet 18

Exercise 1. Is the gradient of $f(x, y) = \sqrt{1 + 2x^2 + y^2}$ orthogonal to it's level curve through $(1, 1)$?

$$f_x(x, y) = \frac{(1 + 2x^2 + y^2)^{-1/2}}{2} (4x) = \frac{2x}{\sqrt{1 + 2x^2 + y^2}}$$

$$f_y(x, y) = \frac{(1 + 2x^2 + y^2)^{-1/2}}{2} (2y) = \frac{y}{\sqrt{1 + 2x^2 + y^2}}$$

$$\nabla f = \left\langle \frac{2x}{\sqrt{1 + 2x^2 + y^2}}, \frac{y}{\sqrt{1 + 2x^2 + y^2}} \right\rangle \quad (1, 1) \quad N = \langle 2, 1 \rangle$$

$$\nabla f(1, 1) = \left\langle \frac{2(1)}{\sqrt{1 + 2(1) + 1}}, \frac{1}{\sqrt{1 + 2(1) + 1}} \right\rangle = \left\langle \frac{2}{2}, \frac{1}{2} \right\rangle$$

$$\nabla f(1, 1) \langle 1, 1/2 \rangle \cdot \vec{v} = 0$$

$$1a + 1/2 b = 0$$

$$\sqrt{a^2 + (2a)^2} = 1 \quad \vec{v} = \left\langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \right\rangle \quad a = 1/2 b$$

$$3a^2$$

$$a\sqrt{3} = 1$$

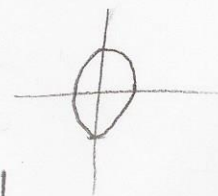
$$a = 1/\sqrt{3}$$

$$2 = \sqrt{1 + 2x^2 + y^2} = 2 = 1 + 2x^2 + y^2 \quad 3 = 2x^2 + y^2$$

$$= \frac{4/y}{2y} = \frac{2x}{y} = \frac{1}{1} = -2$$

$$\vec{v} = \langle 1, -2 \rangle$$

So is orthogonal



Exercise 2. Find the equation of the plane tangent to the surface $F(x, y, z) = \frac{x^2}{9} + \frac{y^2}{25} + z^2 - 1 = 0$ at the point $P_0(0, 4, \frac{3}{5})$.

$$F_x = \frac{2x}{9} \text{ at } P_0 = 0(x-0)$$

$$F_y = \frac{2y}{25} \text{ at } P_0 = \frac{8}{25}(y-4)$$

$$F_z = 2z \text{ at } P_0 = \frac{6}{5}(z - \frac{3}{5})$$

$$\text{The plane: } 0(x-0) + \frac{8}{25}(y-4) + \frac{6}{5}(z - \frac{3}{5}) = 0$$

$x, y, z=0$

y

x

$x, z, y=0$

z

x

$y, z, x=0$

z

y

