Jörn's Blog

Science, code and stuff...

SciPy Hierarchical Clustering and Dendrogram Tutorial

This is a tutorial on how to use scipy's hierarchical clustering.

One of the benefits of hierarchical clustering is that you don't need to already know the number of clusters k in your data in advance. Sadly, there doesn't seem to be much documentation on how to actually use scipy's hierarchical clustering to make an informed decision and then retrieve the clusters.

In the following I'll explain:

- how to use scipy's hierarchical clustering
- how to plot a nice dendrogram from it
- how to use the dendrogram to select a distance cut-off (aka determining the number of clusters k in your data)
- how to retrieve the k clusters
- how to visualize the clusters (2D case)

Other works:

Some short shameless self-advertising:

- I teach machines to associate like humans. In that project I used hierarchical clustering to group similar learned graph patterns together.
- I'm always searching for good students, contact me.

Naming conventions:

Before we start, as i know that it's easy to get lost, some naming conventions:

- X samples (n x m array), aka data points or "singleton clusters"
- n number of samples
- m number of features
- Z cluster linkage array (contains the hierarchical clustering information)
- k number of clusters

So, let's go.

Imports and Setup

```
In [1]:
# needed imports
from matplotlib import pyplot as plt
from scipy.cluster.hierarchy import dendrogram, linkage
import numpy as np
In [2]:
# some setting for this notebook to actually show the graphs inline
# you probably won't need this
%matplotlib inline
np.set_printoptions(precision=5, suppress=True) # suppress scientific float notation
```

Generating Sample Data

You'll obviously not need this step to run the clustering if you have own data.

The only thing you need to make sure is that you convert your data into a matrix X with n samples and m features, so that X. shape == (n, m).

```
In [3]:
```

```
# generate two clusters: a with 100 points, b with 50:
np.random.seed(4711) # for repeatability of this tutorial
a = np.random.multivariate_normal([10, 0], [[3, 1], [1, 4]], size=[100,])
b = np.random.multivariate_normal([0, 20], [[3, 1], [1, 4]], size=[50,])
X = np.concatenate((a, b),)
print X.shape # 150 samples with 2 dimensions
plt.scatter(X[:,0], X[:,1])
plt.show()
(150, 2)

30
25
20
15
0
-5
10
```

Perform the Hierarchical Clustering

Now that we have some very simple sample data, let's do the actual clustering on it:

```
In [4]:
# generate the linkage matrix
Z = linkage(X, 'ward')
```

Done. That was pretty simple, wasn't it?

Well, sure it was, this is python;), but what does the weird 'ward' mean there and how does this actually work?

As the <u>scipy linkage docs</u> tell us, 'ward' is one of the methods that can be used to calculate the distance between newly formed clusters. 'ward' causes linkage() to use the <u>Ward variance minimization algorithm</u>.

I think it's a good default choice, but it never hurts to play around with some other common linkage methods like <u>'single', 'complete', 'average'</u>, ... and the different <u>distance metrics</u> like <u>'euclidean'</u> (<u>default</u>), <u>'cityblock' aka Manhattan</u>, <u>'hamming'</u>, <u>'cosine'</u>... if you have the feeling that your data should not just be clustered to minimize the overall intra cluster variance in euclidean space. For example, you should have such a weird feeling with long (binary) feature vectors (e.g., word-vectors in text clustering).

As you can see there's a lot of choice here and while python and scipy make it very easy to do the clustering, it's you who has to understand and make these choices. If i find the time, i might give some more practical advice about this, but for now i'd urge you to at least read up on the mentioned linked methods and metrics to make a somewhat informed choice. Another thing you can and should definitely do is check the <u>Cophenetic Correlation Coefficient</u> of your clustering with help of the <u>cophenet(.)</u> function. This (very very briefly) compares (correlates) the actual pairwise distances of all your samples to those implied by the hierarchical clustering. The closer the value is to 1, the better the clustering preserves the original distances, which in our case is pretty close:

In [5]:

```
from scipy.cluster.hierarchy import cophenet
from scipy.spatial.distance import pdist

c, coph_dists = cophenet(Z, pdist(X))
c
Out[5]:
```

0.98001483875742679

No matter what method and metric you pick, the linkage() function will use that method and metric to calculate the distances of the clusters (starting with your n individual samples (aka data points) as singleton clusters)) and in each iteration will merge the two clusters which have the smallest distance according the selected method and metric. It will return an array of length n - 1 giving you information about the n - 1 cluster merges which it needs to pairwise merge n clusters. Z[i] will tell us which clusters were merged in the i-th iteration, let's take a look at the first two points that were merged:

We can see that each row of the resulting array has the format [idx1, idx2, dist, sample_count].

In its first iteration the linkage algorithm decided to merge the two clusters (original samples here) with indices 52 and 53, as they only had a distance of 0.04151. This created a cluster with a total of 2 samples.

In the second iteration the algorithm decided to merge the clusters (original samples here as well) with indices 14 and 79, which had a distance of 0.04914. This again formed another cluster with a total of 2 samples.

The indices of the clusters until now correspond to our samples. Remember that we had a total of 150 samples, so indices 0 to 149. Let's have a look at the first 20 iterations:

```
In [8]:
Z[:20]
Out[8]:
array([[ 52. , 53. , 0.04151, 2.
                                         ],
     [ 14.
             , 79. , 0.05914, 2.
                                         ],
             , 68.
, 73.
                          0.07107, 2.
0.07137, 2.
     [ 33.
                                          1,
     ſ 17.
                                          ],
                          0.07543, 2.
     f 1.
                 8.
                                          1.
     [ 85.
             , 95.
                         0.10928, 2.
     ſ 108.
            , 131. , 0.11007, 2.
                                         1.
             , 66.
, 69.
     [ 9.
                          0.11302,
                                    2.
                                          ],
                          0.11429, 2.
     [ 15.
                                          ],
             , 98.
     [ 63.
                      , 0.1212 , 2.
                                         ],
     [ 107.
            , 115. , 0.12167, 2.
                                         ],
             , 74.
, 61.
                          0.1249 , 2.
0.14028, 2.
     ſ 65.
                                         1.
     [ 58.
                                          ],
             , 152.
                          0.1726 , 3.
     ſ 62.
                                         1,
     [ 41.
                      , 0.1779 , 3.
             , 158.
                                         ],
     ſ 10.
             , 83.
                          0.18635,
                                   2.
                                         1.
             , 139.
     [ 114.
                          0.20419,
                                    2.
                                         ],
                          0.20628,
             , 88.
     [ 39.
                                    2.
                                          ],
             , 96.
     [ 70.
                          0.21931, 2.
                                          1,
     [ 46.
             , 50.
                           0.22049, 2.
                                        ]])
```

We can observe that until iteration 13 the algorithm only directly merged original samples. We can also observe the monotonic increase of the distance.

In iteration 14 the algorithm decided to merge cluster indices 62 with 152. If you paid attention the 152 should astonish you as we only have original sample indices 0 to 149 for our 150 samples. All indices idx >= len(X) actually refer to the cluster formed in Z[idx - len(X)].

This means that while idx 149 corresponds to X[149] that idx 150 corresponds to the cluster formed in Z[0], idx 151 to Z[1], 152 to Z[2], ...

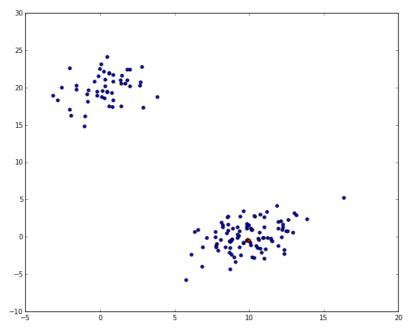
Hence, the merge iteration 14 merged sample 62 to our samples 33 and 68 that were previously merged in iteration 3 corresponding to Z[2] (152 - 150).

Let's check out the points coordinates to see if this makes sense:

Seems pretty close, but let's plot the points again and highlight them:

In [10]:

```
idxs = [33, 68, 62]
plt.figure(figsize=(10, 8))
plt.scatter(X[:,0], X[:,1])  # plot all points
plt.scatter(X[idxs,0], X[idxs,1], c='r')  # plot interesting points in red again
plt.show()
```

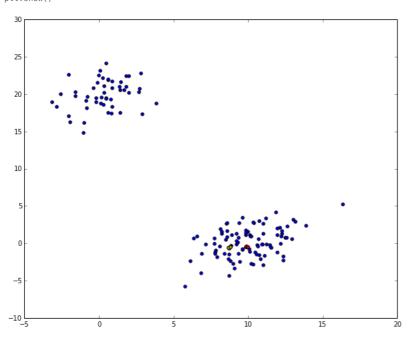


We can see that the 3 red dots are pretty close to each other, which is a good thing.

The same happened in iteration 15 where the alrogithm merged indices 41 to 15 and 69: $\frac{1}{2}$

In [11]:

```
idxs = [33, 68, 62]
plt.figure(figsize=(10, 8))
plt.scatter(X[:,0], X[:,1])
plt.scatter(X[idxs,0], X[idxs,1], c='r')
idxs = [15, 69, 41]
plt.scatter(X[idxs,0], X[idxs,1], c='y')
plt.show()
```



Showing that the 3 yellow dots are also quite close.

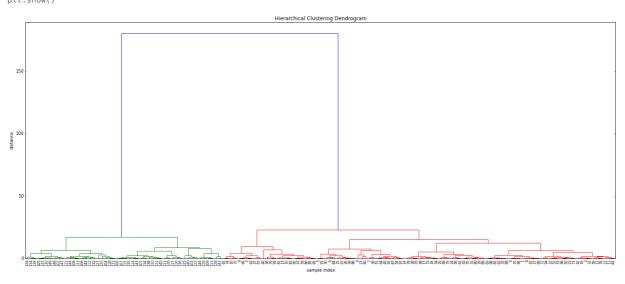
We'll later come back to visualizing this, but now let's have a look at what's called a dendrogram of this hierarchical clustering first:

Plotting a Dendrogram

A dendrogram is a visualization in form of a tree showing the order and distances of merges during the hierarchical clustering.

In [12]:

```
# calculate full dendrogram
plt.figure(figsize=(25, 10))
plt.title('Hierarchical Clustering Dendrogram')
plt.xlabel('sample index')
plt.ylabel('distance')
dendrogram(
    Z,
    leaf_rotation=90., # rotates the x axis labels
    leaf_font_size=8., # font size for the x axis labels
)
plt.show()
```



(right click and "View Image" to see full resolution)

If this is the first time you see a dendrogram, it's probably quite confusing, so let's take this apart...

- On the x axis you see labels. If you don't specify anything else they are the indices of your samples in X.
- On the y axis you see the distances (of the 'ward' method in our case).

Starting from each label at the bottom, you can see a vertical line up to a horizontal line. The height of that horizontal line tells you about the distance at which this label was merged into another label or cluster. You can find that other cluster by following the other vertical line down again. If you don't encounter another horizontal line, it was just merged with the other label you reach, otherwise it was merged into another cluster that was formed earlier.

Summarizing:

- horizontal lines are cluster merges
- vertical lines tell you which clusters/labels were part of merge forming that new cluster
- heights of the horizontal lines tell you about the distance that needed to be "bridged" to form the new cluster

You can also see that from distances > 25 up there's a huge jump of the distance to the final merge at a distance of approx. 180. Let's have a look at the distances of the last 4 merges:

```
In [13]:
Z[-4:,2]
Out[13]:
array([ 15.11533,  17.11527,  23.12199,  180.27043])
```

Such distance jumps / gaps in the dendrogram are pretty interesting for us. They indicate that something is merged here, that maybe just shouldn't be merged. In

other words: maybe the things that were merged here really don't belong to the same cluster, telling us that maybe there's just 2 clusters here.

Looking at indices in the above dendrogram also shows us that the green cluster only has indices >= 100, while the red one only has such < 100. This is a good thing as it shows that the algorithm re-discovered the two classes in our toy example.

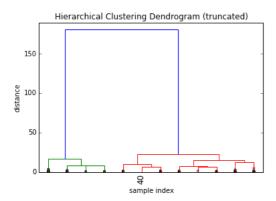
In case you're wondering about where the colors come from, you might want to have a look at the <u>color_threshold</u> argument of dendrogram(), which as not specified automagically picked a distance cut-off value of 70 % of the final merge and then colored the first clusters below that in individual colors.

Dendrogram Truncation

As you might have noticed, the above is pretty big for 150 samples already and you probably have way more in real scenarios, so let me spend a few seconds on highlighting some other features of the <u>dendrogram()</u> function:

In [14]:

```
plt.title('Hierarchical Clustering Dendrogram (truncated)')
plt.xlabel('sample index')
plt.ylabel('distance')
dendrogram(
    Z,
    truncate_mode='lastp', # show only the last p merged clusters
    p=12, # show only the last p merged clusters
    show_leaf_counts=False, # otherwise numbers in brackets are counts
    leaf_rotation=90.,
    leaf_font_size=12.,
    show_contracted=True, # to get a distribution impression in truncated branches
)
plt.show()
```



The above shows a truncated dendrogram, which only shows the last p=12 out of our 149 merges.

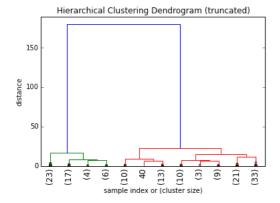
First thing you should notice are that most labels are missing. This is because except for X[40] all other samples were already merged into clusters before the last 12 merges.

The parameter show_contracted allows us to draw black dots at the heights of those previous cluster merges, so we can still spot gaps even if we don't want to clutter the whole visualization. In our example we can see that the dots are all at pretty small distances when compared to the huge last merge at a distance of 180, telling us that we probably didn't miss much there.

As it's kind of hard to keep track of the cluster sizes just by the dots, dendrogram() will by default also print the cluster sizes in brackets () if a cluster was truncated:

In [15]:

```
plt.title('Hierarchical Clustering Dendrogram (truncated)')
plt.xlabel('sample index or (cluster size)')
plt.ylabel('distance')
dendrogram(
    Z,
    truncate_mode='lastp', # show only the last p merged clusters
    p=12, # show only the last p merged clusters
    leaf_rotation=90.,
    leaf_font_size=12.,
    show_contracted=True, # to get a distribution impression in truncated branches
)
plt.show()
```



We can now see that the right most cluster already consisted of 33 samples before the last 12 merges.

Eye Candy

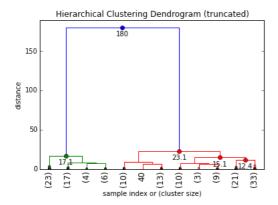
Even though this already makes for quite a nice visualization, we can pimp it even more by also <u>annotating the distances inside the dendrogram</u> by using some of the useful return values <u>dendrogram()</u>:

In [16]:

```
def fancy_dendrogram(*args, **kwargs):
    max_d = kwargs.pop('max_d', None)
   if max_d and 'color_threshold' not in kwargs:
        kwargs['color_threshold'] = max_d
   annotate_above = kwargs.pop('annotate_above', 0)
   ddata = dendrogram(*args, **kwargs)
   if not kwargs.get('no_plot', False):
       plt.title('Hierarchical Clustering Dendrogram (truncated)')
       plt.xlabel('sample index or (cluster size)')
        plt.ylabel('distance')
        for i, d, c in zip(ddata['icoord'], ddata['dcoord'], ddata['color_list']):
           x = 0.5 * sum(i[1:3])
            y = d[1]
           if y > annotate_above:
               plt.plot(x, y, 'o', c=c)
               plt.annotate("%.3g" % y, (x, y), xytext=(0, -5),
                            textcoords='offset points',
                            va='top', ha='center')
        if max_d:
           plt.axhline(y=max_d, c='k')
    return ddata
```

In [17]:

```
fancy_dendrogram(
    Z,
    truncate_mode='lastp',
    p=12,
    leaf_rotation=90.,
    leaf_font_size=12.,
    show_contracted=True,
    annotate_above=10, # useful in small plots so annotations don't overlap
)
plt.show()
```



Selecting a Distance Cut-Off aka Determining the Number of Clusters

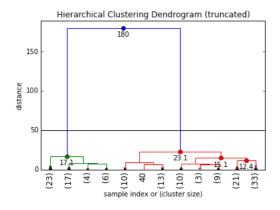
As explained above already, a huge jump in distance is typically what we're interested in if we want to argue for a certain number of clusters. If you have the chance to do this manually, i'd always opt for that, as it allows you to gain some insights into your data and to perform some sanity checks on the edge cases. In our case i'd probably just say that our cut-off is 50, as the jump is pretty obvious:

```
In [18]:
# set cut-off to 50
max_d = 50  # max_d as in max_distance
```

Let's visualize this in the dendrogram as a cut-off line:

In [19]:

```
fancy_dendrogram(
    Z,
    truncate_mode='lastp',
    p=12,
    leaf_rotation=90.,
    leaf_font_size=12.,
    show_contracted=True,
    annotate_above=10,
    max_d=max_d, # plot a horizontal cut-off line
)
plt.show()
```

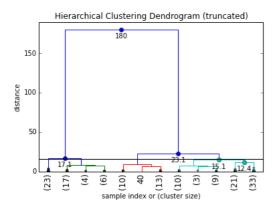


As we can see, we ("surprisingly") have two clusters at this cut-off.

In general for a chosen cut-off value max_d you can always simply count the number of intersections with vertical lines of the dendrogram to get the number of formed clusters. Say we choose a cut-off of max_d = 16, we'd get 4 final clusters:

In [20]:

```
fancy_dendrogram(
    Z,
    truncate_mode='lastp',
    p=12,
    leaf_rotation=90.,
    leaf_font_size=12.,
    show_contracted=True,
    annotate_above=10,
    max_d=16,
)
plt.show()
```



Automated Cut-Off Selection (or why you shouldn't rely on this)

Now while this manual selection of a cut-off value offers a lot of benefits when it comes to checking for a meaningful clustering and cut-off, there are cases in which you want to automate this.

The problem again is that there is no golden method to pick the number of clusters for all cases (which is why i think the investigative & backtesting manual method is preferable). Wikipedia lists a <u>couple of common methods</u>. Reading this, you should realize how different the approaches and how vague their descriptions are.

I honestly think it's a really bad idea to just use any of those methods, unless you know the data you're working on really really well.

Inconsistency Method

For example, let's have a look at the "inconsistency" method, which seems to be one of the defaults for the fcluster() function in scipy.

The question driving the inconsistency method is "what makes a distance jump a jump?". It answers this by comparing each cluster merge's height h to the average avg and normalizing it by the standard deviation std formed over the depth previous levels:

$$inconsistency = \frac{h - avg}{std}$$

The following shows a matrix of the avg, std, count, inconsistency for each of the last 10 merges of our hierarchical clustering with depth = 5

In [21]:

from scipy.cluster.hierarchy import inconsistent

Now you might be tempted to say "yay, let's just pick 5" as a limit in the inconsistencies, but look at what happens if we set depth to 3 instead:

 $Oups!\ This\ should\ make\ you\ realize\ that\ the\ inconsistency\ values\ heavily\ depend\ on\ the\ depth\ of\ the\ tree\ you\ calculate\ the\ averages\ over.$

Another problem in its calculation is that the previous d levels' heights aren't normally distributed, but expected to increase, so you can't really just treat the current level as an "outlier" of a normal distribution, as it's expected to be bigger.

Elbow Method

Another thing you might see <u>out there</u> is a variant of the "elbow method". It tries to find the clustering step where the acceleration of distance growth is the biggest (the "strongest elbow" of the blue line graph below, which is the highest value of the green graph below):

In [23]:

```
last = Z[-10:, 2]
last_rev = last[::-1]
idxs = np.arange(1, len(last) + 1)
plt.plot(idxs, last_rev)

acceleration = np.diff(last, 2)  # 2nd derivative of the distances
acceleration_rev = acceleration[::-1]
plt.plot(idxs[:-2] + 1, acceleration_rev)
plt.show()
k = acceleration_rev.argmax() + 2  # if idx 0 is the max of this we want 2 clusters
print "clusters:", k
```

clusters: 2

-50

While this works nicely in our simplistic example (the green line takes its maximum for k=2), it's pretty flawed as well.

One issue of this method has to do with the way an "elbow" is defined: you need at least a right and a left point, which implies that this method will never be able to tell you that all your data is in one single cluster only.

Another problem with this variant lies in the np.diff(Z[:, 2], 2) though. The order of the distances in Z[:, 2] isn't properly reflecting the order of merges within one branch of the tree. In other words: there is no guarantee that the distance of Z[i] is contained in the branch of Z[i+1]. By simply computing the np.diff(Z[:, 2], 2) we assume that this doesn't matter and just compare distance jumps from different branches of our merge tree.

If you still don't want to believe this, let's just construct another simplistic example but this time with very different variances in the different clusters:

In [24]:

20 -0 --20 -40

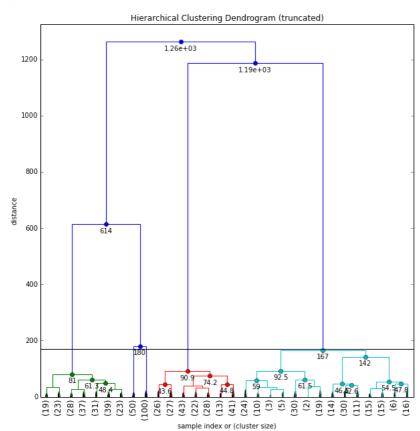
```
c = np.random.multivariate_normal([40, 40], [[20, 1], [1, 30]], size=[200,])
d = np.random.multivariate_normal([80, 80], [[30, 1], [1, 30]], size=[200,])
e = np.random.multivariate_normal([0, 100], [[100, 1], [1, 100]], size=[200,])
X2 = np.concatenate((X, c, d, e),)
plt.scatter(X2[:,0], X2[:,1])
plt.show()
140
120
100
80
60
40
```

60

100

As you can see we have 5 clusters now, but they have increasing variances... let's have a look at the dendrogram again and how you can use it to spot the problem:

In [25]:



When looking at a dendrogram like this and trying to put a cut-off line somewhere, you should notice the very different distributions of merge distances below that cut-off line. Compare the distribution in the cyan cluster to the red, green or even two blue clusters that have even been truncated away. In the cyan cluster below the cut-off we don't really have any discontinuity of merge distances up to very close to the cut-off line. The two blue clusters on the other hand are each merged below a distance of 25, and have a gap of > 155 to our cut-off line.

The variant of the "elbow" method will incorrectly see the jump from 167 to 180 as minimal and tell us we have 4 clusters:

In [26]:

```
last = Z2[-10:, 2]
last_rev = last[::-1]
idxs = np.arange(1, len(last) + 1)
plt.plot(idxs, last_rev)

acceleration = np.diff(last, 2) # 2nd derivative of the distances
acceleration_rev = acceleration[::-1]
plt.plot(idxs[:-2] + 1, acceleration_rev)
plt.show()
k = acceleration_rev.argmax() + 2 # if idx 0 is the max of this we want 2 clusters
print "clusters:", k
1500
1000
```

clusters: 4

500

0

-500

The same happens with the inconsistency metric:

In [27]:

```
print inconsistent(Z2, 5)[-10:]
[[ 13.99222 15.56656 30.
                                 3.865851
[ 16.73941 18.5639 30.
                                3.45983]
[ 19.05945 20.53211 31.
                                3.49953]
[ 19.25574 20.82658 29.
[ 21.36116 26.7766 30.
                                 3.51907]
                                 4.50256]
[ 36.58101 37.08602 31.
                                 3.507611
[ 12.122 32.15468 30.
                                 5.22936]
[ 42.6137 111.38577 31.
                                 5.13038]
[ 81.75199 208.31582
                       31.
                                  5.30448]
[ 147.25602 307.95701 31.
                                  3.6215 ]]
```

I hope you can now understand why i'm warning against blindly using any of those methods on a dataset you know nothing about. They can give you some indication, but you should always go back in and check if the results make sense, for example with a dendrogram which is a great tool for that (especially if you have higher dimensional data that you can't simply visualize anymore).

Retrieve the Clusters

Now, let's finally have a look at how to retrieve the clusters, for different ways of determining k. We can use the fcluster function.

Knowing max_d:

Let's say we determined the max distance with help of a dendrogram, then we can do the following to get the cluster id for each of our samples:

Knowing k:

Another way starting from the dendrogram is to say "i can see i have k=2" clusters. You can then use:

Using the Inconsistency Method (default):

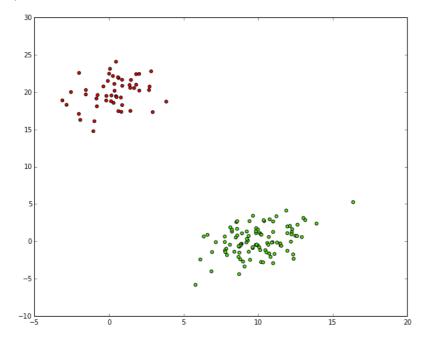
If you're really sure you want to use the <u>inconsistency method</u> to determine the number of clusters in your dataset, you can use the default <u>criterion</u> of <u>fcluster()</u>, and hope you picked the correct values:

Visualizing Your Clusters

 $If you're \ lucky \ enough \ and \ your \ data \ is \ very \ low \ dimensional, \ you \ can \ actually \ visualize \ the \ resulting \ clusters \ very \ easily:$

In [31]:

```
 plt.figure(figsize=(10, 8)) \\ plt.scatter(X[:,0], X[:,1], c=clusters, cmap='prism') \textit{ \# plot points with cluster dependent colors } \\ plt.show()
```



I hope you enjoyed this tutorial. Feedback welcome;)

Further Reading:

- The scipy hierarchical clustering module: http://docs.scipy.org/doc/scipy/reference/cluster.hierarchy.html
- $\bullet \ \ The scipy distance computation docs: \underline{http://docs.scipy.org/doc/scipy/reference/spatial.distance.html}\\$

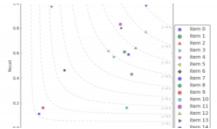
In "Coding"

• The scipy hierarchical clustering module docs often refer to the MATLAB docs saying that a certain function is similar to the MATLAB one. Here's their hierarchical clustering tutorial: http://mathworks.com/help/stats/hierarchical-clustering.html

Related



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128 thoughts on "SciPy Hierarchical Clustering and Dendrogram Tutorial"

 $[https://joernhees.de/blog/2015/08/26/scipy-hierarchical-clustering-and-dendrogram-tutorial/]\ .$

