Q4) Solve the system of equation by using gauss-elimination method

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

Sol)
$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

$\Rightarrow AX = B$

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

$$\therefore [A\,B] = \left[\begin{array}{ccc|c} 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \\ 1 & 2 & 1 & 4 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & -3 & -1 & -3 \\ 3 & 1 & 2 & 3 \end{array}\right]$$

$R_2 : R_2 - 2R_1 \; ; \; R_3 : R_3 - 3R_1$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & -5 & -1 & -9 \end{array}\right]$$

$R_2 : 5R_2 , \; R_3 : 7R_3$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 35 & 15 & 55 \\ 0 & -35 & -7 & -63 \end{array}\right]$$

$R_3 : R_3 + R_2$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 35 & 15 & 55 \\ 0 & 0 & 8 & -8 \end{array}\right]$$

$\therefore AX = B$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 35 & 15 \\ 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 55 \\ -8 \end{bmatrix}$$

$\Rightarrow x + 2y + z = 4 \rightarrow (1)$

$\Rightarrow 35y + 15z = 55$

$7y + 3z = 11 \rightarrow (2)$

$\Rightarrow -8z = -8$

$\boxed{z = -1}$

② ⇒ $7y+3(-1)=4$

$7y-3=11$

$7y=11+3$

$y=\frac{14}{7}=2$

⇒ $x+2(2)-1=4$

$x+4-1=4$

$x+3=4$

$x=1$

∴ The sol $\boxed{x=1,\ y=2,\ z=-1}$

Q) Gauss elimination method:- $x+2y+z=3$

$3x+2y+z=3$

$x-2y-5z=1$

Sol)
$$\begin{bmatrix}1&2&1\\3&2&1\\1&-2&-5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\3\\1\end{bmatrix}$$

$$\therefore [AB]=\left[\begin{array}{ccc:c}1&2&1&3\\3&2&1&3\\1&-2&-5&1\end{array}\right]$$

$R_2: R_2-3R_1$; $R_3: R_3-R_1$

$$\left[\begin{array}{ccc:c}1&2&1&3\\0&-4&-2&-6\\0&-4&-6&-2\end{array}\right]$$

$R_3: R_3-R_2$

$$=\left[\begin{array}{ccc:c}1&2&1&3\\0&-4&-2&-6\\0&0&-4&4\end{array}\right]$$

$$=\left[\begin{array}{ccc:c}1&2&1&3\\0&-4&-2&-6\\0&0&-4&4\end{array}\right]\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\-6\\4\end{bmatrix}$$

⇒ $x+2y+z=3$ ⇒ ①

⇒ $-4y-2z=-6$

$2y+z=3$ ⇒ ②

⇒ $-4z=4$

$z=-1$

② ⇒ $2y-1=3$

$y=2$

① ⇒ $x+4-1=3$

$x=0$

$\boxed{\therefore x=0;\ y=2;\ z=-1}$

Q) Gauss-Jordan method:- $x+2y+z=3$

$2x+3y+2z=5$

$3x-5y+5z=2$

Sol)
$$\begin{bmatrix}1&2&1\\2&3&2\\3&-5&5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\5\\2\end{bmatrix}$$

$$\therefore [AB]=\left[\begin{array}{ccc:c}1&2&1&3\\2&3&2&5\\3&-5&5&2\end{array}\right]$$

$R_2: R_2-2R_1$; $R_3: R_3-3R_1$

$$=\left[\begin{array}{ccc:c}1&2&1&3\\0&-1&0&-1\\0&-11&2&-7\end{array}\right]$$

$R_1: R_1+2R_2$; $R_3: R_3-11R_2$

$$=\left[\begin{array}{ccc:c}1&0&1&1\\0&-1&0&-1\\0&0&2&4\end{array}\right]$$

$R_3 : \frac{1}{2} R_3$

$$= \begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & -1 & 0 & : & -1 \\ 0 & 0 & 1 & : & 2 \end{bmatrix}$$

$R_1 : R_1 - R_3 ; \; R_2 : -(R_2)$

$$= \begin{bmatrix} 1 & 0 & 0 & : & -1 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 1 & : & 2 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} -1 \\ 1 \\ 2 \end{vmatrix}$$

$x = -1 ; \; y = 1 ; \; z = 2$

6) Gauss Jordan Method :- $x + y + z = 9$
$x - 2y + 3z = 8$
$2x + y - z = 3$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ 3 \end{bmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 & : 9 \\ 1 & -2 & 3 & : 8 \\ 2 & 1 & -1 & : 3 \end{pmatrix}$$

$R_2 : R_2 - R_1 ; \; R_3 : R_3 - 2R_1$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -3 & 2 & : & -1 \\ 0 & -1 & -3 & : & -15 \end{bmatrix}$$

$R_1 : 3R_1 + R_2 ; \; R_3 : 3R_3 - R_2$

$$= \begin{bmatrix} 1 & 0 & 5 & : & 26 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & -11 & : & -44 \end{bmatrix}$$

$3 - 18$ $\quad -3 \quad$ $15-$ $\quad 3 + 2 \quad 27 - 1 \quad -9 - 12 \quad -45 + 2$

$R_2 \leftrightarrow R_3$

$$= \begin{bmatrix} 1 & 0 & 5 & : & 26 \\ 0 & & & & \end{bmatrix}$$

$R_3 \rightarrow \frac{R_3}{-11}$

$$= \begin{pmatrix} 1 & 0 & 5 & : & 26 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & 1 & : & 4 \end{pmatrix}$$

$R_2 \leftrightarrow R_3$

$$= \begin{pmatrix} 1 & 1 & 1 & : 9 \\ 0 & -1 & -3 & : -15 \\ 0 & -3 & 2 & : -1 \end{pmatrix}$$

$R_1 : R_1 + R_2 , \; R_3 : R_3 - 3R_2$

$$= \begin{vmatrix} 1 & 0 & -2 & : -6 \\ 0 & -1 & -3 & : -15 \\ 0 & 0 & 11 & : 44 \end{vmatrix}$$

$R_3 : \frac{1}{4} R_3$

$$= \begin{bmatrix} 1 & 0 & -2 & : -6 \\ 0 & -1 & -3 & : -15 \\ 0 & 0 & 1 & : 5 \end{bmatrix}$$

$R_1 : R_1 + 2R_3 , \; R_2 : R_2 + 3R_3$

$$= \begin{pmatrix} 1 & 0 & 0 & : 2 \\ 0 & -1 & 0 & : 3 \\ 0 & 0 & 1 & : 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & : 2 \\ 0 & 1 & 0 & : 3 \\ 0 & 0 & 1 & : 4 \end{pmatrix}$$

$x = 2 ; \; y = 3 ; \; z = 5$

Q) Find the eigen value of square matrix

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

let $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{bmatrix}$

$\therefore$ W.K.T $\Rightarrow |A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0$

$\Rightarrow (1-\lambda)(4-\lambda) - 6 = 0$

$\Rightarrow 4 - \lambda - 4\lambda + \lambda^2 - 6 = 0$

$\Rightarrow \lambda^2 - 5\lambda - 2 = 0$

$\therefore \lambda = \frac{(-5) \pm \sqrt{(-5)^2 - (4)1(-2)}}{2(1)}$

$\lambda = \frac{5 \pm \sqrt{25+8}}{2}$

$\lambda = \frac{5}{2} \pm \frac{\sqrt{33}}{2}$

$\lambda = \frac{5}{2} - \frac{\sqrt{33}}{2}, \frac{5}{2} + \frac{\sqrt{33}}{2}$

Q) Find the eigen values & eigen vectors of the matrix

$A = \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix}$

$\boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

let $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} -1-\lambda & 3 \\ -3 & 4-\lambda \end{bmatrix}$

$\therefore$ W.K.T

$|A - \lambda I| = 0$

$\cancel{\lambda(1+\lambda)} \Rightarrow \begin{vmatrix} -(\lambda+1) & 3 \\ -3 & (4-\lambda) \end{vmatrix} = 0$

$\Rightarrow -(\lambda+1)(4-\lambda) + 9 = 0$

$\Rightarrow (\lambda+1)(4-\lambda) - 9 = 0$

$\Rightarrow 4\lambda - \lambda^2 + 4 - \lambda - 9 = 0$

$\Rightarrow -\lambda^2 + 3\lambda - 5 = 0$

$\Rightarrow$

$\Rightarrow \lambda^2 - 3\lambda + 5 = 0$

$\lambda = \frac{3 \pm \sqrt{9 - 4(1)(5)}}{2(1)} \Rightarrow 3\lambda + 5 = 0$

$$\therefore \lambda = \frac{3 \pm \sqrt{9 - 4(1)(5)}}{2(1)}$$

$$\lambda = \frac{3 \pm \sqrt{9-20}}{2}$$

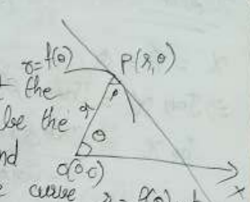
$$\lambda = \frac{3 \pm \sqrt{-11}}{2}$$

$$\lambda = \frac{3 \pm \sqrt{11}\, i}{2}$$

$$\lambda = \frac{3}{2} \pm \frac{\sqrt{11}}{2} i$$

$$\lambda = \frac{3}{2} - \frac{\sqrt{11}}{2} i,\ \frac{3}{2} + \frac{\sqrt{11}}{2} i$$

Polar Curves



$\Rightarrow$ let $\vec{OX}$ be a initial line, let $r = f(\theta)$ be a polar curve at the point 'P', let $OP = r$, let θ be the angle b/w OP & initial line and 'T' be a tangent to the curve $r = f(\theta)$ at 'P'. Then the co-ordinates of 'P' of (r, θ) are called the polar ~~co~~ co-ordinates

$\Rightarrow$ $O(0,0)$ is called the pole, $r = f(\theta)$ is called the polar curve and the entire system is called polar system

$\Rightarrow$ Angle b/w the radius of vector and tangent

$\Rightarrow$ let $\vec{OX}$ be the initial line, let P be any point on the plane with $OP = r$ be the radius of vector, let $r = f(\theta)$ be a polar curve at he point 'P', let 'T' be a tangent to the curve at 'P', let 'χ' be the angle b/w the radius of vector & tangent to the curve & finally ϕ χ be angle b/w the tangent & initial line

Theorem

$\chi = \phi + \theta$

$\Rightarrow \tan\chi = \tan(\phi + \theta)$

$\tan\chi = \dfrac{\tan\theta + \tan\phi}{1 - \tan\theta \cdot \tan\phi}$ — ①

$\therefore$ The slope of T in $m = \tan\chi$

$\therefore m = \tan\chi = \dfrac{dy}{dx}$ — ②

$\therefore$ let $x = r\cos\theta$, $y = r\sin\theta$

$\dfrac{dx}{d\theta} = \dfrac{dr}{d\theta}\cos\theta - r\sin\theta$

$\therefore \dfrac{dy}{d\theta} = \dfrac{dr}{d\theta}\sin\theta + r\cos\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\dfrac{\frac{dr}{d\theta}\sin\theta}{\frac{dr}{d\theta}\cos\theta} + \dfrac{r\cos\theta}{\frac{dr}{d\theta}\cos\theta}}{1 - \dfrac{r\sin\theta}{\frac{dr}{d\theta}\cos\theta}}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\tan\theta + r\frac{d\theta}{dr}}{1 - (\tan\theta)(r\frac{d\theta}{dr})}$

$\Rightarrow \dfrac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \dfrac{\tan\theta + r\frac{d\theta}{dr}}{1 - (\tan\theta(r\frac{d\theta}{dr})}$

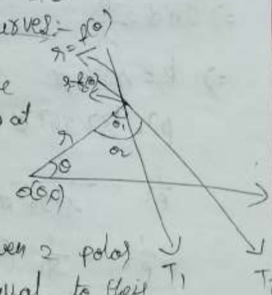
$\Rightarrow \boxed{\therefore \tan\phi = r\dfrac{d\theta}{dr}}$

$\dfrac{1}{\tan\phi} = \dfrac{1}{r\frac{d\theta}{dr}}$

$\Rightarrow \cot\phi = \dfrac{1}{r}\dfrac{dr}{d\theta}$

Angle b/w two polar curves:-

$r = f_1(\theta)$, $r = f_2(\theta)$ be the 2 polar curves intersecting at 'p' and making angles with the radius of vector

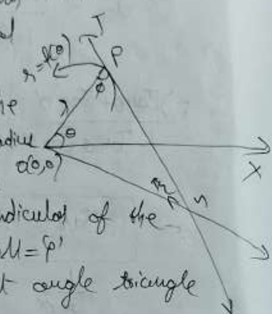


$\therefore$ The angle b/w the given 2 polar curves should be equal to their tangents

$\therefore \phi = |\phi_2 - \phi_1|$

Perpendicular distance of a pole to a tangent:-

Let $r = f(\theta)$ be a polar curve, a polar curve at capital 'P' with $OP = r$ & let ϕ be the angle b/w the tangent to the curve & radius to the curve. Let 'M' be the foot of the perpendicular of the pole to the tangent $OM = p$



$\therefore \Delta OPM$ be a right angle triangle

we have

$\sin\phi = \frac{OM}{OP}$

$\Rightarrow \sin\phi = \frac{p}{r}$

$\Rightarrow p = r\sin\phi$

$p^2 = r^2\sin^2\phi$

$\frac{1}{p^2} = \frac{1}{r^2\sin^2\phi}$

$\frac{1}{p^2} = \frac{1}{r^2}(\operatorname{cosec}^2\phi)$

$\frac{1}{p^2} = \frac{1}{r^2}(1+\cot^2\phi)$

$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$

where the above eq's are called pedal eq's (or) 'p-r' equations

Some Important results:-

~~1) $\sin(A+B) = S$~~

1) $\sin 2\theta = 2\sin\theta\cos\theta$

2) $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

3) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

4) $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

5) $1+\cos 2\theta = 2\cos^2\theta$ (or) $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

6) $1+\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)$ (or) $\cos^2\left(\frac{\theta}{2}\right) = \frac{1+\cos\theta}{2}$

7) $2\sin^2\theta = 1-\cos 2\theta$ (or) $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

8) $\sin^2\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{2}$

Q) Find the angle b/w the radius of vector and tangent to the given polar at the angle indicated

$r = a(1+\cos\theta)$; $\theta = \frac{\pi}{3}$

$r = a(1-\cos\theta)$; $\theta = \frac{\pi}{6}$

$r = 3\cos\theta$; $\theta = \frac{\pi}{4}$

$r^2 = 2\sin\theta$; $\theta = \frac{\pi}{2}$

Sol) $r = a(1+\cos\theta) \to ①$

differentiate ① w.r.t θ

$\therefore ① \Rightarrow \frac{dr}{d\theta} = a(0-\sin\theta)$

$\Rightarrow \frac{dr}{d\theta} = -a\sin\theta$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{a\sin\theta}{r}$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{\cancel{a}\sin\theta}{\cancel{a}(1+\cos\theta)}$

$\Rightarrow \cot\phi = \frac{-\sin\theta}{1+\tan\theta}$

$\Rightarrow \cot\phi = \frac{-\cancel{2}\sin\frac{\theta}{2}\cot\frac{\theta}{2}}{\cancel{2}\cos^2\frac{\theta}{2}}$

$\Rightarrow \cot\phi = -\tan\left(\frac{\theta}{2}\right)$

$\Rightarrow \cot\phi = \cot\left(\frac{\pi}{2}+\frac{\theta}{2}\right)$

$\Rightarrow \phi = \frac{\pi}{2}+\frac{\theta}{2}$

$\therefore$ At $\theta = \frac{\pi}{3}$

$\therefore \phi = \frac{\pi}{2} + \frac{\pi/3}{2}$

$\Rightarrow \phi = \frac{\pi}{2}+\frac{\pi}{6}$

$\Rightarrow \phi = \frac{\pi}{2}+\frac{\pi}{6}$

$\Rightarrow \phi = \frac{3\pi+\pi}{6}$

$\Rightarrow \phi = \frac{4\pi}{6}$

$\Rightarrow \phi = \frac{2\pi}{3}$

Given $r = a(1-\cos\theta) - ①$

diff ① w.r.t θ

$\therefore ① \Rightarrow \frac{dr}{d\theta} = a(0+\sin\theta)$

$\Rightarrow \frac{dr}{d\theta} = a\sin\theta$

$\Rightarrow \frac{1\,dr}{r\,d\theta} = \frac{a\sin\theta}{r}$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{\cancel{a}\sin\theta}{\cancel{a}(1-\cos\theta)}$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{\sin\theta}{1-\cos\theta}$

$\Rightarrow \cot\phi = 2\sin(\theta/2)\cos\left(\frac{\theta}{2}\right)$

$\Rightarrow \cot\phi = \cot(\theta/2)$

$\therefore \phi = \frac{\theta}{2}$

$\therefore$ At $\theta = \pi/6$

$\therefore \phi = \frac{\pi/6}{2}$

$\Rightarrow \phi - \frac{\pi}{12}$

$r = 3\cos\theta \rightarrow ①$

diff ① w.r.t θ

$① \Rightarrow \frac{dr}{d\theta} = -3\sin\theta$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = -\frac{3\sin\theta}{r}$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{-\cancel{3}\sin\theta}{\cancel{3}\cos\theta}$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = -\tan\theta$

$\Rightarrow \cancel{\cot}\,\phi = \cancel{\cot}\left(\frac{\pi}{2}+\theta\right)$

$\Rightarrow \phi = \frac{\pi}{2}+\theta$

$\Rightarrow \phi = \frac{\pi}{2}+\frac{\pi}{4}$

$\Rightarrow \phi = \frac{4\pi+2\pi}{8}$

$\Rightarrow \phi = \frac{6\pi}{8}$

$\Rightarrow \phi = \frac{3\pi}{4}$

$r^2 = 2\sin^2\theta \rightarrow ①$

diff ① w.r.t θ

$① \Rightarrow \frac{dr}{d\theta} = 2\sin^2\theta$

$\Rightarrow \cancel{2}r\frac{dr}{d\theta} = \cancel{2}\cos\theta$

$\Rightarrow r\frac{dr}{d\theta} = 2\cos\theta$

$\Rightarrow \frac{\cancel{r}}{r\cancel{r}}\frac{dr}{d\theta} = \frac{2\cos\theta}{r^2}$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{\cancel{2}\cos\theta}{\cancel{2}\sin\theta}$

$\Rightarrow \cancel{\cot}\,\phi = \cancel{\cot}\,\theta$

$\Rightarrow \phi = \frac{\pi}{4}$

Q) Show that given polar curves can intersect orthogonally

(i) $r = a(1-\cos\theta)$, $r = b(1+\cos\theta)$

(ii) $r(1-\cos\theta) = 2a$, $r(1+\cos\theta) = 2b$

(iii) $r^n = a^n \cos n\theta$, $r^n = b^n \sin n\theta$

(iv) $\cancel{r^n = \cos n} \; r^n\cos n\theta = a^n$, $r^n \sin n\theta = b^n$

① $r = a(1-\cos\theta) \to ①$, $r = b(1+\cos\theta) \to ②$

$\therefore ① \Rightarrow \frac{dr}{d\theta} = a\sin\theta$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{a\sin\theta}{r}$

$\Rightarrow \cot\phi_1 = \frac{\cancel{a}\sin\theta}{\cancel{a}(1-\cos\theta)}$

$\Rightarrow \cot\phi_1 = \frac{\sin\theta}{1-\cos\theta}$

$\Rightarrow \cot\phi_2 = \frac{2\sin\theta/2 \cdot \cos\theta/2}{2\sin^2\theta/2}$

$\Rightarrow \cot\phi_1 = \cos(\frac{\theta}{2})$

$\Rightarrow \phi_1 = \frac{\theta}{2}$

$\therefore ② \Rightarrow \frac{dr}{d\theta} = -b\sin n\theta$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{-b\sin n\theta}{r}$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{-b\sin n\theta}{b(1+\cos\theta)}$

$\Rightarrow \cot\frac{\theta}{2} = \frac{-2\sin\theta/2\cos\theta/2}{2\cos^2\theta/2}$

$\Rightarrow \cot\frac{\theta}{2} = -\tan\theta/2$

$\Rightarrow \cot\frac{\theta}{2} = \cot(\frac{\pi}{2}+\frac{\theta}{2})$

$\Rightarrow \frac{\theta}{2} = \frac{\pi}{2}+\frac{\theta}{2}$

$\therefore |\phi_2-\phi_1|$

$= |\frac{\pi}{2}+\cancel{\frac{\theta}{2}}-\cancel{\frac{\theta}{2}}|$

$\Rightarrow |\phi_2-\phi_1|$

$= \frac{\pi}{2}$

② Given $r(1-\cos\theta) = 2a \to ①$; $r(1+\cos\theta) = 2b \to ②$

diff ① w.r.t $\to \theta$

$\therefore ① \Rightarrow (1-\cot\theta)\frac{dr}{d\theta} + r\sin\theta = 0$

$\Rightarrow (1-\cot\theta)\frac{dr}{d\theta} = -r\sin\theta$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{-\sin\theta}{1-\cos\theta}$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{-2\sin\theta/2 \cdot \cos\theta/2}{2\sin^2\theta/2}$

$\Rightarrow \cot\phi_1 = -\cot\theta/2$

$\Rightarrow \cot\phi_1 = \cot(-\frac{\theta}{2})$

$\Rightarrow \phi_1 = -\theta/2$

② $\Rightarrow (1+\cos\theta)\frac{dr}{d\theta} - r\sin\theta = 0$

$\Rightarrow (1+\cos\theta)\frac{dr}{d\theta} = r\sin\theta$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{\sin\theta}{1+\cos\theta}$

$\Rightarrow \cot\frac{\phi}{2} = \frac{2\sin\theta/2\cos\theta/2}{2\cos^2\theta/2}$

$\Rightarrow \cot\frac{\phi}{2} = \tan\theta/2$

$\Rightarrow \cot\frac{\phi}{2} = \cot(\frac{\pi}{2}-\frac{\theta}{2})$

$\Rightarrow \phi_2 = \frac{\pi}{2}-\frac{\theta}{2}$

$\therefore |\phi_2-\phi_1| = |\frac{\pi}{2}-\frac{\theta}{2}+\frac{\theta}{2}|$

$= \frac{\pi}{2}-\cancel{\frac{\theta}{2}}+\cancel{\frac{\theta}{2}}$

$= \frac{\pi}{2}$

Given $r^n = a^n \cos n\theta \to (1)$, $r^n = b^n \sin n\theta \to (2)$

$\therefore$ diff (1) w.r.t 'θ'

$\therefore (1) \Rightarrow n r^{n-1} \frac{dr}{d\theta} = a^n$

$\Rightarrow \frac{r^n}{r}\frac{dr}{d\theta} = -a^n \sin n\theta$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{-a^n \sin n\theta}{r^n}$

$\Rightarrow \cot\phi_1 = \frac{-a^n \sin n\theta}{a^n \cos n\theta}$

$\Rightarrow \cot\phi_1 = -\tan n\theta$

$\Rightarrow \cot\phi_1 = \cot\left(\frac{\pi}{2} + n\theta\right)$

$\Rightarrow \phi_1 = \frac{\pi}{2} + n\theta \to (3)$

$\therefore (2) \Rightarrow \cancel{n} r^{n-1} \frac{dr}{d\theta} = b^n (\cos n\theta)(\cancel{n})$

$\Rightarrow \frac{r^n}{r}\frac{dr}{d\theta} = b^n \cos n\theta$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{b^n \cos n\theta}{r^n}$

$\Rightarrow \cot\phi_2 = \frac{b^n \cos n\theta}{b^n \sin n\theta}$

$\Rightarrow \cot\phi_2 = \cot n\theta$

$\phi_2 = n\theta$

$\therefore |\phi_2 - \phi_1| = |\cancel{n\theta} - \frac{\pi}{2} + \cancel{n\theta}|$

$= |\phi_2 - \phi_1| = \frac{\pi}{2}$

iv) $r^n \cos n\theta = a^n$, $r^n \sin n\theta = b^n$

$\therefore$ diff (1) w.r.t 'θ'

$\cancel{\therefore (1) \Rightarrow n b^{n-1} = r^n \cos n\theta +}$

$\cancel{n r^{n-1} \frac{dr}{d\theta} = \cos\theta \cos n\theta \times b^n}$

$\cancel{\frac{r^n}{r}\frac{dr}{d\theta} = \cos n\theta \times b^n}$

$\cancel{\frac{1}{r}\frac{dr}{d\theta} = \cos n\theta \times b^n}$

$\therefore (1) \Rightarrow n r^{n-1} \frac{dr}{d\theta} \cdot \cos n\theta + r^n(-\sin n\theta)(n) = 0$

$\frac{r^n}{r}\frac{dr}{d\theta} \cdot \cos n\theta = r^n \sin n\theta$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{\sin n\theta}{\cos n\theta}$

$\cot\phi_1 = \tan n\theta$

$\cot\phi_1 = \cot\left(\frac{\pi}{2} - n\theta\right)$

$\phi_1 = \frac{\pi}{2} - n\theta$

$\therefore (2) \Rightarrow n r^{n-1} \frac{dr}{d\theta} \sin n\theta + r^n (\cos n\theta) n = 0$

$\Rightarrow \frac{r^n}{r}\frac{dr}{d\theta} \sin n\theta = -r^n \cos n\theta$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = -\frac{\cos n\theta}{\sin n\theta}$

$\Rightarrow \cot\phi_2 = -\cot n\theta$

$\Rightarrow \cot\phi_2 = \cot(-n\theta)$

$\phi_2 = -n\theta$

$\therefore |\phi_2 - \phi_1| = |\cancel{-n\theta} - \frac{\pi}{2} + \cancel{n\theta}|$

$= \frac{\pi}{2}$

$r = 2\cos\theta,\ r = 3\sin\theta$

$r = 2\cos\theta$

$\frac{dr}{dx} = -2\sin\theta$

$\frac{1}{r}\frac{dr}{dx} = \frac{-2\sin\theta}{r}$

$\frac{1}{r}\frac{dr}{dx} = \frac{-\cancel{2}\sin\theta}{\cancel{2}\cos\theta}$

$\cot\phi_1 = -\tan\theta$

$\cancel{\cot}\phi_1 = \cancel{\cot}\left(\frac{\pi}{2}+\theta\right)$

$\phi_1 = \frac{\pi}{2}+\theta$

$r = 3\sin\theta$

$\frac{dr}{dx} = 3\cos\theta$

$\frac{1}{r}\frac{dr}{dx} = \frac{3\cos\theta}{r}$

$\cot\phi_1 = \frac{\cancel{3}\cos\theta}{\cancel{3}\sin\theta}$

$\cancel{\cot}\phi_1 = \cancel{\cot}\theta$

$\phi_1 = \theta$

$|\phi_2 - \phi_1| = \left|\frac{\pi}{2} + \cancel{\theta} - \cancel{\theta}\right|$

$= \frac{\pi}{2}$

Q) Find the angle b/w ① $r = a\log\theta$, ② $r = \frac{a}{\log\theta}$

diff ① w.r.t θ

$\therefore$ ① $\Rightarrow \frac{dr}{d\theta} = \frac{a}{\theta}$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{a/\theta}{r}$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{\cancel{a}/\theta}{\cancel{a}\log\theta}$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{\theta\log\theta}$

$r\frac{d\theta}{dr} = \theta\log\theta$

$\Rightarrow \tan\phi_1 = \theta\log\theta \rightarrow$ ③

$\therefore$ ② $\Rightarrow r\log\theta = a$

$\log\theta\cdot\frac{dr}{d\theta} + r\cdot\left(\frac{1}{\theta}\right) = 0$

$\log\theta\cdot\frac{dr}{d\theta} = -\frac{r}{\theta}$

$\frac{1}{r}\frac{dr}{d\theta} = -\frac{1}{\log\theta}$

$r\frac{d\theta}{dr} = -\theta\log\theta$

$\tan\phi_2 = -\theta\log\theta \rightarrow$ ④

w.K.T

$\tan(\phi_1-\phi_2) = \frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\tan\phi_2}$

$\tan(\phi_1-\phi_2) = \frac{2\theta\log\theta}{1-\theta^2(\log\theta)^2}$

$\therefore$ From ① & ②

$r = a/\log\theta$, $r = \frac{a}{\log\theta}$

$a\log\theta = \frac{a}{\log\theta}$

$(\log\theta)^2 = 1$

$\log_2 \theta = 1$

$\theta = e^1 = e$

$\therefore \tan(\phi_1 - \phi_2) = \frac{2e\theta}{1 - e^2\theta^2}$

$\Rightarrow \tan(\phi_2 - \phi_1) = \frac{2e}{1-e^2}$

$\Rightarrow |\phi_1 - \phi_2| = \tan^{-1}\left(\frac{2e}{1-e^2}\right)$

Q) Solve the angle b/w polar curves

$r^2 \sin 2\theta = 4$, $r^2 = 16\sin 2\theta$

A) Given $r^2 \sin 2\theta = 4 \rightarrow ①$

$r^2 = 16\sin 2\theta \rightarrow ②$

diff ① w.r.t ⓞ

$\therefore ① \Rightarrow 2r\frac{dr}{d\theta}\sin 2\theta + r^2(\cos 2\theta)2 = 0$

$\Rightarrow r\frac{dr}{d\theta}\sin 2\theta = -r^2\cos 2\theta$

$\Rightarrow \frac{r}{r^2}\frac{dr}{d\theta} = \frac{-\cos 2\theta}{\sin 2\theta}$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = -(\cot 2\theta)$

$\Rightarrow \cot\phi_1 = \cot(-2\theta)$

$\Rightarrow \phi_1 = -2\theta$

$\therefore ② \Rightarrow 2r\frac{dr}{d\theta} = 16\cos(2\theta)2$

$\Rightarrow r\frac{dr}{d\theta} = 16\cos 2\theta$

$\Rightarrow \frac{r}{r^2}\frac{dr}{d\theta} = \frac{16\cos 2\theta}{r^2}$

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{\cancel{16}\cos 2\theta}{\cancel{16}\sin 2\theta}$

$\Rightarrow \cot\phi_2 = \cot 2\theta$

$\Rightarrow \phi_2 = 2\theta$

$\therefore |\phi_1 - \phi_2| = |-2\theta - 2\theta|$

$= |-4\theta|$

$= 4\theta$

①= $r^2 = \frac{4}{\sin^2\theta}$, $r^2 = 16\sin 2\theta$

$\frac{4}{\sin 2\theta} = 16\sin 2\theta$

$1 = 4\sin^2 2\theta$

$\sin^2 2\theta = \frac{1}{4}$

$\sin 2\theta = \frac{1}{2}$

$2\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$2\theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{12}$

Subs θ in eq①

$4 \times \frac{\pi}{12} = \frac{\pi}{3}$

Q) Given Find the angle b/w the curves

$r = \sin\theta + \cos\theta$; $r = 2\sin\theta$

Sol) $r = \sin\theta + \cos\theta$

$r = 2\sin\theta$

$$\frac{dr}{d\theta} = \cos\theta - \sin\theta$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}$$

$$\cancel{\cot\phi_1 = \frac{\cos\theta}{\sin\theta} - \sin\theta}$$

$$\cot\phi_1 = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$\cot\phi_1 = \frac{\cos\theta/\cos\theta - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$$

$$\cot\phi_1 = \frac{1 - \tan\theta}{1 + \tan(\theta)}$$

$$\cot\phi_1 = \frac{(1) - \tan\theta}{1 + (1)(\tan\theta)}$$

$$= \frac{\tan(\pi/4) - \tan\theta}{1 + \tan(\pi/4)\cdot\tan\theta}$$

$$= \tan\left(\frac{\pi}{4} - \theta\right)$$

$$= \cot\left(90 - \left(\frac{\pi}{4} - \theta\right)\right)$$

$$\phi_1 = \frac{\pi}{2} - \frac{\pi}{4} + \theta$$

$$= \frac{\pi}{4} + \theta$$

$\therefore$ ② $\Rightarrow \frac{dr}{d\theta} = 2\cos\theta$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{2\cos\theta}{r}$$

$$\cot\phi_2 = \frac{2\cos\theta}{2\sin\theta}$$

$$\cot\phi_2 = \cot\theta$$

$$\phi_2 = \theta$$

$$|\phi_2 - \phi_1| = \left|\cancel{\theta} - \frac{\pi}{4} - \cancel{\theta}\right|$$

$$|\phi_2 - \phi_1| = \frac{\pi}{4}$$

Q) Find the angle b/w given polar curves

~~$r = 3\sin\theta \cos\theta$~~ $r = a(1-\sin\theta)$

$r = b(1+\sin 2\theta)$

$$\frac{dr}{d\theta} = a(0-\cos\theta)$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{-a\cos\theta}{r}$$

$$\frac{1}{r}\frac{dr}{d\theta} = -\frac{-\cancel{a}\cos\theta}{\cancel{a}(1-\sin\theta)}$$

$$\cot\phi_1 = \frac{-\cos\theta}{1-\sin\theta}$$

$$\cancel{\cot\phi = \frac{\cos^2\theta/2 - \sin^2\theta/2}{\cos\theta}}$$

$$\cancel{\cot\phi_1 = \frac{\cos^2\theta/2 - \sin^2\theta/2}{(\cos\theta/2 - \sin\theta/2)^2}}$$

$$\cot\phi_1 = \frac{-[\cos^2\frac{\theta}{2} - \sin^2\theta/2]}{\cos^2\theta/2 + \sin^2\theta/2 - 2\sin\theta/2\cos\theta/2}$$

$$\cot\phi_1 = \frac{-(\cos\frac{\theta}{2} + \sin\frac{\theta}{2})\cancel{(\cos\frac{\theta}{2} - \sin\frac{\theta}{2})}}{(\cos\theta/2 - \sin\theta/2)^{\cancel{2}}}$$

$$\cot\phi_1 = \left|\frac{\cos\theta/2 + \sin\frac{\theta}{2}}{\cos\theta - \sin\theta/2}\right|$$

$$\cot\phi_1 = -\left|\frac{1+\tan\theta/2}{1-\tan\theta/2}\right|$$

$$\cot\phi_1 = -\tan\left(\frac{\pi}{4}+\theta\right)$$

$$\phi_1 = \frac{\pi}{2} + \frac{\pi}{4} + \theta$$

$r = b(1+\sin 2\theta)$

$$\frac{dr}{d\theta} = b\cos\theta$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{b\cos\theta}{r}$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{\cancel{b}\cos\theta}{\cancel{b}(1+\sin 2\theta)}$$

$$\cot\phi_2 = \frac{\cos\theta}{1+\sin 2\theta}$$

$$\cot\phi_2 = \frac{\cos^2\theta/2 - \sin^2\theta/2}{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} + 2\cos\theta/2\sin\theta/2}$$

$$= \frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}} \quad = \frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} = \tan\left(\frac{\pi}{4} - \theta\right)$$

$\cot \phi/2 = \cot\left(\frac{\pi}{2} - \frac{\pi}{2} - \theta\right)$

$|\theta_2 - \theta_1| = \left(\cancel{\frac{\pi}{2}} + \cancel{\frac{\pi}{4}} + \cancel{\theta} - \cancel{\frac{\pi}{2}} + \cancel{\frac{\pi}{4}} + \cancel{\theta}\right)$

$|\theta_2 - \theta_1| = \frac{\pi}{2}$

Q) Find the pedal eq for the given polar curve

(i) $r = a \times 1 - \cos\theta$

ii) $r = a \times 1 + \cos\theta$

iii) $r^n = a^n \times \cos^n\theta$

iv) ~~$r^n = \cos^n\theta$ $r^n \cos n\theta$~~ $r^n = a^n \sin n\theta$

Q) Given $r = a(1-\cos\theta) - ①$

diff ① w.r.t θ

$\frac{dr}{d\theta} = a\,|0 - (-\sin\theta)|$

$\frac{dr}{d\theta} = a\sin\theta$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{a\sin\theta}{r}$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{\cancel{a}\sin\theta}{\cancel{a}(1-\cos\theta)}$

$\cot\phi = \frac{\sin\theta}{1-\cos\theta}$

$\cot\phi = \frac{2\sin\theta/2 \cdot \cos\theta/2}{2\sin^2\theta/2}$

$\cot\phi = \cot\left(\frac{\theta}{2}\right)$

w.r.t $\frac{1}{p^2} = \frac{1}{r^2}\left(1+\cot^2\phi\right]$

$\frac{1}{p^2} = \frac{1}{r^2}\left|1+\cot^2\left(\frac{\theta}{2}\right)\right|$

$\frac{1}{p^2} = \frac{1}{r^2}\operatorname{cosec}^2\left(\frac{\theta}{2}\right)$

$\frac{1}{p^2} = \frac{1}{r^2\sin^2\left(\frac{\theta}{2}\right)}$

$p^2 = r^2\sin^2\left(\frac{\theta}{2}\right)$

$p^2 = r^2\left|\frac{1-\cos\theta}{2}\right|$

$p^2 = \frac{r^2}{2}\left(\frac{r}{a}\right)$

$2ap^2 = r^3$

ii) $r = a \times 1 + \cos\theta$

diff ① w.r.t θ

$\frac{dr}{d\theta} = a\left(0 - \sin\theta\right)$

$\frac{dr}{d\theta} = a(-\sin\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{-a\sin\theta}{r}$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{-\cancel{a}\sin\theta}{\cancel{a}(1+\cos\theta)}$

$\cot\phi = \frac{-\sin\theta}{2\cos^2\theta/2}$

$\cot\phi = \frac{-2\sin\frac{\theta}{2}\cancel{\cos\frac{\theta}{2}}}{2\cos^{\cancel{2}}\frac{\theta}{2}}$

$\cot\phi = -\frac{\cancel{2}\sin\frac{\theta}{2}}{\cancel{2}\cos\frac{\theta}{2}}$

$\cot\phi = -\tan\frac{\theta}{2}$

$\cot\phi = -\tan\frac{\theta}{2}$

W.K.T :- $\frac{1}{p^2} = \frac{1}{r^2}(1+\cot^2\phi)$

$$\frac{1}{p^2} = \frac{1}{r^2}\left(1+\tan^2\left(\frac{\theta}{2}\right)\right)$$

$$\frac{1}{p^2} = \frac{1}{r^2}\sec^2\frac{\theta}{2}$$

$$\frac{1}{p^2} = \frac{1}{r^2\cos^2\frac{\theta}{2}}$$

$$p^2 = r^2\cos^2\frac{\theta}{2}$$

$$p^2 = r^2\left(\frac{1+\cos\theta}{2}\right)$$

$$p^2 = \frac{r^2}{2}(1+\cos\theta)$$

$$p^2 = \frac{r^2}{2}\frac{r}{a}$$

$$2ap^2 = r^3$$

iii) $r^n = a^n \times \cos n\theta$

diff ① w.r.t θ

$$n r^{n-1} = -a^n \sin n\theta$$

$$\frac{r^n}{r}\frac{dr}{d\theta} = -a^n \sin n\theta$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{-a^n \sin n\theta}{r^n}$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{-\cancel{a^n}\sin n\theta}{\cancel{a^n}\cos n\theta}$$

$$\cot\phi = -\tan n\theta$$

W.K.T :- $\frac{1}{p^2} = \frac{1}{r^2}\left(1+\tan^2\frac{\theta}{2}\right)$

$$\frac{1}{p^2} = \frac{1}{r^2}\sec^2\frac{\theta}{2}$$

$$p^2 = r^2\cos^2\frac{\theta}{2}$$

$$p^2 = \frac{r^2}{2}\frac{r}{a}$$

$$2ap^2 = r^3$$

iv) ~~$r^n \cos n\theta = a^n$~~ $r^n = a^n \sin n\theta$

diff ① w.r.t θ

$$n r^{n-1} = a^n \cos n\theta$$

$$\frac{r^n}{r}\frac{dr}{d\theta} = a^n \cos n\theta$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{a^n \cos n\theta}{r^n}$$

$$\cot\phi = \frac{\cancel{a^n}\cos n\theta}{\cancel{a^n}\sin n\theta}$$

$$\cot\phi = \cot n\theta$$

W.K.T $\frac{1}{p^2} = \frac{1}{r^2}(1+\cot^2\phi)$

$$\frac{1}{p^2} = \frac{1}{r^2}\left(\operatorname{cosec}^2\frac{\theta}{2}\right)$$

$$\frac{1}{p^2} = \frac{1}{r^2\sin^2\frac{\theta}{2}}$$

$$p^2 = r^2\sin^2\left(\frac{\theta}{2}\right)$$

$$p^2 = r^2\left(\frac{1-\cos\theta}{2}\right)$$

$$p^2 = \frac{r^2}{2}\left(\frac{r}{a}\right)$$

$$2ap^2 = r^3$$

$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix}$ 2×2

$m = 2$
$n = 2$

Q) Find the characteristics rules or Eigen values of the matrix $A \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}_{3\times 3}$

Sol) Given matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

Write the characteristic equation of A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$\Rightarrow (2-\lambda)[(3-\lambda)(2-\lambda)-2] - 2[(2-\lambda)-1] + 1[2-(3-\lambda)] = 0$

$\Rightarrow (2-\lambda)[6-5\lambda+\lambda^2-2] - 2[1-\lambda] + 1[\lambda-1] = 0$

$\Rightarrow 12-10\lambda+2\lambda^2-4-6\lambda+5\lambda^2-\lambda^3+2\lambda-2+2\lambda+\lambda-1$

$\Rightarrow -\lambda^3+12\lambda^2-11\lambda+3 = 0$

$\Rightarrow \lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$$\begin{array}{r|rrrr} \lambda = 1 & 1 & -7 & 11 & -5 \\ & 0 & 1 & -6 & 5 \\ \hline & 1 & -6 & 5 & 0 \end{array}$$

$(\lambda-1)(\lambda^2-6\lambda+5) = 0$

$\lambda^2 - 5\lambda + 5$

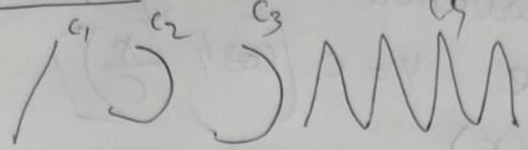
$\lambda^2 - 5\lambda - \lambda + 5$

$\lambda(\lambda-5) - 1(\lambda-5)$

$(\lambda-1)(\lambda-1)(\lambda-5)$

$\lambda = 1, 1, 5$

Curvature :-



Curvature is a concept we calculate the degree of bending of a curvature at any point

=> In C^1, which is a straight, the curvature $= 0^\circ$

=> In C^2, curvature is parabolic shape, having degree of bending

=> In C^3, we observe more shaped bending than C_2.

=> In C^4, The curve is quickly reverting back & has more sharp bending ECG format

=> Radius of curvature in cartition ~~form~~ form

i) If $y = f(x)$ then radius of curvature at any point P is given by

$$\rho \text{ at } P = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

=> ii) If $x = g(y)$, then radius of curvature at any point P is given by

$$\rho \text{ at } P = \frac{\left[1 + \left(\frac{dx}{dy}\right)^2\right]^{3/2}}{\frac{d^2x}{dy^2}}$$

Q) Radius of curvature @ any point on the curve $y = c\left[\cosh\left(\frac{x}{c}\right)\right]$

Ans) $y = c\cosh\left(\frac{x}{c}\right)$

$y = c\cosh\left(\frac{x}{c}\right) \rightarrow ①$

$\frac{dy}{dx} = \frac{d}{dx}\left(c\cosh\left(\frac{x}{c}\right)\right)$

$= c\left[\sinh\left(\frac{x}{c}\right)\cdot\frac{1}{c}\right)$

$\frac{dy}{dx} = \sinh\left(\frac{x}{c}\right) \rightarrow ②$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left[\sinh\left(\frac{x}{c}\right)\right]$

$\frac{d^2y}{dx^2} = \cosh\left(\frac{x}{c}\right)\cdot\frac{1}{c}$

Radius of curvature at any point P =

ρ at $P = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$

$= \frac{d^2y}{dx^2}$

$= \frac{\left[1+\sinh^2\left(\frac{x}{c}\right)\right]^{3/2}}{\cosh\left(\frac{x}{c}\right)\frac{1}{c}}$

$\Rightarrow \frac{c\left[\cosh^2\left(\frac{x}{c}\right)\right]^{3/2}}{\cosh\frac{x}{c}} = c$

$\sin^2\theta+\cos^2\theta = 1$

$\sinh^2x - \cosh^2x = 1$

@ any point
$\left[\cosh\left(\frac{x}{c}\right)\right]$

x

$\cdot\frac{1}{c}$

(2)

$\left[\sinh\left(\frac{x}{c}\right)\right]$

$\cdot\frac{1}{c}$

point P

$\sin^2\theta+\cos^2\theta=1$

$\sinh^2 x-\cosh^2 x=1$

$$= c\,\frac{\cosh^{\cancel{3}\,2}(x/c)}{\cancel{\cosh(x/c)}}$$

$$= c\cosh^2(x/c)$$

From eq (1)

~~y = c.f~~

$$y = c\cdot\cosh\left(\frac{x}{c}\right)$$

$$\cosh(x/c) = y/c$$

$$= \cancel{c}\cdot\frac{y^2}{c^{\cancel{2}}}$$

$$= \frac{y^2}{c}$$

$$\rho \text{ at } P = \frac{y^2}{c}$$

Q) Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the curve $x^3+y^3=3axy$

$$= c\,\frac{\cosh^{\cancel{3}\,2}(x/c)}{\cancel{\cosh(x/c)}}$$

$$= c\cosh^2(x/c)$$

From eq ①

~~$y = c\,c$~~

$$y = c\cdot\cosh\left(\frac{x}{c}\right)$$

$$\cosh(x/c) = y/c$$

$$= \cancel{c}\cdot\frac{y^2}{c^2}$$

$$= \frac{y^2}{c}$$

$$\rho \text{ at } p = \frac{y^2}{c}$$

Q) Find the radius of curvature ~~at~~ the ~~point~~ curve ~~$\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the curve $x^3+y^3=3axy$~~ $a^2y = x^3 - y^3$ at the point where the curve cuts x axis

Given $a^2y = x^3 - y^3 \rightarrow ①$ the x-axis $(x, 0)$

let p be a point cuts the x-axis

$\therefore$ substitute in eq ① $(y = 0)$

$$0 = x^3 - 0$$

Q) Find the radius of curvature at any point of the curve $y = a\log(\sec x/a)$

Sol) Given curve $y = a\log(\sec x/a)$ on both sides we diff eq ① w.r.t x on both sides we

get $\frac{dy}{dx} = \frac{d}{dx}\left(a\log\left(\sec(x/a)\right)\right)$

$$= a\frac{1}{\cancel{\sec(x/a)}}\cancel{\sec(x/a)}\tan(x/a)\cdot\frac{1}{a}$$

$$\frac{dy}{dx} = \tan(x/a) \rightarrow ②$$

Again diff eq ② w.r.t x on both sides

we get $\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left(\tan(x/a)\right)$

$$\frac{d^2y}{dx^2} = \sec^2(x/a)\frac{1}{a}$$

$$= \frac{\sec^2(x/a)}{a}$$

The radius of curvature at any point 'P' is given by ρ at $P = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$

$$= \frac{\left[1+\left(\tan\left(\frac{x}{a}\right)\right)^2\right]^{3/2}}{\frac{\sec^2(x/a)}{a}}$$

=

=

= c

= a

Q) Prove that

curve $xy^2 =$

Sol) Given curve

diff

get $\frac{d}{dx}$

$x \cdot 2y$

$2xy \cdot$

$2x$

any point

sides we

$\left(\frac{x}{a}\right) \cdot \frac{1}{a}$

both sides

$\left(\frac{x}{a}\right)\Big)$

any point

$\Big]^{3/2}$

$$= \frac{a\left[1+\tan^2\left(\frac{x}{a}\right)\right]^{3/2}}{\sec^2\left(\frac{x}{a}\right)}$$

$\sec^2 x - \tan^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

$$= \frac{a\left[\sec^{\cancel{2}}\left(\frac{x}{a}\right)\right]^{3/\cancel{2}}}{\sec^2\left(\frac{x}{a}\right)}$$

$$= \frac{a \sec^{\cancel{3}}\left(\frac{x}{a}\right)}{\cancel{\sec^2\left(\frac{x}{a}\right)}}$$

$$= a\sec\left(\frac{x}{a}\right)$$

Q) Prove that the radius of curvature of the curve $xy^2 = a^3 - x^3$ at the point $(a,0)$ is $\frac{3a}{2}$

Sol) Given curve $xy^2 = a^3 - x^3 \rightarrow ①$

diff eq ① w.r.t x on both sides we get

$$\frac{d}{dx}(x \cdot y^2) = \frac{d}{dx}(a^3 - x^3)$$

$$x \cdot 2y \cdot \frac{dy}{dx} + y^2 \cdot 1 = (0 - 3x^2)$$

$$2xy \cdot \frac{dy}{dx} + y^2 = -3x^2$$

$$2xy\left(\frac{dy}{dx}\right) = -3x^2 - y^2$$

$$\frac{dy}{dx} = \frac{-(3x^2 + y^2)}{2xy}$$

Again diff w.r.t 'x' on both sides we get

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = -\frac{d}{dx}\left(\frac{3x^2+y^2}{2xy}\right)$$

$$\frac{d^2y}{dx^2} = \frac{(2xy)(6x+2y.y') - (3x^2+y^2)}{4x^2y^2}$$

[3.

we get $(3x^2+y^2)$

Q) The radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of curve $x^3+y^3=3axy$

Sol) The radius of curvature $x^3+y^3=3axy \rightarrow (1)$

diff eq (1) w.r.t x on both sides we get,

$$\frac{d}{dx}\left[x^3+y^3\right]=\frac{d}{dx}\left[3axy\right]$$

$$3x^2+3\cdot y^2\cdot y'=3a\left[x\cdot y'+y\cdot 1\right]$$

$$3x^2+3y^2y'=3axy'+3ay$$

$$3y^2y'-3axy'=3ay-3x^2$$

$$\cancel{3}y'\left[y^2-ax\right]=\cancel{3}\left[ay-x^2\right]$$

$$y'=\frac{dy}{dx}=\frac{ay-x^2}{y^2-ax} \rightarrow (2)$$

diff on both sides

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{ay-x^2}{y^2-ax}\right)$$

$$\frac{d^2y}{dx^2}=\frac{(y^2-ax)(ay'-2x)-(ay-x^2)(2yy'-a)}{(y^2-ax)^2}$$

$$\frac{d^2y}{dx^2}=\frac{(y^2-ax)(ay'-2x)-(ay-x^2)(2y\cdot y'-a)}{(y^2-ax)^2}$$

$$\frac{d^2y}{dx^2}=\frac{\cancel{(y^2-ax)(ay'-2x)}\; ay'y^2-2xy-a^2x\cdot y'+2x^2-2ay^2\cdot y'+a^2y+2yx^2\cdot y'-ax^2}{(y^2-ax)^2}$$

$$= \frac{-ay'y^2 + ax^2 + a^2(y-x) + 2xy(xy'-2y) + a^2y}{(y^2 - ax)^2}$$

$$\cancel{= -ay'y^2 + ax^2 + a^2\left(y + \frac{y-x}{?}\right)}$$

$$= \frac{-a \cdot y'y^2 + ax^2 + a^2\left(\frac{y+y}{2y} - x\right) + 2xy(xy'-y)}{(y^2 - ax)^2}$$

$$\rho \text{ at } P = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$