Hyperplane, Subspace and Halfspace

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1. Hyperplane :
Geometrically, a hyperplane is a geometric entity whose dimension is one less than that of its ambient space.

What does it mean?
It means the following. For example, if you take the 3D space then hyperplane is a geometric entity that is 1 dimensionless. So it's going to be 2 dimensions and a 2-dimensional entity in a 3D space would be a plane. Now if you take 2 dimensions, then 1 dimensionless would be a single-dimensional geometric entity, which would be a line and so on.

- The hyperplane is usually described by an equation as follows

$$X^T n + b = 0$$

- If we expand this out for n variables we will get something like this

$$X_1 n_1 + X_2 n_2 + X_3 n_3 + \ldots\ldots\ldots + X_n n_n + b = 0$$

- In just two dimensions we will get something like this which is nothing but an equation of a line.

$$X_1 n_1 + X_2 n_2 + b = 0$$

Example:

Let us consider a 2D geometry with

$$n = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \ and \ b = 4$$

$$x_1 + 3x_2 + 4 = 0$$

So as you can see from the solution the hyperplane is the equation of a li
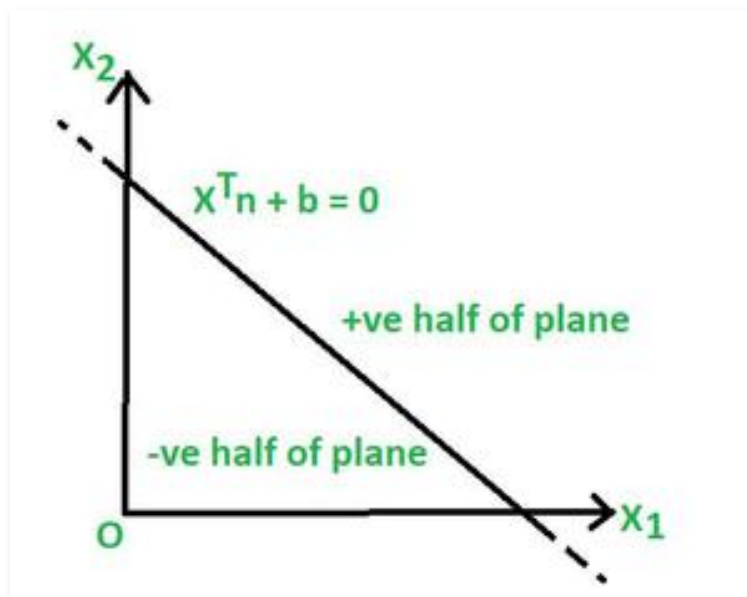
2. Subspace :

Hyper-planes, in general, are not sub-spaces. However, if we have hyper-planes of the form,

$$X^T n = 0$$

That is if the plane goes through the origin, then a hyperplane also becomes a subspace.

3. Half-space :

Consider this 2-dimensional picture given below.

So, here we have a 2-dimensional space in X_1 and X_2 and as we have discussed before, an equation in two dimensions would be a line which would be a hyperplane. So, the equation to the line is written as

$$X^T n + b = 0$$

So, for this two dimensions, we could write this line as we discussed previously

Let's consider the same example that we have taken in hyperplane case. So by solving, we got the equation as

$$x_1 + 3x_2 + 4 = 0$$

There may arise 3 cases. Let's discuss each case with an example.

Case 1:

$$x_1 + 3x_2 + 4 = 0 : On\ the\ line$$

Let consider two points (-1,-1). When we put this value on the equation of line we got 0. So we can say that this point is on the hyperplane of the line.

Case 2:

Similarly,

$$x_1 + 3x_2 + 4 > 0 : Positive\ half-space$$

Consider two points (1,-1). When we put this value on the equation of line we got 2 which is greater than 0. So we can say that this point is on the positive half space.

Case 3:

$$x_1 + 3x_2 + 4 < 0 : Negative\ half-space$$

Consider two points (1,-2). When we put this value on the equation of line we got -1 which is less than 0. So we can say that this point is on the negative half-space.

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Article Contributed By :

AmiyaRanjanRout

@AmiyaRanjanRout

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