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Shear Strength of Steel Fiber Reinforced Concrete (SFRC) Slender Beams

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Abstract

In the last four decades, many equations have been proposed to estimate the shear strength of Steel Fiber Reinforced Concrete (SFRC) beams. However, in terms of accuracy and uniformity of the prediction, there is considerable diversity between existing test results and researchers' predictions. In this study, by using the basic principle of mechanics and considering the slenderness effect of SFRC beams without stirrups, a new design expression is proposed for the shear strength of SFRC beams. The proposed equation and researchers' predictions are compared to the test results of 170 SFRC beams without stirrups. It is found that the proposed equation shows good agreement with regard to the existing test results.

Keywords: *steel fibers, reinforced concrete, shear strength, slenderness effect, beam*

1. Introduction

Although Steel Fiber Reinforced Concrete (SFRC) is often considered a relatively new material, it has been developed gradually over many years. During the past four decades, numerous studies regarding to experimental and analytical methods for evaluating mechanical properties of SFRC have been reported. SFRC has been developed to improve the behaviour of concrete which is a brittle material with low tensile strength. Fibers are intended to enhance the mechanical properties, particularly toughness, ductility and energy absorbing capacity under impact strength, durability, etc. (Li *et al.*, 2001; Nelson *et al.,* 2002; Balendran *et al.*, 2002; Naaman, 2003; Balaguru and Najm, 2004; Wang *et al.*, 2010).

It is well established from many previous studies (Narayanan and Darwish, 1987; Li *et al.*, 1992; Shin *et al.,* 1994; Adebar *et al.,* 1997) that the addition of steel fibers in concrete significantly increases its shear strength. However, the contribution of steel fibers to the shear strength was observed to be in a wide range, such as the shear strength increase for normal strength concrete was observed to be 258% by adding steel fibers to the plain reference concrete in a study by Adebar *et al.* (1997), whereas a 9% increase was observed in a study by Shin *et al.* (1994). Since the interaction between the steel fibers and the concrete matrix is a complex phenomenon, it is difficult to predict the increase in shear strength accurately (Dinh *et al.*, 2010). Accurate shear strength prediction of SFRC beams is needed for using steel fiber in the construction industry widely.

The objective of this study is to develop semi-empirical equation for accurately predicting the shear strength of SFRC beams that can be implemented in design codes. An equation for predicting the shear strength of SFRC beams is proposed using the basic principle of mechanics and considering the factor of slenderness ratio. The proposed equation has been statistically evaluated and compared to those developed previously by other researchers.

2. Existing Shear Strength Models

Various models (Sharma, 1986; Narayanan and Darwish, 1987; Ashour *et al.*, 1992; Swamy *et al.*, 1993; Imam *et al.*, 1997; Khuntia *et al.*, 1999; Kwak *et al.*, 2002; RILEM, 2003; Yakoub, 2011; Gandomi *et al.*, 2011; Dinh *et al.*, 2011) have been proposed to predict shear strength of SFRC beams without stirrups. The primary parameters that affect the shear strength of SFRC beams are the compressive strength of concrete (f_c), fiber length (L_f), fiber diameter (D_f), bond factor (d_f), volume fraction of steel fibers (V_f), flexural reinforcement ratio (ρ), slenderness ratio (a/d), and effective depth of the beam (d), and yield strength of flexural reinforcement (f_y).

The models considered in this study are summarized in Table 1. RILEM (2003) recommended an analytical approach for the prediction of the fiber reinforcement contribution in terms of shear resistance of concrete beams. Sharma (1986), Narayanan and Darwish (1987) and Ashour *et al.* (1992) proposed an empirical equation for predicting the shear strength of SFRC beams based on the test results. Imam *et al.* (1997) modified an expression developed by Bažant and Kim (1984) to predict the shear strength of normal strength conventional concrete beams. The Imam *et al*. (1997) equation differs from the Bažant and

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Table 1. Existing Shear Strength Models for SFRC Beams without Stirrups

Reference	Shear strength models (MPa)
Sharma (1986)	$v_u = kf_t(d/a)^{0.25}$ $k = 1$ and 2/3 for direct and indirect tension tests, respectively; $k = 4/9$ if f_t is obtained using modulus of rupture; or $f_t = 0.79f_c^{0.5}$
Narayanan and Darwish (1987)	$v_u = e[0.24f_{sp} + 80\rho d/a] + v_b$, $f_{sp} = f_{cuf}/(20 - \sqrt{F}) + 0.7 + 1.0\sqrt{F}$ $v_b = 0.41\,\tau F$, $e = 1$ for $a/d > 2.8$, $e = 2.8\,d/a$ for $a/d \le 2.8$ $d_f = 0.5$ for round, 0.75 for crimped, 1.0 for indented fibers.
Ashour *et al.* (1992)	$v_u = (2.11f_c^{1/3} + 7F)(\rho d/a)^{1/3}$ for $(a/d) \ge 2.5$
Swamy *et al.* (1993)	$v_u = 0.37\,\tau V_f\, L_f/D_f + 0.167\sqrt{f_c}$
Imam *et al.* (1997)	$v_u = 0.6\dfrac{1+\sqrt{(5.08/d_a)}}{\sqrt{1+d/(25d_a)}}\sqrt[3]{\rho(1+4F)}\left[f_c^{0.44} + 275\sqrt{\dfrac{\rho(1+4F)}{(a/d)^5}}\right]$ $d_f = 0.50$ for smooth, 0.9 for deformed, 1.0 for hooked fibers.
Khuntia *et al.* (1999)	$v_u = (0.167 + 0.25F)\sqrt{f_c}$ $d_f = 2/3$ for plain and round, 1.0 for hooked or crimped fibers.
Kwak *et al.* (2002)	$v_u = 2.1ef_{sp}^{0.7}(\rho d/a)^{0.22} + 0.8(0.41\,\tau F)^{0.97}$ $e = 1$ for $a/d > 3.5$; $e = 3.5d/a$ for $a/d \le 3.5$
RILEM (2003)	$v_{Rd,3} = v_{cd} + v_{fd}$, $v_{cd} = 0.12k(100\rho f_c)^{1/3}$, $k = (1 + \sqrt{200/d}) \le 2$, $\rho \le 0.02$, $v_{fd} = 0.7k_f\, k_l\, \tau_{fd}$, $k_1 = (1600 - d)/1000 \ge 1$, $\tau_{fd} = 0.12f_{eqk,3}$, $f_{eq\,,3}$= characteristic value of the equivalent tensile strength, $k_f = 1$ for rectangular sections
Yakoub (2011)	$v_u = \beta\sqrt{f_c}(1 + 0.70V_f\, L_f\, d_f/D_f)$ for $a/d \ge 2.5$ $\beta = \dfrac{0.40}{1+1500\varepsilon_x}\dfrac{1300}{1000+s_{xe}}$, $\varepsilon_x = \dfrac{M/d_v + V}{2E_sA_s}$ $s_{xe} = \dfrac{35s_x}{16+d_a} \ge 0.85s_x$ M and V are the external failure moment and shear acting on the section, s_x = crack spacing parameter ($\cong d_v$, flexural lever arm), $d_v = 0.9d$ or $d_v = 0.72h$, ε_x = longitudinal strain at the middepth, d_f = 0.83 for crimped, 0.89 for duoform, 1.00 for hooked, 0.91 for rounded fibers.
Gandomi *et al.* (2011)	$v_u = \dfrac{2d}{a}(\rho f_c + v_b) + \dfrac{d}{2a}\dfrac{\rho}{(288\rho - 11)^4} + 2$
Dinh *et al.* (2011)	$v_n = 0.13\rho f_y + 1.2\left(\dfrac{V_f}{0.0075}\right)^{1/4}\left(1 - \dfrac{c}{d}\right)$, $c = 0.1h$

f_{sp} = computed value of split-cylinder strength of fiber concrete; f_{cuf} = cube strength of fiber concrete; d_a = maximum aggregate size; h = beam height; F = fiber factor $(= L_fV_fd_f/D_f)$; v_u = ultimate shear strength; v_c = shear resistance of concrete, v_{fb} = contribution of steel fibers to shear strength; τ = average interfacial bond stress of fiber matrix; τ_{fd} = design shear stress; e = arch action factor.

Kim equation only in that the reinforcement factor (ρ(1+4F)) was substituted in place of the flexural reinforcement ratio and the constants were calibrated to the result of statistical analysis. Kwak *et al.* (2002) proposed shear strength equation by introducing additional terms that account for the influence of tensile strength on arching action and the direct contribution of the fibers to shear resistance to Zsutty's (1971) equation. Yakoub (2011) modified the CSA Committee A23.3 (2004) general shear design method and Bažant and Kim equation to predict the shear strength of SFRC beams without stirrups. Gandomi *et al.* (2011) developed a nonlinear model to evaluate the shear strength of SFRC beams using linear genetic programming. Dinh *et al.* (2011) proposed a model which assumes that the shear strength of SFRC beams is composed of shear stress carried in the compression zone and tension transferred across diagonal cracks by steel fibers.

3. The Prediction of Shear Strength of SFRC Beams Without Stirrups

The failure of a shear critical Reinforced Concrete (RC) beam without stirrups occurs usually with the sudden formation of a critical diagonal tension crack. After its formation over a large distance of the beam, the compression zone crushes and the crack continues at the bottom side as a large horizontal crack along the tensile reinforcement bars through the end of the beam. Fig. 1 shows the shear stress and strain distributions of a RC

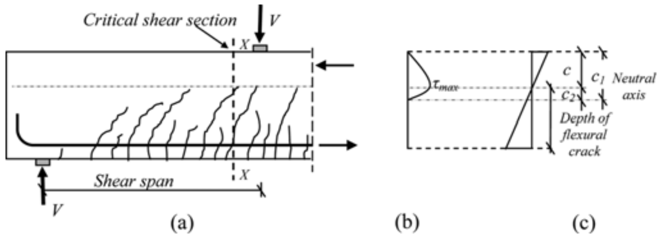

Fig. 1. Shear Stress and Strain Distribution in a RC Beam with Flexural Cracks: (a) Typical Crack Pattern, (b) Shear Stress Distribution, (c) Longitudinal Strain Distribution

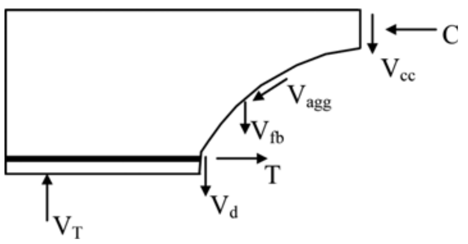

Fig. 2. Components of Shear Resistance

beam with flexural cracks (Arslan, 2012). In SFRC beams, control of cracking resulting from normal stress is more effective in comparison with the ones without fibers and the fibers provide increased stiffness after cracking (Furlan and De Hanai, 1997). The steel fibers help to form bridges through developing cracks in the concrete and provide more resistance to the crack growth (Li *et al*., 1993; Imam *et al*., 1994; Lim and Oh, 1999; Choi *et al*., 2007; Dinh *et al*., 2010). This eliminates the possibility of a sudden failure in concrete and allows for a more progressive failure.

Shear force in SFRC beams is transferred in various ways as indicated in Fig. 2. For slender beams, where shear span-to-depth ratio $(a/d) \geq 2.5$ (Zsutty, 1971; Kim and Park, 1996; Zararis, 2003; Park *et al*., 2006; Arslan, 2012), shear force is carried by the shear resistance of uncracked concrete in the compression zone (V_{cc}), the interlocking action of aggregates along the rough concrete surfaces on each side of the crack (V_{agg}), the additional bridging action due to steel fibers (V_{fb}), and the dowel action of the tensile reinforcement and steel fibers (V_d). The longitudinal forces T and C, are related to the flexural resistance of the member.

The mechanical characteristics of the concrete change significantly when a large volume of fiber is added. However, the tensile strength of the concrete is not modified considerably when the volume of fiber added is lower than 2% (ACI Committee 544, 1986; Hannant, 1978; Bentur and Mindess, 1990; Shah, 1991). It was noted that in the beams without stirrups failure occurred soon after the appearance of the diagonal crack, except in the beams containing 2% steel fibers (Junior and Hanai, 1997). The shear stress distribution for concrete without steel fiber can be modelled as a parabolic distribution over the effective shear depth with the maximum value at the neutral axis (Khuntia and Stojadinovic, 2001); Fig. 1(b). The shear stress at the neutral axis, τ_{max}, can be taken as:

$$\tau_{max} = \frac{V_{o,c}}{\frac{2}{3}b_w c_1} = f_t \tag{1}$$

in which b_w is the width of section, c_1 is the effective shear depth, f_t is the tensile strength of concrete and $V_{o,c}$ is the shear force of concrete without steel fiber, respectively. Using the method of satisfaction of strain compatibility and equilibrium conditions (Khuntia and Stojadinovic, 2001), c_1 can be calculated as (Fig. 1(c)):

$$c_1 = c\left(1 + \frac{\varepsilon_{cr}}{\varepsilon_c}\right) \tag{2}$$

in which c is the depth of compression zone above the tip of the diagonal crack, ε_c is the compressive strain in concrete, taken as 0.002, and ε_{cr} is the cracking strain in concrete.

$$\varepsilon_{cr} = \frac{f_t}{E_c} \tag{3}$$

The tensile strength of concrete f_t is assumed as $f_t = 0.3 f_c^{2/3}$ (Eurocode 2, 2004) and the modulus of elasticity E_c is taken as $4700\sqrt{f_c}$ (ACI Committee 318, 2008) for all levels of concrete strength. Considering that the modulus of elasticity of steel E_s = 200 GPa, the depth of the neutral axis can be calculated from force equilibrium, that is, the concrete compression force equals to the tensile force in steel, as has been previously proposed (Zararis and Papadakis, 2001):

$$\left(\frac{c}{d}\right)^2 + 600\frac{\rho c}{f_c d} - 600\frac{\rho}{f_c} = 0 \tag{4}$$

Substituting Eq. (3) and c/d obtained from Eq. (4) into Eq. (2), c_1/d is obtained in terms of c/d. Introducing c_1/d into the relationship:

$$V_{o,c} = \frac{2}{3} f_1 b_w c_1 = v_{o,c} b_w d \tag{5}$$

the shear strength of concrete without steel fiber can be calculated as (Arslan, 2012):

$$v_{o,c} = 0.2 f_c^{2/3}\left(\frac{c}{d}\right)(1 + 0.032 f_c^{1/6}) \tag{6}$$

Shear displacement across a crack is resisted in part by the flexural stiffness of the reinforcement that crosses the crack. Based on the test results, Junior and Hanai (1997) indicated that the strain measured was slightly less in the beams with fibers. At the points near the supports, normally there was an increase of stress when shear failure was imminent, due to the tied-arch mechanism. The highest stress in the reinforcement occurred in the beams with fibers, possibly because dowel action was more effective. It has been suggested by Swamy and Bahia (1979) that fibers increase the stiffness of the dowel zone helping to contain dowel crack growth, resulting in more efficient dowel contribution. Steel fiber contribution to shear strength of SFRC beams ($v_{d,fb}$) is defined by:

$$v_{d,fb} = 0.405\frac{L_f}{D_f}V_f\sqrt{f_c} \tag{7}$$

in which L_f, D_f and V_f are fiber length, fiber diameter and volume fraction of steel fibers, respectively. Imam *et al.* (1994) defined $v_{d,fb}$ for slender beams ($a/d \geq 2.5$) as:

$$v_{d,fb} = 0.162F\sqrt{f_c} \tag{8}$$

in which F is defined as fiber factor and equal to $\frac{L_f}{D_f}V_f d_f \cdot d_f$ is the fiber shape bond factor 0.5 for round fibers, 0.75 for crimped fibers, 1.0 for indented fibers, taken from Narayanan and Darwish's (1987) study.

Zsutty (1971) and EN1992:2004 shear procedure suggest that the influence of the amount of tensile reinforcement is proportional to $\rho^{1/3}$. Based on the experimental study involving high strength concrete (Kim and Park, 1994), concrete shear strength is proportional to $\rho^{0.31}$. Cladera and Mari (2005) and Arslan (2010) suggest that the tensile reinforcement has a greater influence ($\rho^{0.5}$) and the dowel strength of tensile reinforcement $v_{d,l}$, can be expressed approximately as follows (Arslan, 2010):

$$v_{d,l} = 0.3\sqrt{\rho f_c} \tag{9}$$

If the dowel strength of tensile reinforcement and steel fiber contribution to shear strength of SFRC beams are combined as:

$$v_d = 0.3\sqrt{\rho f_c} + 0.162F\sqrt{f_c} \tag{10}$$

Eq. (10) can be replaced approximately by the following simpler expression within the practical range of $20\ \text{MPa} \leq f_c \leq 105\ \text{MPa}$, $\rho \cong 0.5\%$, $F \cong 0.7$ (Fig. 3).

$$v_d = \sqrt{\rho(1+4F)f_c} \tag{11}$$

The shear strength of SFRC slender beams based on the primary shear transfer mechanisms can be represented as:

$$v_u = v_{o,c} + v_d \tag{12}$$

Substituting Eq. (6) and Eq. (11) into Eq. (12), the shear strength becomes:

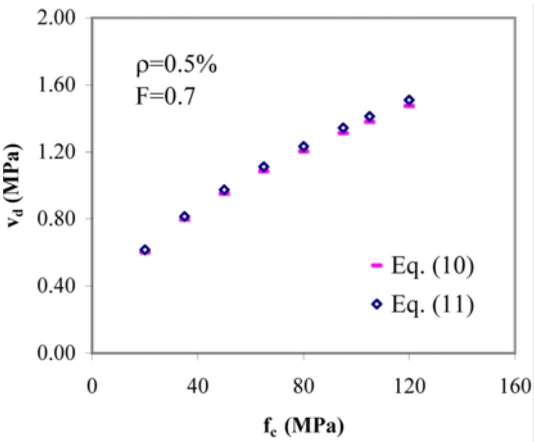

Fig. 3. Comparison of Eq. (10) and (11) by Changing f_c

$$v_u = 0.2f_c^{2/3}\left(\frac{c}{d}\right)(1+0.032f_c^{1/6}) + \sqrt{\rho(1+4F)f_c} \tag{13}$$

The weak influence of slenderness ratio (a/d) is neglected in some shear procedures (Eurocode 2, 2004; ACI Committee 318, 2008, TS500, 2000; Collins and Kuchma, 1999; Arslan, 2008). Tension stiffening causes a minor influence of a/d, which is described with a coefficient $k(a/d)$ in some design formulas (Bazant and Kim, 1984; Zsutty, 1971; Khuntia and Stojadinovic, 2001; CEB-FIP, 1990). CEB-FIP90 equation is proposed to determine the relationship between a/d and the shear strength, leading to the following expression for slender beams: $v_c \propto (3d/a)$. According to Zsutty (1971), the relationship between a/d and the shear strength is set to $(d/a)^{1/3}$. Based on the regression analysis, v_u is taken as proportional to $(3d/a)^{1/3}$ to identify the slenderness effect, which is similar to CEB-FIP90 equation. Considering the influence of the slenderness ratio (a/d), the following equation is derived for the ultimate shear strength of SFRC beams without stirrup:

$$v_u = \left(0.2f_c^{2/3}\left(\frac{c}{d}\right)(1+0.032f_c^{1/6}) + \sqrt{\rho(1+4F)f_c}\right)\left(\frac{3.0}{a/d}\right)^{1/3} \tag{14}$$

For practical design, a simple and accurate equation deduced from the simplification of the originally proposed equation is derived as:

$$v_u = \left(0.2f_c^{2/3}\left(\frac{c}{d}\right) + \sqrt{\rho(1+4F)f_c}\right)\left(\frac{3.0}{a/d}\right)^{1/3} \tag{15}$$

4. Evaluation of Proposed Shear Strength Equation

In the present study, the shear strengths of simply-supported beams with rectangular cross sections subjected to concentrated loads were studied. The deformation of a beam varies according to the types of loading and boundary conditions. In high shear regions of a simply supported beam, the effect of moment is relatively small. However, in a continuous beam, the negative moment region is subjected to both high shear and high moment and, thus, the effect of moment can be significant. The supporting conditions were not included in the analysis since the available databases are for simply-supported beams.

The proposed shear strength equation of SFRC beam was applied to the specimens that had been tested in previous experiments. In the comparison, 170 specimens tested by 17 researchers were considered. The specimens have a broad range of design parameters: $20.6 \leq f_c \leq 68.6$ MPa, $2.5 \leq a/d \leq 5.0$, $125 \leq d \leq 610$ mm, $0.79 \leq \rho \leq 5.72\%$, $45 \leq L_f / D_f \leq 133$, $0.22 \leq V_f \leq 3.00\%$.

From the comparisons of the observed shear strength with the eleven different predictions in Table 2, it can be seen that the proposed Eq. (15) has the lowest Coefficient of Variation (COV) and hence Eq. (15) provides better results than the eleven different methods for the prediction of shear strength of SFRC beams.

Table 2. Comparison of Predictions

Exp. / Prediction	MV	SD	COV
Exp. / Proposed Eq. (15)	1.04	0.16	0.15
Exp. / Sharma (1986)	1.60	0.36	0.23
Exp. / Narayanan and Darwish (1987)	1.26	0.24	0.19
Exp. / Ashour *et al.* (1992)	1.32	0.21	0.16
Exp. / Swamy *et al.* (1993)	1.43	0.28	0.19
Exp. / Imam *et al.* (1997)	1.04	0.26	0.25
Exp. / Khuntia *et al.* (1999)	1.44	0.30	0.21
Exp. / Kwak *et al*. (2002)	1.15	0.22	0.19
Exp. / RILEM (2003)	1.45	0.35	0.24
Exp. / Yakoub (2011)	2.36	0.76	0.32
Exp. / Gandomi *et al*. (2011)	0.84	0.15	0.17
Exp. / Dinh *et al*. (2011)	1.06	0.24	0.23

Table 3. Statistical Results of Each Type of Fiber

Fiber type	No. of beams	MV	SD
Hooked	95	1.05	0.17
Crimped	54	1.00	0.13
Indented	3	1.05	0.09
Sheared	3	0.83	0.05
Round	11	1.08	0.08
Duoform	4	1.28	0.10
Total	170	1.04	0.16

Figure 4 compares the proposed shear strength (Eq. (15)) and test results (Batson *et al.*, 1972; Swamy and Bahia, 1985; Niyogi and Dwarakanathan, 1985; Kadir and Saeed, 1986; Uomoto *et al.,* 1986; Mansur *et al.*, 1986; Lim *et al.*, 1987; Narayanan and Darwish, 1987; Swamy *et al*., 1993; Tan *et al*., 1993; Rosenbusch and Teutsch, 2002; Kwak *et al.*, 2002; Dupont and Vandewalle, 2003; Cucchiara *et al.*, 2004; Para-Montesinos *et al.*, 2006; Dinh *et al*., 2010; Ding *et al.,* 2011). The Mean Value (MV) and Standard Deviation (SD) for the ratio of the experimental to the proposed shear strength are 1.04 and 0.16, respectively.

Table 3 presents the statistical results of the ratio of the test results available in the literature to the estimates by the proposed equation for each type of fiber. While comparing among the previously proposed equations, the proposed equation can more accurately predict the shear strength of SFRC beams with hooked, crimped and round fibers with its COV value of 0.16, 0.13 and 0.07, respectively. The equation proposed by Narayanan and Darwish (1987), can accurately predict the shear strength of SFRC beams with indented and duoform fibers with its COV of 0.04. The equation proposed by Kwak *et al.* (2002), can accurately predict the shear strength of SFRC beams with indented and sheared fibers with its COV of 0.04. Contrarily, the equation proposed by Yakoub (2011) is less effective than the other equations since it has the highest COV value.

The equation proposed by Gandomi *et al.* (2011) overestimates the shear strength of beams with hooked, indented, dueform, crimped, round and sheared fibers with a low MV of 0.82, 0.90, 0.90, 0.87, 0.88 and 0.72, respectively. The equation proposed by Imam *et al.* (1997) overestimates the shear strength of beams with crimped, indented and sheared fibers due to their low MV of 0.94, 0.94 and 0.76, respectively. Similarly, the equation proposed by Kwak *et al.* (2002) overestimates the shear strength of beams with sheared fibers due to their low MV of 0.93. On the other hand, the equation proposed by Yakoub (2011) underestimates the shear strength of beams with hooked, indented, dueform, crimped, round and sheared fibers with a high MV of 2.61, 2.63, 2.45, 2.05, 1.91 and 1.69, respectively.

Prediction equations overestimating shear strength can be dangerous for designers as the amount of stirrups needed to prevent shear failure contains much more uncertainty. Although a small shear strength overestimation can be tolerated as in the case of Imam *et al.* (1997) models for beams with crimped, indented and sheared fibers, lower shear strength underestimations as in the case of Yakoub (2011) models for SFRC beams cannot be used in practice. Current design provisions are found to be conservative enough in predicting the shear strength of SFRC beams without stirrups, but large scatter in the ratio of the experimental to the predicted shear strength for existing test data is observed.

The effects of concrete compressive strength (f_c'), slenderness ratio (a/d) and fiber factor (F) on the proposed shear strength are

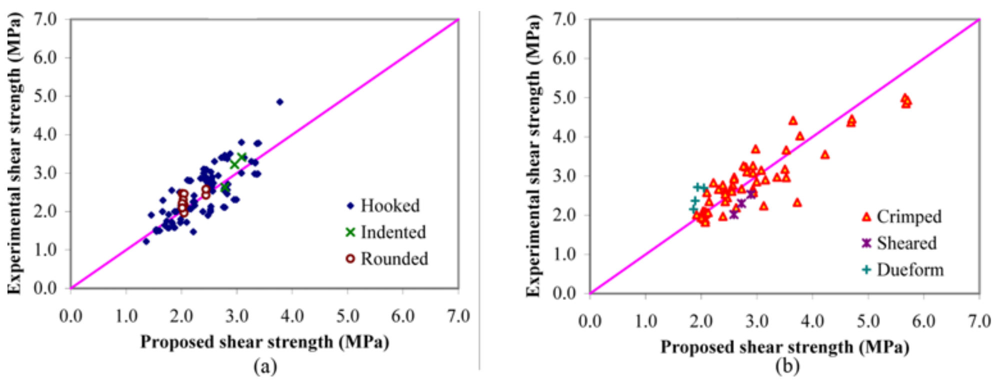

Fig. 4. Shear Strength Values using Eq. (15) versus Experimental Results: (a) For Hooked, Indented, Round, (b) For Crimped, Sheared, Duoform

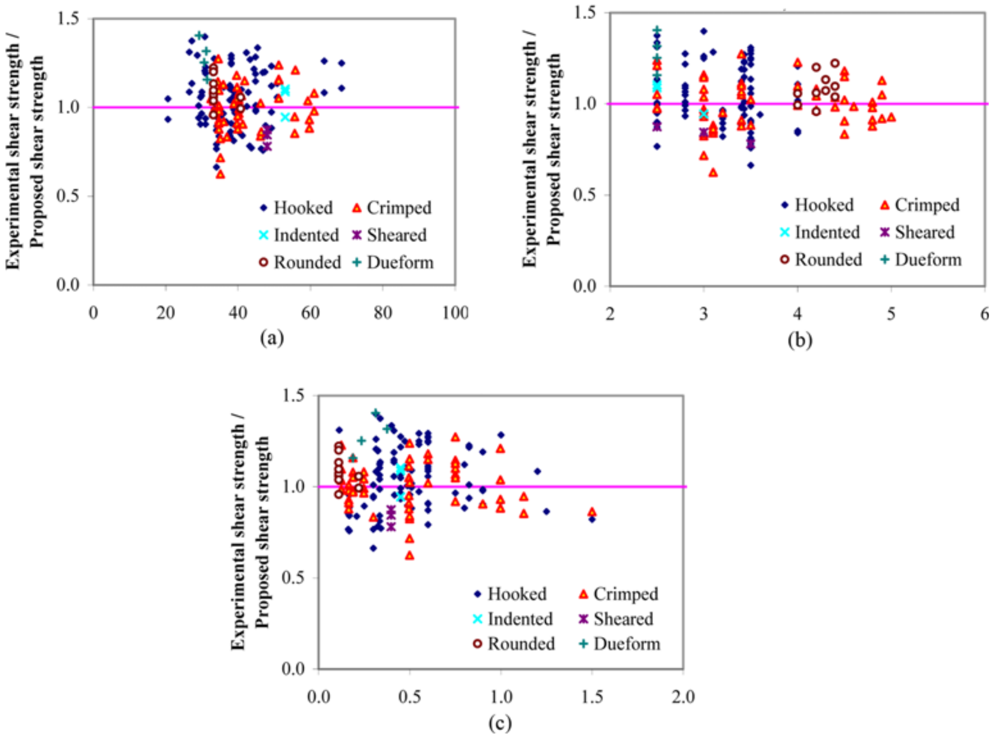

Fig. 5. The Effects of: (a) f_c (MPa), (b) *a*/*d*, (c) *F*

discussed as follows.

Figure 5(a) shows the effect of concrete compressive strength on the ratio of the experimental to the proposed shear strength for existing test data. Only 10.6% of the beam tests (18 of 170 tests) were conducted for $f_c \leq 30$ MPa. The ratio of the experimental to the proposed shear strength is not influenced significantly by f_c for beams. Only 12.9% of the SFRC beam tests (22 of 170 tests) were conducted for $f_c \geq 50$ MPa. Since, the test data for high strength concrete members are very limited, further research is required to verify the proposed equation for SFRC beams.

As shown in Fig. 5(b), the experimental results of beams with slenderness ratios higher than 4 ($a/d > 4.0$) are limited, consequently further research is required to verify the proposed equations. 77.6% of the beam tests (132 of 170 tests) were conducted for a relatively low slenderness ratio ($a/d < 4.0$). Large scatter in the ratio of the experimental to the proposed shear strength for existing test data is observed, when the slenderness ratio is lower than 4.0.

Figure 5(c) shows the effect of fiber factor on the ratio of the experimental to the proposed shear strength for existing test data. Only 15.3% of the beam tests (26 of 170 tests) were conducted for a relatively high fiber factor of $F \geq 0.75$, which may explain the unevenly spread data. The ratio of the experimental to the proposed shear strength for existing test data decreases with an increase in the fiber factor, indicating a pronounced beneficial effect of the fiber factor.

5. Conclusions

The proposed shear strength equation of SFRC slender beams without stirrup and eleven different predictions were compared with the existing test results and the following conclusions are drawn.

By comparing the proposed shear strength Eq. (15) against 170 available experimental results, the COV obtained for the ratio of the experimental to the proposed equation is 94% of that obtained for Ashour et al.'s equation, 88% of that obtained for Gandomi *et al.*'s equation, 79% of that obtained for Narayanan and Darwish, Swamy et al. and Kwak et al's equations, 71% of that obtained for Khuntia *et al.*'s equation, 65% of that obtained for Sharma and Dinh *et al.*'s equation, 63% of that obtained for RILEM prediction and 60% of that obtained for Imam *et al.*'s equation.

When the slenderness ratio is lower than 4.0, large scatter in the ratio of the experimental to the proposed shear strength for existing test data is observed.

It can also be noted that the proposed shear strength Eq. (15) is in good agreement with the test results. It provides better results than eleven different predictions, when compared with test data for beams without stirrups. However, the test data for high strength concrete members and $a/d > 4.0$ are very limited, therefore further research is required to verify the proposed equation for SFRC beams.

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