Name: Erik Sundblad **Worksheet 14**

Exercise 1. Evaluate the following limits or show they do not exist.

a. $\lim_{(x,y)\to(-3,3)}(4x^2-y^2)$ Continous at all points (x,y)

$4(-3)^2-(3)^2$

$36-9=27$

$\lim_{(x,y)\to(-3,3)}(4x^2) - \lim_{(x,y)\to(-3,3)} y^2$

$4(-3)^2 \quad 3^2$

$36 - 9 = 27$

b. $\lim_{(x,y)\to(0,0)} \frac{x+2y}{x-2y}$

$\lim_{x,y\to 0,0} \frac{x+2y}{x-2y} = \underline{DNE}$

$\frac{x}{x-2y} + \frac{2y}{x-2y}$

$y=0, \lim_{(x,0)\to(0,0)} \frac{x+2(0)}{x-2(0)} = \boxed{1}$ $\neq$

The limit does not exist!

$x=y, \lim_{(x,y)\to 0,0} \frac{x+2x}{x-2x} = \lim_{x\to 0} \frac{3x}{-x} = \boxed{-3}$

$\neq$

$y=2x \lim_{x,y\to 0,0} \frac{x+2y}{x-2x} = \lim_{x\to 0} \frac{x+2(2x)}{x-2(2x)} = \frac{5x}{3x} = \boxed{-5/3}$

Exercise 2. Determine the points at which the following function is continuous.

$$f(x,y) = \begin{cases} \frac{3xy^2}{x^2+y^4} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$\frac{3xy^2}{x^2+y^4}$ $\{(x,y) \mid (x,y) \neq (0,0)\}$

domain of f is $\mathbb{R}^2$

0 $\{(x,y) \mid (x,y) = (0,0)\}$

Continuous! by Polar Conversion

$y = 0$ $\lim_{x,0 \to 0,0} \frac{3xy^2}{x^2+0} = 0$

$x = y^2$ $\lim_{x,x^2 \to 0,0} \frac{3x(x^2)}{x^2+x^2}$ $\frac{3x^3}{2x^2} = \frac{3}{2}$

$\boxed{\text{The limit does not exist}}$

Convert $\frac{3xy^2}{x^2+y^2} \xrightarrow{\text{Polar}} \lim_{r \to 0} \frac{3r\cos\theta\, r^2\sin^2\theta}{r^2\cos\theta + r^2\sin\theta\, r^2\sin\theta} = \frac{3r^3\cos\theta\sin^2\theta}{r^2(\cos^2\theta + r^2\sin^4\theta)}$