Name: Erik Sundblad **Worksheet 9**

Exercise 1. Plot the function $\mathbf{r}(t) = \langle \cos(\pi t), \sin(\pi t), e^{-t} \rangle$, $t \geq 0$. Then,

a) Evaluate $\lim_{t \to 2} \mathbf{r}(t)$.

b) Evaluate $\lim_{t \to \infty} \mathbf{r}(t)$.

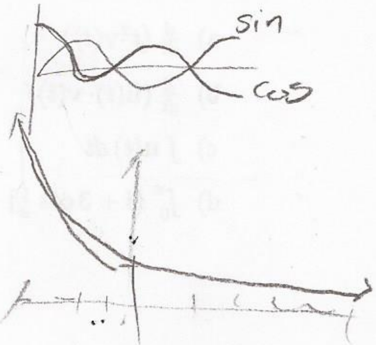
c) At what points is $\mathbf{r}$ continuous?

$r(0) = \langle 1, 0, 1 \rangle$

$r\,\tfrac{1}{2}\ \langle 0, 1, e^{-1/2}$

$r(1)\ \langle -1, 0, e^{-1}$

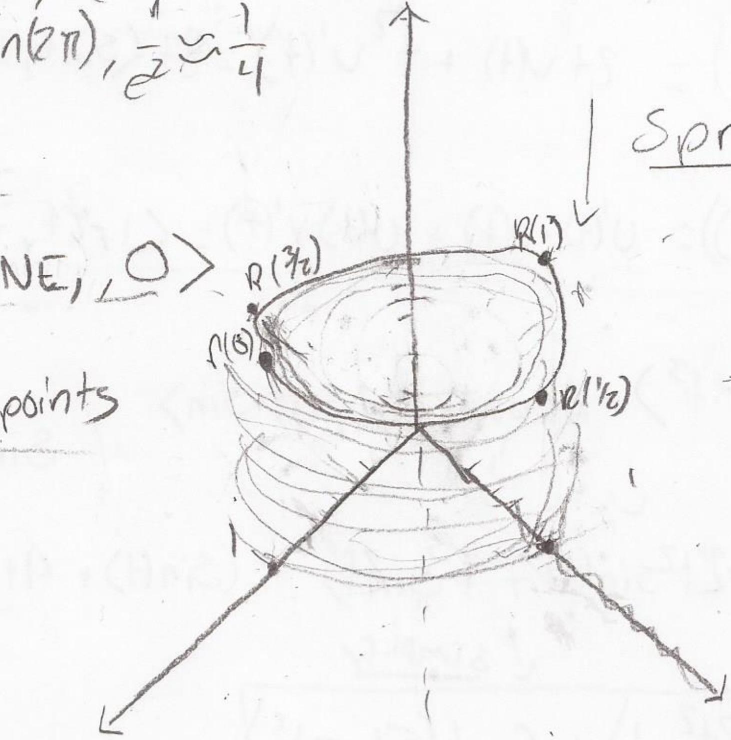
$r(3/2)\ \langle 0, -1, e^{-3/2}$



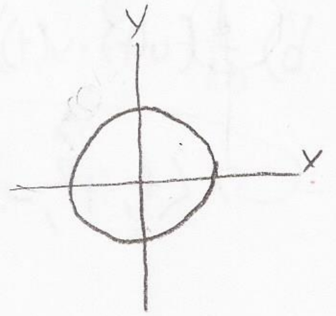
$\langle 1, \ 0, \ e^{-2} \rangle$

$r(t) = \text{Cos}(2\pi), \text{Sin}(2\pi), \frac{1}{e^2} \approx \frac{1}{4}$

$\lim t \to 2$



Spring



$\lim t \to \infty\ r(t) = \langle \text{DNE}, \text{DNE}, 0 \rangle$ DNE

r is continous at all points

Exercise 2. Given $\mathbf{u}(t) = t\mathbf{i} + t^2\mathbf{j} - t^3\mathbf{k}$ and $\mathbf{v}(t) = \langle \sin t, 2\cos t, \cos t\rangle$, evaluate

a) $\frac{d}{dt}\left(t^2\mathbf{v}(t)\right)$

b) $\frac{d}{dt}\left(\mathbf{u}(t)\cdot\mathbf{v}(t)\right)$

c) $\int \mathbf{u}(t)\,dt$

d) $\int_0^\pi \left(\mathbf{i} + 3\cos\frac{t}{2}\mathbf{j} - 4t\mathbf{k}\right)\,dt$

a) $\frac{d}{dt}\left(t^2 v(t)\right) = 2t\,v(t) + t^2 v'(t) = 2t\langle \sin t, 2\cos t, \cos t\rangle + t^2\langle \cos t, -2\sin, -\sin\rangle$

b) $\frac{d}{dt}\left(u(t)\cdot v(t)\right) = u'(t)\cdot v(t) + u(t)\,v'(t) = \langle 1, 2t, -3t^2\rangle \cdot \langle \sin, 2\cos, \cos\rangle +$

$\langle t, t^2, -t^3\rangle \cdot \langle \cos t, -2\sin(t), -\sin\rangle$

$\left(\sin(t) + 4t\cos(t) - 3t^2\cos t\right)$

$\left(t\cos t - 2t^2\sin(t) + t^3\sin t\right) + \left(\sin(t) + 4t\cos(t) - 3t^2\cos(t)\right)$

simplify

$\boxed{\sin t\left(t^3 - 2t^2 - 1\right) + \cos t\left(5t - 3t^2\right)}$

c) $\int u(t)\,dt = t, t^2, -t^3 = \boxed{\left\langle \frac{t^2}{2}, \frac{t^3}{3}, -\frac{t^4}{4}\right\rangle + \langle C_1, C_2, C_3\rangle} \to \vec{C}$

d) $\int_0^\pi \langle 1, 3\cos t/2, -4t\rangle\,dt = \langle t, -6\sin(t/2), -2t^2\rangle \Big|_0^\pi$

$u = t/2$

$du = \frac{1}{2}dt$

$\langle \pi, -6, -2\pi^2\rangle - \langle 0,0,0\rangle = \langle \pi, -6, -2\pi^2\rangle$