

Q4) Solve the system of equation by using gauss-elimination method

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4$$

Sol)
$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

$$\Rightarrow AX = B$$

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

$$\therefore [AB] = \begin{bmatrix} 3 & 1 & 2 & : & 3 \\ 2 & -3 & -1 & : & -3 \\ 1 & 2 & 1 & : & 4 \end{bmatrix}$$

$$R_1 \leftrightarrow R_3 = \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 2 & -3 & -1 & : & -3 \\ 3 & 1 & 2 & : & 3 \end{bmatrix}$$

$$R_2 = R_2 - 2R_1; R_3 = R_3 - 3R_1$$

$$= \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & -7 & -3 & : & -11 \\ 0 & -5 & -1 & : & -9 \end{bmatrix}$$

$$R_2 : 5R_2, R_3 : 7R_3$$

$$= \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & 35 & 15 & : & 55 \\ 0 & -35 & 7 & : & -63 \end{bmatrix}$$

$$R_3 : R_3 + R_2$$

$$= \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & 35 & 15 & : & 55 \\ 0 & 0 & 8 & : & -8 \end{bmatrix}$$

$$\therefore AX = B$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 35 & 15 \\ 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 55 \\ -8 \end{bmatrix}$$

$$\Rightarrow x + 2y + z = 4 \rightarrow \textcircled{1}$$

$$\Rightarrow 35y + 15z = 55$$

$$\Rightarrow 7y + 3z = 11 \rightarrow \textcircled{2}$$

$$\Rightarrow -8z = -8$$

$$\boxed{z = -1}$$

$$\textcircled{2} \Rightarrow 7y + 3(-1) = 4$$

$$7y - 3 = 11$$

$$7y = 11 + 3$$

$$y = \frac{14}{7} = 2$$

$$\Rightarrow x + 2(2) = 1$$

$$x + 4 - 1 = 1$$

$$x + 3 = 1$$

$$x = 1$$

$$\therefore \text{The sol } (x=1, y=2, z=-1)$$

Q) Gauss elimination method:- $x + 2y + z = 3$
 $3x + 2y + z = 3$
 $x - 2y - 5z = 1$

$$\text{Sol) } \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} \quad R_3: R_3 - R_1$$

$$\therefore [AB] = \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 3 & 2 & 1 & : & 3 \\ 1 & -2 & -5 & : & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & -4 & -2 & : & -6 \\ 0 & -4 & -4 & : & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ 4 \end{bmatrix}$$

$$R_2: R_2 - 3R_1; R_3: R_3 - R_1$$

$$\begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & -4 & -2 & : & -6 \\ 0 & -4 & -6 & : & -2 \end{bmatrix}$$

$$\Rightarrow x + 2y + z = 3 \Rightarrow \textcircled{1}$$

$$\Rightarrow -4y - 2z = -6$$

$$2y + z = 3 \Rightarrow \textcircled{2}$$

$$\Rightarrow -4z = 4$$

$$z = -1$$

$$\textcircled{2} \Rightarrow 2y - 1 = 3$$

$$y = 2$$

$$\textcircled{1} \Rightarrow x + 4 - 1 = 3$$

$$x = 0$$

$$\therefore x = 0; y = 2; z = -1$$

Q) Gauss-Jordan method:- $x + 2y + z = 3$
 $2x + 3y + 2z = 5$
 $3x + 5y + 5z = 2$

$$\text{Sol) } \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 5 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

$$\therefore [AB] = \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 2 & 3 & 2 & : & 5 \\ 3 & 5 & 5 & : & 2 \end{bmatrix}$$

$$R_2: R_2 - 2R_1; R_3: R_3 - 3R_1$$

$$= \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & -1 & 0 & : & -1 \\ 0 & -1 & 2 & : & -7 \end{bmatrix}$$

$$R_1: R_1 + 2R_2; R_3: R_3 - 11R_2$$

$$= \begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & -1 & 0 & : & -1 \\ 0 & 0 & 2 & : & 4 \end{bmatrix}$$

$$R_3: \frac{1}{2} R_3$$

$$= \begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 1 & : & 2 \end{bmatrix}$$

$$R_1: R_1 - R_3; R_2: -R_2$$

$$= \begin{bmatrix} 1 & 0 & 0 & : & -1 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 1 & : & 2 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} -1 \\ 1 \\ 2 \end{vmatrix}$$

$$x = -1; y = 1; z = 2$$

6) Gauss Jordan Method:- $x + y + z = 9$
 $x - 2y + 3z = 8$
 $2x + y - z = 3$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 1 & -2 & 3 & : & 8 \\ 2 & 1 & -1 & : & 3 \end{bmatrix}$$

$$R_2: R_2 - R_1; R_3: R_3 - 2R_1$$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -3 & 2 & : & -1 \\ 0 & -1 & -3 & : & -15 \end{bmatrix}$$

$$R_1: R_1 + R_2; R_3: 3R_3 - R_2$$

$$= \begin{bmatrix} 1 & 0 & 3 & : & 8 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & -11 & : & -46 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3$$

$$= \begin{bmatrix} 1 & 0 & 3 & : & 8 \\ 0 & -1 & -3 & : & -15 \\ 0 & -3 & 2 & : & -1 \end{bmatrix}$$

$$R_3 \rightarrow \frac{R_3}{-11}$$

$$= \begin{bmatrix} 1 & 0 & 3 & : & 8 \\ 0 & -1 & -3 & : & -15 \\ 0 & 0 & 1 & : & 4 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3$$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -1 & -3 & : & -15 \\ 0 & -3 & 2 & : & -1 \end{bmatrix}$$

$$R_1: R_1 + R_2; R_3: R_3 - 3R_2$$

$$= \begin{bmatrix} 1 & 0 & -2 & : & -6 \\ 0 & -1 & -3 & : & -15 \\ 0 & 0 & 11 & : & 44 \end{bmatrix}$$

$$R_3: \frac{1}{11} R_3$$

$$= \begin{bmatrix} 1 & 0 & -2 & : & -6 \\ 0 & -1 & -3 & : & -15 \\ 0 & 0 & 1 & : & 4 \end{bmatrix}$$

$$R_1: R_1 + 2R_3; R_2: R_2 + 3R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & : & 2 \\ 0 & -1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & : & 2 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 4 \end{bmatrix} \quad x=2; y=3; z=4$$

Q) Find the eigen values of square matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\text{let } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{bmatrix}$$

$$\therefore \text{W.K.T} \Rightarrow |A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(4-\lambda) - 6 = 0$$

$$\Rightarrow 4 - \lambda - 4\lambda + \lambda^2 - 6 = 0$$

$$\Rightarrow \lambda^2 - 5\lambda - 2 = 0$$

$$\therefore \lambda = \frac{(-5) \pm \sqrt{(-5)^2 - 4(1)(-2)}}{2(1)}$$

$$\lambda = \frac{5 \pm \sqrt{25+8}}{2}$$

$$\lambda = \frac{5}{2} \pm \frac{\sqrt{33}}{2}$$

$$\lambda = \frac{5}{2} - \frac{\sqrt{33}}{2}, \frac{5}{2} + \frac{\sqrt{33}}{2}$$

Q) Find the eigen values & eigen vectors of the matrix

$$A = \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix}$$

$$\text{let } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A - \lambda I = \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} -1-\lambda & 3 \\ -3 & 4-\lambda \end{bmatrix}$$

$$\therefore \text{W.K.T}$$

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -(1+\lambda) & 3 \\ -3 & (4-\lambda) \end{vmatrix} = 0$$

$$\Rightarrow -(1+\lambda)(4-\lambda) - 9 = 0$$

$$\Rightarrow (\lambda+1)(4-\lambda) - 9 = 0$$

$$\Rightarrow 4\lambda - \lambda^2 + 4 - \lambda - 9 = 0$$

$$\Rightarrow -\lambda^2 + 3\lambda - 5 = 0$$

$$\Rightarrow \lambda^2 - 3\lambda + 5 = 0$$

$$\lambda = \frac{3 \pm \sqrt{9 - 4(1)(5)}}{2(1)} \Rightarrow 3\lambda + 5 = 0$$

$$\therefore \lambda = \frac{3 \pm \sqrt{4 \pm (1)5}}{2(1)}$$

$$\lambda = \frac{3 \pm \sqrt{9 \pm 20}}{2}$$

$$\lambda = \frac{3 \pm \sqrt{-1}}{2}$$

$$\lambda = \frac{3 \pm \sqrt{11} i}{2}$$

$$\lambda = \frac{3 \pm \sqrt{11} i}{2}$$

$$\lambda = \frac{3}{2} - \frac{\sqrt{11} i}{2}, \frac{3}{2} + \frac{\sqrt{11} i}{2}$$

Polar Curves

\Rightarrow let OX be a initial line let $r = f(\theta)$

$r = f(\theta)$ be a polar curve at the point P , let $OP = r$, let θ be the angle b/w OP & initial line and $O(0,0)$

P be a tangent to the curve $r = f(\theta)$ at P . Then the co-ordinates of P are (r, θ) or $(f(\theta), \theta)$

are called the polar co-ordinates

$\Rightarrow O(0,0)$ is called the pole, $r = f(\theta)$ is called the polar curve and the entire system is called the polar system

\Rightarrow angle b/w the radius of vector and tangent

\Rightarrow let OX be the initial line let $r = f(\theta)$

P be any point on the curve with $OP = r$, let θ be the radius of OP

vector, let $r = f(\theta)$ be a polar curve at the point P , let T be a tangent to the curve at P , let X be the angle b/w the radius of vector & tangent to the curve & finally O be the angle b/w the tangent & initial line

Problem

$$\chi = \phi + \theta$$

$$\Rightarrow \tan \chi = \tan (\phi + \theta)$$

$$\tan \chi = \frac{\tan \phi + \tan \theta}{1 - \tan \phi \cdot \tan \theta} \quad \text{--- (1)}$$

\therefore The slope of T in m = $\tan \chi$

$$\therefore m = \tan \chi = \frac{dy}{dx} = 0$$

$$\therefore \text{let } x = r \cos \theta, y = r \sin \theta$$

$$\frac{dx}{d\theta} = \frac{dr}{d\theta} \cos \theta - r \sin \theta$$

$$\therefore \frac{dy}{d\theta} = \frac{dr}{d\theta} \sin \theta + r \cos \theta$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$$

$$1 - \frac{r \sin \theta}{\frac{dr}{d\theta} \cos \theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\tan \theta + r \frac{d\theta}{dr}}{1 - (\tan \theta) (r \frac{d\theta}{dr})}$$

$$\Rightarrow \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi} = \frac{\tan \theta + r \frac{d\theta}{dr}}{1 - (\tan \theta) (r \frac{d\theta}{dr})}$$

$$\therefore \tan \phi = r \frac{d\theta}{dr}$$

$$\tan \phi = \frac{1}{r} \frac{dr}{d\theta}$$

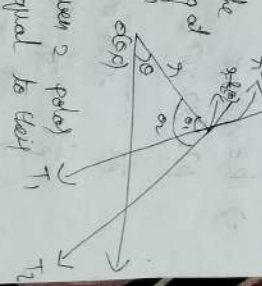
$$\Rightarrow \cot \phi = \frac{1}{r} \frac{dr}{d\theta}$$

Angle b/w two polar curves = 90°

$r_1 = f_1(\theta)$, $r_2 = f_2(\theta)$ be the 2 polar curves intersecting at P and making angle with the radius of vector

\therefore The angle b/w the given 2 polar curves should be equal to their tangents

$$\therefore \phi = |P_2 - P_1|$$



The vertical distance of a pole to a tangent
 can $s = f(\theta)$ be a polar curve
 a polar curve at capital
 P with $OP = s$ & let
 N be the angle θ then the
 tangent to the curve & radius
 to the curve. Let N be
 the foot of the perpendicular of the
 pole to the tangent $OM = p'$
 $\therefore \triangle OPM$ be a right angle triangle
 we have

$$\sin \theta = \frac{OM}{OP}$$

$$\Rightarrow \sin \theta = \frac{p'}{s}$$

$$\Rightarrow p = s \sin \theta$$

$$p' = s^2 \sin^2 \theta$$

$$\frac{1}{p'} = \frac{1}{s^2 \sin^2 \theta}$$

$$\frac{1}{p'} = \frac{1}{s^2} (\sec^2 \theta)$$

$$\left(\frac{1}{p'} = \frac{1}{s^2} (1 + \cot^2 \theta) \right)$$

$$\frac{1}{p'} = \frac{1}{s^2} + \frac{1}{s^2} \left(\frac{dy}{dx} \right)^2$$

where the above eqs are called pedal eqs
 (or) $p-r$ equations

Some important results:-

$$1) \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$2) \sin 2\theta = 2 \sin \theta \cos \theta$$

$$3) \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$4) \cos \theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$$

$$5) 1 + \cos 2\theta = 2 \cos^2 \theta \quad \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$6) 1 - \cos 2\theta = 2 \sin^2 \theta \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$7) \sin^2 \theta = 1 - \cos^2 \theta \quad \cos^2 \theta = 1 - \sin^2 \theta$$

$$8) \sin \left(\frac{\theta}{2} \right) = \frac{1 - \cos \theta}{2}$$

$$9) \cos \left(\frac{\theta}{2} \right) = \frac{1 + \cos \theta}{2}$$

$$10) \sin \left(\frac{\theta}{2} \right) = \frac{1 - \cos \theta}{2}$$

$$11) \cos \left(\frac{\theta}{2} \right) = \frac{1 + \cos \theta}{2}$$

$$12) \sin \left(\frac{\theta}{2} \right) = \frac{1 - \cos \theta}{2}$$

$$13) \cos \left(\frac{\theta}{2} \right) = \frac{1 + \cos \theta}{2}$$

$$14) \sin \left(\frac{\theta}{2} \right) = \frac{1 - \cos \theta}{2}$$

$$15) \cos \left(\frac{\theta}{2} \right) = \frac{1 + \cos \theta}{2}$$

$$16) \sin \left(\frac{\theta}{2} \right) = \frac{1 - \cos \theta}{2}$$

$$17) \cos \left(\frac{\theta}{2} \right) = \frac{1 + \cos \theta}{2}$$

$$18) \sin \left(\frac{\theta}{2} \right) = \frac{1 - \cos \theta}{2}$$

$$19) \cos \left(\frac{\theta}{2} \right) = \frac{1 + \cos \theta}{2}$$

$$20) \sin \left(\frac{\theta}{2} \right) = \frac{1 - \cos \theta}{2}$$

$$21) \cos \left(\frac{\theta}{2} \right) = \frac{1 + \cos \theta}{2}$$

$$22) \sin \left(\frac{\theta}{2} \right) = \frac{1 - \cos \theta}{2}$$

$$23) \cos \left(\frac{\theta}{2} \right) = \frac{1 + \cos \theta}{2}$$

$$24) \sin \left(\frac{\theta}{2} \right) = \frac{1 - \cos \theta}{2}$$

Q) find the angle b/w the radius of vector and tangent to the given polar at the angle indicated

1) $r = a(1 + \cos \theta)$; $\theta = \frac{\pi}{3}$

differentiate w.r.t θ $r = a(1 - \cos \theta)$; $\theta = \frac{\pi}{6}$

$\therefore \Rightarrow \frac{dr}{d\theta} = a(0 - (-\sin \theta))$ $r = 3 \cos \theta$; $\theta = \frac{\pi}{4}$

$\Rightarrow \frac{dr}{d\theta} = -a \sin \theta$ $r = 2 \sin \theta$; $\theta = \frac{\pi}{2}$

$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{a \sin \theta}{r}$

$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{a \sin \theta}{r(1 + \cos \theta)}$

$\Rightarrow \cot \phi = \frac{-\sin \theta}{1 + \cos \theta}$

$\Rightarrow \cot \phi = \frac{-2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{1 + 2 \cos^2 \frac{\theta}{2} - 1} \Rightarrow \phi = \frac{\pi}{2} + \frac{\pi}{3}$

$\Rightarrow \cot \phi = -\tan \left(\frac{\theta}{2} \right)$ $\Rightarrow \phi = \frac{\pi}{2} + \frac{\pi}{6}$

$\Rightarrow \cot \phi = \cot \left(\frac{\pi}{2} + \frac{\theta}{2} \right)$

$\Rightarrow \phi = \frac{\pi}{2} + \frac{\theta}{2}$

$\therefore \text{At } \theta = \frac{\pi}{3}$

$\therefore \phi = \frac{\pi}{2} + \frac{\pi}{3}$

Given $r = a(1 - \cos \theta)$ - ①

diff w.r.t θ

$\therefore \Rightarrow \frac{dr}{d\theta} = a(0 + \sin \theta)$

$\Rightarrow \frac{dr}{d\theta} = a \sin \theta$

$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{a \sin \theta}{r}$

$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{a \sin \theta}{a(1 - \cos \theta)}$

$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{\sin \theta}{1 - \cos \theta}$

$\Rightarrow \cot \phi = 2 \sin \left(\frac{\theta}{2} \right) \cot \left(\frac{\theta}{2} \right)$

$\Rightarrow \cot \phi = \cot \left(\frac{\theta}{2} \right)$

$\therefore \phi = \frac{\theta}{2}$

$\therefore \text{At } \theta = \frac{\pi}{6}$

$\therefore \phi = \frac{\pi}{6}$

$\Rightarrow \phi = \frac{\pi}{12}$

$$r = 3\cos\theta \rightarrow \textcircled{1}$$

diff. w.r.t. θ

$$\Rightarrow \frac{dr}{d\theta} = -3\sin\theta$$

$$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = -\frac{3\sin\theta}{r}$$

$$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = -\frac{3\sin\theta}{3\cos\theta}$$

$$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = -\tan\theta$$

$$\Rightarrow \ln r = -\left(\frac{\pi}{2} + \theta\right)$$

$$\Rightarrow r = \frac{\pi}{2} + \theta$$

$$\Rightarrow r = \frac{\pi}{2} + \frac{\pi}{4}$$

$$\Rightarrow r = \frac{4\pi + 2\pi}{8}$$

$$\Rightarrow r = \frac{6\pi}{8}$$

$$\Rightarrow r = \frac{3\pi}{4}$$

$$r = 2\sin^2\theta \rightarrow \textcircled{2}$$

diff. w.r.t. θ

$$\Rightarrow \frac{dr}{d\theta} = 2\sin\theta \cos\theta$$

$$\Rightarrow \frac{dr}{d\theta} = \sin 2\theta$$

$$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{\sin 2\theta}{2\sin^2\theta}$$

$$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{2\cos\theta}{2\sin\theta}$$

$$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \cot\theta$$

$$\Rightarrow \ln r = \frac{1}{2} \ln \theta$$

$$\Rightarrow r = \frac{\pi}{4}$$

Q Show that given polar curves intersect orthogonally

(i) $r = a(1 - \cos \theta)$, $r = b(1 + \cos \theta)$

(ii) $r(1 - \cos \theta) = 2a$, $r(1 + \cos \theta) = 2b$

(iii) $r^n = a^n \cos n\theta$, $r^n = b^n \sin n\theta$

(iv) $r^n = \cos^n n\theta = a^n$, $r^n \sin n\theta = b^n$

① $r = a(1 - \cos \theta) \rightarrow ①$, $r = b(1 + \cos \theta) \rightarrow ②$

$\therefore ① \Rightarrow \frac{dr}{d\theta} = a \sin \theta$ $\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{a \sin \theta}{r}$

$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{a \sin \theta}{r}$

$\Rightarrow \cot \phi_1 = \frac{a \sin \theta}{r(1 - \cos \theta)}$

$\Rightarrow \cot \phi_1 = \frac{\sin \theta}{1 - \cos \theta}$

$\Rightarrow \cot \phi_2 = \frac{2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}}$

$\Rightarrow \cot \phi_1 = \cot \left(\frac{\theta}{2} \right)$

$\Rightarrow \phi_1 = \frac{\theta}{2}$

$\therefore ② \Rightarrow \frac{dr}{d\theta} = -b \sin \theta$

$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{-b \sin \theta}{r}$

$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{-b \sin \theta}{r(1 + \cos \theta)}$

$\Rightarrow \cot \frac{\phi_2}{2} = \frac{-2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}}$

$\Rightarrow \cot \frac{\phi_2}{2} = -\tan \frac{\theta}{2}$

$\Rightarrow \cot \frac{\phi_2}{2} = \cot \left(\frac{\pi}{2} + \frac{\theta}{2} \right)$

$\Rightarrow \frac{\phi_2}{2} = \frac{\pi}{2} + \frac{\theta}{2}$

$\therefore |\phi_2 - \phi_1| = \left| \frac{\pi}{2} + \frac{\theta}{2} - \frac{\theta}{2} \right|$

$= \frac{\pi}{2}$

$\Rightarrow |\phi_2 - \phi_1| = \frac{\pi}{2}$

$= \frac{\pi}{2}$

② Given $r(1 - \cos \theta) = 2a \rightarrow ①$; $r(1 + \cos \theta) = 2b \rightarrow ②$

diff ① w.r.t $\theta \rightarrow 0$

$\therefore ① \Rightarrow (1 - \cos \theta) \frac{dr}{d\theta} + r \sin \theta = 0$

$\Rightarrow (1 - \cos \theta) \frac{dr}{d\theta} = -r \sin \theta$

$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{-\sin \theta}{1 - \cos \theta}$

$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{-2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}}$

$\Rightarrow \cot \phi_1 = -\cot \frac{\theta}{2}$

$\Rightarrow \cot \phi_1 = \cot \left(\frac{\pi}{2} - \frac{\theta}{2} \right)$

$\Rightarrow \phi_1 = \frac{\pi}{2} - \frac{\theta}{2}$

② $\Rightarrow (1 + \cos \theta) \frac{dr}{d\theta} - r \sin \theta = 0$

$\Rightarrow (1 + \cos \theta) \frac{dr}{d\theta} = r \sin \theta$

$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{\sin \theta}{1 + \cos \theta}$

$\Rightarrow \cot \frac{\phi_2}{2} = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}}$

$\Rightarrow \cot \frac{\phi_2}{2} = \tan \frac{\theta}{2}$

$\Rightarrow \cot \frac{\phi_2}{2} = \cot \left(\frac{\pi}{2} - \frac{\theta}{2} \right)$

$\Rightarrow \frac{\phi_2}{2} = \frac{\pi}{2} - \frac{\theta}{2}$

$\therefore |\phi_2 - \phi_1| = \left| \frac{\pi}{2} - \frac{\theta}{2} - \left(\frac{\pi}{2} - \frac{\theta}{2} \right) \right|$

$= \frac{\pi}{2} - \frac{\theta}{2} + \frac{\theta}{2}$

$= \frac{\pi}{2}$

Given $x^n = a^n \cosh u \rightarrow 0$, $x^n = b^n \sinh u \rightarrow 0$

\therefore diff ① w.r.t 'u'

$$\therefore ① \Rightarrow n x^{n-1} \frac{dx}{du} = a^n$$

$$\Rightarrow \frac{x^n}{x} \frac{dx}{du} = -a^n \sinh u$$

$$\Rightarrow \frac{1}{x} \frac{dx}{du} = \frac{-a^n \sinh u}{x^n}$$

$$\Rightarrow \cot \phi_1 = \frac{-a^n \sinh u}{a^n \cosh u}$$

$$\Rightarrow \cot \phi_1 = -\tanh u$$

$$\Rightarrow \cot \phi_1 = \cot\left(\frac{\pi}{2} + u\right)$$

$$\Rightarrow \phi_1 = \frac{\pi}{2} + u \rightarrow ②$$

$$\therefore ② \Rightarrow n x^{n-1} \frac{dx}{du} = b^n (\cosh u) (u)$$

$$\Rightarrow \frac{x^n}{x} \frac{dx}{du} = b^n \cosh u$$

$$\Rightarrow \frac{1}{x} \frac{dx}{du} = \frac{b^n \cosh u}{x^n}$$

$$\Rightarrow \cot \phi_2 = \frac{b^n \cosh u}{b^n \sinh u}$$

$$\Rightarrow \cot \phi_2 = \coth u \quad (|\phi_2 - \phi_1| = \frac{\pi}{2})$$

$$\phi_2 = u$$

$$\therefore |\phi_2 - \phi_1| = \left| u - \frac{\pi}{2} + u \right|$$

$$iv) x^n \cosh u = a^n, x^n \sinh u = b^n$$

\therefore diff ① w.r.t 'u'

$$\therefore ① \Rightarrow n b^{n-1} = x^n \cosh u +$$

$$n x^{n-1} \frac{dx}{du} = \cosh u \times b^n$$

$$\frac{x^n}{x} \frac{dx}{du} = \cosh u \times b^n$$

$$\frac{1}{x} \frac{dx}{du} = \cosh u \times b^n$$

$$\therefore ② \Rightarrow n x^{n-1} \frac{dx}{du} \cdot \cosh u + x^n (-\sinh u) (u) = 0$$

$$\frac{x^n}{x} \frac{dx}{du} \cdot \cosh u = \sinh u$$

$$\frac{1}{x} \frac{dx}{du} = \frac{\sinh u}{\cosh u}$$

$$\cot \phi_1 = \tanh u$$

$$\cot \phi_1 = \cot\left(\frac{\pi}{2} - u\right)$$

$$\phi_1 = \frac{\pi}{2} - u$$

$$\therefore ② \Rightarrow n x^{n-1} \frac{dx}{du} \sinh u + x^n (\cosh u) (u) = 0$$

$$\Rightarrow \frac{x^n}{x} \frac{dx}{du} \sinh u = -x^n \cosh u$$

$$\Rightarrow \frac{1}{x} \frac{dx}{du} = -\frac{\cosh u}{\sinh u}$$

$$\Rightarrow \cot \phi_2 = -\coth u$$

$$\Rightarrow \cot \phi_2 = \cot(-u)$$

$$\phi_2 = -u$$

$$|\phi_2 - \phi_1| = \left| -u - \frac{\pi}{2} + u \right| = \frac{\pi}{2}$$

$$r = 2 \cos \theta, \quad r = 3 \sin \theta$$

$$r = 2 \cos \theta$$

$$\frac{dr}{d\theta} = -2 \sin \theta$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{-2 \sin \theta}{r}$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{-2 \sin \theta}{2 \cos \theta}$$

$$\cot \phi_1 = -\tan \theta$$

$$\cot \phi_1 = \cot \left(\frac{\pi}{2} + \theta \right)$$

$$\phi_1 = \frac{\pi}{2} + \theta \quad \left| \phi_2 - \phi_1 \right| = \left| \frac{\pi}{2} + \theta - \theta \right|$$

$$= \frac{\pi}{2}$$

$$r = 3 \sin \theta$$

$$\frac{dr}{d\theta} = 3 \cos \theta$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{3 \cos \theta}{r}$$

$$\cot \phi_1 = \frac{3 \cos \theta}{3 \sin \theta}$$

$$\cot \phi_1 = \cot \theta$$

$$\phi_1 = \theta$$

a) Find the angle b/w \odot & \odot $r = a \log \theta, r = \frac{a}{\log \theta}$
diff \odot w.r.t θ

$$\therefore \odot \Rightarrow \frac{dr}{d\theta} = \frac{a}{\theta}$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{a/\theta}{r}$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{a/\theta}{a \log \theta}$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{1}{\theta \log \theta}$$

$$r \frac{d\theta}{dr} = \theta \log \theta$$

$$\Rightarrow \odot \tan \phi_1 = \theta \log \theta \rightarrow \odot$$

$$\therefore \odot \Rightarrow r \log \theta = a$$

$$\log \theta \cdot \frac{dr}{d\theta} + r \cdot \left(\frac{1}{\theta} \right) = 0$$

$$\log \theta \cdot \frac{dr}{d\theta} = -\frac{1}{\theta}$$

$$\frac{1}{r} \frac{dr}{d\theta} = -\frac{1}{\log \theta}$$

$$r \frac{d\theta}{dr} = -\theta \log \theta$$

$$\tan \phi_2 = -\theta \log \theta \rightarrow \odot$$

w.k.T

$$\tan (\phi - \alpha_2) = \frac{\tan \phi_1 - \tan \phi_2}{1 + \tan \phi_1 \tan \phi_2}$$

$$\tan (\phi_1 - \phi_2) = \frac{2 \theta \log \theta}{1 - 2 \theta \log \theta}$$

\therefore From (1) & (2)

$$r = \frac{1}{\log 2}, \quad r = \frac{2}{\log 2}$$

$$x \log 2 = \frac{2}{\log 2}$$

$$(\log 2)^2 = 1$$

$$\log_2 2 = 1$$

$$2 = 2^1 = 2$$

2) Solve the angle b/w polar curves

$$r^2 \sin 2\theta = 4, \quad r^2 = 16 \sin 2\theta$$

3) Given $r^2 \sin 2\theta = 4 \rightarrow \textcircled{1}$

$$r^2 = 16 \sin 2\theta \rightarrow \textcircled{2}$$

diff (1) w.r.t (2)

$$\therefore \textcircled{1} \Rightarrow 2r \frac{dr}{d\theta} \sin 2\theta + r^2 (\cos 2\theta) \cdot 2 = 0$$

$$\Rightarrow 2r \frac{dr}{d\theta} \sin 2\theta = -r^2 \cos 2\theta$$

$$\Rightarrow \frac{r}{r^2} \frac{dr}{d\theta} = \frac{-\cos 2\theta}{\sin 2\theta}$$

$$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = -\cot 2\theta$$

$$\Rightarrow \cot \phi_2 = \cot (-2\theta)$$

$$\Rightarrow \phi_1 = -2\theta$$

$$\therefore \tan(\phi_1 - \phi_2) = \frac{2\theta}{1 - \cot 2\theta}$$

$$\Rightarrow \tan(\phi_2 - \phi_1) = \frac{2\theta}{1 - \cot 2\theta}$$

$$\Rightarrow |\phi_1 - \phi_2| = \tan^{-1} \left(\frac{2\theta}{1 - \cot 2\theta} \right)$$

$$\therefore \textcircled{1} \Rightarrow r \frac{dr}{d\theta} = 16 \cos 2\theta \cdot r$$

$$\Rightarrow r \frac{dr}{d\theta} = 16 \cos 2\theta$$

$$\Rightarrow \frac{r}{r^2} \frac{dr}{d\theta} = \frac{16 \cos 2\theta}{r^2}$$

$$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = \frac{16 \cos 2\theta}{16 \sin 2\theta}$$

$$\Rightarrow \cot \phi_2 = \cot 2\theta$$

$$\Rightarrow \phi_1 = 2\theta$$

$$\therefore |\phi_1 - \phi_2| = |-2\theta - 2\theta| = -4\theta$$

$$\textcircled{2} r^2 = \frac{4}{\sin^2 \theta}, \quad r^2 = 16 \sin 2\theta$$

$$\frac{r^2}{\sin^2 \theta} = 16 \sin 2\theta$$

$$1 = 4 \sin^2 2\theta$$

$$\sin^2 2\theta = \frac{1}{4}$$

$$\sin 2\theta = \frac{1}{2}$$

$$2\theta = \sin^{-1} \left(\frac{1}{2} \right)$$

$$2\theta = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{12}$$

Subs θ in eq (1)

$$\phi_1 = \frac{\pi}{12} = \frac{\pi}{3}$$

Q) Given Find the angle for the curve

$$r = \sin \theta + \cos \theta ; r = 2 \sin \theta$$

$$\text{Q)} \quad r = \sin \theta + \cos \theta$$

$$r = 2 \sin \theta$$

$$\frac{dr}{d\theta} = \cos \theta - \sin \theta$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{\cos \theta - \sin \theta}{\sin \theta + \cos \theta}$$

$$\cot \phi_1 = \frac{\cos \theta - \sin \theta}{\sin \theta}$$

$$\cot \phi_1 = \frac{\cos \theta - \sin \theta}{\sin \theta + \sin \theta}$$

$$\cot \phi_1 = \cot \theta - \frac{\sin \theta}{\cos \theta}$$

$$\frac{\cos \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta}$$

$$\cot \phi_1 = \frac{1 - \tan \theta}{1 + \tan \theta}$$

$$\cot \phi_1 = \frac{1 - \tan \theta}{1 + \tan \theta}$$

$$= \frac{\tan(\pi/4) - \tan \theta}{1 + \tan(\pi/4) \cdot \tan \theta}$$

$$= \tan(\pi/4 - \theta)$$

$$= \cot(\pi/2 - (\pi/4 - \theta))$$

$$\phi_1 = \frac{\pi}{2} - \frac{\pi}{4} + \theta$$

$$= \frac{\pi}{4} + \theta$$

$$\therefore \text{Q)} \Rightarrow \frac{dr}{d\theta} = 2 \cos \theta$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{2 \cos \theta}{2 \sin \theta}$$

$$\cot \phi_2 = \frac{2 \cos \theta}{2 \sin \theta}$$

$$\cot \phi_2 = \cot \theta$$

$$\phi_2 = \theta$$

$$|\phi_2 - \phi_1| = |\theta - \frac{\pi}{4} + \theta|$$

$$|\phi_2 - \phi_1| = \frac{\pi}{4}$$

a) Find the angle b/w given polar curves

$$r = 3\sin\theta \quad r = a(1 - \sin\theta)$$

$$r = b(1 + \sin 2\theta)$$

$$\frac{dr}{d\theta} = a(0 - \cos\theta)$$

$$\frac{1}{r} \frac{dr}{d\theta} = -\frac{a\cos\theta}{r}$$

$$\frac{1}{r} \frac{dr}{d\theta} = -\frac{a\cos\theta}{a(1 - \sin\theta)}$$

$$\cot\phi_1 = \frac{\cos\theta}{1 - \sin\theta}$$

$$\cot\phi = \frac{\cos^2\theta/2 - \sin^2\theta/2}{\cos\theta}$$

$$\cot\phi_1 = \frac{\cos^2\theta/2 - \sin^2\theta/2}{(\cos\theta/2 - \sin\theta/2)}$$

$$\cot\phi_1 = -\frac{(\cos^2\theta/2 - \sin^2\theta/2)}{(\cos\theta/2 - \sin\theta/2)}$$

$$\cos\theta/2 + \sin\theta/2 - 2\sin\theta/2 \cos\theta/2$$

$$\cot\phi_1 = -\frac{(\cos\theta/2 + \sin\theta/2)(\cos\theta/2 - \sin\theta/2)}{(\cos\theta/2 - \sin\theta/2)^2}$$

$$\cot\phi_1 = \frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2}$$

$$\cot\phi_1 = -\frac{1 + \tan\theta/2}{1 - \tan\theta/2}$$

$$\cot\phi_1 = -\tan\left(\frac{\pi}{4} + \theta\right)$$

$$\phi_1 = \frac{\pi}{2} + \frac{\pi}{4} + \theta$$

$$r = b(1 + \sin 2\theta)$$

$$\frac{dr}{d\theta} = b\cos\theta$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{b\cos\theta}{b(1 + \sin 2\theta)}$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{\cos\theta}{1 + \sin 2\theta}$$

$$\cot\phi_2 = \frac{\cos\theta}{1 + \sin 2\theta}$$

$$\cot\phi_1 = \frac{\cos^2\theta/2 - \sin^2\theta/2}{\cos\theta/2 + \sin\theta/2 + 2\cos\theta/2 \sin\theta/2}$$

$$= \frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2} = \frac{1 + \tan\theta/2}{1 - \tan\theta/2} = \tan\left(\frac{\pi}{4} + \theta\right)$$

$$\cot \theta/2 = \cot \left(\frac{\pi}{2} - \frac{\pi}{2} - 0 \right)$$

$$|\theta_2 - \theta_1| = \left(\frac{\pi}{2} + \frac{\pi}{2} + 0 - \frac{\pi}{2} + \frac{\pi}{2} + 0 \right)$$

$$|\theta_2 - \theta_1| = \frac{\pi}{2}$$

Q) Find the pedal eq for the given polar curve

i) $r = a(1 - \cos \theta)$

ii) $r = a(1 + \cos \theta)$

iii) $r^n = a^n \cos^n \theta$

iv) ~~$r^n = a^n \cos^n \theta$~~ ~~$r^n = a^n \sin^n \theta$~~ $r^n = a^n \sin^n \theta$

Q) Given $r = a(1 - \cos \theta)$ — (1)

diff w.r.t θ

$$\frac{dr}{d\theta} = a(0 - (-\sin \theta))$$

$$\frac{dr}{d\theta} = a \sin \theta$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{a \sin \theta}{r}$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{a \sin \theta}{a(1 - \cos \theta)}$$

$$\cot \phi = \frac{\sin \theta}{1 - \cos \theta}$$

$$\cot \phi = \frac{2 \sin \theta/2 \cdot \cos \theta/2}{2 \sin^2 \theta/2}$$

$$\cot \phi = \cot \left(\frac{\theta}{2} \right)$$

$$\text{w.k.T } \frac{1}{p^2} = \frac{1}{r^2} (1 + \cot^2 \phi)$$

$$\frac{1}{p^2} = \frac{1}{r^2} (1 + \cot^2 \left(\frac{\theta}{2} \right))$$

$$\frac{1}{p^2} = \frac{1}{r^2} \operatorname{cosec}^2 \left(\frac{\theta}{2} \right)$$

$$\frac{1}{p^2} = \frac{1}{r^2 \sin^2 \left(\frac{\theta}{2} \right)}$$

$$p^2 = r^2 \sin^2 \left(\frac{\theta}{2} \right)$$

$$p^2 = r^2 \left| \frac{1 - \cos \theta}{2} \right|$$

$$p^2 = \frac{r^2}{2} \left(\frac{1 - \cos \theta}{2} \right)$$

$$2ap^2 = r^3$$

ii) $r = a(1 + \cos \theta)$

diff w.r.t θ

$$\frac{dr}{d\theta} = a(0 - \sin \theta)$$

$$\frac{dr}{d\theta} = a(-\sin \theta)$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{-a \sin \theta}{r}$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{-a \sin \theta}{a(1 + \cos \theta)}$$

$$\cot \phi = \frac{-\sin \theta}{1 + \cos \theta}$$

$$\cot \phi = \frac{-2 \sin \theta/2 \cos \theta/2}{2 \cos^2 \theta/2}$$

$$\cot \phi = \frac{-\cancel{2} \sin \theta/2}{\cancel{2} \cos \theta/2}$$

$$\cot \phi = \frac{-\cancel{2} \sin \theta/2}{\cancel{2}}$$

$$\cot \phi = -\tan \frac{\theta}{2}$$

$$w.k.T := \frac{1}{p^2} = \frac{1}{s^2} (1 + \cos^2 \theta)$$

$$\frac{1}{p^2} = \frac{1}{s^2} \left(1 + \tan^2 \left(\frac{\theta}{2} \right) \right)$$

$$\frac{1}{p^2} = \frac{1}{s^2} \sec^2 \frac{\theta}{2}$$

$$\frac{1}{p^2} = \frac{1}{s^2 \cos^2 \frac{\theta}{2}}$$

$$p^2 = s^2 \cos^2 \frac{\theta}{2}$$

$$p^2 = s^2 \left(\frac{1 + \cos \theta}{2} \right)$$

$$p^2 = \frac{s^2}{2} (1 + \cos \theta)$$

$$p^2 = \frac{s^2}{2} \frac{s}{a}$$

$$2ap^2 = s^3$$

$$iii) \gamma^h = a^h \times \cosh \theta$$

diff w.r.t θ

$$h \gamma^{h-1} = -a^h \sinh \theta$$

$$\frac{h \gamma^h}{\gamma} d\gamma = -a^h \sinh \theta$$

$$\frac{1}{\gamma} \frac{d\gamma}{d\theta} = \frac{-a^h \sinh \theta}{\gamma^h}$$

$$\frac{1}{\gamma} \frac{d\gamma}{d\theta} = \frac{-a^h \sinh \theta}{a^h \cosh \theta}$$

$$\cot \phi = -\tanh \theta$$

$$w.k.T := \frac{1}{p^2} = \frac{1}{s^2} (1 + \tan^2 \frac{\theta}{2})$$

$$\frac{1}{p^2} = \frac{1}{s^2} \sec^2 \frac{\theta}{2}$$

$$p^2 = s^2 \cos^2 \frac{\theta}{2}$$

$$p^2 = \frac{s^2}{2} \frac{s}{a}$$

$$2ap^2 = s^3$$

$$iv) \gamma^h \cosh \theta = a^h \cosh \theta$$

diff w.r.t θ

$$h \gamma^{h-1} = a^h \cosh \theta$$

$$\frac{h}{\gamma} \frac{d\gamma}{d\theta} = a^h \cosh \theta$$

$$\frac{1}{\gamma} \frac{d\gamma}{d\theta} = \frac{a^h \cosh \theta}{\gamma^h}$$

$$\cot \phi = \frac{a^h \cosh \theta}{a^h \sinh \theta}$$

$$\cot \phi = \coth \theta$$

$$w.k.T \frac{1}{p^2} = \frac{1}{s^2} (1 + \cot^2 \phi)$$

$$\frac{1}{p^2} = \frac{1}{s^2} (\coth^2 \frac{\theta}{2})$$

$$\frac{1}{p^2} = \frac{1}{s^2 \sinh^2 \frac{\theta}{2}}$$

$$p^2 = s^2 \sinh^2 \frac{\theta}{2}$$

$$p^2 = s^2 \left(\frac{1 - \cosh \theta}{2} \right)$$

$$p^2 = \frac{s^2}{2} \left(\frac{s}{a} \right)$$

$$2ap^2 = s^3$$

$$\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 3 & 5 \\ 2 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}$$

$m=2$
 $n=2$

Q) Find the characteristics rule or Eigen Value of the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}_{3 \times 3}$

Sol) Given matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

Write the characteristic equation of A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)[(3-\lambda)(2-\lambda)-2] - 2[(2-\lambda)-1] + 1[(2-(3-\lambda))]$$

$$\Rightarrow (2-\lambda)[6-5\lambda+\lambda^2-2] - 2[1-\lambda] + 1[\lambda-1] = 0$$

$$\Rightarrow 12-10\lambda+2\lambda^2-8+6\lambda+5\lambda^2-\lambda^3+2\lambda-2+2\lambda+\lambda-1$$

$$\Rightarrow -\lambda^3+12\lambda^2-11\lambda+3=0$$

$$\Rightarrow \lambda^3-7\lambda^2+11\lambda-5=0$$

$$\lambda=1 \mid \begin{array}{ccc|c} 1 & -7 & 11 & -5 \\ 0 & 1 & -6 & 5 \\ 0 & 0 & 0 & 0 \end{array}$$

$$(\lambda-1)(\lambda^2-6\lambda+5)=0$$

$$\lambda^2-6\lambda+5$$

$$\lambda^2-5\lambda-\lambda+5$$

$$\lambda(\lambda-5)-1(\lambda-5)$$

$$(\lambda-1)(\lambda-1)(\lambda-5)$$

$$(\lambda=1, 1, 5)$$