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Introduction to Computation Theory

Quiz 1 – Review (10 pts)

Answer all questions

[3 pts]

1. Write formal descriptions of the following sets.
   a. The set containing all integers that are greater than 5
   b. The set containing the empty string
   c. The set containing nothing at all

$A = \{a : a \in \mathbb{Z} \text{ and } a > 5\}$

$\{\text{" "}\} = S$

$S = \emptyset$

[5 pts]

2. Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. The unary function $f: X \longrightarrow Y$ and the binary function $g: X \times Y \longrightarrow Y$ are described in the following tables.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

a. What is the value of $f(2)$? 7

b. What are the range and domain of f? Domain $= X = \{1,2,3,4,5\}$, Range $= \{6,7\}$

c. What is the value of $g(2, 10)$? 6

d. What are the range and domain of g? Domain of $g = X \times Y$, Range $= Y$

e. What is the value of $g(4, f(4))$? 8

$g(4,7) = 8$

[2 pts]

3. If A has a elements and B has b elements, how many elements are in A × B? Explain your answer

$|A| = a$ $|B| = b$

$|A \times B| = a \cdot b$

When calculating the cross product we yield a set of ordered pairs of all the elements of A. Matched to all elements of B.

$A \times B = \{a_1 b_1, a_1 b_2 \ldots a_1 b_b, a_2 b_1 \ldots a_a b_b\}$

Formal Proof by induction

Base Case: is true $A=\{1\}$ $B=\{1\}$

$|A|=1$ $|A\times B|=1$ $A\times B=\{(1,1)\}$

$|B|=1$

Hypothesis is for any sets A, and B with cardinalidy a, b : the Cordinality of $A\times B = a\cdot b$!

I will show that $C = A+\{\ \}$, and $|C\times B|=(a+1)b$.

For any additional element added to A the cross adss that element and evey element of B to the set $A\times B$. Incresing the set by b so that the final set equals $ab+b$ therefore for any sets where there cardinality is $\in \mathbb{Z}^+$. The cardirulity of the cross of the two sets equals the product of the Cardinclity of those sets $\blacksquare$