How Rational Are the Option Prices of Hong Kong Dollar Exchange Rate?

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October 25, 2019

Abstract

In this paper, we study the prices of the options on Hong Kong's linked exchange rate. The study was motivated by the apparent contradiction that options with strike prices outside the narrow trading band have positive prices. We developed a simple regime-switching model of the exchange rate and provided a formula for the prices of its options. The option pricing formula allows us to back out the implied probability of the failure of the linked exchange rate regime. With the option price data for the period from June 1, 2005 to July 31, 2018, we find that the market's belief about the likelihoods of the failure, as implied by the option prices, is too high to be justified by the possibility of the failure alone. This is first shown qualitatively, without using any model, with ratio of empirical over implied volatilities and the implied range of strike prices. Then, using the model, we find that, close to 40% of the sample period, the market thinks the failure probability is greater than 10%. When contrasted with the fact that the regime has not failed since 1983, the p-value of that event is less than 0.1%. Our finding suggests that, while the failure of the linked exchange rate regime is a significant risk factor, it cannot explain the risk premium seen in the option prices.

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1 Introduction

Many countries/regions, especially small developing countries/regions, maintain in one form or another fixed exchange rates. In Hong Kong, for example, one unit of US dollar has been targeted to trade for 7.80 Hong Kong dollars with a narrow band $[7.75, 7.85]$ of variation. Interestingly, even though the exchange rate are targeted with a very narrow band, there is an active option market on the exchange rate. This seems illogical since the exchange rate is essentially fixed. It is even more so as options with strike prices outside the band have been seen to have non-zero values. The conventional explanation is that investors in markets with a pegged foreign exchange face the uncertainty that at some point in the future the monetary authority may not be able to maintain the pegged exchange rate. Options are insurance contracts for that uncertainty.

Figure 1 shows the ratio of the empirical volatility of the USD-HKD exchange rate over the volatility implied from six-month at-the-money (ATM) options in comparison with the same ratio for three free-floating exchange rates EUR-USD, USD-JPY and GBP-USD.

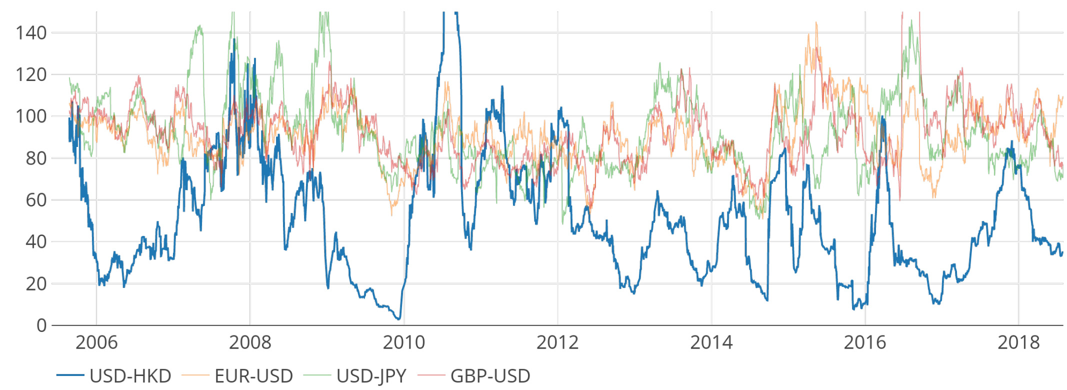


Figure 1: Ratio of Empirical over implied volatilities for the period of June 2005 to July 2018. The blue line corresponds to the USD-HKD pair while the other three corresponds to the EUR-USD, USD-JPY and GBP-USD. The empirical volatilities are 60 days annualized, while the implied volatilties are the 6-month ATM volatilities.

A casual look at this figure suggests that the average ratio of empirical over implied volatilities for USD-HKD exchange rate is lower than those for the free floating currencies while the volatility is significantly higher. Indeed, the mean of and standard deviation of the ratio for USD-HKD exchange rate are 51% and 30%, respectively, while the mean and standard deviation of the ratio for EUR-USD exchange rate are 91% and 13%, respectively.

This contrast seems to lend some empirical support to the conventional wisdom. The lower average and higher standard deviation of the ratio of empirical over implied volatilities for USD-HKD suggest that the market sees more uncertainty than the normal daily up and down of the exchange rate. That uncertainty could be about the potential failure of the pegged exchange rate. However, the pegged exchange rate of USD-HKD has never failed in the 35 years since 1983. In light of that, many questions arise. Are the difference in average ratio and volatility of the ratio quantitatively justified by that uncertainty? What more can the option prices tell us? What do they tell us about the market's belief about the likelihood of the failure of the currency peg? What do they tell us about the market's belief about the true volatility after the failure of the pegged exchange rate? The purpose of this paper is to provide a simple model and an empirical analysis that speaks to these two aspects of the investors' belief. The empirical results will raise some deep questions. In that respect, it is hoped that this study will lay the ground work for future studies that will examine at a more fundamental level the economics behind the stylized empirical patterns documented in this paper.

The literature on derivatives on fixed exchange rate is scant. This is not surprising because, in theory, options on truly fixed exchange rates are of zero value. In practice, however, fixed exchange rates are typically allowed to move within a narrow band. Because of that, the study of fixed exchange rate is related to target zone models. Krugman (1991) and Svensson (1991) are the pioneer target zone models of exchange rate. That literature is large and some of the earlier research includes Froot and Obstfeld (1991), Bertola and Caballero (1992), Delgado and Dumas (1993), Dumas and Svensson (1994), and Dumas, Jennergren, and Naslund (1995). In Krugman (1991), Froot and Obstfeld (1991), Delgado and Dumas (1993), and Dumas and Svensson (1994), it is assumed that the target-zone regime can always be maintained. Svensson (1991), Bertola and Caballero (1992), and Dumas, Jennergren, and Naslund (1995) considered the possibility that the original target zone may not be maintained and may be subject to adjustment to new target zone. Some of the earlier empirical research includes Bekaert and Gray (1998). Dumas, Jennergren, and Naslund (1995) studied the pricing of options on exchange rate with target zones that is subject to realignment risk. See Duarte, Andrade, and Duarte (2013) for a survey of the literature. There is, however, an important difference between a fixed exchange rate with a narrow trading band and a target zone regime of exchange rate. In a target zone regime of exchange rate, there is risk of realignment. However, when the exchange rate is realigned, it is realigned into a new target zone. In a fixed exchange regime, if the band is broken, it is often the case that the exchange rate regime is moved from the fixed rate regime to a free-floating rate regime.[1] The purpose of our study is to introduce a basic regime-switching model for the fixed exchange rate which, with the help of the option data, can be used to make a first attempt in understanding fixed exchange rates by addressing some of the questions raised above.

The rest of the paper is organized as follows: in Section 2 we provide a simple regime-switching model of exchange rate that allows the exchange rate to begin in the fixed rate regime and then, possibly switch to the free-floating rate regime. In Section 3, we document a set of stylized facts without using our option pricing model. In Section 4, using the model as a lens, we study the option prices. Finally, Section 5 concludes.

2 Model

In this section, we provide a stylized regime switching model of exchange rate that will be used for analyzing the option data on USD-HKD exchange rate. It should be clarified at the beginning that the purpose of the model is not to provide an explanation of why the data exhibit the documented characteristics, but rather to serve as a lens for looking at the data quantitatively and to document further from the quantitative perspective of the characteristics of the exchange rate data.

There are two currencies in the economy, the domestic currency called HKD and a foreign currency called USD. The currencies can be traded in the spot market where investors can trade the domestic currency HKD for the foreign currency USD and vice versa. Let $S(t)$ denote the price, that is the exchange rate, at time t of one unit of foreign currency in terms of domestic currency. Let r_e, where $e = d$ or f, denote the risk free rate in the domestic or foreign country. We assume for simplicity that both rates are constant. There is also a market of options where investors buy and sell call and put options on the exchange rate. We will also assume that investors have homogeneous belief.

[1] As shown in the case of the Swiss Franc (EUR-CHF) in 2015, the Thai Bath (USD-THB) in 1997, the Czech Koruna (EUR-CZK) in 2017, or the Argentinean Pesos (USD-ARS) in 2001-2002 for instance.

2.1 Frictionless Market

To set the benchmark, we start with the frictionless spot market where the spot price of the foreign currency fluctuates according to the demand and supply of the investors. We assume that the floating spot price follows the dynamic[2]

$$\frac{dS(t)}{S(t)} = \mu(t)dt + \sigma dW_0(t), \quad S(0) = S > 0 \tag{1}$$

where $\mu(t)$ is a deterministic function, σ is a constant and $W_0(t)$ is a Brownian motion. As the markets are complete, the following theorem is standard (Garman and Kohlhagen (1983)).

Theorem 2.1. *Let Φ be the indicator of the type of the option such that $\Phi = C$ or P if the option is a call or put option. Let $\phi = 1$ or -1 if $\Phi = C$ or P. Let*

$$GK(x, y, \sigma, \Phi) = \phi\Big[xN\big(\phi d_+(x, y, \sigma)\big) - yN\big(\phi d_-(x, y, \sigma)\big)\Big] \tag{2}$$

where

$$d_\pm(x, y, \sigma) = \frac{\ln(x/y)}{\sigma} \pm \frac{\sigma}{2}$$

If the domestic and foreign interest rates are r_d and r_f, respectively, and the maturity of the option contract is T, then the price of a vanilla option is given by

$$GK(Se^{-r_f T}, Ke^{-r_d T}, \sigma\sqrt{T}, \Phi) \tag{3}$$

Clearly, under this frictionless model, the implied volatility is constant which is clearly rejected by the data documented in Section 3.

2.2 Exchange Rate with Regime Switching

We now turn to the model of pegged USD-HKD exchange rate. It is well understood that for any fixed exchange rate regime there is a non-zero probability that the band cannot be held. In the case of USD-HKD exchange rate, the reason could be that Hong Kong Monetary Authority (HKMA) has finite amount of foreign currency reserve and may run out of the reserve in trying to defend the

[2] On some filtrated probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ carrying a Brownian motion W. We assume that the filtration $\mathbb{F}$ is generated by W and completed, henceforth, satisfies the usual assumptions.

band. Or it could be that the economic condition of the economy warrants a new level of average exchange rate which is inconsistent with the band. We model fixed exchange rate by a regime switching model.

The exchange rate $S(t)$ is assumed to follow the following dynamic,

$$d\underset{\sim}{S}(t) = \underset{\sim}{S}(t)\left(\underset{\sim}{\mu}(t)dt + \underset{\sim}{\sigma} dW_0(t)\right), \quad S(0) = S > 0 \quad \text{for } t < \tau \tag{4}$$

$$d\bar{S}(t) = \bar{S}(t)\left(\bar{\mu}(t)dt + \bar{\sigma} dW_0(t)\right), \quad \bar{S}_\tau = (1+\kappa)\underset{\sim}{S}(\tau-) \quad \text{for } t \geq \tau \tag{5}$$

where τ is an independent random Poisson time at which the exchange rate is no longer pegged, the jump intensity of which is λ_0. Here we have modeled the fixed exchange rate movement with a basic regime-switching model with only two regimes. Before time τ, the spot exchange rate $S(t)$ given in equation (4) is as that in equation (1) with a low volatility $\underset{\sim}{\sigma}$. This is to capture the fact that in a fixed regime, the exchange rate is maintained by the central bank within a narrow band.[3] At the time when the fixed rate regime is abandoned, the exchange rate jumps from S to $S(1+\kappa)$, where κ is the relative change from old to new spot rate. The probability of a regime switch before time T is $p = 1 - e^{-\lambda_0 T}$. We are agnostic about the direction in which the band will be first broken. It is assumed to break only once. A negative κ means a depreciation, while a positive κ means an appreciation. Once the fixed rate regime is abandoned, it will not be restored. After time τ, the exchange rate is given by equation (5) where $\bar{\sigma}$ is the volatility of the spot rate in the free-floating rate regime.

The model is a departure from the standard target zone models in the literature (Krugman (1991), Svensson (1991), Froot and Obstfeld (1991), Bertola and Caballero (1992), Delgado and Dumas (1993), Dumas and Svensson (1994), and Dumas, Jennergren, and Naslund (1995)). In a target zone regime of exchange rate, there is risk of realignment. However, when the exchange rate is realigned, it is realigned into a new target zone. In a fixed exchange regime, if the band is broken, it is often the case that the exchange rate regime is moved from the fixed rate regime to a free-floating rate regime.

[3]Technically, in our model the exchange rate can get outside of the band with positive probability. The USD-HKD rate does touch the upper and lower bounds from time to time. It has never been outside the band in a manner that can be taken as the violation of the band. The low volatility level $\underset{\sim}{\sigma}$ is chosen such that the probability that the exchange rate according to equation (4) goes outside the band is small. One obvious alternative is to model the exchange by a target-zone model before the switch of the regime as Krugman (1991) and then model it as a free-floating rate regime. The advantage of our model is that it is analytically simpler. As we will argue later in Section 4, it would under-estimate the switching intensity λ_0 and hence strength the empirical results documented.

For our purpose, our basic regime-switching model is meant to extract three important pieces of information from the options prices in a simplest possible way. That is, the jump intensity λ_0, the jump size of the exchange rate κ, and the future volatility after the regime switch $\bar{\sigma}$. These are three important pieces of information concerning the market belief about the potential failure of the current fixed exchange rate regime. In Section 4 we will see that these three parameters will allow us assess qualitatively what the market belief is.

In equilibrium, in the absence of arbitrage, there exists a risk-neutral probability measure such that the spot rate follows the dynamic:

$$\frac{dS(t)}{S(t-)} = (r_d - r_f)\,dt + \sigma(\tau,t)dW(t) + d(L(t) - \lambda\kappa t \wedge \tau)$$

where $W(t)$ and

$$L(t) = \begin{cases} 0 & \text{for } t < \tau \\ \kappa & \text{for } \tau \leq t \end{cases} \quad \text{and} \quad \sigma(\tau,t) = \begin{cases} \underline{\sigma} & \text{for } t < \tau \\ \bar{\sigma} & \text{for } \tau \leq t \end{cases} \tag{6}$$

are Brownian motion and stopped Poisson process under the risk neutral probability measure and τ is the first time that an independent Poisson process jumps. Denote by λ the jump intensity under the risk neutral probability measure which can be different from λ_0, the intensity under the objective probability measure. The risk premium can be written as

$$\mu(t) - (r_d - r_f) = \xi_B \sigma + \lambda_0 \xi_P \kappa 1_{[0,\tau)}(t) \tag{7}$$

where ξ_B is the price for diffusion risk and ξ_P is the price for Poisson jump risk and $\lambda = \lambda_0 \xi_P$.

A classical application of Dolean-Dade stochastic exponential yields

$$S(t) = S(0)\exp\left((r_d - r_f)\,t - \lambda\kappa(t \wedge \tau) - \frac{1}{2}\Sigma^2(\tau,t) + \int_0^t \sigma(\tau,s)dW(s)\right)\left(1 + \kappa 1_{[\tau,\infty)}(t)\right) \tag{8}$$

where the quadratic variations – given the jump event τ – are given by

$$\Sigma^2(\tau,t) = \begin{cases} \underline{\sigma}^2 t & \text{if } \tau < t \\ \underline{\sigma}^2\tau + \bar{\sigma}^2(t-\tau) & \text{otherwise} \end{cases} \tag{9}$$

The spot rate (8) simplifies therefore

$$S(t) = \begin{cases} S(0)e^{-\lambda\kappa t}\exp\left(\left(r_d - r_f - \frac{\underline{\sigma}^2}{2}\right)t + \underline{\sigma}W(t)\right) & \text{for } t < \tau \\ S(\tau-)(1+\kappa)\exp\left(\left(r_d - r_f - \frac{\bar{\sigma}^2}{2}\right)(t-\tau) + \bar{\sigma}\left(W(t) - W(\tau)\right)\right) & \text{for } t \geq \tau \end{cases} \tag{10}$$

In equilibrium, the option prices are given by

Theorem 2.2. *Let*

$$V(x, y, \underline{\sigma}, \bar{\sigma}, \kappa, \lambda, \Phi) = e^{-\lambda} GK\left(xe^{-\lambda\kappa}, y, \underline{\sigma}, \Phi\right) + \lambda \int_0^1 GK\left(x(1+\kappa)e^{-\lambda\kappa s}, y, \Sigma(s), \Phi\right) e^{-\lambda s} ds$$

where $\Sigma(s) = \sqrt{\underline{\sigma}^2 s + \bar{\sigma}^2(1-s)}$. Then given the domestic and foreign interest rates, r_d and r_f, respectively, and the maturity T, the price of the vanilla option is given by

$$V(Se^{-r_f T}, Ke^{-r_d T}, \underline{\sigma}\sqrt{T}, \bar{\sigma}\sqrt{T}, \kappa, \lambda T, \Phi) \tag{11}$$

2.3 Volatility Smile under Regime Switching

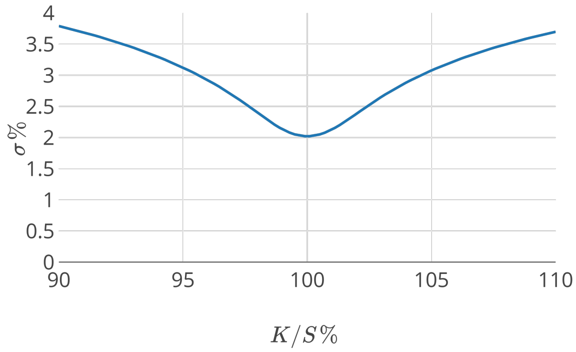
Using (11), the implied volatility $\sigma(m;\theta)$ can be computed as a function of moneyness and the market parameters $\theta = (\underline{\sigma}, \bar{\sigma}, \kappa, \lambda)$. It is implicitly given by

$$GK(1, m, \sigma(m;\theta)\sqrt{T}, \Phi) = V(1, m, \underline{\sigma}\sqrt{T}, \bar{\sigma}\sqrt{T}, \kappa, \lambda T, \Phi) \tag{12}$$

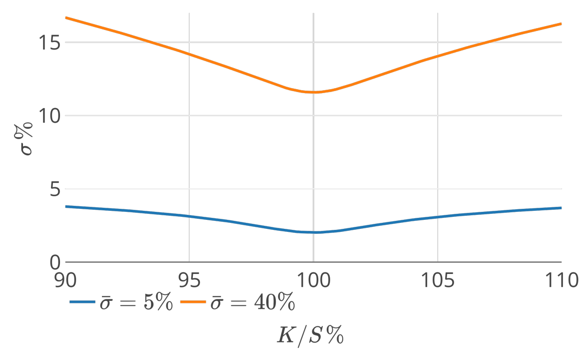
where $m = Ke^{-r_d T}/(Se^{-r_f T})$ is the moneyness of the option adjusted for the interest rates.

The regime switching model, Equation (12) in particular, will be used as a lens to examine the empirical stylized facts documented in Section 4. Before doing that, however, it is useful to have some intuitive understanding of the effect of the parameters on the shape of the option smile curve.

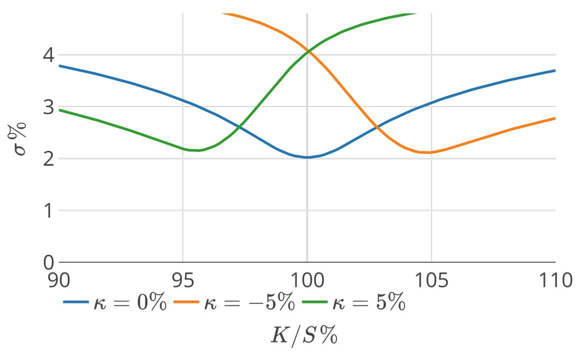
Figures 2(a)-(d) show the effect of the parameters on the shape of the smile curve. Figure 2(a) shows a smile curve with parameter $\theta = (\underline{\sigma}, \bar{\sigma}, \kappa, \lambda) = (0.01, 0.05, 0, 0.5)$, which is the baseline smile curve for the other three figures. The interest rates r_d and r_f, are assumed to be zero. The current volatility of the rate is $\underline{\sigma} = 0.01$. With $\lambda = 0.5$, the probability that a regime switching happens before time t is $1 - e^{-\lambda t}$. The expected time of regime switch is in $1/\lambda = 2$ years. When the switch occurs, the volatility of the rate will jump to $\bar{\sigma} = 0.05$, but with $\kappa = 0$. In other words, the rate itself will not jump. Compared to the GK model of constant implied volatility, the introduction of a regime switching introduces upward curvature in implied volatility. This is because when there is positive probability of switching to the future regime of higher volatility, a move of the strike price away from the current level of exchange rate does not reduce the option price as much as when there is no chance of a regime switch.



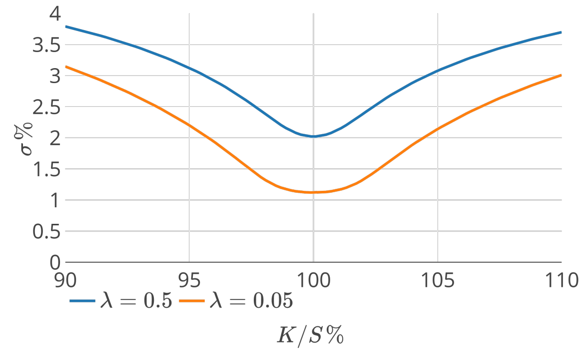
(a) A typical smile curve.



(b) Effect of $\bar{\sigma}$ on convexity of smile.



(c) Effect of κ on skewness of smile.



(d) Effect of λ on level of smile.

Figure 2: Effect of $\bar{\sigma}$, κ and λ on the smile curve. The blue smile curve in the four figures is the base case with parameters $(\underline{\sigma}, \bar{\sigma}, \kappa, \lambda) = (0.01, 0.05, 0, 0.5)$ and $r_d = r_f = 0$.

Figure 2(b) shows the effect of an increase in the volatility parameter $\bar{\sigma}$. Higher $\bar{\sigma}$ has two effects. On the one hand, it makes the smile curve more convex while, on the other hand, it also shifts the smile curve upward. This is intuitive. With higher volatility in the future regime, the current option price is higher and the increase in price is more pronounced for options with strike price that is a distance away from the current level of the rate. Figure 2(c) shows the effect of κ on the smile curve. A negative jump decreases the value of call options. The effect is, however, asymmetric. Downside protection is more expensive. This leads to a change in the skewness of the smile curve. Figure 2(d) shows the effect of the jump intensity parameter λ on the shape of the smile curve. As λ increases, the smile curve shifts upward, producing a level effect.

3 The Stylized Empirical Facts

In this section, we document some stylized empirical facts on the fixed Hong Kong dollar and US dollar exchange rates. The focus is on possibility of the failure seen by the market. The source of our data on the exchange rate and its derivatives is Bloomberg for the period from June 1, 2005 to July 31, 2018.

The Hong Kong dollar has been pegged in one form or the other throughout its history. Its current system started in 1983 with a peg to US dollar with the target of 7.80 USD-HKD. In May of 2005, the Hong Kong Monetary Authority (HKMA) introduced strong-side and weak-side Convertibility Undertaking and set the lower and upper limits of the exchange rate to 7.75 and 7.85, respectively. The Hong Kong dollar has traded against the US dollar in the range of 7.75 to 7.85 Hong Kong dollars since then (HKMA (2005)). This means a deviation of $\pm 0.64\%$ from the target of 7.8.

3.1 Spot Rates and Volatilties

To begin with, Figure 3 shows the exchange of rate of Hong Kong dollar to the US dollar over the period from June 2005-July 2018. The target is 7.80 with the band of $[7.75, 7.85]$. The figure

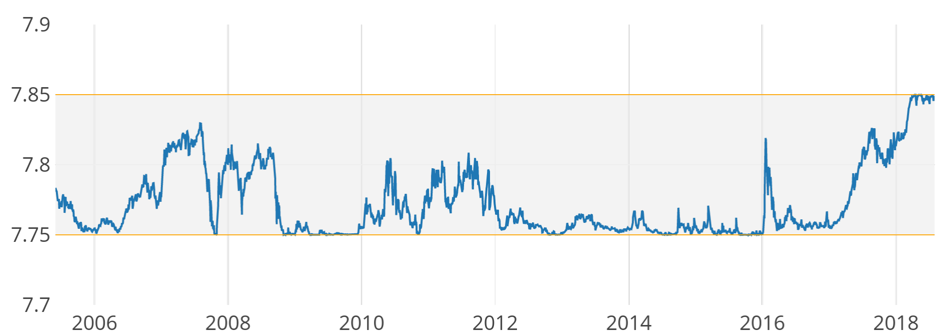


Figure 3: USD-HKD Exchange Rates over the period from June 2005-July 2018.

shows that the exchange rates are below the target level for most of the time. There are several subperiods in which the exchange rates touched or were close to the lower bound. Table 1 provides the summary statistics of the exchange rates.

Mean	Volatility	Below Target	Close to Lower Bound	Close to Upper Bound
7.7726	0.52%	83.20%	30.48%	2.62%

Table 1: Descriptive statistics of daily USD-HKD exchange rate for the period of June 2005 to July 2018. The target rate is 7.80. Volatility is calculated as the standard deviation of the difference of the natural log of the exchange rates in two successive days. The volatility is annualized. In the table, "Close to Lower Bound" is defined as being in the lowest 5% of the band (≤ 7.755). "Close to Upper Bound" is defined as being in the highest 5% of the band (≥ 7.845).

Its mean is 7.7726 which is 27.4% below the targeted rate with respect to the band. As Figure 3 suggests, the rates are below the target most of the time, 83.20% for the period. The exchange rates are in the lowest 5% of the band 30.48% of time while in the highest 5% of the band 2.62% of the time. The closeness to the upper bound occurred during the sub-period from March 21 of 2018 to July 31 of 2018. For the longer period we have data on the exchange rate, from January 2 of 1992 to July 31 of 2018, the rates are below the target 90.37% for the period. The exchange rates 42.08% of the time in the lowest 5% of the band and 1.28% of the time in the upper 5%. Once again, the closeness of the exchange rate to the upper bound happens during the same period from March 21 of 2018 to July 31 of 2018. Note that this longer period time include the 1997 Asian financial crisis period and the 2008 global financial crisis period. Thus the data suggest that the pressure of the appreciation of Hong Kong dollar against US dollar is frequent, while the pressure of depreciation is much less frequent, which is contradictory to the common wisdom that the insufficiency of foreign reserve is the greater potential cause of the failure of the pegged exchange rate.

To contrast, Table 2 reports the volatilities of six pairs of exchange rates, USD-HKD, EUR-USD, USD-JPY, GBP-USD, USD-THB and EUR-CHF rates for the specified periods.

FX	Period	Volatility
USD-HKD	Jun 2005 — Jul 2018	0.52%
EUR-USD	Jun 2005 — Jul 2018	9.71%
USD-JPY	Jun 2005 — Jul 2018	10.39%
GBP-USD	Jun 2005 — Jul 2018	9.64%
USD-THB	Jan 1992 — May 1997	3.26%
USD-THB	Jun 2005 — Jul 2018	5.10%
EUR-CHF	Oct 2011 — Dec 2014	3.38%
EUR-CHF	Feb 2015 — Jul 2018	11.24%

Table 2: Annualized volatilities of USD-HKD, EUR-USD, GBP-USD, USD-JPY, USD-THB, and EUR-CHF rates for the specified periods.

The EUR-USD, USD-JPY, and GBP-USD rates are free floating ones. Thai Baht was pegged to the US dollar during the period from January 1992 to June 1997. After its devaluation in the Asian financial crisis, it has been a managed-float rate. In response to strong appreciation pressure, the Swiss National Bank set in the course of September 2011 a lower bound of 1.2 Swiss francs for one Euro. They revert to a free floating rate in mid January 2015. The most striking comparison in the table is perhaps that between the volatilities of the three floating currency pairs and that of the fixed currency pair of USD-HKD. The volatilities of the floating pairs are all higher than that of USD-HKD by a magnitude of over 18. Admittedly, the comparison is across different exchange rates and hence is not exactly apple to apple. However, the volatilities of the three floating rates are fairly close to each other, which gives us some comfort in the comparison. The EUR-CHF rate experienced both a one sided fixed regime and the fully floating regime. Its volatility during the one-sided fixed regime period is higher than that of USD-HKD by a magnitude of 6.5. After the drop of the lower bound, the volatility of EUR-CHF went back in line with the other free-floating currency pairs. Thai Baht is a currency that has gone through both the fixed exchange regime and the floating exchange rate regime. Due to its wider band around its target rate, USD-THB has a volatility 6.5 time larger than that of USD-HKD. In its floating rate regime, its volatility is lower than the other three floating exchange rate, due likely to its manage-float regime. The comparison in Table 2 suggests that for USD-HKD, the band around its target is tight, and there is nothing that is obviously abnormal about the volatility in comparison with other exchange rates.

3.2 Implied Volatilities

We turn next to the options market. There has been an active market for options on the USD-HKD exchange rate since January 2003 where both call and put options are traded.[4] In this section, we focus on the at-the-money (ATM) options. The ATM options can have three different maturities: 3 months, 6 months and one year. We will focus on the 6-month ATM options.

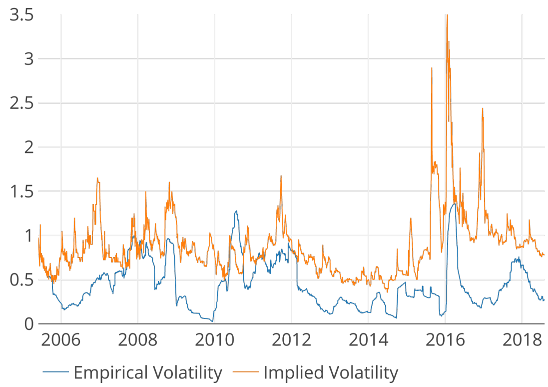
Figure 4 shows that the empirical volatility and the 6-month implied volatility for USD-HKD as well as the ratio of the two for EUR-USD, USD-JPY and GBP-USD markets. The average ratios of the empirical volatility over the implied volatility for the three floating rates are remarkably close to each other, all at around 91%. In contrast, in the case of USD-HKD, the ratio of the two volatilities averages about 51%. It is well known that in free floating currency markets, the implied volatility of the exchange rate deviates from its empirical counterpart. The figure shows, in addition, that the implied volatility of the USD-HKD rate is not only on average higher than its empirical volatility, but also almost twice as higher than that of the three floating exchange rates. Table 3 reports the statistics for the ratios.

	USD-HKD	EUR-USD	USD-JPY	GBP-USD
Mean	51%	91%	92%	92%
Std	30%	13%	18%	20%

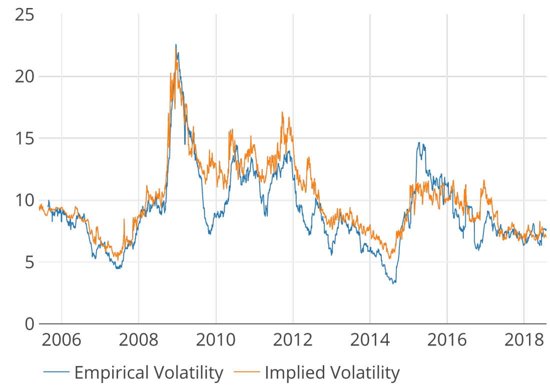
Table 3: Mean and standard deviation of the ratio of empirical volatility to implied volatility of USD-HKD pair in comparison to EUR-USD, USD-JPY and GBP-USD pairs.

We recognize that the underlying economic factors that drive the different exchange rates may be different. Nevertheless, the fact that the ratios of empirical over implied volatilities of those floating exchange rates are similar, while that of USD-HKD is about half of that for floating rate, suggests that the market is charging or the investor is willing to pay a significant premium over and

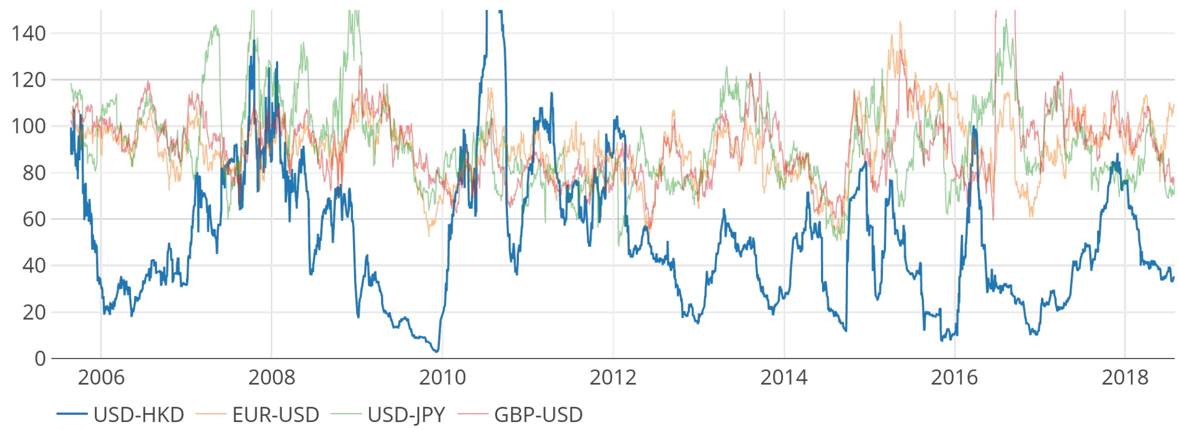
[4] A limited options market for USD-HKD existed since January 1999 with limited choice of strike prices.



(a) Empirical and ATM volatility of USD-HKD



(b) Empirical and ATM volatility of EUR-USD



(c) Ratio of empirical over implied volatility USD-HKD, EUR-USD, USD-JPY and GBP-USD

Figure 4: Empirical and implied volatilities for the period of June 2005 to July 2018. Figure (a) is the empirical and implied volatilities of USD-HKD. Figure (b) is the empirical and implied volatilities of EUR-USD. Figure (c) is the ratio of the empirical volatility over the implied volatility of USD-HKD, EUR-USD, USD-JPY and GBP-USD. The implied volatilities are the 6-month ATM volatilities.

above that in the floating rate market. If the failure of the regime is the unique risk factor, then the premium could be the evidence of the price for that risk.

3.3 Volatility smile

The foreign exchange options are useful financial instruments for risk management. The inherent skewed leverage allows investors to hedge foreign exchange rate risk more efficiently than by using other more primitive instruments. Because of that the prices of options also convey more information on the risk perceived by the investors and the level of tolerance of the risk by the investors. Volatility smile is widely recognized as descriptive of that information. In this section, we will provide some stylized facts from volatility smiles.

Unlike in stock markets where vanilla call and put option prices are quoted directly in option premiums to be paid for given strike prices and maturities, options in foreign exchange market are often quoted in implied volatilities for given option deltas and maturities. Three types of quotes are standard, the at-the-money (ATM) volatility σ_{ATM}, the butterfly (BF) volatility σ_{BF} and the risk reversal (RR) volatility σ_{RR}. The deltas are also often standardized at 25% and 10%. In terms of their effects on volatility smile curves, the ATM volatility has the level effect. It raises or lowers the smile curve. The BF volatility affects the convexity of the smile curve while the RR volatility introduces skewness into the smile curve. A positive RR implies skewness to the right while a negative RR implies the skewness to the left. The 25% delta options have more effect on the middle part of the smile curve while the 10% options have more effect on the tails of the smile curve. The relation between the risk reversal and butterfly delta volatility can be converted into call/put delta volatility by means of the relation[5]

$$\sigma_C = \sigma_{ATM} + \sigma_{BF} + \frac{\sigma_{RR}}{2} \quad \text{and} \quad \sigma_P = \sigma_{ATM} + \sigma_{BF} - \frac{\sigma_{RR}}{2}$$

The Black-Scholes option pricing formula is then used to convert the resulting Volatility-Delta smile into the corresponding conventional volatility-strike smile. Figure 5 illustrate this principle.

[5]Quotations on FX market do have many different, sometimes confusing, conventions. We refer to Reiswich and Wystup (2010) and Clark (2011) for more information. The data provided by Bloomberg, however, uses the conventions presented is this paper.

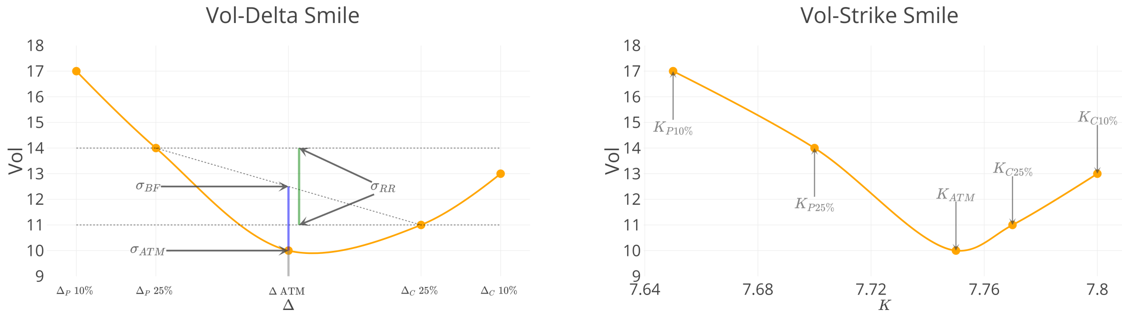


Figure 5: On the left, a typical Delta-Volatility smile in terms of ATM, Butterfly and Risk Reversal quotations as well as the corresponding delta call and put volatility quotations. On the right, the conversion with Black-Scholes formula in the conventional strike volatility smile.

Figure 6 shows the average volatility smile curves for USD-HKD, EUR-USD, GBP-USD, and USD-JPY for the period of June 2005 to July 2018 for 6 months and one year maturity. The average

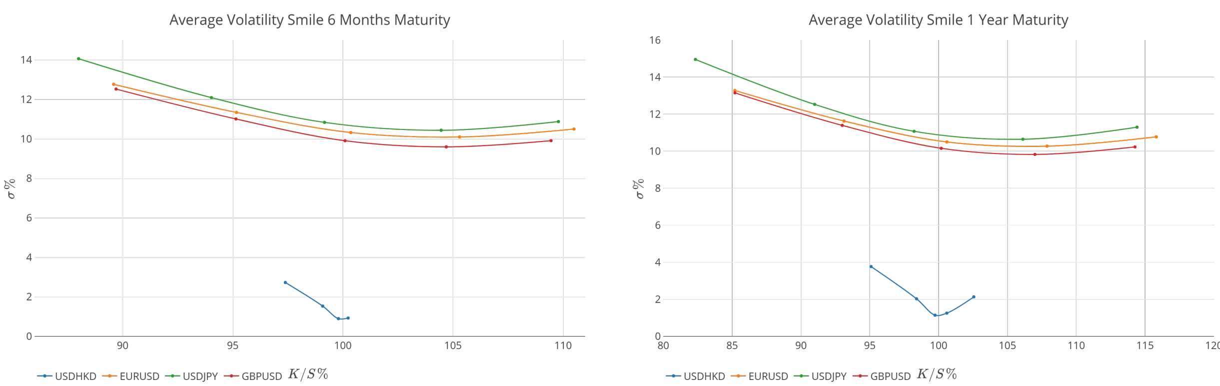


Figure 6: Average volatility smile curves for USD-HKD, EUR-USD, GBP-USD, and USD-JPY for the period of June 2005 to July 2018. On the left 6 months maturity and on the right one year maturity.

volatility smile curves for USD-HKD, EUR-USD, GBP-USD, and USD-JPY are all skewed to the left. The smile curve for USD-HKD is significantly lower than those for EUR-USD, GBP-USD, and USD-JPY, which are fairly close to each other.

Table 4 shows the statistics on the level, convexity and skewness of the average smile curves. First, the levels of ATM for EUR-USD, GBP-USD, USD-JPY rates are all more than 10 times

	ATM	25% BF	10% BF	25% RR	10% RR
USD-HKD	0.88%	0.34%	1.24%	-0.58%	-1.12%
EUR-USD	10.01%	0.36%	1.20%	-1.04%	-1.89%
GBP-USD	9.58%	0.36%	1.19%	-1.20%	- 2.22%
USD-JPY	10.43%	0.40%	1.52%	-1.57%	-3.00%

Table 4: The average of ATM, 25% BF, 10% BF, 25% RR and 10% RR for the period of June 2005 to July 2018.

greater than that for USD-HKD rate, while the ATM levels for the floating exchange rates are fairly similar, at around 10%. That the levels of ATM for floating exchange rates are significantly higher than that the fixed exchange rate of USD-HKD is not surprising. What the difference in magnitude tells us, however, is not obvious. Such quantitative question cannot be answered without a model. We will revisit this question in Section 4. Second, the levels of BF are similar in magnitude for all four exchange rates. As the ATM for USD-HKD rate is much smaller, the similar levels of BF implies that the average smile curve for USD-HKD is more convex, as seen in Figure 6. Translating to prices, it means that for the same level of moneyness, investors are willing to pay relatively more for insurance against the swing of USD-HKD rate. Third, the RR for all four exchange rates are all negative, which is consistent with the left skewness of the average smile curves. The levels of RR for USD-HKD, however, are about half of that of the other three currency pairs.

	K_{10P}		K_{25P}		K_{ATM}		K_{25C}		K_{10C}	
	< 7.75	> 7.85	< 7.75	> 7.85	< 7.75	> 7.85	< 7.75	> 7.85	< 7.75	> 7.85
3 Months	97.41%	0.00%	77.70%	0.00%	11.33%	0.00%	0.00%	3.11%	0.00%	29.39%
6 Months	100.00%	0.00%	94.30%	0.00%	21.79%	0.00%	0.42%	5.21%	0.36%	56.91%
12 Months	100.00%	0.00%	98.96%	0.00%	39.37%	0.00%	1.07%	17.73%	0.94%	79.59%

Table 5: The strike prices implied from the option data for the period of June 2005 to July 2018. Maturities 3, 6 and 12 months.

What is most significant, however, is that the range of strike prices of the smile curves for USD-HKD significantly exceeds that implied by the band of the pegged exchange rate. This means that there are options whose strike prices are outside the band of $[7.75, 7.85]$ and still have positive prices. As an illustration, Figure 7 shows randomly selected smile curves on four days in the

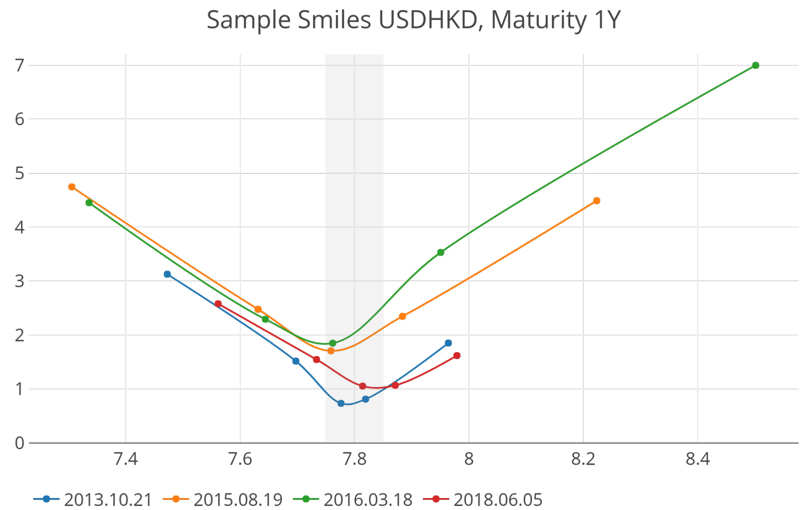


Figure 7: Randomly selected smile curves in the sample period for one year maturity. The grey area represents the band $[7.75, 7.85]$.

sample period. The dots are (K, σ) from the option price data on the four days, the curves are generic interpolations. On each curve, there are three to four data points that lie outside the band. The figure also shows how far the data points can be outside of the band. Table 5 shows that this happens frequently, even for options with shorter maturities of 3 months. For example, for 6 months maturity, K_{10P} is less than 7.75 100% of the time during the sample period, while K_{10C} is greater than 7.85 56.91% of the time. Thus the options data speaks unambiguously that the market believes that the currency band can not be held. Moreover, it is more likely that the lower bound of the band will be broken than the upper bound of the band.

A common argument for the vulnerability of the pegged exchange rate regime is that HKMA has limited reserve and when HKMA runs out its reserve, the band will be broken, in which case it is the upper bound of the band that will be broken. The empirical evidence given above, that the market believes it is more likely that the lower bound of the band will be broken than the upper bound, seems to suggest that limited reserve is not what the market concerns the most. It is rather that the HKD is too expensive relative to USD and HKMA for some reason is unwilling to accommodate the view of the market.

	Right Skewed	Left Skewed
USD-HKD	4.61%	93.46%
EUR-USD	11.79%	68.70%
USD-JPY	8.77%	81.95%
GBP-USD	4.54%	83.61%

Table 6: Percentage of time when the smile curves for 6 months maturity are left and right skewed for the currency pairs, USD-HKD, EUR-USD, USD-JPY and GBP-USD for the period of June 2005 to July 2018. A smile curve is defined to be right (left) skewed if the ratio of the volatility of its right tail over the volatility of its left tail is greater than 1.05 (0.95).

To corroborate, we examined whether overall the smile curves for the pegged USD-HKD rate differ from those of the three floating rates. We looked at the daily smile curves of the four currency pairs and the percentage of time when the smile curves are left and right skewed. Table 6 suggests that while the smile curves for all four exchange rates skewed to the left far more often than to the right, the skewness of USD-HKD rate is similar to the other floating rates with somewhat higher left skewness, which is consistent with the data characteristic shown in Table 5.

4 Stylized Facts based on Model Calibration

Armed with the model developed in Section 2, we now move to the analysis of probability of regime switching, jump size and the volatility of the exchange rate in the new regime.

4.1 Calibration of the model to the data

We first calibrate our model to the data to let the data tell us what the parameters of our model should be. On each day t, we calibrate our model to the option smile of the day by minimizing the mean square of the errors of the option quotes of the day and the implied volatilities of our model. There are four parameters in the model, $\underaccent{\bar}{\sigma}$, $\bar{\sigma}$, κ and λ. As $\underaccent{\bar}{\sigma}$ is the volatility of the exchange rate under the current regime, one way of calibrating $\underline{\sigma}$ is to estimating it by the standard deviation of the log of the ratio of the exchange rate at time t over the rate at time $t-1$ of the past 60-days. The drawback of this approach is that the estimation may not be accurate. An alternative approach is to calibrate $\underline{\sigma}$, together with the other three parameters, to the option data and let the data tell us about $\underline{\sigma}$. Obviously, the $\underline{\sigma}$ obtained will be the implied volatility. As will be seen the two approaches produce similar $\underline{\sigma}$. However, the overall fitting of the option smile curve is much better with the second approach. As we are mostly interested in the jump intensity, we will focus mostly on the second approach.

4.2 The Fitting of the Model

To provide some intuition of the calibration, lets revisit Figure 7 which shows four sample smiles. The gray area is the trading band. One of the smile curve is skewed more to the left, one more to the right, one balanced, and one skewed to the left with the bottom of the curve closer to the upper bound of the band than the other three. As shown in Figure 2, the shapes and levels of the smile curves provide information about the information on λ, κ and $\bar{\sigma}$. For instance, the green smile curve in Figure 2 skews more to the right and its is relatively high, suggesting a higher probability of positive jump. Thus the fitting of the day-by-day fitting of the smile curve tells us what the market belief is on daily basis.

As noted in Section 2, no matter how small $\underline{\sigma}$ is, there is a non-zero probability in our model that the exchange rate may move outside the band before the regime switch. Compared to a model with zero probability before the regime switch, the smile curve in our model is higher. In other words, for the same smile curve, the λ and κ are smaller compare to those calibrated from the model in which the exchange rate never moves outside the band before the regime switch. The reason is that in our model $\underline{\sigma}$ contributes to bigger current volatility of the exchange rate which increases option price and hence lower levels λ and κ are needed to match the level of the smile curve. Thus, our model may to some extent under-estimate λ and κ.

Before presenting the results from the calibrated model, it is helpful to provide some summary statistics about the goodness of fit of the calibrated model. As described above, on each day, the model is calibrated to the option prices of 3-months, 6-month and 12-months on that day. On that day, the error of the calibrated model is calculated as follow:

$$err = \frac{1}{5} \sum_{k \in \{K_{10P}, K_{25P}, K_{ATM}, K_{25C}, K_{10C}\}} 100 \frac{|\sigma_k - \hat{\sigma}_k|}{\sigma_k}$$

which is average of the percentage error at each of the strike prices. Figure 8 shows the time series of the errors for 3 months, 6 months (emphasized) and one year.

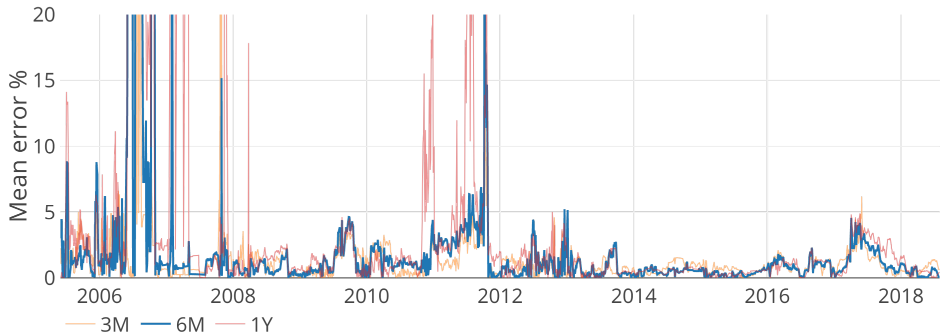


Figure 8: Time series of the errors of calibrate model for 3 months, 6 months and one year maturity.

	Mean	Std.	q 25%	**Median**	q 75%	Maximum
3 Months	1.37%	2.44%	0.43%	**0.87%**	1.56%	47.79%
6 Months	2.32%	11.46%	0.36%	**0.77%**	1.56%	266.92%
12 Months	4.57%	20.36%	0.48%	**1.08%**	2.23%	421.39%

Table 7: Mean, standard deviation of quantiles of the errors of calibrated model, in percentage for maturities 3 months, 6 months and one year.

Table 7 shows, in percentage, the mean, standard deviation and quantiles of the errors of the calibrated model. The error is very skewed mostly due to the period before 2008 as showed by the standard deviation and mostly by the 75% quantile which is 1.56%, 1.56% and 2.23% in comparison to the mean of percentage error of 1.37%, 2.32% and 4.57% for 3 months, 6 months and one year maturity, respectively. Overall, the fit of the model is quite well as showed by the better indicator in this respects which is the median percentage error of 0.87%, 0.77% and 1.08% for 3 months, 6 months and one year maturity.

4.3 Anomalous Arrival Intensity

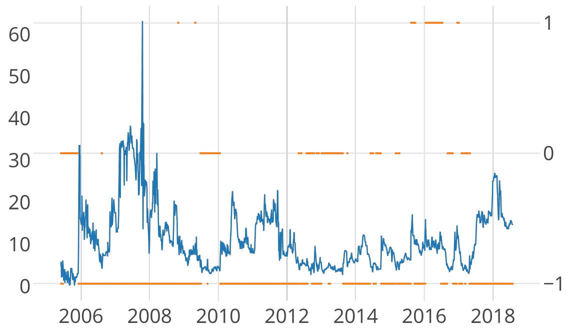
Now we turn to the analysis of the option price data with the model. Our data is from 3435 trading days for the period from June 1 of 2005 to July 31 of 2018.

Figure 9 plots the time series of implied regime switching probability, $1 - e^{-\lambda T}$, and jump size, κ for each maturity 3 months, 6 months and one year, that is $T = 0.25, 0.5$ and 1. For better viewing, we have superimposed an indicator function in Figure 9 (a), (c) and (e). It indicates the days when probability of regime switching is significant. We have chosen 1.5% as the threshold level of significance for 3 months, 6 months and one year respectively. The three possible values of the indicator are -1, 0 and 1. When the implied probability is greater than the threshold level of significance, the value of the indicator is set to 1 if $\kappa > 0$, it is set to -1 if $\kappa < 0$, otherwise it is set to 0. So if on a day the value of the indicator is 1, the market thinks that the upper bound of the trading band is likely to be broken.

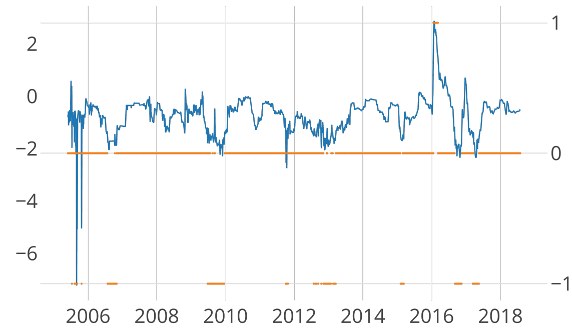
Figure 9 (a), (c) and (e) show first that the implied probability of regime switching is greater than zero for the entire period, which is consistent with the result in Table 5 in Section 3. With the help of the indicator function, it further shows that on majority of the days, the market thinks that the lower bound of the trading band is likely to be broken, which is also consistent with Table 5. There is only one period, the first half of year 2016, when the market thinks that the upper bound of the trading band is likely to be broken.

The message of Figure 9(a), (c) and (e) is confirmed in Figure 9(b), (d) and (f), where it is seen that κ is negative most of the time. The indicator function on Figures 9(b), (d) and (f) is set to 1 if the implied probability of switching is greater than 5% and 0 otherwise.

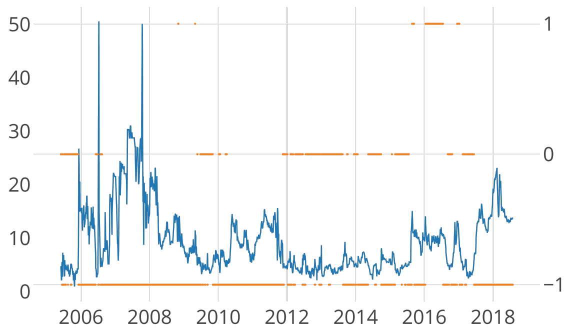
That on majority of days, the market thinks that the lower bound, as opposed to the upper bound, of the trading band is likely to be broken is quite interesting. Textbook argument typically tells us the main challenge in maintaining the exchange rate trading band is that the monetary authority runs out US dollar reserve and fail to defend the upper band. Here the data on HKDUSD option prices seem to suggest that is not what the market worries most of the time. Instead the



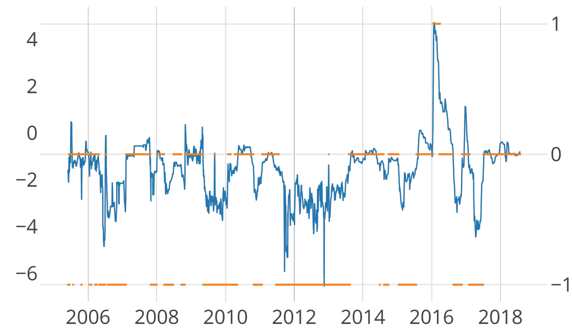
(a) Implied probability of switching 3 months.



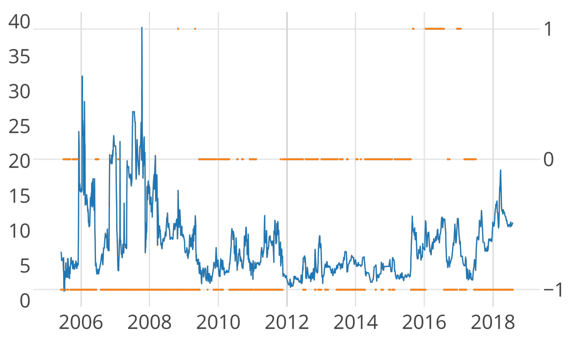
(b) Implied jump size 3 months.



(c) Implied probability of switching 6 months.



(d) Implied jump size 6 months.



(e) Implied probability of switching one year.

(f) Implied jump size one year.

Figure 9: Implied probability of switching and jump size κ for, $T = 0.25, 0.5$ and 1. The step functions in the figures are indicator functions.

market worries most of the time whether the monetary authority is willing to accommodate the demand in the currency market with the appropriate interest rate policy. That is, the trilemma faced by the monetary policy makers may force their hand and the lower bound may have to be given up to accommodate internal policy goals.

An even more striking result of the calibration is the magnitude of the implied probability of regime switching. Over the entire period, the minimum implied probability is 2.11% and it ran as high as 50.11%. Panel A of Table 8 provides the frequency for various levels of implied

Frequency of Regime Switching

			A			
probability (mean = 9.73%)	30%	25%	20%	15%	10%	5%
fraction of the sample	0.8%	3.1%	8.2%	14.6%	39.5%	75.5%
N	3428	3428	3428	3428	3428	3428
			B			
λ (mean = 21%)	40%	30%	20%	15%	10%	5%
fraction of the sample	9.9%	19.1%	42.4%	54.2%	77.1%	99.8%
N	3428	3428	3428	3428	3428	3428

Table 8: Sample frequency of various levels of implied probabilities of regime switching during the next six months, and various levels of regime switching arrival intensity annualized during the next six months, over the period from June 1 of 2005 to July 31 of 2018.

probabilities over the sample period. Panel B provides the frequency for various levels of arrival intensity λ.

Such high levels of arrival intensity is anomalous in the framework of standard asset pricing models. As shown in (7), the regime switching risk premium is given by

$$\lambda\kappa 1_{[0,\tau)}(t). \tag{13}$$

The price of jump risk when the investor has power utility with risk aversion γ is given by (see the appendix. Liu, Pan, and Wang (2004))

$$\lambda_0\kappa\left(1-(1+\kappa)^{-\gamma}\right)1_{[0,\tau)}(t).$$

Thus $\lambda = \lambda_0\left(1-(1+\kappa)^{-\gamma}\right) < \lambda_0$. In other words, the risk neutral arrival intensity is always less than the physical arrival intensity. This is intuitive because, the risk neutral probability for the switching is always higher than the physical probability as it has the risk premium built in it. Such relation can be readily tested. Table 9 shows the probabilities of no regime switching for the

p Values for the Hypothesize Values of λ_0

λ_0	40%	30%	20%	15%	10%	5%
p value	0.00%	0.00%	0.09%	0.52%	3.02%	17.38%
N	3428	3428	3428	3428	3428	3428

Table 9: p values for the hypothesize values of λ_0 for the sample from June 1 of 2005 to July 31 of 2018.

period from 1983 to 2018 (35 years) for various levels of hypothesized values of λ_0. For example, if $\lambda_0 = 0.15$, the probability of no regime switching in 35 years is 0.52%, a very small probability event. However, during the period the period from June 1 of 2005 to July 31 of 2018, the market thinks the λ_0 is no less than 15% over 50% of the time.

The official exchange rate of 7.80 HK dollar per US dollar has been managed by the Hong Kong Monetary Authority since 1983. Over the time, the world went through the Asian financial crisis of the 90s and the 2007-2008 financial crisis. The pegged exchange rate has never been broken.[6] So the true probability, $1 - e^{-0.5\lambda_0}$, of regime switching in six months is practically zero, which mean λ_0 is practically zero. Therefore the high frequencies provided in Table 8 for various levels of implied regime switching probabilities are very puzzling. Very similar results occurs for the other two maturities. They would imply that the price of regime switching risk is irrationally high. It is even more puzzling given that it is the lower bound that the market is concerned about, as argued above, which in principle can be held by policies of the Hong Kong Monetary Authority.

Of course, regime switching is a rare event. Is it possible that because there is a lot of ambiguity regarding the true likelihood the event, investors are ambiguity averse and therefore will ask for a high ambiguity premium in addition to the risk premium? As seen in Liu, Pan, and Wang (2004), indeed in that case, there is an additional premium. However, $\lambda < \lambda_0$ still holds. It should in fact be more so as there is an additional ambiguity premium built into the risk neutral probability. Therefore, even taking ambiguity regarding the likelihood of regime switching and investors'

[6]There are times when the exchange rates were outside the band, but they are all very brief and they were not considered as the switching of the regimes by the Hong Kong Monetary Authority.

aversion to it, the levels of λ shown in Table 8 are still anomalous.

4.4 Implied Volatility — $\bar{\sigma}$

The implied volatility $\bar{\sigma}$ is shown in Figure 10. The red step function in the right figure is also an indicator function. Its benchmark level is 0. When $\kappa > 2\%$, the value of the indicator is 1 if $\kappa < -2\%$ and it is -1.

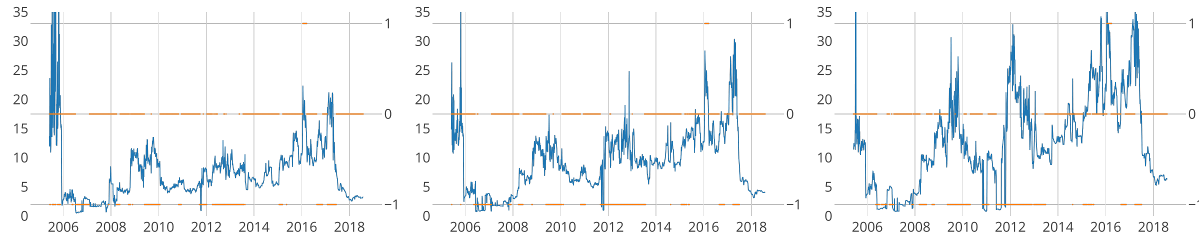


Figure 10: Time series of implied $\bar{\sigma}$. From left to right, 3 months, 6 months and one year. The red step function in the right figure is the same indicator function as in Figure 9(b).

In comparison with the floating exchange rate, EUR-USD, shown in Figure 4, the implied volatility $\bar{\sigma}$ after the linked exchange system is abandoned is quite similar in range. Also, the pairing of it with the indicator function suggests that there is no clear correlation between the implied volatility $\bar{\sigma}$ and κ. More importantly, that the implied volatility $\bar{\sigma}$ after the linked exchange system is abandoned is quite similar in range with EUR-USD suggests that there may not be significant mispricing in terms of implied volatility $\bar{\sigma}$.

The similarity of $\bar{\sigma}$ with that for floating rate seems to suggest that the market believes that once the regime switching occurs, it will switch to a floating exchange rate regime. One implication here is that there is no abnormally high or low $\bar{\sigma}$ to distort the estimate of λ and κ.

5 Conclusion

In this paper, we provided a study of the prices of the options on Hong Kong's linked exchange rate. The study was motivated by the apparent contradiction seen in the exchange rate markets. First, the USD-HKD exchange rate has been managed essentially as a pegged exchange rate with a very narrow trading band for practical operational reasons. Second, the exchange rate has been maintained in that pegged exchange rate regime since 1983. Third, there is an active options market on the exchange rate and there are options with strike prices outside the narrow trading band that have positive prices.

One possible explanation of the apparent contradiction is the potential failure of the linked exchange rate regime. The paper provides a study of that explanation. While our study does provide empirical results that speaks to the qualitative aspect of the explanation, our focus is on its quantitative justification. We developed a simple regime-switching model of the pegged exchange rate and provided a formula for the prices of its options. The option pricing formula allows us to back out the implied probability of the failure of the regime. With option price data for the period from June 1, 2005 to July 31, 2018, our main finding is that the market belief about the likelihood of the failure implied by the option prices is irrationally high to be justified by the possibility of the failure alone. This is first shown, without using any model, with ratio of empirical over implied volatility and the implied range of strike prices. While we cannot make any strong quantitative statement about the physical probability of failure of the pegged exchange rate regime, the evidence indicates a strong belief by the market. Then, using the model developed, we find that, close to 40% of the sample period, the market thinks the failure probability is greater than 10%. When contrasted with the fact that the regime has not failed since 1983, the p-value of that event is less than 0.1%, which suggests that the market belief seems to be irrational.

The empirical results documented in our study would be useful for further study of Hong Kong's linked exchange rate. Any successful theoretical model of the exchange rate would have to make predications consistent not only qualitatively but also quantitatively with the empirical characteristics of the option prices documented in this study. Based on the evidence, we think, while the failure of the linked exchange rate regime is a significant risk factor, it alone cannot rationally explain the risk premium seen in the option prices.

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