Trading Events

Smiles, Frowns and Moustaches – The Many Faces of the Options Market

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Abstract

Financial markets are becoming increasingly event driven. These can take the form of economic releases, speeches and elections. The arrival of this new information often causes the underlying asset to jump. Consequently, the expectation of these known announcements can produce significant distortions in option volatility surfaces. This presents a challenge, or opportunity, for pricing and risk management. We therefore undertake an examination of large events. Firstly, we perform an empirical analysis of the UK referendum in 2016. This is in order to give market practitioners intuition for these extreme scenarios and to guide us towards a quantitative approach. As a result, we introduce a new bimodal jump model. By varying the two jump parameters, we show how to parsimoniously produce the aforementioned smile distortions – specifically: skews, frowns and W-shapes. Furthermore, we demonstrate the model's calibration to the observed volatility surface leading up to the referendum, and the French election in 2017. If jump parameters are frozen with time, the model can be used in the temporal volatility interpolation. Finally, we discuss how the jump probabilities are becoming increasingly observable via alternative data, such as betting markets, election polls and research reports.

1 Motivation

Modern financial markets are eventful; there is a daily stream of economic releases, earnings and data. These are the very mechanisms with which we obtain information on the macro and microeconomic environment – in addition to irregular, but significant, political results. Consequently, changes to event information can have a considerable impact on an asset. They provide risk or reward for financial practitioners that need to hedge or seek trading opportunities. In this way, a single event can generate disproportionate trading activity. After all, equities, such as IBM, often have more variance on their earnings days than the rest of year [6]. Moreover, the result of the UK referendum saw a 10% change in the GBPUSD exchange rate in a single day. In this paper, we examine the impact of such events on option markets, in particular on the FX option implied volatility surface.

While one might easily recognise the instantaneous magnitude of an election, their long term impact on the implied market is easily ignored. In practice, as soon as a voting date is announced, volatility surfaces are dramatically deformed. Put another way, even with a modest one election per annum in the G10 economies, currency traders will spend a meaningful fraction of their year dealing with its pricing and risk implications. Furthermore, a thorough understanding of an event's relationship with the market – in advance – can be particularly critical in cases where liquidity becomes sparse and trading costs high. This will prevent businesses acquiring challenging positions that are costly to manage or exit. These are real life scenarios that risk managers and regulators strive to contain.

Modern markets are also competitive; the bid-offer price difference in options can be extremely tight. For example, an FX option market maker can quote a vanilla contract with a volatility spread of 10 basis points (that is, 0.1%). This is the domain of precision options. In this way, for a successful trader, it is not sufficient for their standard tenors and quotes to match the market: Everything has to be correct. Events meaningfully change the interpolation of the volatility surface between liquid points. Especially in an increasingly electronic world, clients are able to request prices, continuously, with expiries that exploit this time resolution. In addition, exotics can contain path dependent features, which expose the exact time and weight of a market artefact. A known failure to model events by a market maker causes a widening of spreads, a loss of opportunity and decrease

*The opinions in this paper are those of the authors and do not necessarily represent those of their employer.

in market share. Alternatively, being unaware of the subtleties of Non-Farm Payrolls or inflation numbers, for example, will impart a random component on their P&L, which will silently be driven negative by adverse selection. Naturally, all of the aforementioned challenges are opportunities for the more informed quant and trading team.

We hope to have advertised, or at least have reiterated, the importance of events. Our main focus will be illustrating these effects using a bimodal model. In the next section, we build up the intuitions necessary for trading these scenarios and assess the existing literature. The future bimodal jump model is introduced in Section 3. This model generates a wide range of implied volatility smiles (frowns and moustaches) as illustrated in 3.3. In Section 4 we suggest how to calibrate the future jump model to the implied market. Some case studies of calibrated jump parameters and illustrations are given in Section 5. Finally, we conclude with a brief summary of our research in Section 6, but give additional illustrations, and a comparison with a related mixture model, in Appendix B.

2 Intuitions From Time Series Data

In this Section, our intention is to provide a qualitative understanding of the market's behaviour using observations. To do this, we analyse the spot market in the hours leading up to the UK's referendum result. This is insightful, not just because it was so significant, but because it enables us to measure the market at the whole range of event probabilities. The event swung rapidly between almost certain remain, to leaving. An easy trick to gain intuition with any concept is to consider the limits. Furthermore, when we are so close to the event, we are isolating its impact (signal) and minimising other unmodelled market factors (noise). The results of this experiment are shown in Figure 1.

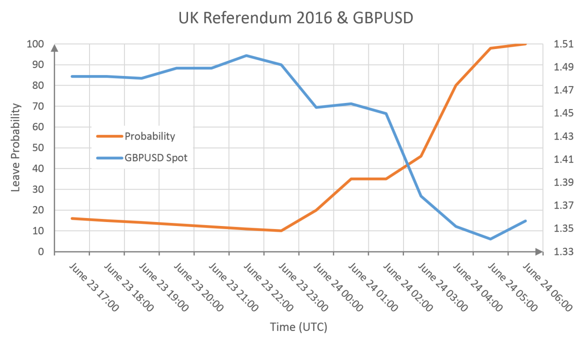
Figure 1a illustrates the impact of the referendum's result on the GBPUSD currency pair. As the probability of a leave outcome increases, the underlying spot rate drops. It is worth noting that this probability is not directly related to the trading of sterling currency; it is the result of bets made by the public on the BetFair Exchange. It is interesting that political betting has become a viable market. In this way, insights between traditional finance and the bookmakers can be utilised. The other beneficial component is that we see almost the entire range of event expectations in several hours.

In order to understand the impact on the options market, we now observe the empirical relationship between spot (and therefore probability) and the 1W ATM in Figure 1b. As results become public, and as the spot falls, the volatility rallies. However, they are not negatively correlated. There is a point of peak volatility. Figure 1c shows that this corresponds to a probability of 50% – not the leave scenario being 100%. Of course, under this latter outcome the market might remain in a turbulent state, with much hedging and rebalancing. However, when the chance tends to certainly leave or certainly remain, one is not expecting much of an event in the future. This is analogous to the concept of information entropy[1] [21], where the expected surprise is maximal at 50% and drops to zero at $p = 0\%$ and $p = 100\%$. Similarly, the excess ATM peaks at 50% and as the outcome of the referendum becomes clearer, starts to fall.

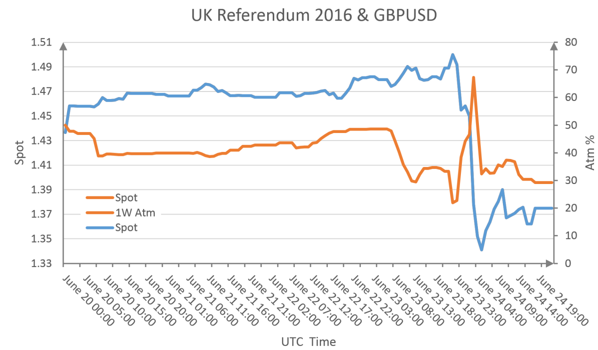
Figure 1d illustrates that the impending event also induces concavity with a time series of the 1W GBPUSD 25-Delta Strangle quote. The Strangle is a measure of convexity of the implied volatility smile[2]. Therefore, the Strangle has become negative. At first sight, this can appear counter intuitive. However, the behaviour is easy to understand. Again, let us consider the limits. If we had only two possible GBPUSD spot levels of 1.33 and 1.50 at a hypothetical expiry, what would the implied volatility be for a call struck at 1.50 or put struck at 1.33? They must be zero. This is because they are guaranteed to expire worthless. Equally, the implied volatility anywhere less than the lower level or greater than the upper level is zero. Alternatively, between the levels, the contracts have value and therefore the implied volatility is positive. In this referendum example, as the main component of the event passes, the Strangle snaps back towards positive territory.

[1] In information theory, Shannon's (information) entropy $H(p) = -p\log_2(p) - (1-p)\log_2(1-p)$ gives the expected surprise for an outcome where p is the probability for one of the two outcomes.

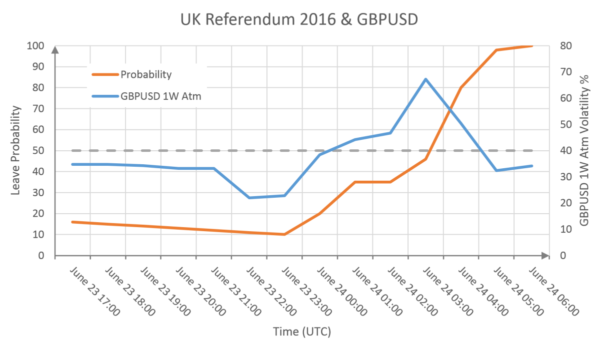
[2] In FX options, the Strangle is the extra volatility needed to price a portfolio of a 25-delta call and put option correctly, compared with the ATM struck volatility.



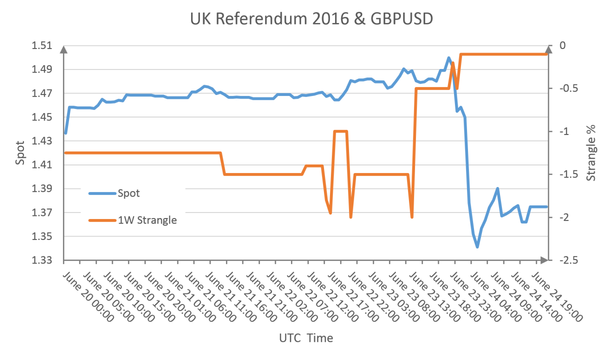
(a) Spot and probability, GBPUSD



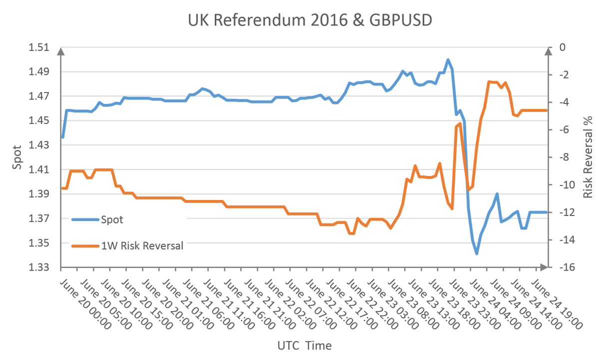
(b) Spot and 1W ATM, GBPUSD



(c) Probability and 1W ATM, GBPUSD



(d) Spot and 1W Strangle, GBPUSD



(e) Spot and 1W Risk Reversal, GBPUSD

Figure 1: (a) shows the GBPUSD exchange rate and the leave probability, which is sourced from the BetFair Exchange, in the hours leading up to the UK's referendum result. Their negative correlation and the event's impact on the market are clear. What is particularly useful, is being able to observe almost the entire range of probability in several hours. With two possible outcomes, we can start to infer what we might already expect: the spot market evolves between two levels. (b) demonstrates the relationship between GBPUSD's spot and the 1W GBPUSD ATM volatility. As voting results filter through, the exchange rate starts to fall and the ATM rallies. However, they are not negatively correlated. Moreover, the move is not a consequence of realised volatility. The ATM peaks half way between the range and begins to fall as the outcome becomes more certain. The event is maximally uncertain when leave and remain are equally likely. This is made especially clear in (c), where the dotted line indicates a 50% BetFair probability. (d) illustrates a feature that can initially appear counter-intuitive in the G10 FX option market. This is concavity. As a sign of an impending jump, the Strangle is negative until the event is over. (e) highlights GBPUSD's significantly negative 1W Risk Reversal. This is indicative of the perceived downside risk, which can be expressed as a negative spot/volatility correlation. Source: BAML.

The perceived downside risk associated to the referendum can be seen further in Figure 1e. This shows the

GBPUSD 1W 25-Delta Risk Reversal quote[3]. GBPUSD's Risk Reversal significantly favours GBP puts leading up to the results. Dynamically, this can be expressed as a negative correlation between spot and volatility; if spot were to fall to the leave outcome, the market would be left with more turbulence than spot rallying to a remain state. In this way, a market participant expects the value of GBP puts to be more valuable than equivalent upside options: after the underlying spot move, the participant gets additional gains from the volatility because of being long vega. Of course, this skew can also be thought as the supply and demand resulting from investors protecting against downside GBP as an asset. This thought pervades equities to the extent that it is commonly known as the leverage effect.

To conclude this Section, the data makes it quite clear what the model should do in this situation. A two outcome event is obviously bimodal. Consequently, there will be two effective spot levels, where the underlying will jump on the arrival of information. When striving to isolate the impact of the event in the forward volatility surface, we should be able to observe an elevated ATM, a pronounced Risk Reversal and negative Strangle. These are conditional on the probability of a certain outcome, and the magnitude of the expected change in spot. Technically, we have not completely isolated the event above. The 1W tenor is a compromise between an entity liquid enough to be observable and therefore reliable, but short enough in maturity to be dominated by the jump.

2.1 Related Studies

In FX, some economies have floored and/or capped, exchange rates due to enforced economic policies. A change in policy can cause the exchange rate to increase, or devalue, in a jump like move. It is possible to imply jump levels and probabilities from the implied option markets on such "pegged" currency pairs [22]. Several studies have similarly focused on the EUR/CHF exchange rate floor [10, 11] which was removed on January 15th, 2015. The problem tackled in this note differs as we assume the time of the jump to be known in advance, the so called known unknowns. A similar argument applies to other jump diffusion models in finance [1, 17]. Such jump diffusion models investigates option prices as the underlying may jump at unkown future times, rather than at a known future time.

Large events, such as elections and national referendums, can cause drastic changes to the FX exchange rates. Even though there can be a range of outcomes in large events, the events can often be well described as having a positive, 'up', or negative 'down', impact on the related economy. In this note, we model such events as having bimodal outcomes. Extensions to multinomial models are straight forward. For ease of exposition, we concentrate on the bimodal market.

The closest related study we have found, is a brief discussion on the implied volatility smile when a single large jump is anticipated [13]. The anticipated jump adjusts the underlying S at time T up to level S_u with probability p_u, or down to S_d with probability $p_d = 1 - p_u$, and the price of a vanilla option maturing after the jump is set as

$$PV(S, K) = p_u PV(S_u, K) + p_d PV(S_d, K). \tag{1}$$

In the presence of an implied volatility smile, this model predicts concavity in the implied volatility.

The studies in [4, 8] infers probabilities of certain outcomes of large events from the implied options market. To do so, they use a mixture model of two standard log-normal volatility processes, where the log normal parameters are conditional on the bimodal outcomes. The vanilla option price just after the event is defined similarly to (1) as the probability weighted sum of the prices under the individual dynamics. The mixture model is calibrated to the implied volatility smile maturing after the event. When calibrating only to one smile maturity, the mixture probabilities reflect all market reactions from present time up to the smile maturity. Typically, there are a number of poll releases, or data estimates released prior to large events. Such pre-event events will impact the mixture probabilities, and the resulting values will not be for only the event in question. To investigate a specific event one needs to isolate the event by looking at how the implied volatility smile changes for two maturities straddling the event in time. Further details of this mixture model, and a longer discussion, is given in Appendix B, where we compare the mixture model with our contribution.

A short well written summary of recent research relating to de-pegging, and mixture model usage can be found in [20]. [9] further considers the role of the bookmakers' odds, and makes the observation that the odds, even for international and online exchanges, should not be equal for those betting in different currencies. This is

[3]The 25-Delta Risk Reversal is the volatility difference of a call and put option with strikes such that the option deltas are 0.25 and -0.25, respectively.

because, unlike the outcome of a sporting event, elections and referenda have the power to affect the exchange rate (the very motivation for this document), and thus the relative value of the payout. Put succinctly, there is a dependence between the bet probability and bet-currency. Empirically, the odds implied probability does not differ between currencies, which indicates a possible arbitrage opportunity.

3 A Simple Model of A Future Event

The FX option market has a set of standard tenor maturities that are frequently traded on a wide range of currency pairs. At the close of trading in New York, the market date typically changes to the next business day. During this roll of market date, the maturity dates of the standard tenors roll forward according to market convention rules [5]. The implied smile is related to the expected realised variance up until the maturity. Since the uncertainty of events generate an increase in realised volatility, the change in the implied volatility smile is related to events that comes into the tenor maturity bucket. Market makers thus have to predict how the new smile will look, which is a non trivial problem.

As an example, consider the case of the UK referendum coming into the 1W maturity date. A 1W GBPUSD option as of June 16th 2016, had an expiry date of June 23rd. The 1W GBPUSD maturity as of June 17th 2016 is one business day later, on June 24th. Even though the time to maturity is identical between the two 1W options, the implied 1W volatility moved from approximately 21% to 50%. This large change in volatility vastly distorts the 1W implied volatility smiles observed on consecutive days [4].

In the precision world of making the FX options market, being able to predict such distortions of the implied volatility smile is crucial. We present here a model that describes the changes in skew and volatility level by an expected jump in spot. A similar model was used in [9] to illustrate the numeraire sensitivity of betting probabilities on events that move FX exchange rates.

3.1 A Future Jump Model

Let T be the time of a future event, which is expected to cause a large change in the spot rate S. The maturity time after the change in the underlying due to the event will be denoted T_+, and the maturity before the event T_-. The time scale considered in this article is short, and we assume throughout this note that all interest rates are 0. With the future change in the underlying as $dS = S_{T_+} - S_{T_-}$, we require there to be no arbitrage, that is

$$E\left[dS\right] = 0. \tag{2}$$

However, prior information may suggest either a large "down" move, or a more likely smaller "up" move. We assume the log-normal change in S to be independent of the spot level. Our parametrization of the change in spot is chosen so that

$$\begin{aligned} S_{T_+} &= S_u = S_{T_-} e^{r_u} \quad \text{with probability } p_u \text{, and} \\ S_{T_+} &= S_d = S_{T_-} e^{r_d} \quad \text{with probability } p_d = 1 - p_u. \end{aligned}$$

Since the process S remains a martingale, the process remains arbitrage free. The martingale property (2) gives us the probability for the up move as

$$p_u = \frac{1 - e^{r_d}}{e^{r_u} - e^{r_d}}. \tag{3}$$

The expected *absolute* change in spot due to the event is

$$\begin{aligned} E\left[|dS|\right] &= (S_u - S_{T_-})p_u + (S_{T_-} - S_d)p_d, \\ &= 2S_{T_-} p_u p_d (e^{r_u} - e^{r_d}) = 2S_{T_-} \frac{(1 - e^{r_d})(e^{r_u} - 1)}{e^{r_u} - e^{r_d}}. \end{aligned} \tag{4}$$

The variance of the binomial spot jump is

$$\begin{aligned} V\left(dS\right) &= S_{T_-}^2 \left(e^{r_u} - 1\right)\left(1 - e^{r_d}\right) \\ &= (S_u - S_{T_-})(S_{T_-} - S_d). \end{aligned}$$

[4]This smile deformation is visualised in Section 5.

The log-returns can be thought of as stochastic variable R taking values r_u, r_d with probabilities p_u, p_d, respectively. The variance of the binomial log-returns are

$$V(R) = p_u r_u^2 + p_d r_d^2 - (p_u r_u + p_d r_d)^2. \tag{5}$$

This binomial log-return variance corresponds to the change in the fair variance of a continuously fixing variance swap.

As an interesting note, the High-Low-Open-Close (HLOC) realised variance estimator [18], maps the high and low spot values observed over an interval to a realised log-normal variance RV_{OC} as

$$RV_{OC} = \ln(H/O)\ln(H/C) + \ln(L/O)\ln(L/C). \tag{6}$$

The HLOC estimator is adjusted for drifts in the underlying process [16]. During a news announcement, the underlying moves largely in one direction. Therefore, the spot will either move from high to low (H to L), or from low to high (L to H). The expected realized variance due to the information release according to HLOC is therefore

$$E(RV_{OC}) = r_u^2 p_u + r_d^2 p_d. \tag{7}$$

Note that this is closely related to the variance of the log-return R, in equation (5). The change to the expected fair variance attributable to the event, can be estimated with either (5) or (6). The log normal variance contribution of the single expected jump comes from the binomial R component. We find the fair volatility just after the jump, $\hat{\sigma}_{T_+}$, as

$$\hat{\sigma}_{T_+}^2 T_+ = \hat{\sigma}^2 T_- + V[R]$$

where $\hat{\sigma}^2 T_-$ is the expected total variance before the event. That is, it is the fair variance from a continuously fixing variance swap.

3.2 Jump Price By Replication

The forward price of a vanilla call option is such that

$$C(T, K) = E[(S_T - K)^+],$$

where the expectation is under the discounted risk neutral measure. With R denoting the binomial log returns for the expected jump at T, we have

$$C(T_+, K) = E\left[(S_{T_-} e^R - K)^+\right],$$

and by conditioning on the two outcomes of R, we find the replication price as

$$\begin{aligned} C(T_+, K) &= p_u e^{r_u} E[(S_{T_-} - Ke^{-r_u})^+] + p_d e^{r_d} E[(S_{T_-} - Ke^{-r_d})^+] \\ &= p_u e^{r_u} C(T_-, Ke^{-r_u}) + p_d e^{r_d} C(T_-, Ke^{-r_d}). \end{aligned} \tag{8}$$

Given the knowledge of the jump distribution, the option price can be replicated with two options maturing before the event. This replication pricing formula holds also in the case of an implied volatility smile at time T_-, since the implied smile is defined such that the option price $C(T_-, K)$ matches the market price. Note that this replication is exact. To manage the risk of an option maturing just after a jump, we can instead look at the replication on an underlying that does not jump. Since the processes are identical up until the moment of the jump, we can build our hedging strategy on the portfolio of the replicating contracts, instead for the sold contract. The replication is used throughout this article to value contracts maturing after a future jump.

Does a jump introduce arbitrage? No, because the underlying remains a martingale by design[5]. There are also the following arguments regarding option calendar, and butterfly, arbitrage: Calendar arbitrage is not introduced since adding uncertainty to a convex vanilla pay-off increases the price, following Jensens' inequality. Since prices are increasing in volatility, a jump at time T will cause the implied volatility to increase from time T_-

[5] In the case of non-zero interest rates, one instead consider the process discounted by the forward, $S_t/F_{t,T}$.

to T_+, for all strikes, but less so very far out in the smile wings, where call and put prices are linear for a change in strike. This monotonic increase in implied volatility ensures that there is no calendar arbitrage introduced with the jump. The implied volatility smile can be concave [21], as long as prices are increasing. Similarly, if the implied volatility smile at maturity T_- is free of butterfly arbitrage, then so is the implied smile at maturity T_+. This follows directly from the price replication formula (8), and the fact that the sum of monotonic functions is itself a monotonic function.

Is the "Up" Probability Currency Dependent? Yes, if the future jump is parametrized by expected jump returns, the corresponding risk neutral domestic event probability will differ from that of the foreign risk neutral probability. Interestingly, the international betting market seems to largely ignore this observation [9], which can lead to arbitrageable opportunities.

The reason for a numeraire and denominator difference in probabilities follows from the exchange rate being a martingale in both of the risk neutral measures. Since the jump is parametrized in fixed log-returns, the risk neutral probabilities differ in the numeraire and denominator risk neutral measures.

3.3 Jump Smile – Frowns And Moustaches

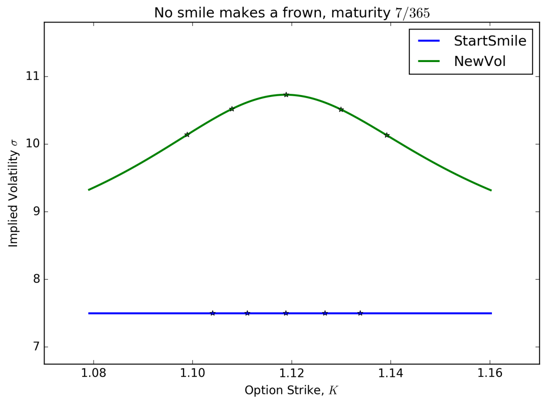
With a parameterized jump and the implied smile just prior the jump, at maturity T_-, we can compute the option prices with maturity just after the expected jump, at maturity T_+. At the forward-strike, the implied volatility after the jump can be found analytically. The level, skew and convexity of the implied smile after the jump are straight forward to derive via a Taylor expansion of the T_+ replication price for a strike struck at the forward. With the call prices at maturity T_+ we can invert for the corresponding implied volatility smile for maturity T_+ using standard numerical techniques[6].

Figure 2 shows how a future smile is deformed when a jump in spot is expected. The Figures on the left hand side take a starting point in a flat smile world, with volatility of 10%, whereas the right hand side use a EURUSD implied 1W smile, as of May 23rd, 2017. In Figures (a) and (b), spot is expected to move either up or down by 1%, resulting in a move up occurring with similar likelihood as a move down. As a result, the now less likely at the money forward strike becomes expensive. In the far wings, the vanilla payoff is linear, and we expect the smile after a jump to approach the pre-jump price since assumption (2) ensures the jump does not change the value of a linear position in the underlying. The T_- smile therefore increases the T_+ volatility at extreme strikes, which results in the moustache, or "W" smile in Figure 2b.

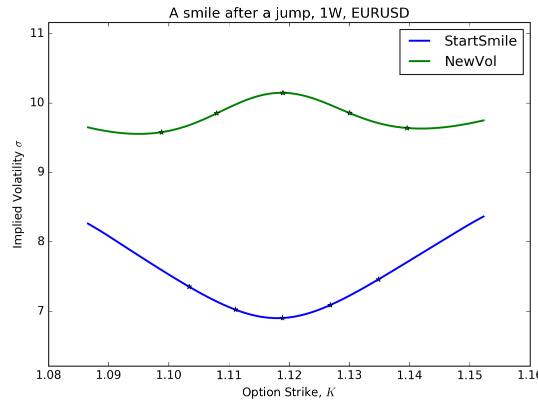
In the lower Figures (c) and (d), the large down jump of 2% happens with approximately 9% likelihood. As a result, the risk of a large down move is associated with an increase in price and implied volatility. The very likely small up move only results in a small increase in implied volatility, resulting in a change in the smile's skew.

In the two jump assumptions, the difference between the up and down returns are comparable, 2% versus 2.2%. Still, the change in ATM volatility is approximately three times larger in the symmetric jump, than in the skew jump. This is reflected in the expected surprise which is 44.252% in the skew case, and 99.998% in the symmetric case. Similarly, the expected absolute change in spot (4) is 2.75 times larger in the symmetric case than the skew case.

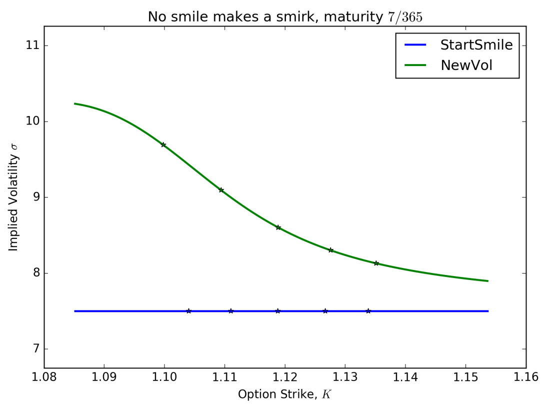
[6] See for instance the accurate algorithm outlined in [14], with its freely available implementations.



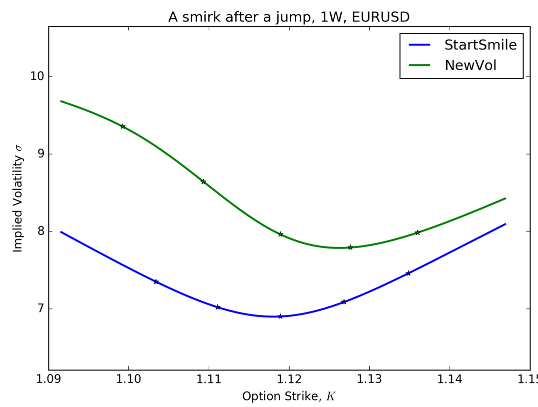
(a) No smile makes a frown, $R \in \{\pm 1\%\}$



(b) EURUSD, 1W Jump $R \in \{\pm 1\%\}$ (Source: BAML)



(c) No smile makes a smirk, $R \in \{+0.2\%, -2\%\}$

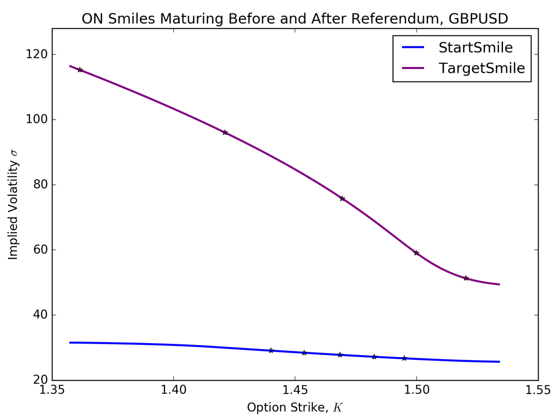


(d) EURUSD, 1W Jump, $R \in \{+0.2\%, -2\%\}$ (Source: BAML)

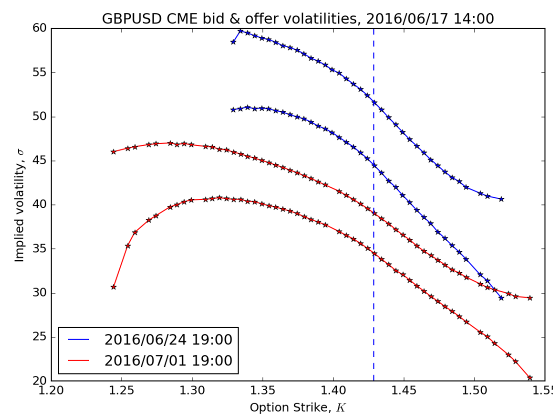
Figure 2: Illustrations of changes in implied volatility smiles as a jump is anticipated. The lower blue smile is the implied volatility smile before the jump T_-. The green curve is the smile with maturity just after the jump T_+. Strikes where the call delta is $0.1, 0.25, 0.5, 0.75, 0.9$ have been highlighted with stars. (a) shows the smile when a jump of $\pm 1\%$ is expected in 7 business days, starting with a flat implied volatility of 7.5%. (b) shows the smiles for a jump of $\pm 1\%$ in EURUSD spot in 7 business days, using an implied smile from May 23rd, 2017. (c) shows the smirky-smile when a jump of $+0.2\%$ or -2% is expected in 7 business days, starting with a flat implied volatility of 7.5%. (b) shows the smiles for a jump of $+0.2\%$ or -2% in EURUSD spot in 7 business days, using an implied smile from May 23rd, 2017.

Are these smiles realistic? Yes, large changes in single stocks are expected regularly during earning announcements [6]. This results in moustache / "W" implied volatility smiles for short maturities, as illustrated in Figure 3. The induced "W" shape is most clear in the shorter expiry, where it dominates any expected information, but interestingly can still be seen out to three weeks. Consequently, understanding these effects is crucial for single stock options. Tim Klassen and the *Vola Dynamics* company have a series of interesting presentations, with many illustrations of moustache smiles available at the Vola Dynamics website [15].

(a) Apple, 1W, 2W, 3W, Smiles (Source: Bloomberg)



(b) UK Referendum, Overnight Smiles (Source: BAML)



(c) UK Referendum, 1W and 2W Smiles (Source: CME)

Figure 3: (a) The first three listed implied volatility smiles on Apple's shares that expire after its 2018 Q1 earnings. This was released on 1st May 2018, with the smiles observed on 27th April 2018. The vertical dashed line denotes the implied 1W forward. In equities, regular earnings announcements for single stocks often impose a "W" shape, or moustache, onto the skew. (b) Two GBPUSD overnight implied volatility smiles, one maturing just before, the other just after, the UK referendum. The implied smiles are interpolated between the strikes marked with $\star$'s. As shown, the smile is concave around the ATM strike (center $\star$), and the two smiles have vastly different skew. (c) The first two GBPUSD bid and offer implied volatility smiles traded at the CME FX options exchange, as of June 17th, 2016. Both smiles expiry after the UK referendum. The dashed vertical line shows the position of the spot at the observation time. Strikes quoted are shown with $\star$'s.

Variance swaps are liquid instruments in the equity market. If the underlying is expected to jump over an earnings announcement, the fair variance should be noticeably higher after the announcement than before. This is in fact the observation shown in [6].

An anticipated symmetric jump results in a decrease in the smile convexity. However, frowns are not commonly observed in the FX options market. On rare occasions, the Strangle can become negative, as seen for instance leading up to the UK referendum as shown in Figure 1d. As a further illustration, Figure 3b show the overnight GBPUSD implied volatility smiles with maturity just before, and just after, the UK referendum resulting in a smile change similar to Figure 2c. In the FX options market, the implied volatility smile is typically quoted at effectively 5 strikes. That is, the smile resolution is much lower than in equity markets. With a resolution of only 5 points, the W-smiles can be difficult to detect. The CME FX option exchange quotes prices at many strikes. Figure 3b shows bid and offer implied volatility levels for two GBPUSD maturities shortly after the UK referendum, where observable strikes are highlighted with full stars. The wide difference between bid and offer volatilities illustrate the uncertainty around the event. At this strike resolution, we start to see the moustache shape. However, we need a wider range of strikes to see the the wings increasing further since the expected jump was so large.

4 Implying Probabilities From Changes In Forward Skew

To accurately capture the change due to a single event, one must compare a smile with maturity just after the event with a smile maturing just before the event. In theory, this is straight forward to do. However, the data tend to be especially uncertain in this region, as seen in the widened bid ask spreads. To get data of good quality is difficult since prices are interpolated between standard tenors, and the temporal interpolation itself depends on how the event is described.

In this section we focus on what jump-parameters can be inferred from the implied market as the standard broker tenors cross the event. However, the CME also has a plethora of option expiries throughout the term structure. The exchange has moved to quoting two fixed expiries per week (a Wednesday and a Friday contract). When either of these CME expiries and a standard tenor maturity straddles an event, the jump parameters can be calibrated to the smile differences.

In addition to calibrating the parameters to the implied market, the jump parameters can be treated as partially observable parameters. Jump probabilities can sometimes be observed on betting exchanges, election polls, or other news releases. That is, one could tune the the jump parameters after expectations of the event. As the event information changes, the jump parameters are adjusted to meet the new information. From a market makers perspective; if the jump parameters are known, the future jump can be included in the temporal interpolation.

4.1 Calibrating a Jump to Implied Volatility

In the FX options market, options are typically traded with standard tenor maturities. The corresponding standard maturities are defined as a period, or tenor, from the business day[7] At the end of the business day, the business day rolls to the following day. In this day-roll, also the maturity times of standard tenors roll forward. Rather than using interpolated data, we suggest comparing using standard maturity smiles from two different dates; one where the expiry is just prior the event, T_-, and one where the maturity is after the event, T_+. To make sure the data is of good quality, we focus on the deformation of the standard maturity smiles, as the event moves from after to before, the smile maturity. When using FX data as of different times, the spot can move in between the data capture time. Following standard FX quoting conventions, we look at smiles defined in terms of 'sticky delta' convention [3], and reset the spot levels holding the smile in delta terms constant.

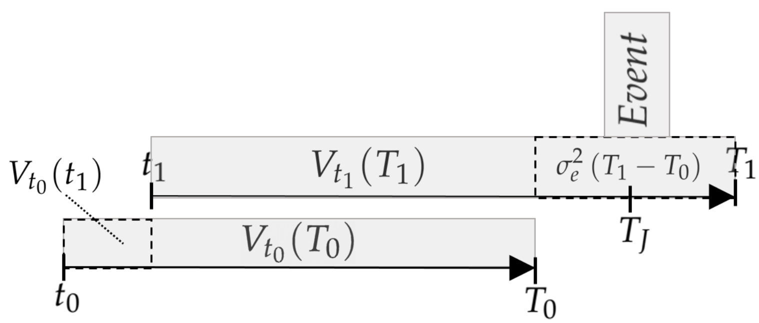


Figure 4: Illustration of variance contributions, smile origins, and maturities considered in the jump parameter calibration. Two smiles are used, observed as of time t_0 and t_1, with standard FX maturities at times T_0 and T_1. Grey areas illustrate expected forward variance. An event causing the spot to jump is expected at time T_J, between the standard maturities $T_0 < T_J < T_1$. To isolate the event variance contribution, the differences between the forward variances implied by the smiles is isolated by subtracting the variance highlighted by dashed regions. The jump parameters are calibrated to the forward variance changes at time T_J.

Since we use volatility surfaces taken as of different dates and with different maturities, the time between the surface origin and the smile maturity might differ. The smiles can therefore not easily be directly compared with each other. We introduce our notation in Figure 4 and below with an example by calibrating to the impact of the UK referendum. The referendum results are expected at time T_J, the evening of June 23rd, 2016, around 22:00 UTC time. To investigate the impact as the event comes into the 1W maturity we use the following GBPUSD smiles:

[7] The standard tenors start with the over night (ON), one and two week (1W, 2W), and then monthly (1M, 2M, 3M, 6M) tenors.

- Smile with origin t_0 on June 16th, 2016. The 1W maturity time T_0 is June 23rd, 2016, at 14:00 UTC time. This is *before* the referendum results are released.
- Smile with origin t_1 on June 17th, 2016. The 1W maturity time T_1 is June 24th, 2016, at 14:00 UTC time. This is *after* the referendum results are released.

Given an implied volatility smile observed at time t, we associate the Black Scholes total variance V_t with each of the strikes,

$$V_t(T,K) = (T-t)\sigma^2(K,T,t),$$

where T is the maturity time. The implied total variance is related to the local volatility σ_t as elegantly shown in the most likely path arguments [7]. The total variance is the integral of the square local volatility,

$$V_t(T,K) = \int_t^T E\left[\sigma_s^2 \,\middle|\, S_T = K, \mathcal{F}_t\right] ds,$$

where the conditional expectation measure weights the possible spot paths from S_0 to $S_T = K$. We may therefore write the implied total variance of the t_0 smile as

$$V_{t_0}(T_0,K) = \int_t^T E\left[\sigma_s^2 \,\middle|\, S_{T_0} = K, \mathcal{F}_{t_0}\right] ds.$$

This variance does not contain the event associated variance. However, the expected variance for the t_1 surface, $V_{t_1}(T_1,K)$, contains the additional variance from the referendum. The difference between the V_{t_0} and V_{t_1} variances can be attributed to:

1. Variance between time t_0 and t_1 goes from being expected (implied) to being realized.
2. Variance between T_0 and T_1 includes a jump and period of heightened market activity.

The variance that goes from expected to realized as time pass from t_0 to t_1, can be estimated from the t_0 surface as

$$V_{t_0}(t_1,F_t) = (t_1-t_0)\sigma(F_{t_1},t_1,t_0)^2.$$

In order to better align the two smiles, we subtract this variance uniformly across strikes from the t_0 surface, and approximate the forward smile from t_1 to expiry T_0 implied by the t_0 surface as

$$\left(\sigma_K^-\right)^2 (T_0-t_1) \approx V_{t_0}(T_0,K) - V_{t_0}(t_1,F_t).$$

The σ_K^- smile is a proxy of the smile as of time t_1 with maturity just before the event, expected just after T_0. Using the jump-replication price formula we can define the smile for just after the event, σ_K^+, since

$$C^{BS}\left(T_0-t_1,K,\sigma_K^+\right) = p_u e^{r_u} C^B S\left(T_0-t_1, Ke^{-r_u}, \sigma^-_{Ke^{-r_u}}\right) + p_d e^{r_d} C^B S\left(T_0-t_1, Ke^{-r_d}, \sigma^-_{Ke^{-r_d}}\right).$$

At each of our calibration strikes, we can efficiently invert the replication prices for the σ_K^+ volatilities. The σ_K^+ smile is entirely built out of the t_0-surface information and jump parameters. In the view of implied volatility as the conditional expected variance, we pose the approximation as

$$\int_{t_1}^{T_0^+} E\left[\sigma_s^2 \,\middle|\, S_{T_0^+} = K, \mathcal{F}_{t_0}\right] ds \approx \left(\sigma_K^+\right)^2 (T_0^+ - t_1). \tag{9}$$

The main difference of the above with V_{t_1} is the diffusive variance contribution between T_0 and T_1. We approximate the non-event variance between T_0 and T_1 with a constant strike independent volatility of σ_e. At a first approximation, this level is taken as the average implied volatility at the forward strike at T_0,

$$\sigma_{e_0} = \sigma(F_{T_0},t_0,T_0).$$

Most large events will come with a sudden shift in spot as well as a period of high variance. Because of this uncertainty, we allow the extrapolation σ_e as a parameter that can take values close to σ_{e_0}. The most likely path approximation is to approximate the conditional expectation with the local volatility of a single strike along one (the most likely) the possible paths. We subtract the local variance in the extrapolated region, $\sigma_e^2(T_1 - T_0)$, to approximate the forward variance implied by the market up until just after the jump, as

$$\int_{t_1}^{T_0^+} E\left[\sigma_s^2 \,\middle|\, S_{T_0^+} = K, \mathcal{F}_{t_1}\right] ds \approx V_{t_1}(T_1, K) - \sigma_e^2(T_1 - T_0), \tag{10}$$

$$= \left(\sigma_K^{(t)}\right)^2 (T_0 - t_1) \tag{11}$$

where the introduced $\sigma_K^{(t)}$ is our calibration target implied volatility. The subtracted forward variance is depicted in Figure 4.

The jump model has two parameters, r_u and r_d, and together with σ_e, we have three parameters to calibrate. The component σ_e is a volatility with which we extrapolate the t_0 smile. In order to explain most of the changes with a jump component, we encourage σ_e to be close to the current average implied volatility by adding a regularizing term to the calibration target. The parameters are calibrated to minimize the following expression,

$$E = \sum_i \nu(\Delta_i)^2 \left(\sigma_{K_i}^{(t)} - \sigma_{K_i}^{+}\right)^2 + \nu(0.01)^2 \left(\sigma_e - \sigma_{e_0}\right)^2, \tag{12}$$

where $\nu(\Delta) = \exp\left(-\mathcal{N}^{-1}(\Delta)^2/2\right)$ scales the error proportionally to the Black Scholes vega, and $\mathcal{N}^{-1}$ is the inverse cumulative standard normal distribution function. The regularization weight is chosen proportional to the vega at the 1-delta strikes. Having a larger regularization weight forces more of the smile changes to be explained by the jump process. This is illustrated in Appendix A where parameters with different weights are shown. From empirical tests, the above choice gives a reasonable balance of jump-to-level volatility.

Note: Following a leave outcome of the referendum, there would follow a longer period of high volatility than after a remain outcome. Assuming different extrapolation levels further generates a change in skew, as illustrated for a jump from a Black Scholes model in [12]. Similar extensions in the presence of implied volatility smiles is left for future investigations.

5 Smile Rolling Numerical Examples

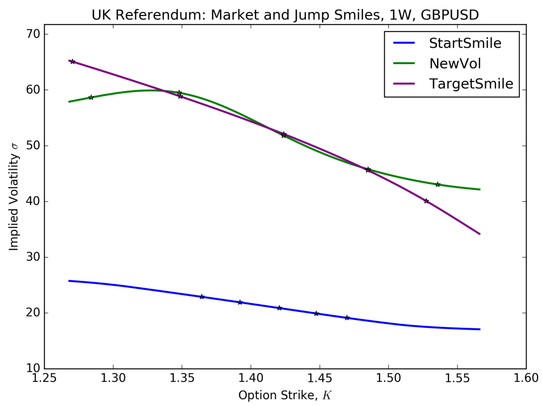
In this section we investigate the future jump model by performing an analysis of the UK referendum 2016, and the French election 2017. We calibrate the jump parameters to the deformations of the implied volatility smiles as the event comes into the standard FX tenors.

Even though we show the calibration of parameters on only a few days, it is important to realise that the jump model is useful for market makers (and takers) for intermediate days. Firstly, as alluded to earlier, one can also use CME option maturities to increase the dates where jump parameters are calibrated. Furthermore, one might have a view on the event probability itself, from surveys, polls or observations of the bookmakers, as shown in Appendix B. In this way, the model is therefore a decomposition of the event-surface into a physical, intuitive and tradable parameter. Freezing the jump-parameters when time passes and the event is in between standard tenors, the changes caused by the jump can be used in the price interpolation between standard tenors

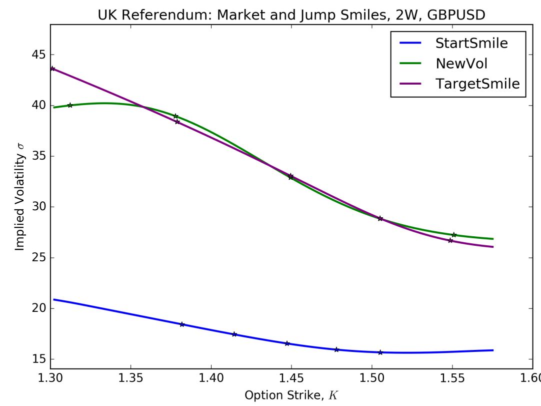
In the examples in this section, we used five strikes in the calibration. These strikes were taken as the average of the t_0 and t_1 surface strikes corresponding to the ATM, 10- and 25- delta strikes for maturity T_0 and T_1, respectively. This choice is to have a balance between the most liquid strikes of the base, and target, smiles.

5.1 UK Referendum With Skew Explained By A Single Jump

As we have seen, the UK referendum held 23rd of June 2016 is an excellent case study for a bimodal event. The public were given the simple choice of whether to remain or leave the EU. However, each of these decisions were estimated to have significantly different outcomes for the future of the economic and political landscape. Consequently, the sterling exchange rate, and therefore option market, were highly sensitive to the unknown outcome.



(a) GBPUSD 1W smiles, and calibrated Jump Smile



(b) GBPUSD 2W smiles, and calibrated Jump Smile

Figure 5: Illustrations showing the implied volatility of standard maturities for two consecutive business days, where the UK referendum occurs before (blue), and after (purple), maturity time. The green line shows the calibrated jump-smile using the pre-referendum smile as a basis, and the calibration target in Section 4.1. (a) 1W maturity as of June 16th (17th), 2016 in blue (purple). (b) 2W maturity as of June 9th (10th), 2016 in blue (purple). Source: BAML.

Being an event of significant economic significance, the implied volatility smiles deformed as the maturities came to include the referendum results, as illustrated for the 1W and 2W tenors of GBPUSD in Figure 5.

The jump parameters can be calibrated to the difference of the implied smiles as suggested in Section 4.1. The green lines show the implied volatility smiles of the calibrated parameters, when the jump is assumed to happen shortly after the maturity of the pre-event (blue) smile. We do not expect a perfect fit, as market moves between the observations, and other factors outside of a jump are expected to impact the market. A future bimodal jump in the underlying can explain much of the change in implied volatility observed between the two consecutive market days.

Pair	Tenor	p_u	r_u	r_d	$E\left[\frac{\lvert dS\rvert}{S}\right]$	σ_e
GBPUSD	1W	69%	3.7%	−8.9%	5.2%	55.8%
GBPUSD	2W	74%	3.2%	−9.3%	4.7%	23.6%
GBPUSD	1M	85%	1.6%	−9.6%	2.7%	12.3%
GBPUSD	2M	88%	1.5%	−11.0%	2.6%	10.9%
EURUSD	1W	76%	1.0%	−3.4%	1.6%	24.4%
EURUSD	2W	79%	0.9%	−3.5%	1.4%	11.2%
EURUSD	1M	94%	0.3%	−5.0%	0.6%	8.6%
EURUSD	2M	96%	0.4%	−9.7%	0.8%	8.4%
GBPAUD	1W	58%	4.0%	−5.8%	4.7%	27.2%
GBPAUD	2W	66%	2.8%	−5.7%	3.7%	16.7%
GBPAUD	1M	76%	1.7%	−5.6%	2.6%	13.1%
GBPAUD	2M	83%	1.3%	−6.4%	2.1%	12.0%
GBPJPY	1W	74%	3.3%	−10.1%	4.9%	44.1%
GBPJPY	2W	77%	2.8%	−10.1%	4.4%	24.0%
GBPJPY	1M	88%	1.4%	−11.4%	2.5%	14.4%
GBPJPY	2M	92%	1.3%	−16.0%	2.3%	15.9%
EURGBP	1W	38%	6.8%	−4.4%	5.3%	25.4%
EURGBP	2W	32%	7.1%	−3.6%	4.7%	16.6%
EURGBP	1M	13%	8.4%	−1.3%	2.3%	11.3%
EURGBP	2M	12%	8.4%	−1.2%	2.2%	9.8%

Table 1: Calibrated jump parameters as the UK referendum came into standard tenor maturities and deformed the implied volatility smile, for a range of currencies. The risk neutral probability p_u is for an increase in the value of the currency pair over the event. The smaller expected jump in EURUSD shows that the major risk/uncertainty is on GBP, not EUR.

The jump parameters were calibrated to the implied smile deformations for several currency pairs and deformations for maturities up to two months. The resulting parameter values are shown in Table 1.

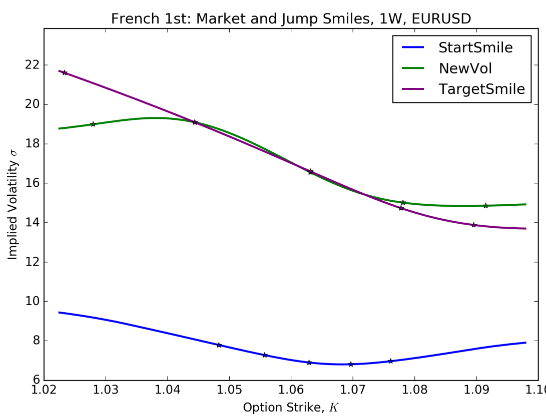
The jump probabilities are implied from the risk neutral smile, and differ as expected between currencies. Across all exchange rates, the downside outweigh the upside, in that $r_u < |r_d|$. A leave outcome was perceived by the market as more uncertain for both the EUR and GBP economies, than a remain outcome. The smaller expected jump in EURUSD shows that the major uncertainty is with GBP, not EUR. The magnitude of the move in EURGBP is similar in size to the GBPUSD move, which seem a bit high. As the actual results became clear, the GBPUSD and EURGBP exchange rates moved by -9.4% and 7.3% before 4AM on June 24th[8]. The implied moves from the 1W smile deformations align well with the realised outcomes of -8.9% and 6.8%, respectively.

5.2 French Election (Both Rounds) With Skew Explained By A Single Jump

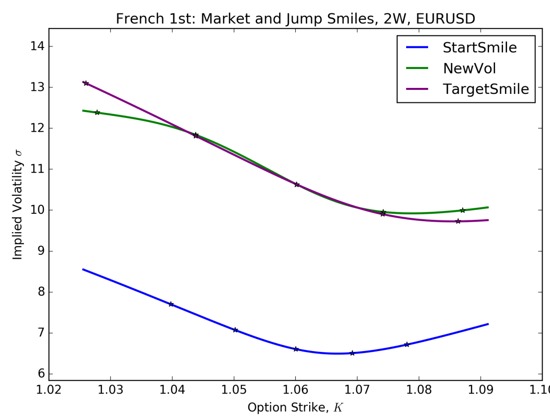
The French election 2017 consisted of two national voting sessions, the first held on April 23rd 2017, and the second on May 7th 2017. Both rounds of the election were held on Sundays, which forces the smiles used in our calibration to have maturity days that differ by 3 days ($T_1 - T_0 = 3$ days).

5.2.1 French Election, First Round

For the first round of the French election, there were several candidates. Some of the candidates' campaigns suggested the holding of a referendum to allow France to leave the EU. Therefore, some of the outcomes of the election would significantly increase the uncertainty around the EUR currency. The uncertainty is clearly seen in the implied volatility, as shown in Figure 6.



(a) EURUSD 1W smiles, and calibrated Jump Smile



(b) EURUSD 2W smiles, and calibrated Jump Smile

Figure 6: Illustrations showing the implied volatility of standard maturities for two consecutive business days, where the first round of the French Election 2017, April 23rd, occurs before (blue), and after (purple), the smiles maturity time. The green line show the calibrated jump-smile using the pre-election smile as a basis, and the calibration target in Section 4.1. (a) 1W maturity as of April 7th (10th), 2017 in blue (purple). (b) 2W maturity as of April 14th (17th), 2017 in blue (purple). Source: BAML.

Parameters calibrated to the smile deformations for EURJPY, EURUSD, EURAUD and EURGBP as the 1W, 2W, 1M and 2M tenors received the French election, are shown in Table 2.

[8]See `https://www.bbc.co.uk/news/business-36611512` for market observations following release of the referendum results.

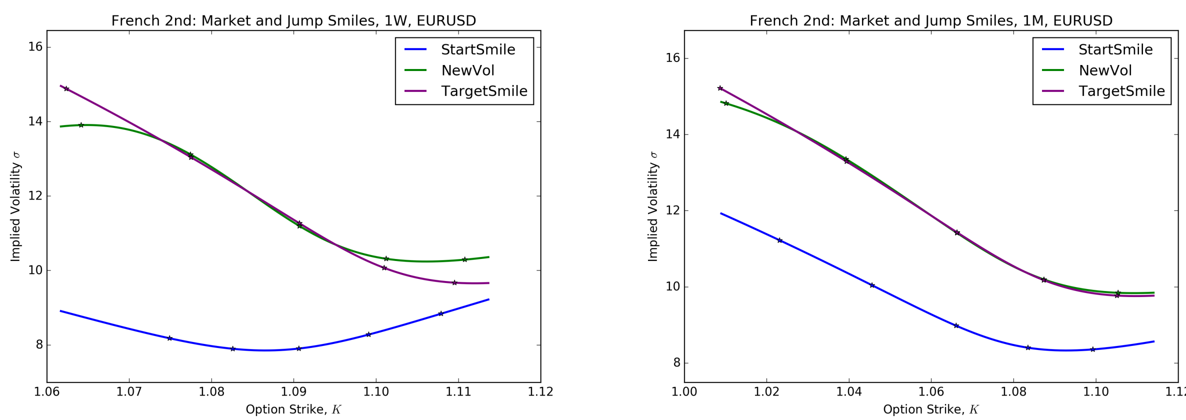
Pair	Tenor	p_u	r_u	r_d	$E\left[\frac{\|dS\|}{S}\right]$	σ_e
EURJPY	1W	78%	1.1%	−3.9%	1.7%	33.4%
EURJPY	2W	80%	0.9%	−3.7%	1.4%	16.3%
EURJPY	1M	90%	0.6%	−5.5%	1.1%	9.5%
EURUSD	1W	83%	0.6%	−3.3%	1.1%	17.6%
EURUSD	2W	82%	0.6%	−2.8%	1.0%	12.0%
EURUSD	1M	92%	0.3%	−4.3%	0.6%	7.0%
EURAUD	1W	77%	0.5%	−1.5%	0.7%	13.1%
EURAUD	2W	77%	0.5%	−1.9%	0.8%	9.6%
EURAUD	1M	96%	0.2%	−4.8%	0.4%	7.2%
EURGBP	1W	85%	0.5%	−3.0%	0.9%	18.1%
EURGBP	2W	82%	0.5%	−2.4%	0.8%	10.0%
EURGBP	1M	93%	0.3%	−3.8%	0.5%	7.8%

Table 2: Calibrated jump parameters as the first round of the French election 2017 came into standard tenor maturities and deformed the implied volatility smile, for a range of currencies. The risk neutral probability p_u is for an increase in the value of EUR over the event.

5.2.2 French Election, Second Round

For the second round of the French Election 2017, May 7th, there were two candidates to choose among. One of these candidates campaigned with suggestions of holding to allow France to leave the EU. Such a referendum would increase the uncertainty around the value of the EUR currency.

There were some difficulties in calibrating to the 2W smile deformation. The pre-election 2W maturity was taken at April 21st, and the target data is taken as of April 24th. That is, dates straddling the first round of the election. Since the events are strongly correlated, the expectations of the second round change significantly as the results of the first round are released. Odds from the betting market implied a likelihood of over 83% for one of the candidates to win the election, following results of the first round[9].



(a) EURUSD 1W smiles, and calibrated Jump Smile

(b) EURUSD 1M smiles, and calibrated Jump Smile

Figure 7: Illustrations showing the implied volatility of standard maturities for two consecutive business days, where the second round of the French Election 2017, May 7th, occurs before (blue), and after (purple), the smiles maturity time. The green line shows the calibrated jump-smile using the pre-election smile as a basis, and the calibration target in Section 4.1. (a) 1W maturity as of May 1st (April 28th), 2017 in blue (purple). (b) 1M maturity as of April 5th (6th), 2017 in blue (purple). Source: BAML.

The changes to the 1W and 2M smiles as the maturity comes to include the second round of the French election, are shown in Figures 7. The green line shows the calibrated Jump smile using the pre (blue) smile as a starting ground. The single jump assumption is capable of explaining the majority of the changes.

[9]The betting probabilities are sourced via Oddschecker on Bloomberg.

Pair	Tenor	p_u	r_u	r_d	$E\left[\frac{\lvert dS\rvert}{S}\right]$	σ_e
EURJPY	1W	86%	0.6%	−3.7%	1.0%	11.0%
EURJPY	1M	86%	1.2%	−7.6%	2.0%	12.9%
EURUSD	1W	86%	0.5%	−2.9%	0.8%	4.0%
EURUSD	1M	90%	0.7%	−6.7%	1.2%	10.2%
EURAUD	1W	83%	0.3%	−1.3%	0.4%	8.7%
EURAUD	1M	81%	0.7%	−3.0%	1.1%	9.3%
EURGBP	1W	88%	0.3%	−2.1%	0.5%	8.2%
EURGBP	1M	88%	0.6%	−4.7%	1.1%	9.2%

Table 3: Calibrated jump parameters as the second round of the French election 2017, May 7th, came into standard tenor maturities and deformed the implied volatility smile, for a range of currencies. The risk neutral probability p_u is for an increase in the value of EUR. The calibration failed for the 2W calibration, since the expected variance reduced with the release of the outcome from the first round of the election.

All parameters calibrated for the smile deformations are given in Table 3, for smile deformations calibrated to EURGBP, EURAUD, EURUSD, and EURJPY as the 1W and 1M smiles receives the event. After the results of the first round, the absolute expected jump decreased.

6 Summary and Future Work

The value of an asset is often strongly related to a company, or economy. As new information regarding the company or economy is released, the asset can shift in value significantly. Trading therefore intensifies around such events. Since many events are scheduled far in advance, the associated uncertainty is continuously visible in the implied options market.

Even though events rarely have two outcomes, one can describe the information as being positive or negative for the asset. We have introduced a bimodal model that describes the impact on the asset as an expected jump. By varying the bimodal scenario probabilities, the future jump assumption generates a change in implied volatility skew, and can generate concave – frowning – implied volatility smiles. In the presence of an implied volatility smile the frowns becomes moustaches, or "W", smiles, showing the many faces of the implied options market.

By looking at two smiles with maturities straddling the event we can calibrate the jump to the changes in the implied volatility smiles. The change in implied volatility skew is modelled as being generated from only the future jump. The uncertainty in the UK referendum 2016, and French election 2017, on the implied FX options market, had significant impact on the implied volatility. The single jump model was calibrated to these two events, and can be used to explain much of the observed changes in the implied volatility surfaces.

There are several ways to extend the simple future jump model in this document. For example, we may describe the spot dynamic after the jump differently depending on outcome. The conditional log returns, r_u and r_d, can be described as stochastic. By giving them some uncertainty, the shift in concavity of the implied volatility is reduced. When the event occurs between two tenors, the temporal interpolation of volatility needs to change at the time of the event. As a jump distorts the smile significantly, any interpolation method has to ensure the forward variance from after the event to the next tenor is positive. Such interpolation investigations are left to be detailed.

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A Sensitivity To Regression Weight

The regularization weight impacts the flexibility of the level of volatility extrapolation level, σ_e. In (12) we suggest using an extrapolation weight proportional to the 2Δ strike. This choice give a good balance of explaining the change in implied volatility between the jump, and the diffusive σ_e level. Table 4 illustrates parameters when other regularization weights are used than the suggested of Table 1, on the example of the UK referendum.

Pair	Tenor	p_u	r_u	r_d	$E\left[\frac{\lvert dS\rvert}{S}\right]$	σ_e
GBPUSD	1W	66%	4.3%	−8.9%	5.8%	21.4%
GBPUSD	2W	73%	3.2%	−9.3%	4.8%	16.7%
GBPUSD	1M	85%	1.6%	−9.6%	2.8%	11.1%
GBPUSD	2M	88%	1.5%	−11.0%	2.6%	10.8%
EURUSD	1W	70%	1.4%	−3.3%	1.9%	11.3%
EURUSD	2W	78%	0.9%	−3.5%	1.5%	9.3%
EURUSD	1M	94%	0.3%	−5.0%	0.6%	8.6%
EURUSD	2M	96%	0.4%	−9.8%	0.8%	8.7%
GBPAUD	1W	58%	4.1%	−5.9%	4.8%	19.1%
GBPAUD	2W	66%	2.8%	−5.7%	3.7%	15.0%
GBPAUD	1M	74%	1.8%	−5.5%	2.7%	11.8%
GBPAUD	2M	83%	1.3%	−6.4%	2.1%	11.9%
GBPJPY	1W	73%	3.5%	−10.1%	5.2%	27.0%
GBPJPY	2W	77%	2.9%	−10.1%	4.5%	20.1%
GBPJPY	1M	88%	1.5%	−11.2%	2.6%	13.6%
GBPJPY	2M	92%	1.3%	−15.7%	2.4%	14.5%
EURGBP	1W	38%	6.8%	−4.4%	5.4%	19.2%
EURGBP	2W	32%	7.1%	−3.6%	4.8%	14.8%
EURGBP	1M	14%	8.4%	−1.4%	2.4%	10.5%
EURGBP	2M	12%	8.4%	−1.2%	2.2%	9.8%

(a) Referendum, GBP move. Regression vega-weight at 10Δ strike.

Pair	Tenor	p_u	r_u	r_d	$E\left[\frac{\lvert dS\rvert}{S}\right]$	σ_e
GBPUSD	1W	83%	1.9%	−9.6%	3.2%	100.4%
GBPUSD	2W	84%	1.8%	−10.5%	3.1%	74.1%
GBPUSD	1M	86%	1.5%	−9.9%	2.6%	14.6%
GBPUSD	2M	88%	1.4%	−11.2%	2.5%	19.0%
EURUSD	1W	88%	0.5%	−4.0%	0.9%	35.3%
EURUSD	2W	90%	0.5%	−4.2%	0.9%	25.1%
EURUSD	1M	94%	0.3%	−5.0%	0.6%	8.6%
EURUSD	2M	96%	0.4%	−9.4%	0.8%	4.0%
GBPAUD	1W	81%	1.3%	−5.7%	2.1%	90.4%
GBPAUD	2W	80%	1.5%	−6.2%	2.4%	57.0%
GBPAUD	1M	84%	1.2%	−6.3%	2.0%	19.3%
GBPAUD	2M	86%	1.1%	−6.7%	1.9%	20.9%
GBPJPY	1W	87%	1.7%	−12.1%	2.9%	90.0%
GBPJPY	2W	88%	1.6%	−12.1%	2.8%	71.2%
GBPJPY	1M	89%	1.3%	−11.8%	2.4%	16.4%
GBPJPY	2M	94%	1.0%	−18.1%	1.9%	21.7%
EURGBP	1W	20%	6.7%	−1.8%	2.8%	96.5%
EURGBP	2W	20%	7.5%	−2.0%	3.2%	72.6%
EURGBP	1M	12%	8.6%	−1.3%	2.2%	13.1%
EURGBP	2M	12%	8.4%	−1.2%	2.2%	9.5%

(b) Referendum, GBP move. Regression vega-weight at 0.1Δ strike.

Table 4: Jump parameters for Calibrated jump parameters as the UK referendum came into standard tenor maturities and deformed the implied volatility smile, for a range of currencies. The risk neutral probability p_u is for an increase in the value of the currency pair over the event. Table (a) show parameters when the extrapolation level is forced to be close to the average volatility level, as the regression weight is large. Table (b) show parameters when the extrapolation level is flexible, since the regression weight is small.

Table (a) show parameters when the regularization weight is large, proportional to the vega at the 10Δ-strike. Table (b) have a small regularization weight, proportional to the vega at the 0.1Δ-strike. Comparing the two tables we see that the extrapolation level σ_e increase as the regularization weight is reduced. More of the smile deformation is explained by the extrapolation level than by the jump. Conversely, the expected jump length, $E\left[\frac{|dS|}{S}\right]$, is larger for a larger weight. This is because the jump is forced to explain more of the smile changes since the range of viable values of σ_e is smaller. The tables can be compared with Table 1, which uses our suggested regularization weight proportional to the vega at 1Δ.

B Comparison with Mixture Model

Typically, one of the two bimodal outcomes is followed by a prolonged period of heightened realised volatility. Describing the event only by differences in volatility levels does also generate a implied volatility skew.

In this appendix, we compare results from the anticipated jump model, with results from the mixture model detailed shortly in Section B.1. The mixture model is a combination of two standard log-normal volatility processes, where the model parameters are conditional on the bimodal outcomes. That is, mainly the conditional volatility level shapes the smile. Both the mixture model and jump model can be calibrated to market data to imply probabilities of outcomes of bimodal events. The single jump model is calibrated on dates where the standard tenor comes to incorporate the event, whereas the mixture model is calibrated to interpolated implied volatility smiles for a range of dates leading up to the event. The future jump model, and the mixture model, are both simple bimodal models while being very different in nature.

The data can be considered of good quality, especially for the standard tenor maturities, 1W, 2W, 1M, 2M, and so forth, where options are liquidly traded and implied volatilities are directly observable in the market. The market data t When the event is within the next week of the calculation date, the spread is larger, and the implied volatility less certain. Any results for shorter maturities should therefore be assumed uncertain. By looking at model results for a range of dates, the trending impact may yield some understanding of the event priced by the market. The market data used in this study is provided by Bank of America Merrill Lynch unless explicitly mentioned.

B.1 Mixture Model – Separate Up and Down Log-normal Processes

The prices of options maturing after a large event include the market expectation of the move in the exchange rate due to the event. The mixture model describes the underlying conditional on the 'up' or 'down' outcome, as following a standard log-normal process. With a subscript of d and u for the condition being on the down and up scenarios respectively, the dynamics for the underlying S are

$$\begin{aligned} dS_u/S_u &= \mu_u dt + \sigma_u dW, \\ dS_d/S_d &= \mu_d dt + \sigma_d dW, \end{aligned} \tag{13}$$

for times t up until the maturity. That is, the underlying is assumed to follow a Black Scholes dynamic conditional on either of the two possible outcomes. The vanilla call option expected value in the mixture model is

$$C_{mix}(K) = p_u C(\mu_u, \sigma_u, K) + (1 - p_u) C(\mu_d, \sigma_d, K), \tag{14}$$

where $C(\mu, \sigma, K) = E\left[(S-K)^+\right]$ is the call option's expected value given that S follows a log-normal distribution with parameters μ and σ. For an arbitrage free model, the forward must be repriced. As in [4], we suggest using this condition to express the down drift in terms of the market data and the probability, $\mu_d = f(T, p_u, F_{0,T}, S_0)$. To further constrain the parameter fitting, it is possible to take the probability as given by the betting market [8]. However, we have chosen to calibrate the probability using option data only, and then compare it with probabilities from the betting market. The remaining parameters, $\mu_u, \sigma_u, \sigma_d, p_u$, are calibrated to option prices.

Mixture Model Calibration

In our numerical experiments, we calibrated the mixture model parameters, such that prices (14) are close to market prices. The four parameters, $\mu_u, \sigma_u, \sigma_d, p_u$, are calibrated to minimize the implied volatility differences at five strikes. Option prices at strikes where the call (put) option delta is $0.10(-0.10)$, and $0.25(-0.25)$, as well as the call price for the At-The-Money strike [5], were matched by minimizing the sum of squared price differences.

B.1.1 Predicting Events From One Smile Only?

The mixing-parameters are calibrated to the implied volatilities of only one smile. If we use the same parameters for other maturities, the prices produced by the model do not match the market. The mixture model is very useful for many purposes, as illustrated in the enjoyable book [2]. However, using a mixture model for anything but pure vanilla contracts is dangerous [19]. When predicting event probabilities, the change in forward skew and forward variance is key, and the mixture model is again not appropriate.

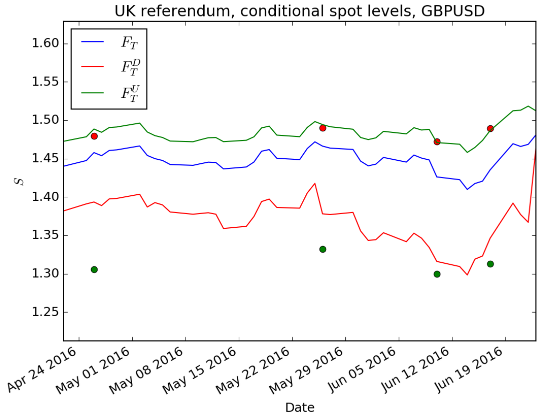
The mixture model can be calibrated to any implied smile, also when no significant event is expected. For instance, when calibrated to a typically skew AUDJPY smile, the mixture model predicts an 'up' jump occurring with $\sim 60\%$ likelihood. A skew implied smile generates a bimodal distribution with a skew probability. The fact that other factors and unrelated events, produces a skew implied smile is not taken in consideration. Typically, there are a multitude of poll releases and political discussions leading up to the election or referendum. All such events affect the smile with maturity after the event, and therefore also the calibrated parameters of the mixing model. Some of these events will be unrelated to the event we are investigating, since regular news releases, as EU meetings, NFP releases etc, also affect the implied volatility smile. All these events influences the mixing model parameters. The French election 2017, is a typical example of such an event. Blatantly ignoring (or forgetting) that the election had two rounds, the mixing-model still produces interesting results. The results, however, do not concern the actual second round of the election, but instead the first election round.

An event causes a change to the implied volatility surface in the temporal/maturity direction. The additional variance attributed to news announcements causes a jump in the forward variance [6]. An event is rarely the dominant variance contributor when the event is more than a few days away. Still, the mixture model is typically used to imply levels of future spot conditional only on the terminal event, using implied volatility quotes with maturities over a week.

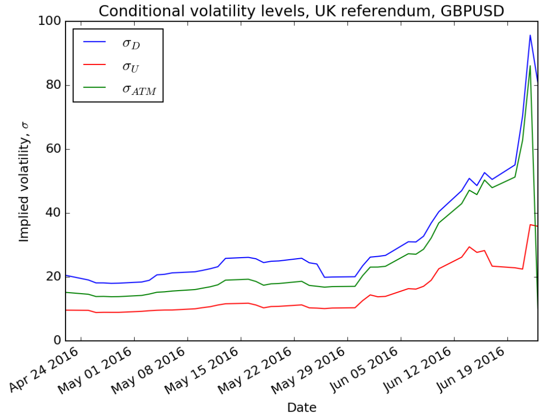
To capture the impact due to the event, one must compare the implied volatility smile with maturity after the event, with the smile with maturity before the event. If the smile with maturity after the event is considered by itself, the model predictions does not relate to the event in isolation, but rather everything that can occur up until the post-event maturity.

B.2 UK Referendum With The Mixture Model and A Future Single Jump

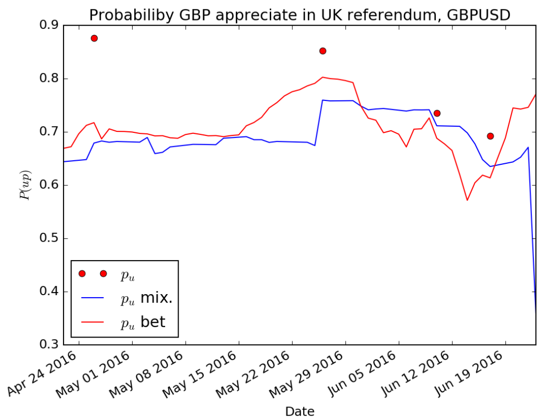
Illustrations of the mixing model parameters are shown with lines in Figure 8. Results from the single jump model are shown with filled circles in the same figures.



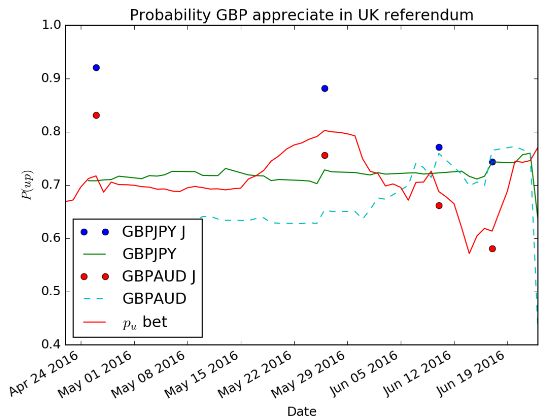
(a) Referendum Predicted Spot Levels, GBPUSD



(b) Referendum conditional volatility, GBPUSD



(c) Referendum Probability, GBPUSD



(d) Referendum Probability, GBPJPY & GBPAUD

Figure 8: Model parameters and predictions when the model is calibrated to the implied volatility smile with maturity just after the UK referendum 2016, using market data for a range of dates, from both the single jump, and mixing models. Full circles "•" shows predictions from the single jump model. (a) shows the expected forward of GBPUSD spot conditional on the up, or down, scenarios. (c) shows the probability of a leave outcome of the referendum predicted by the implied GBPUSD market, and the average odds available on betting exchanges. (d) shows the probability of a leave outcome of the referendum predicted by the implied GPBAUD and GBPJPY market, and the average odds available on betting exchanges. The betting probabilities are sourced via Oddschecker on Bloomberg.

The underlying GBPUSD spot rate, as well as the predicted conditional spot levels for the GBPUSD exchange rate, are shown in Figure 8a. The spread between the upper and lower levels increases as the event comes closer. The conditional implied volatility levels are shown in Figure 8b. For a long time period, the levels are quite stable, but as the event comes closer, the ATM volatility and conditional downside volatility, increase significantly. Figure (c) shows the model estimated probability of a remain outcome using GBPUSD implied volatilities, and the corresponding probability implied from the betting markets given by Oddschecker via Bloomberg. Comparing (a) and (c) we see a strong correlation between the betting market probabilities and the spot level. As the event comes into the 1W bucket, the estimated remain and leave spot levels for GBPUSD are 1.4866 and 1.3463, compared with the reference spot at 1.4354. The spot range predicted is closely reflected to the exchange rate, which moved from 1.49 to 1.37 as the results were released to the public.

The predicted probability for a leave-outcome is significantly smaller for the jump assumption than both the betting market and the mixing-model. This seems to be a bad indication for the jump model, but the observation is in-line with the market temperament leading up to the election. A leave outcome was not given enough GBP-risk by the market prices. The risk neutral jump probabilities implied from GBPJPY and GBPAUD differ noticeably, as illustrated with the blue and red circles in Figure (d).

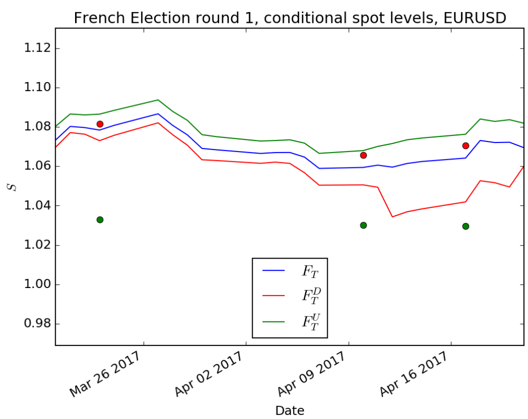
Figure 8a shows the spot scaled with the jump length. As seen, the differences between the up and down jump levels are large. Since the leave outcome is deemed unlikely, the expected change in spot is considerable smaller than the difference between the up and down levels.

B.3 French Election (Both Rounds) With Mixture Model and an Expected Jump

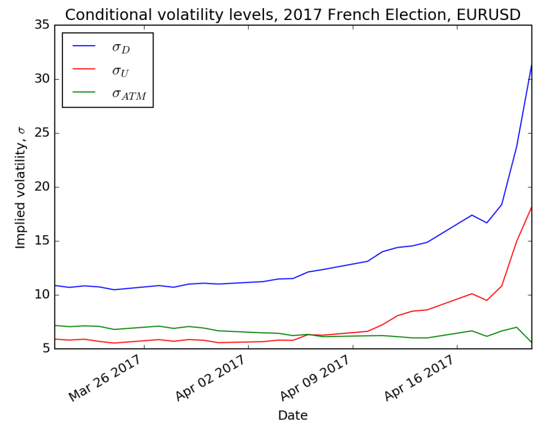
The jump and mixing-models was also calibrated to both rounds of the french election.

B.3.1 French Election, First Round

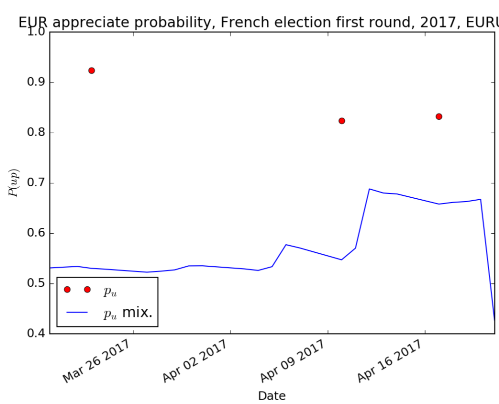
Figure 9 shows the jump and mixing model predictions for the first round of the French Election taking place on April 23rd 2017, predicted by implied volatility and spot data for EURUSD. Figure (a) shows the predicted exchange rate levels conditional on the two outcomes together with the spot traded in the market. Figure (c) shows the model estimated probability for a EUR increase as the results are released.



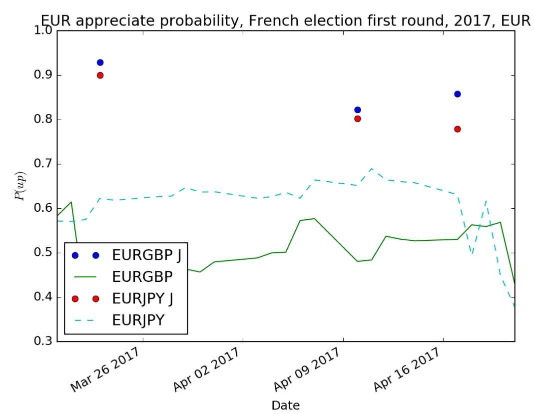
(a) French election, 1st round, Predicted Spot Levels, EURUSD



(b) French election, 1st round, conditional volatility, EURUSD



(c) French election, 1st round, Probability, EURUSD



(d) French election, 1st round, Probability, EURJPY & EURGBP

Figure 9: Jump and mixture model parameters and predictions when the model is calibrated to the implied volatility smile with maturity just after the first round of the French Election 2017, April 23rd. •'s show the jump model predictions. (a) shows the expected forward of EURUSD spot conditional on the up, or down, outcome. (b) shows the conditional log-normal volatility levels σ_u, σ_d, predicted by the EURUSD implied volatility surface. (c) shows the models implied probability of a outcome of the French 2017 election that would decrease the value of the EUR currency, using EURUSD implied volatility data. (d) shows the models implied probability of a outcome of the French 2017 election that would decrease the value of the EUR currency as predicted using EURJPY and EURGBP implied volatility data.

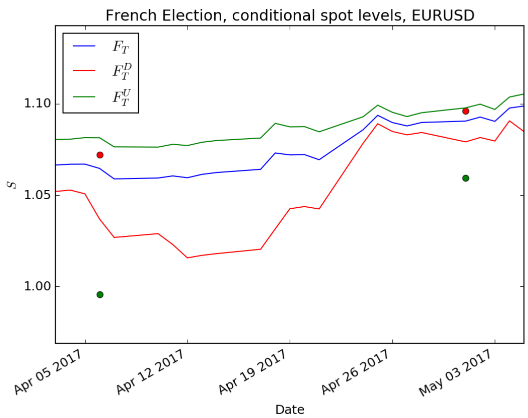
The results look similarly plausible, but the spread between the spot levels goes from 1.7% difference in April, to 0.73% during the first few days in May. This stems from the results of the first round of the French election, which was held April 23rd, 2017.

Still, our analysis of the second round seems to indicate a wildly different forward level in the case of an up or down move, when in reality there being a much less relevant event around the second event.

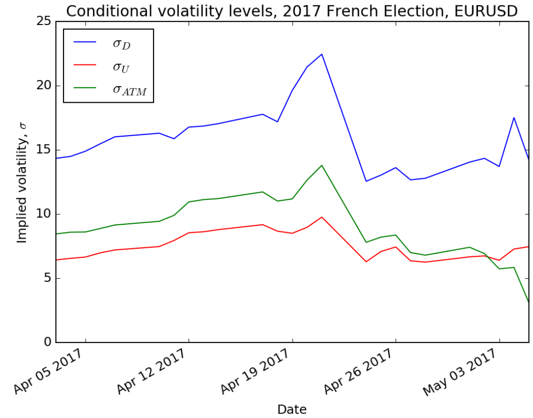
B.3.2 French Election, Second Round

There were some difficulties in calibrating to the second round of the election. The jump model has problems calibrating the 2W tenor roll, since the forward variance did not increase as predicted by to decrease between the dates. This is due to the pre-event data coming from t_0 at April 21st, and the target data is taken as of April 24th. That is, dates straddling the first round of the election. Since the events are strongly correlated, the expectations of the second round change significantly as the results of the first round are released.

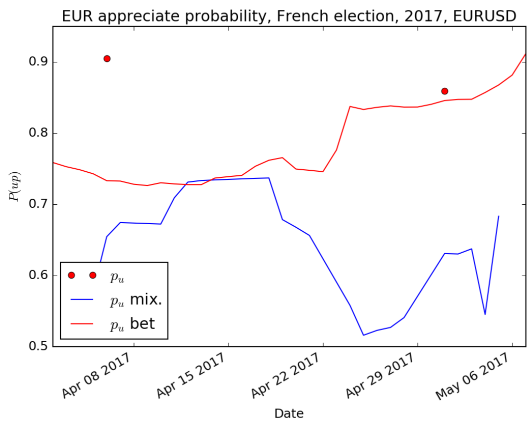
Figure 10 shows the model predictions for the second round of the French Election taking place on May 7th 2017, predicted by implied volatility and spot data for EURUSD. Figure (a) shows the predicted exchange rate levels conditional on the two outcomes together with the spot traded on the market. Figure (c) shows the model estimated probability for a EUR increase using EURUSD implied volatility data, and the probability of one of the candidates to win implied from the betting market. The odds are taken from Oddschecker via Bloomberg. Figure (d) similarly shows the model probability using EURJPY and EURGBP implied volatility data.



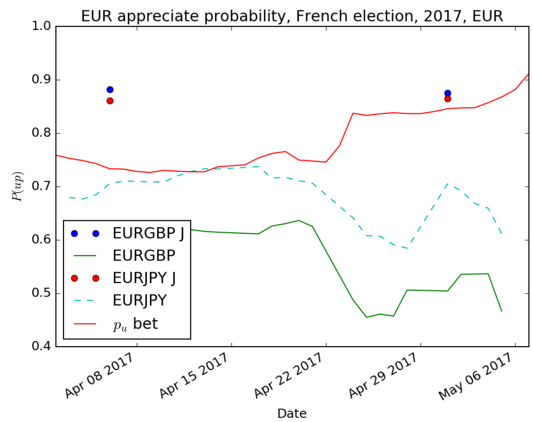
(a) French election, round 2, Predicted Spot Levels, EURUSD



(b) French election, round 2, conditional volatility, EURUSD



(c) French election, round 2, Probability, EURUSD



(d) French election, round 2, Probability, EURJPY & EURGBP

Figure 10: Mixture model parameters and predictions when the model is calibrated to the implied volatility smile with maturity just after the second round of the French Election 2017, on May 7th 2017. (a) shows the expected forward of EURUSD spot conditional on the up, or down, outcome. (b) shows the conditional log-normal volatility levels σ_u, σ_d, predicted by the EURUSD implied volatility surface. (c) shows the probability of a outcome of the French 2017 election that increase the EUR value, using EURUSD implied implied volatility data, and probability for one of the candidates to win implied from the betting market odds. (d) further show the probabilities inferred using EURJPY and EURGBP implied volatility data. The betting probabilities are sourced via Oddschecker on Bloomberg.

The mixture model conditional spot levels indicates a jump of on average 1.9% up, or 1.6% down in value of EUR. The average expected jump is a change of 1.7% in spot. That is, the event is predicted to have a large impact on the spot. The spread between the spot levels decrease significantly as the date of the French election approaches, as the spread between the conditional forward jumps is 0.73% during the first few days in May. This stems from the results of the first round of the French election, which was held April 23rd, 2017. The majority of the uncertainty and implied volatility skew associated with the election was associated with the first round of the election. As the results from the first election round was absorbed by the market, the uncertainty of the actual election outcome was reduced significantly.

Still, our analysis of the second round seems to indicate as causing a large change in future forward levels when in reality there being a much less relevant event around the second event.