

Name: Erik Sundblad

Worksheet 21

Exercise 1. Evaluate the average value of $f(x, y) = ye^{xy}$ on $R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq \ln 2\}$.

$$\int_0^1 \int_0^{\ln 2} ye^{xy} dy dx = \int_0^1 \left[\frac{e^{xy} - 1}{x} \right]_0^{\ln 2} dx = \int_0^1 \frac{e^{x \ln 2} - 1}{x} dx$$

Handwritten calculation: $\frac{1 - \ln(2)}{\ln(2)} \approx 0.41427$

Handwritten note: $e^x - \frac{1}{x} \Big|_0^{\ln 2} = 2 - \ln(2) - 1 - 0 = 1 - \ln(2)$

Exercise 2. Draw the following regions for integration and set up an integral of $f(x, y)$ over the region

a. R in the xy -plane bounded by $x = 0$, $x = \frac{\pi}{4}$, $y = \sin x$, and $y = \cos x$.

$$\iint_R f(x, y) dA = \int_0^{\pi/4} \int_{\sin x}^{\cos x} f(x, y) dy dx$$

Handwritten note: Switch!

b. R in the xy -plane bounded by $x = y^2$ and $x = 1 - y^2$.

$$\int_{-\sqrt{1/2}}^{\sqrt{1/2}} \int_{y^2}^{1-y^2} f(x, y) dx dy$$

Handwritten notes: $y^2 \leq 1 - y^2$, $x y^2 = 1$, $y = \sqrt{1/2}$

c. R in the xy -plane bounded by $y = x$, $y = 8 - x$, and $y = 1$.

$$\int_1^4 \int_y^{8-y} f(x, y) dx dy$$

d. R in the xy -plane bounded by $y = x^2$, $y = -x + 2$, and $y = x + 2$.

$$\int_0^1 \int_{-x+2}^{x+2} f(x, y) dy dx + \int_1^2 \int_{x^2}^{x+2} f(x, y) dy dx$$

Handwritten notes: $x^2 = -x + 2$, $2 - x = x^2$, $2 - x = x(x + 1)$, $1 = \frac{2-x}{x^2}$, $\frac{2}{x^2} - \frac{1}{x}$