

$$2\left(\frac{4}{3}\right) + 5\left(-\frac{4}{3}\right) = -\frac{4}{3}$$

$$\frac{8}{3} - \frac{20}{3} = -\frac{12}{3} = -4$$

$$P = \left(\frac{4}{3}, -\frac{4}{3}, \frac{4}{3}\right)$$

Name: \_\_\_\_\_

# Worksheet 19

Exercise 1. Find the point on the plane  $x + 2y + z = 2$  which is closest to the point  $P(2, 0, 4)$ .

plane  $x + 2y + z = 2$   
 point  $(2, 0, 4)$

$$f(x, y) = 2x^2 + 4xy + 5y^2 + 8$$

$$\begin{cases} f_x = 4x + 4y \\ f_y = 10y + 4x \end{cases} = 0$$

$$\begin{cases} x = -y \\ 5y = -x \end{cases}$$

$$\begin{cases} x + y = 0 \\ 2x + 5y = -4 \end{cases}$$

$$\begin{cases} 3y = -4 \\ y = -\frac{4}{3} \\ x = \frac{4}{3} \end{cases}$$

$$D(x, y) = f_{xx}f_{yy} - (f_{xy})^2 = 4 \times 10 - (4)^2 = 40 - 16 = 24 > 0$$

$x, y, z$  to  $P$

$$d(x, y) = \sqrt{(x-2)^2 + (y-0)^2 + (z-4)^2}$$

$$d(x, y, z-x-2y) = \sqrt{(x-2)^2 + y^2 + (2-x-2y-4)^2}$$

$$f(x, y) = d(x, y)^2$$

$$(x-2)^2 + y^2 + (2-x-2y-4)^2 = (-x-2y-2)(-x-2y-2)$$

$$f(x, y) = x^2 - 4x + 4 + y^2 + x^2 + 2xy + 2x + 4y^2 + 4y + 2x + 4y + 4$$

$$2x^2 + 0x + 8 + 5y^2 + 8y + 4xy$$